HEAVY QUARK PRODUCTION IN pp COLLISIONS

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Abstract

A systematic study of the inclusive single heavy quark and heavy-quark pair production cross sections in pp collisions is presented for RHIC and LHC energies. We compare with existing data when possible. The dependence of the rates on the renormalization and factorization scales is discussed. Predictions of the cross sections are given for two different sets of parton distribution functions.

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†Supported by the U. S. Department of Energy under Contract No. DE-AC03-76SF00515.

INTRODUCTION

Charm and bottom quark production from initial nucleon-nucleon collisions will be copious at the RHIC and LHC colliders. Heavy quark decay into leptons will represent a significant background to dilepton production [1] in heavy ion collisions. A quantitative knowledge of the production cross section in pp collisions is a prerequisite for the detection of collective effects, such as heavy quark production by rescattering and in the quark-gluon plasma, which appears as a deviation from the simple superposition of hadronic collisions.

The lowest order (Born) calculations of the total cross section predict the correct energy dependence but differ from the experimental measurements by a "K factor" of 2-3. While the single-inclusive distributions as well as the mass and rapidity distributions of $Q\overline{Q}$ pairs are also well described to within a K factor by the Born cross section, the p_T and azimuthal double-differential distributions are not calculable at the Born level since the $Q\overline{Q}$ pair is always produced back-to-back in lowest order. For this reason, a next-to-leading order (NLO) calculation is needed. The calculations we present here are done using a Monte Carlo program developed by Nason and collaborators [2, 3, 4]. Similar work on the total cross section and the single inclusive distributions was done by Smith, van Neerven, and collaborators [5].

In this calculation, in addition to the uncertainties in the parton distribution functions, uncertainties arise from the heavy quark mass and the renormalization and factorization scale parameters. At collider energies, the calculations become more uncertain due to the lightness of the heavy quark compared to the center of mass energy, $m_Q/\sqrt{s} \ll 1$. We first discuss the Born calculation in some detail and then outline the NLO calculation with its additional uncertainties. We use the available data on $\sigma_{c\overline{c}}^{\rm tot}(s)$ to fix the charm quark mass and the scale parameters. The resulting parameter set provides a point from which to extrapolate to heavy-ion collider energies. We then compare with single-inclusive and double-differential distributions from charm and bottom data when available. We present estimates of heavy quark production cross sections in proton-proton collisions at RHIC ($\sqrt{s} = 200$ and 500 GeV) and LHC ($\sqrt{s} = 5.5$ TeV and 14 TeV), according to our present theoretical knowledge. We provide both the Born and NLO results for the total $Q\overline{Q}$ production cross section, single inclusive y and p_T distributions, and double differential M, ϕ, y and p_T distributions.

HEAVY QUARK PRODUCTION IN PERTURBATIVE QCD

The most general expression for the double differential cross section for $Q\overline{Q}$ pair production from the collision of hadrons A and B is

$$E_Q E_{\overline{Q}} \frac{d\sigma_{AB}}{d^3p_Q d^3p_{\overline{Q}}} = \sum_{i,j} \int dx_1\, dx_2 F_i^A(x_1, \mu_F) F_j^B(x_2, \mu_F) E_Q E_{\overline{Q}} \frac{d\widehat{\sigma}_{ij}(x_1 P_1, x_2 P_2, m_Q, \mu_R)}{d^3p_Q d^3p_{\overline{Q}}} \ . \quad (1)$$

Here A and B represent the initial hadrons and i, j are the interacting partons, and the functions F_i are the number densities of gluons, light quarks and antiquarks evaluated at momentum fraction x and factorization scale μ_F. The short-distance cross section, $\widehat{\sigma}_{ij}$, is calculable as a perturbation series in $\alpha_s(\mu_R)$ where the strong coupling constant is evaluated at the renormalization scale μ_R. Both scales are of the order of the heavy quark mass. At leading order, $\mu_F = \mu_R = \mu$ where $\mu = 2m_c$ is commonly used. The scale dependence will be discussed in more detail below.

Leading Order

At leading order, $\mathcal{O}(\alpha_s^2)$, $Q\overline{Q}$ production proceeds by two basic processes,

$$\begin{aligned} q + \bar{q} &\rightarrow Q + \overline{Q} && (2) \\ g + g &\rightarrow Q + \bar{Q} \; . && (3) \end{aligned}$$

The invariant cross section for the process $A + B \rightarrow H + \overline{H}$ where the $Q\overline{Q}$ pair has fragmented into hadrons $H(Q\overline{q})$ and $\overline{H}(\overline{Q}q)$ can be written as

$$\begin{aligned} E_H E_{\overline{H}} \frac{d\sigma_{AB}}{d^3p_H d^3p_{\overline{H}}} &= \int \frac{\hat{s}}{2\pi} dx_1 dx_2 dz_Q dz_{\overline{Q}} C(x_1, x_2) \frac{E_H E_{\overline{H}}}{E_Q E_{\overline{Q}}} \\ & \frac{D_{H/Q}(z_Q)}{z_Q^3} \frac{D_{\overline{H}/\overline{Q}}(z_{\overline{Q}})}{z_{\overline{Q}}^3} \delta^4(p_1 + p_2 - p_Q - p_{\overline{Q}}) \; , \end{aligned} \quad (4)$$

where $\sqrt{\hat{s}}$, the parton-parton center of mass energy, is related to $\sqrt{s}$, the hadron-hadron center of mass energy, by $\hat{s} = x_1 x_2 s$. The intrinsic transverse momenta of the incoming partons have been neglected. The sum of the leading order subprocess cross sections convoluted with the parton number densities is contained in $C(x_1, x_2)$ where

$$C(x_1, x_2) = \sum_q [F_q^A(x_1) F_{\overline{q}}^B(x_2) + F_{\overline{q}}^A(x_1) F_q^B(x_2)] \frac{d\hat{\sigma}_{q\overline{q}}}{d\hat{t}} + F_g^A(x_1) F_g^B(x_2) \frac{d\hat{\sigma}_{gg}}{d\hat{t}} \; . \quad (5)$$

Only light quark flavors, those with $m < m_Q$, are included in the sum over q. The dependence on the scale μ_F has been suppressed here.

Fragmentation affects the charmed hadron distributions, not the total $c\overline{c}$ production cross section. The fragmentation functions, $D_{H/Q}(z)$, describe the hadronization of the heavy quarks where $z = |\vec{p_H}|/|\vec{p_Q}|$ is the fraction of the heavy quark momentum carried by the final-state hadron. The D meson x_F distribution is harder than the calculated charmed quark distribution in hadron-hadron interactions. Including a fragmentation function that describes D production in e^+e^- annihilation softens the distribution due to energy lost to light $q\overline{q}$ pair production [6]. Event generators such as PYTHIA [7], based on the Lund string fragmentation model, harden the D distribution. In PYTHIA, the charmed quark is always at the endpoint of a string which pulls the charmed quark in the direction of a beam remnant so that the charmed hadron can be produced at a larger momentum than the charmed quark. Correlations of the produced charmed hadron with the projectile valence quarks, not predicted by perturbative QCD, have been measured. Several possible explanations have been suggested, see *i.e.*, [6, 7, 8]. This interesting high x_F regime will not be measurable at the RHIC and LHC colliders since the center of mass energy is high and the rapidity coverage is mostly confined to the central region. (The PHENIX muon spectrometer at RHIC will have a larger rapidity coverage, $1.5 \leq y \leq 2.5$ [9], but these effects will probably be out of reach at the maximum collider energy.)

If we ignore fragmentation effects for the moment, after taking four-momentum conservation into account, we are left with

$$\frac{d\sigma}{dp_T^2 dy_Q dy_{\overline{Q}}} = x_1 x_2 C(x_1, x_2) \ , \tag{6}$$

where x_1 and x_2 are

$$\begin{aligned} x_1 &= \frac{\widehat{m}_Q}{\sqrt{s}}(e^{y_Q} + e^{y_{\overline{Q}}}) \ , \\ x_2 &= \frac{\widehat{m}_Q}{\sqrt{s}}(e^{-y_Q} + e^{-y_{\overline{Q}}}) \ , \end{aligned} \tag{7}$$

and $\widehat{m}_Q = \sqrt{m_Q^2 + p_T^2}$. At $y_Q = y_{\overline{Q}} = 0$, $x_1 = x_2$. The target fractions, x_2, decrease with rapidity while the projectile fractions, x_1, increase. The subprocess cross sections for $Q\overline{Q}$ production by $q\overline{q}$ annihilation and gg fusion to order $\mathcal{O}(\alpha_s^2)$, expressed as a function of $\widehat{m}_Q$, y_Q, and $y_{\overline{Q}}$ are [10]

$$\frac{d\hat{\sigma}_{q\overline{q}}}{d\hat{t}} = \frac{\pi\alpha_s^2}{9\widehat{m}_Q^4} \frac{\cosh(y_Q - y_{\overline{Q}}) + m_Q^2/\widehat{m}_Q^2}{(1 + \cosh(y_Q - y_{\overline{Q}}))^3} \ , \tag{8}$$

$$\frac{d\hat{\sigma}_{gg}}{d\hat{t}} = \frac{\pi\alpha_s^2}{96\widehat{m}_Q^4} \frac{8\cosh(y_Q - y_{\overline{Q}}) - 1}{(1 + \cosh(y_Q - y_{\overline{Q}}))^3} \left(\cosh(y_Q - y_{\overline{Q}}) + \frac{2m_Q^2}{\widehat{m}_Q^2} - \frac{2m_Q^4}{\widehat{m}_Q^4}\right) \ . \tag{9}$$

Next-to-Leading Order

We now discuss the NLO, $\mathcal{O}(\alpha_s^3)$, corrections to the $Q\overline{Q}$ production cross section. At next-to-leading order, in addition to virtual corrections to these diagrams, production by

$$q + \bar{q} \rightarrow Q + \bar{Q} + g \tag{10}$$

$$g + g \rightarrow Q + \overline{Q} + g \tag{11}$$

$$q(\bar{q}) + g \rightarrow Q + \overline{Q} + (\bar{q})q \ , \tag{12}$$

must also be included. The last process, quark-gluon scattering, is not present at leading order. The quark-gluon graphs can be interpreted at the Born level as the scattering of a heavy quark excited from the nucleon sea with a light quark or gluon and are referred to as flavor excitation [2]. The total short distance cross section $\widehat{\sigma}_{ij}$ for a given production process can be expressed generally as

$$\widehat{\sigma}_{ij}(\widehat{s}, m_Q, \mu_R) = \frac{\alpha_s^2(\mu_R)}{m_Q^2} f_{ij}(\rho, \mu_R^2/m_Q^2) \ , \tag{13}$$

where $\rho = 4m_Q^2/\widehat{s}$. The function f_{ij} can be expanded perturbatively as

$$f_{ij}(\rho, \mu_R^2/m_Q^2) = f_{ij}^0(\rho) + \frac{\alpha_s(\mu_R)}{4\pi}\left[f_{ij}^1(\rho) + \overline{f}_{ij}^1(\rho)\ln(\mu_R^2/m_Q^2)\right] + \mathcal{O}(\alpha_s^2) \ . \tag{14}$$

The leading order part of the cross section is in the function f_{ij}^0. In this case, $f_{qg}^0 = f_{\overline{q}g}^0 = f_{gq}^0 = f_{g\overline{q}}^0 = 0$. Only f_{gg}^0 and $f_{q\overline{q}}^0$ contribute and can be computed from the $\hat{t}$ integration of the cross sections given in (8) and (9). The physical cross section should be independent of the renormalization scale: the dependence in eq. (14) introduces an unphysical parameter in the calculation. If the perturbative expansion is sufficient, *i.e.* if further higher-order corrections are small, at some value of μ the physical $\mathcal{O}(\alpha_s^n)$ and $\mathcal{O}(\alpha_s^{n+1})$ cross sections should be equal‡. If the μ dependence is strong, the perturbative expansion is untrustworthy and the predictive power of the calculation is weak [10]. The rather large difference between the heavy-quark Born and NLO cross sections suggests that further higher-order corrections are needed, particularly for charm and bottom quarks which are rather "light" when $\sqrt{s}$ is large. Usually the renormalization scale in $\widehat{\sigma}_{ij}$ and the factorization scale in the parton distribution functions are chosen to be equal. We follow this prescription in our calculations.

We have used two sets of recent parton distribution functions§, GRV HO [12] and MRS D-′ [13]. The first begins with a low scale, $Q^2_{0,\mathrm{GRV}} = 0.3$ GeV^2, and valence-like parton distributions, therefore evolving very quickly with Q^2. The second, with $Q^2_{0,\mathrm{MRS}} = 5$ GeV^2, has sea quark and gluon distributions that grow as $\sim x^{-1/2}$ when $x \rightarrow 0$. Both are compatible with the recent deep-inelastic scattering data from HERA [14]. We also include estimates of the total cross section using the MRS D0′ [13] distributions. This set assumes a constant value for the sea and gluon distributions at $Q^2_{0,\mathrm{MRS}}$ as $x \rightarrow 0$ and lies below the HERA data. The GRV distributions assume $\overline{u} = \overline{d}$, a symmetric light quark sea, and $x\overline{s}(x, Q^2_{0,\mathrm{GRV}}) = 0$, increasing to give $2\langle x\rangle_{\overline{s}}/(\langle x\rangle_{\overline{u}} + \langle x\rangle_{\overline{d}}) \simeq 0.53$ at $Q^2 = 10$ GeV^2 [12]. The MRS D sets allow $\overline{u} < \overline{d}$ to account for measurements of the Gottfried sum rule and assume $\overline{s} = (\overline{u} + \overline{d})/4$ at $Q^2_{0,\mathrm{MRS}}$ [13]. Thus the MRS distributions, arising from a global fit, provide a somewhat better description of the deep-inelastic scattering data for $x > 0.01$ than the GRV distributions [12, 13].

Since we compare two extreme cases for the nucleon parton distributions as $x \rightarrow 0$, MRS D-′ and GRV HO on one hand and MRS D0′ on the other, our results may be thought of as providing an upper and lower bound to the $Q\overline{Q}$ cross section at heavy-ion collider energies for fixed mass and scale. However, little data exist on the gluon

‡The order of the expansion is represented by n. For $Q\overline{Q}$ production, $n \geq 2$. A calculation to order $\mathcal{O}(\alpha_s^n)$ introduces corrections at the order $\mathcal{O}(\alpha_s^{n+1})$.

§All available parton distribution functions are contained in the package PDFLIB [11], available in the CERN library routines.

distribution function at low x so that it is poorly known, particularly in the x region accessible at RHIC and LHC, $x \approx 10^{-2}$ and 10^{-4} around $y = 0$, respectively. The low x behavior has a significant effect on the shape of the gluon distribution at moderate values of x in the energy range of Fig. 1. Steeply rising gluon distributions at low x are compensated for by a corresponding depletion at moderate x.

Heavy quark production by gluon fusion dominates the $pp \to Q\overline{Q}X$ production cross section in the central region. Thus we show the shape of the gluon distributions of the three parton distribution sets are shown in Fig. 1(a) over the x range of the previous pp data, $0.01 < x < 1$. To facilitate comparison, all three are shown at $\mu = 2.4$ GeV. The solid curve is the GRV HO distribution, the dashed, MRS D0$'$, and the dot-dashed, MRS D-$'$. The GRV distribution at $\mu = 1.2$ GeV is also shown to demonstrate the effect of the Q^2 evolution. Since it has a smaller initial scale, the evolution with μ is quite fast. The D0$'$ distribution can be seen to turn over and begin to flatten as x decreases. However, for much of the range, it is above the D-$'$ distribution, reflected in a larger $\sigma_{c\overline{c}}^{\rm tot}$, as shown in Fig. 3. All three sets, evaluated in the $\overline{MS}$ scheme, have a similar value of $\Lambda_{\rm QCD}$. In Fig. 1(b), we show the running of the two loop value of α_s,

$$\alpha_s(\mu, f) = \frac{1}{b_f \ln(\mu^2/\Lambda_f^2)} \left[1 - \frac{b_f' \ln\ln(\mu^2/\Lambda_f^2)}{b_f \ln(\mu^2/\Lambda_f^2)} \right] \ , \tag{15}$$

where $b_f = (33-2f)/12\pi$, $b_f' = (153-19f)/(2\pi(33-2f))$, f is the number of flavors, and Λ_f is the value of $\Lambda_{\rm QCD}$ appropriate for the number of flavors. In the calculation, the number of flavors depends on the chosen quark mass. For charm, $f = 3$, and for beauty, $f = 4$. At $\mu = m_Q$, $\alpha_s(m_Q, f) = \alpha_s(m_Q, f+1)$. The running of α_s is visible in the renormalization scale dependence, shown in Fig. 2(e). For the NLO $Q\overline{Q}$ production program, Λ_f is chosen by m_Q. Note that $\Lambda_3 > \Lambda_4 > \Lambda_5$. Additional uncertainties may arise because the threshold m_Q for a given parton distribution set can differ from our fitted m_Q.

While it is often possible to use a general prescription like the principle of minimal sensitivity (PMS) [15] to find values of μ_R and μ_F where the scale sensitivity is a minimum, the heavy quark production cross section is very sensitive to changes in μ. In Fig. 2 we show the variation of the $c\overline{c}$ and $b\overline{b}$ production cross sections at RHIC (a), (c) and LHC (b), (d) ion energies. The MRS distributions exhibit an artificial stability for low μ because for $\mu < 2m_c \approx Q_{0,{\rm MRS}}$, the factorization scale is fixed at $Q_{0,{\rm MRS}}$ and only μ_R varies. We use the GRV HO parton distribution functions so that we can show the uncertainty with $\mu = \mu_R = \mu_F$ at lower values of μ since μ_F is not fixed until $\mu_F \approx 0.4m_c \approx Q_{0,{\rm GRV}}$. When $\mu/m_c \approx 0.2$, the cross section diverges since $(\mu/m_c)/\Lambda_{\rm QCD} \approx 1$. In any case, such small scales below 1 GeV, are excluded because a perturbative calculation is no longer assumed to be valid. As μ/m_c increases, the cross section becomes more stable. The behavior we find is similar for RHIC and LHC energies. The $b\overline{b}$ cross section shows a smaller variation with μ, particularly at $\sqrt{s} = 200$ GeV. The variation resembles the running of α_s shown in Fig. 1(b). Indeed, this running is a major source of instability in the NLO $Q\overline{Q}$ cross sections. However, at $\sqrt{s} = 5.5$ TeV the variation with μ at the Born level increases since the cross section becomes more uncertain as $m_Q/\sqrt{s}$ decreases. The NLO results show less variation at this energy. There is no value of μ where the Born and the NLO calculations are equal, suggesting that higher-order corrections are needed for $m_Q/\sqrt{s} \ll 1$.

We show the change of the $c\overline{c}$ cross section at $\sqrt{s} = 200$ GeV induced by fixing $\mu_R = 2m_Q$ and changing μ_F in Fig. 2(e) and fixing $\mu_F = 2m_Q$ and varying μ_R in Fig. 2(f). The running of the coupling constant is clearly shown in 2(e). In 2(f), the increase with μ_F arises because at values of μ_F near $Q_{0,\rm GRV}$ and low x, the sea quark and gluon distributions show a valence-like behavior, decreasing as $x \rightarrow 0$, an effect special to the GRV distributions [12]. The results are quite different for the MRS distributions, especially for the equivalent of Fig. 2(f). There is not much change in the cross section with μ_F, particularly at the Born level, since the parton distribution functions do not change below $Q_{0,\rm MRS}$.

CALCULATIONS OF $\sigma_{Q\overline{Q}}^{\rm tot}$

Previous comparisons of the total charm production cross sections with calculations [16] at leading order suggested that a constant K factor of ~ 2 was needed to reconcile the calculations with data when using $m_c = 1.5$ GeV, but not when $m_c = 1.2$ GeV was chosen. Initial NLO calculations seemed to suggest that the K factor was no longer needed with $m_c = 1.5$ GeV [17]. However, this result is very dependent upon the chosen scale parameters and the parton distribution functions, particularly the shape of the gluon distribution.

Comparison With Current Data

We compare our NLO calculations with the available data [18, 19, 20, 21, 22] on the total $c\overline{c}$ production cross section from pp and pA interactions in Fig. 3. When a nuclear target has been used, the cross section per nucleon is given, assuming an A^α dependence with $\alpha = 1$, supported by recent experimental studies of the A dependence [23]. We assume that we can compare the $c\overline{c}$ production cross section directly with charmed hadron measurements. Often single charmed mesons, denoted $D/\overline{D}$ to include all charge states, in the region $x_F > 0$ are measured. The $c\overline{c}$ production cross section is symmetric around $x_F = 0$ in pp interactions so that $\sigma_{c\overline{c}}^{\rm tot} = 2\sigma_{c\overline{c}}(x_F > 0)$. While the question of how the $c\overline{c}$ pair hadronizes into $D\overline{D}$, $D\overline{\Lambda_c}$, $\Lambda_c\overline{D}$, $\Lambda_c\overline{\Lambda_c}$, *etc.* remains open, some assumptions must be made about how much of $\sigma_{c\overline{c}}^{\rm tot}$ is missing since not all channels are measured. If all single D mesons are assumed to originate from $D\overline{D}$ pairs, ignoring associated $\Lambda_c\overline{D}$ production, then by definition $\sigma(D\overline{D}) = \sigma(D/\overline{D})/2$. Thus the single D cross section for $x_F > 0$ is equal to the $D\overline{D}$ pair cross section over all x_F. However, the contribution to the $c\overline{c}$ total cross section from D_s and Λ_c production has been estimated to be $\sigma(D_s)/\sigma(D^0 + D^+) \simeq 0.2$ and $\sigma(\Lambda_c)/\sigma(D^0 + D^+) \simeq 0.3$. Thus to obtain the total $c\overline{c}$ cross section from $\sigma(D\overline{D})$,

$\sigma(D\overline{D})$ should be multiplied by ≈ 1.5 [24]. This is done in our data comparison. The data exist in the range $19 < \sqrt{s} \leq 63$ GeV, mostly from fixed target experiments. Below the ISR energies, $\sqrt{s} = 53$-63 GeV, the total cross section is primarily inferred from single D or $D\overline{D}$ measurements. At the ISR, the pair production cross section is obtained from lepton measurements, either $e\mu$ and electron pair coincidence measurements or a lepton trigger in coincidence with a reconstructed D or Λ_c. Rather large $c\overline{c}$ cross sections were inferred from the latter analyses due to the assumed shape of the production cross sections: flat distributions in x_F for the Λ_c and $(1-x_F)^3$ for the D. The ISR results must thus be taken with some care.

Modern parton distributions with $\Lambda_{\rm QCD}$ fixed by fits to data cannot explain the energy dependence of the total cross section in the measured energy range when using $m_c = 1.5$ GeV and $\mu_F = \mu_R = m_c$. Since $m_c^2 < Q^2_{0,\rm MRS}$ for the MRS distributions and the scale must be chosen so that $\mu^2 > Q^2_{0,\rm MRS}$ for the calculations to make sense, we take $\mu = 2m_c$ and vary m_c for these distributions. We find reasonable agreement for $m_c = 1.2$ GeV for the D-$'$ and D0$'$ distributions. The results are shown in the solid and dashed curves in Fig. 3 respectively. Since the GRV HO distributions have a much lower initial scale, μ can be fixed to the quark mass. The dot-dashed curve is the GRV HO distribution with $m_c = 1.3$ GeV and $\mu = m_c$. All three curves give an equivalent description of the data. Our "fits" to the low energy data are to provide a reasonable point from which to extrapolate to higher energies. It is important to remember that significant uncertainties still exist which could change our estimates considerably when accounted for. These relatively low values of m_c effectively provide an upper bound on the charm production cross section at high energies. For comparison, we also show the cross section with the GRV distributions and $\mu = m_c = 1.5$ GeV in the dotted curve. It lies a factor of 2-3 below the other calculations. The smaller value of m_c is needed for the MRS distributions even with the larger scale because parton distribution functions at lower values of Q^2 would decrease at low x, as demonstrated by the GRV distributions [12]. Note that such small choices of m_c suggests that the bulk of the total cross section comes from invariant masses less than $2m_D$. In a recent work [24], the total cross section data was found to be in agreement with $m_c = 1.5$ GeV with some essential caveats: the factorization scale was fixed at $\mu_F \equiv 2m_c$ while μ_R was allowed to vary and an older set of parton distribution functions with a range of fits with a different value of $\Lambda_{\rm QCD}$ for each was used. Decreasing μ_R with respect to μ_F and increasing $\Lambda_{\rm QCD}$ both result in a significantly larger cross section for a given m_c. We choose here to use the most up-to-date parton distribution functions and to keep $\mu_F = \mu_R$, facilitating a more direct extrapolation from the current data to the future collider results.

Since data on $c\overline{c}$ and $b\overline{b}$ production by pion beams are also available at fixed target energies, in Fig. 4 we show this data with the same parton distributions where m_c and μ are fixed by the comparison in Fig. 3. The $c\overline{c}$ data [18, 25, 26, 27, 28] is based on the $x_F > 0$ single D cross section. However, the $\pi^- N$ x_F distribution is asymmetric, $\sigma/\sigma(x_F > 0) \sim 1.6$ so that $\sigma(D\overline{D})$ is obtained by dividing by 2 to get the pair cross section and then multiplying by 1.6 to account for the partial x_F coverage. The $b\overline{b}$ data, taken to be over all x_F, are generally obtained from multi-muon studies [29, 30, 31, 32]. The data, especially for $b\overline{b}$ production, are not as extensive and have rather poor statistics. Again, some of the data is from a nuclear target. When a nuclear target has been used, the cross section per nucleon is given, assuming an A^1 dependence.

The GRV HO pion distributions [33] are based on their proton set so that the two distributions are compatible. In Fig. 4(a), the charm production cross section is calculated using the GRV proton and pion distributions. The solid curve shows the result with a nucleon target, the averaged distributions for proton and neutron, while the dashed curve is the result for a proton target alone. The results are consistent at $\sqrt{s} = 30$ GeV; at lower energies, the cross section on a proton target is slightly larger than on a nucleon target. The calculations using the MRS distributions do not have the same consistency as those with GRV because their pion distribution functions, SMRS P1 and P2 [34], are based on an older set of proton distributions than the current MRS distributions used here. The SMRS distributions use $\Lambda_4 = 190$ MeV while the MRS distributions have fixed $\Lambda_4 = 230$ MeV. In the calculations, we fix Λ_4 to the current MRS value. The dot-dashed curve shows the MRS D-$'$ distributions with the SMRS P2 pion distributions while the dotted curve is with the P1 set. Both are for a proton target. The P1 set has a steeper gluon distribution than P2. The two calculations begin to diverge as $\sqrt{s}$ increases since the gluon fusion contribution is becoming dominant. At low $\sqrt{s}$, valence quark annihilation is important for $\pi^- p$ interactions. Although the calculations and data are not in exact agreement, they are close enough to assume that the same parameters are reasonable for both pion and proton projectiles. The comparison to the $b\overline{b}$ production cross section is given in Fig. 4(b). The data is very sparse. We use $m_b = 4.75$ GeV and $\mu = m_b$ for both sets of parton distributions. The solid curve is the GRV distribution, the dashed is the MRS D-$'$ and SMRS P1 result. The agreement is not unreasonable given the quality of the data on the one hand and the theoretical uncertainties on the other.

Extrapolation To RHIC And LHC Energies

The total $c\overline{c}$ and $b\overline{b}$ cross sections at the top ISR energy, $\sqrt{s} = 63$ GeV, and the proton and ion beam energies at RHIC and LHC are given in Tables 1 and 2 respectively. Both the Born and NLO cross sections are given. The theoretical K factor, $\sigma^{\rm NLO}_{Q\overline{Q}}/\sigma^{\rm LO}_{Q\overline{Q}}$, tends to increase with energy and is rather large. There is no *a priori* reason why it should remain constant, rather the increase at collider energies would suggest that the perturbative expansion is becoming less reliable, as discussed below. Note that even though the MRS D-$'$ and GRV HO distributions give an equally valid description of the data at ISR energies and below, they differ at higher energies, partly from the difference in m_c and partly because of our scale difference. The MRS D-$'$ distributions evolve faster since $\mu = 2m_c$ rather than $\mu = m_c$ due to their chosen initial scale $Q_{0,{\rm MRS}}$, resulting in a larger predicted cross section. Less difference is seen between the GRV and MRS D-$'$ distributions for the $b\overline{b}$ cross section since the m_b and μ are used for both. Note that for $b\overline{b}$ production at 14 TeV, the results differ by 30% while the MRS D-$'$ NLO $c\overline{c}$ result is three times larger than the GRV HO result at the same energy. The D0$'$ distributions give smaller cross sections at LHC energies due to the different initial behavior at $x \to 0$. We illustrate this effect using the Born contribution to the production cross section at fixed M and $y = 0$, approximated as

$$\frac{d\sigma}{dMdy}|_{y=0} \approx \frac{\alpha_s^2}{Ms}\left[F_g(M/\sqrt{s})\right]^2 \tag{16}$$

since gluon fusion is the dominant contribution to the Born cross section, $x = M/\sqrt{s}$ at $y = 0$, and at fixed M, σ_{gg} is proportional to $(\alpha_s^2/M^2)F_g^2$. The gluon distribution at low x and $\mu = Q_0$ may be approximated as $F_g(x) = f(x)/x^{1+\delta}$. For a constant behavior at low x, such as in the MRS D0$'$ distribution, $\delta = 0$ and the cross section is independent of $\sqrt{s}$. At the other extreme, the MRS D-$'$ distribution assumes $\delta = 0.5$ at Q_0 so that the cross section grows as $s^\delta \sim \sqrt{s}$.

SINGLE AND DOUBLE DIFFERENTIAL DISTRIBUTIONS

We now compare the NLO calculations with data on Q and $Q\overline{Q}$ distributions. In the presentation of the single inclusive and double differential distributions, we follow the prescription of Nason and collaborators [3, 4] and take $\mu_S = n\widehat{m}_Q$ for the single and $\mu_D = n\sqrt{m_Q + (p_{T_Q}^2 + p_{T_{\overline{Q}}}^2)/2}$ for the double differential distributions. When using MRS distributions for charm production, $n = 2$. For all other cases, $n = 1$. A word of caution is necessary when looking at our predictions for $Q\overline{Q}$ pair distributions. It is difficult to properly regularize the soft and collinear divergences to obtain a finite cross section over all phase space. Soft divergences cancel between real and virtual corrections when properly regularized. The collinear divergences need to be regularized and subtracted. For single inclusive heavy quark production, this is possible because the integration over the partonic recoil variables can be performed analytically and the singularities isolated. In exclusive $Q\overline{Q}$ pair production, the cancellation is performed within the numerical integration. The price paid for this is often a negative cross section near the phase space boundaries, particularly when $p_T \to 0$ for the pair and $\phi \to \pi$ where ϕ is the difference in the azimuthal angle between the heavy quark and antiquark in the plane transverse to the beam axis. A positive differential cross section for $p_T \to 0$ can only be obtained by resumming the full series of leading Sudakov logarithms corresponding to an arbitrary number of soft gluons. This has not been done in the case of heavy quark production [4]. Thus when $m_Q/\sqrt{s} \ll 1$, fluctuations in the cross section due to incomplete numerical cancellations can become very large, resulting in negative components in the mass and rapidity distributions. We have minimized the fluctuations by maximizing the event sampling at low p_T and increasing the number of iterations [35].

Comparison To Current Data

First, we compare with the 800 GeV fixed target data of the LEBC-MPS collaboration [19] in Fig. 5. They measured the x_F and p_T^2 distributions of single D production. The total cross section, $\sigma(D/\overline{D}) = 48 \pm 11\,\mu$b, corresponds to a $D\overline{D}$ production cross section of 24±8 μb. The solid curves are the MRS D-$'$ results, the dashed, the GRV HO calculations. Data on correlated $D\overline{D}$ production is also available at 800 GeV, from pEmulsion studies [36]. The event sample is rather small, only 35 correlated pairs. We compare the mass and p_T^2 of the pair and the azimuthal difference between the pair in Fig. 6 with the calculated NLO distributions. Again the solid curve is MRS D-$'$, the dashed, GRV HO. The Born invariant mass distribution, given by the dashed curve, is parallel to the NLO results shown in the solid curve.

The $p\overline{p}$ data from UA 1, $\sqrt{s} = 630$ GeV, and CDF, $\sqrt{s} = 1.8$ TeV, include single b quark p_T distributions. The measurements are taken in the central region ($|y| < 1.5$ for UA 1 and $|y| < 1$ for CDF) and are integrated over p_T above each $p_{T,\min}$. The comparisons with the NLO calculations are given in Fig. 7(a) for UA 1 [37] and Fig. 7(b) for CDF and D0 [38, 39]. Reasonable agreement is found for both GRV HO and MRS D-$'$ for UA 1 with $\mu_S = \sqrt{m_b^2 + p_T^2}$. However, the results from this same scale choice lie somewhat below the early CDF data where data on J/ψ production was used to determine the B production cross section¶. As reported in Ref. [40], the scale $\mu = \mu_S/4$ was needed for good agreement with the magnitude of the data when the older MRS D0 distributions were used. More recent data using direct measurement of inclusive $b \to J/\psi$ and $b \to \psi'$ decays has shown that the previous results overestimated ψ production from b decays [38]. Better agreement with theory is now found for $\mu = \mu_S$, as shown in Fig. 7(b). Again the GRV HO and MRS D-$'$ distributions look similar, differing primarily for $p_{T,\min} < 10$ GeV. This difference is increased for the lower scale choice where $\mu_S/4 < Q_{0,\mathrm{MRS}}$ for $p_{T,\min} < 7.5$ GeV, cutting off the evolution of the MRS distributions below this $p_{T,\min}$. The GRV calculations evolve over all $p_{T,\min}$ since $\mu_S/4 > Q_{0,\mathrm{GRV}}$, hence the larger difference.

Extrapolation To RHIC And LHC Energies

We now show the predicted heavy quark distributions for RHIC ($\sqrt{s} = 200$ and 500 GeV) and LHC ($\sqrt{s} = 5.5$ and 14 TeV) using the MRS D-$'$ and GRV HO distributions. The results are shown in Figs. 8-23. We use the same scales on the y-axes for both sets of parton distributions as much as possible to facilitate comparison. In each figure we show the single quark p_T (a) and y (b) distributions and the p_T (c), rapidity (d), invariant mass (e), and azimuthal angle (f) distributions of the $Q\overline{Q}$ pair. The Born (LO) results are also given in (b), (d), and (e). All the distributions have been divided by the corresponding bin width. The single and pair p_T distributions are also given with the rapidity cuts $y < |1|$ at the LHC and $y < |0.35|$ at RHIC, corresponding to the planned acceptances of ALICE [41] and the PHENIX central detector [9]. These p_T distributions are also divided by the width of the rapidity interval. In

¶The inclusive decay, $B \to J/\psi X$, has a 1% branching ratio (BR) while the channel $B \to J/\psi K$ has an 0.1% branching ratio.

Tables 3-10 we give the y-integrated single p_T^2 NLO and Born distributions, the pair p_T^2 distributions with the cut on rapidity, and the NLO and Born invariant mass distributions for c and b production at each energy with the MRS D-$'$ partons. Note that all distributions have a 2 GeV bin width and that neither it nor the rapidity bin width has been removed in the tables. The statistical uncertainties are less than 1% at low p_T^2 and M, increasing to 5-6% in the tails. The uncertainty increases slightly with energy.

The development of a rapidity plateau can be seen in both the single and pair rapidity distributions as the energy increases. This plateau is generally broader for the single quarks than the pair since the pair mass enters into the estimate of the maximum pair rapidity while the smaller quark transverse mass gives the maximum single quark rapidity. The plateau is broader for the MRS D-$'$ parton distributions. In the charm rapidity distributions with the MRS D-$'$ partons at 14 TeV, the plateau edge is artificial. The set has a minimum x of 10^{-5}, reached at $y \sim 2.8$ for a single quark and a somewhat larger y for the pair. The GRV HO distributions have a minimum $x = 10^{-6}$, corresponding to $y \sim 4.5$, off the scale of our graphs. The average quark and pair p_T increases with energy. For charmed quarks, $\langle p_T^2 \rangle$ is larger for the pair than for a single quark. The opposite result is seen for b quarks. The GRV distributions result in larger $\langle p_T^2 \rangle$ than the MRS distributions. Near $p_T \rightarrow 0$, the MRS parton distributions show a steeper slope than the GRV distributions. As p_T increases, the slopes become somewhat similar at RHIC energies.

In general, the LO mass and rapidity distributions are nearly equivalent to the NLO results scaled by a theoretical K factor independent of M and y. At LHC energies, the expansion parameter becomes $\alpha_s \log(s/m_Q^2)$, of order 1 for $m_Q/\sqrt{s} \ll 1$, spoiling the convergence of the perturbative expansion [24]. This causes our predictions to be less reliable at these energies. Note that using μ_S for the single inclusive distributions and μ_D for the double differential distributions leads to somewhat different values of the integrated NLO cross sections than given in Tables 1 and 2, calculated with $\mu = n m_Q$, since the correction terms grow with μ. The effect is relatively small for the Born results since the faster evolution of the parton distribution functions is partly compensated by the decrease of α_s with increasing μ.

We also compare to the leading order charm distributions obtained from HIJING [42] for the ion collider energies, 200 GeV (Figs. 8,9) and 5.5 TeV (Figs. 12,13). HIJING uses the same mass and scale parameters and parton distribution functions as the other calculations. Although only a Born level calculation of $Q\overline{Q}$ production, HIJING includes the effect of multiple parton showers which simulates aspects of higher-order production (NLO includes the effect of only one additional parton). The rapidity distributions, shown for $y > 0$ only, closely resemble the NLO calculations. However, the p_T^2 distributions, taken in the rapidity interval $|y| < 2$ for the single c quark and the pair, are softer, especially for the $c\overline{c}$ pair. (Again, the distributions are divided by the rapidity bin width.) The distributions are also not strongly peaked at low p_T, as are the NLO calculations, due to initial state radiation. HIJING also includes fragmentation of the $c\overline{c}$ pair into hadrons. The calculated ϕ distributions are not as sharply peaked at $\phi = \pi$ as the NLO results. Note also that the $D\overline{D}$ pair ϕ distributions from HIJING are more isotropic than the original $c\overline{c}$ pairs.

$Q\overline{Q}$ Decays To Lepton Pairs

Since heavy quark decays are an important contribution to the dilepton continuum, we show $c\overline{c}$ and $b\overline{b}$ decays into dileptons at RHIC and LHC for the MRS D-′ sets. Because heavy quark decays are not incorporated into our double-differential calculation, the heavy quark pairs have been created from the final distributions. The heavy quark decays to leptons are thus calculated using a Monte Carlo program based on data from D decays at SLAC [43] and B decays from CLEO [44]. The inclusive branching ratio for D meson decay into a lepton, averaged over charged and neutral D's is $BR(D^0/D^+ \to l^+X) \sim 12\%$. The corresponding branching ratio for B mesons of unspecified charge is $BR(B \to l^+X) \sim 10.4\%$ [45]. B decays represent a special challenge since lepton pairs of opposite sign can be produced from the decay of a single B by $B \to DlX$ followed by $D \to lX$. Thus the B decays can produce dileptons from the following: a combination of leptons from a single B, two leptons from primary B decays, two leptons from secondary decays, and a primary lepton from one B and a secondary lepton from the opposite sign $\overline{B}$. The measurement of Ref. [44] is assumed to be for primary B decays to leptons. The NLO pair distributions $d\sigma/dM$ and $d\sigma/dy$ agree well with a K factor times the Born results. Therefore the correlated distributions, $d\sigma/dMdy$, are calculated at leading order and multiplied by this K-factor, while the p_T^2 and ϕ distributions, unavailable at leading order, are taken from the NLO results. The heavy quark pair is specified according to the correlated distributions from the calculated cross section. The momentum vectors of the individual quarks are computed in the pair rest frame, using the rapidity gap between the quarks. Once the quark four-momenta have been specified, the decays are calculated in the quark rest frame, according to the measured lepton momentum distributions, and then boosted back to the nucleon-nucleon center of mass, the lab frame for RHIC and LHC. Finally, the pair quantities, M_{ll}, y_{ll}, and $p_{T,ll}$, are computed.

The average number of $Q\overline{Q}$ pairs, $N_{Q\overline{Q}}$, produced in a central nuclear collision is estimated by multiplying the cross section from Tables 1 and 2 by the nuclear thickness $T_{AB}(0)$. If $N_{Q\overline{Q}} < 1$, only correlated production is important. The number of correlated lepton pairs can be estimated by multiplying the number of $Q\overline{Q}$ pairs by the square of the meson, H, branching ratio to leptons: $N_{Q\overline{Q}}BR^2(H/\overline{H} \to l^\pm X)$. However, if $N_{Q\overline{Q}} > 1$, dilepton production from uncorrelated $Q\overline{Q}$ pairs should be accounted for as well. Then two $Q\overline{Q}$ pairs are generated from the production cross section and the Q from one pair is decayed with the $\overline{Q}$ from the other. Thus for uncorrelated $Q\overline{Q}$ production, the average number of lepton pairs is approximately $N_{Q\overline{Q}}(N_{Q\overline{Q}} - 1)BR^2(H\overline{H} \to l^\pm X)$ when $N_{Q\overline{Q}} \gg 1$. If $N_{Q\overline{Q}} \approx 1$, a distribution in $N_{Q\overline{Q}}$ must be considered to calculate the uncorrelated pairs. In the following figures, we show the correlated dilepton cross section in pp collisions, $\sigma_{ll} = BR^2(H/\overline{H} \to l^\pm X)\sigma_{Q\overline{Q}}$. In Fig. 27, showing uncorrelated lepton pairs from $D\overline{D}$ decays at the LHC, we give the uncorrelated distributions with the value of the correlated cross section since $N_{Q\overline{Q}} < 1$ in pp collisions. To find the correct scale in central AB collisions, calculate $N_{Q\overline{Q}}$ and then multiply the lepton pair cross section by $T_{AB}(0)(N_{Q\overline{Q}} - 1)$.

In Figs. 24-25, we show the mass (a), rapidity (b), and p_T (c) distributions for the lepton pairs from $D\overline{D}$ and $B\overline{B}$ pairs respectively. The average mass of the lepton pairs from $D\overline{D}$ decays at RHIC ion energies is $\langle M_{ll} \rangle = 1.35$ GeV and the average lepton pair p_T, $\langle p_{T,ll} \rangle = 0.8$ GeV; from $B\overline{B}$ decays, $\langle M_{ll} \rangle = 3.17$ GeV and $\langle p_{T,ll} \rangle = 1.9$ GeV. A like-sign subtraction should eliminate most of the uncorrelated charm production at RHIC.

At LHC ion energies, the $c\overline{c}$ production cross sections are large enough for uncorrelated charm production to be substantial and difficult to subtract in nuclear collisions. The average mass of the lepton pairs from correlated $D\overline{D}$ decays here is $\langle M_{ll} \rangle = 1.46$ GeV and the $\langle p_{T,ll} \rangle = 0.82$ GeV. When the pairs are assumed to be uncorrelated, then $\langle M_{ll} \rangle = 2.73$ GeV and $\langle p_{T,ll} \rangle = 1$ GeV. The average dilepton mass from uncorrelated $D\overline{D}$ pairs is larger since the rapidity gap between uncorrelated D and $\overline{D}$ mesons is larger on average than between correlated $D\overline{D}$ pairs. The $b\overline{b}$ cross section is still small enough at the LHC for uncorrelated lepton pair production from B meson decays to be small. However, the acceptance for these pairs will be larger than for charm decays since high mass lepton pairs from heavy quark decays have a large rapidity gap. When acceptance cuts are applied, at least one member of a lepton pair will have a large enough rapidity to escape undetected so that high mass pairs from heavy quark decays will have a strongly reduced acceptance. This reduction will occur at larger values of M_{ll} for $B\overline{B}$ than $D\overline{D}$ decays. From all $B\overline{B}$ decays, $\langle M_{ll} \rangle = 3.39$ GeV and $\langle p_{T,ll} \rangle = 2$ GeV. In Figs. 26-28, we show the mass (a), rapidity (b), and p_T (c) distributions for the dilepton pairs from correlated and uncorrelated $D\overline{D}$ and correlated $B\overline{B}$ pairs respectively.

SUMMARY

In this overview, we have attempted to use the theoretical state of the art to predict heavy quark production in pp collisions at RHIC and LHC energies. Although much progress has been made in the higher-order calculations of $Q\overline{Q}$ production, this is not meant to be the final word. Fragmentation and decay effects need to be incorporated into our next-to-leading order calculations. More structure function data from HERA, combined with collider data on jets and prompt photons, will produce further refined sets of parton distribution functions. Theoretical progress may allow resummation at low p_T or produce estimates of next-to-next-to-leading order corrections. New scale fixing techniques may result in a reduction of scale uncertainties. Thus, there is still room for improvement in these calculations. Though the agreement with lower energy data allows us to extrapolate these results to RHIC and LHC energies, major uncertainties still exist, particularly at LHC energies. However, given our mass and scale parameters, the GRV HO and MRS D-′ parton distribution functions provide a rough upper and lower limit on the theoretical predictions. This might be useful in particular for the design of detectors at these facilities.

Acknowledgments

We gratefully acknowledge the help of M. L. Mangano and G. Ridolfi with their program package and G. Schuler for discussions.

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$\sqrt{s}$(GeV)	MRS D0′		GRV HO		MRS D-′	
	$\sigma_{c\bar{c}}^{\mathrm{LO}}$ (μb)	$\sigma_{c\bar{c}}^{\mathrm{NLO}}$ (μb)	$\sigma_{c\bar{c}}^{\mathrm{LO}}$ (μb)	$\sigma_{c\bar{c}}^{\mathrm{NLO}}$ (μb)	$\sigma_{c\bar{c}}^{\mathrm{LO}}$ (μb)	$\sigma_{c\bar{c}}^{\mathrm{NLO}}$ (μb)
63	31.87	75.21	30.41	72.09	26.88	64.97
200	105	244.2	122.6	350.8	139.3	343.7
500	194.8	494	291.6	959	449.4	1138
5500	558.2	1694	1687	6742	7013	17680
14000	742.4	2323	2962	12440	16450	41770

Table 1: Total $c\bar{c}$ production cross sections at collider energies.

$\sqrt{s}$(GeV)	MRS D0′		GRV HO		MRS D-′	
	$\sigma_{b\bar{b}}^{\mathrm{LO}}$ (μb)	$\sigma_{b\bar{b}}^{\mathrm{NLO}}$ (μb)	$\sigma_{b\bar{b}}^{\mathrm{LO}}$ (μb)	$\sigma_{b\bar{b}}^{\mathrm{NLO}}$ (μb)	$\sigma_{b\bar{b}}^{\mathrm{LO}}$ (μb)	$\sigma_{b\bar{b}}^{\mathrm{NLO}}$ (μb)
63	0.0458	0.0884	0.0366	0.0684	0.0397	0.0746
200	0.981	1.82	0.818	1.51	0.796	1.47
500	4.075	8.048	4.276	8.251	3.847	7.597
5500	40.85	112	88.84	202.9	98.8	224
14000	78.46	233.9	222.9	538.4	296.8	687.5

Table 2: Total $b\bar{b}$ production cross sections at collider energies.

$c\bar{c}$ Production $\sqrt{s} = 200$ GeV

$d\sigma_c/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{c\bar{c}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{c\bar{c}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	232.5	102.2	1	30.90			
3	37.93	15.14	3	3.916	3	172.8	76.41
5	12.37	4.589	5	1.548	5	77.05	34.18
7	5.362	1.924	7	0.8435	7	22.60	9.611
9	2.774	0.9704	9	0.4770	9	8.548	3.429
11	1.589	0.5435	11	0.3287	11	3.671	1.427
13	1.003	0.3389	13	0.2203	13	1.863	0.6871
15	0.6715	0.2206	15	0.1608	15	0.9122	0.3438
17	0.4612	0.1542	17	0.1277	17	0.5120	0.1917
19	0.3291	0.1079	19	0.0925	19	0.3154	0.1095
21	0.2399	0.0812	21	0.0786	21	0.1883	0.0651
23	0.1857	0.0602	23	0.0589	23	0.1210	0.0415
25	0.1369	0.0428	25	0.0478	25	0.0689	0.0245
27	0.1088	0.0355	27	0.0356	27	0.0520	0.0166
29	0.0864	0.0280	29	0.0350	29	0.0364	0.0105
31	0.0697	0.0225	31	0.0282	31	0.0257	0.00785
33	0.0574	0.0191	33	0.0206	33	0.0151	0.00538
35	0.0478	0.0160	35	0.0214	35	0.0111	0.00383
37	0.0400	0.0132	37	0.0160	37	0.0678	0.00222
39	0.0343	0.0111	39	0.0135	39	0.0480	0.00198

Table 3:

[The rapidity-integrated p_T^2 distribution is given for single charm (NLO and Born) and the p_T^2 distribution in the range $|y| < 0.35$ is given for $c\bar{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 200$ GeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$c\overline{c}$ Production $\sqrt{s} = 500$ GeV

$d\sigma_c/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{c\overline{c}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{c\overline{c}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	739.7	332.0	1	68.64			
3	134.8	538.7	3	12.01	3	548.1	242.7
5	47.37	17.43	5	4.874	5	259.5	117.2
7	22.19	7.656	7	2.828	7	82.67	35.73
9	12.08	4.054	9	1.809	9	32.71	13.72
11	7.336	2.400	11	1.193	11	15.19	6.223
13	4.658	1.493	13	0.8440	13	7.878	3.108
15	3.281	1.041	15	0.6417	15	4.623	1.734
17	2.343	0.7234	17	0.5002	17	2.555	1.025
19	1.758	0.5370	19	0.3983	19	1.577	0.6242
21	1.328	0.3980	21	0.3345	21	1.143	0.4171
23	1.034	0.3052	23	0.2467	23	0.7373	0.2623
25	0.8118	0.2512	25	0.2098	25	0.4798	0.1905
27	0.6481	0.1950	27	0.1596	27	0.3227	0.1220
29	0.5411	0.1618	29	0.1371	29	0.2817	0.0886
31	0.4544	0.1284	31	0.1283	31	0.2028	0.0673
33	0.3600	0.0997	33	0.1137	33	0.1530	0.0472
35	0.3006	0.0897	35	0.0909	35	0.0997	0.0379
37	0.2701	0.0754	37	0.0758	37	0.0837	0.0293
39	0.2318	0.0643	39	0.0750	39	0.0627	0.0250

Table 4:

[The rapidity-integrated p_T^2 distribution is given for single charm (NLO and Born) and the p_T^2 distribution in the range $|y| < 0.35$ is given for $c\overline{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 500$ GeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$c\bar{c}$ **Production** $\sqrt{s} = 5.5$ **TeV**

$d\sigma_c/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{c\bar{c}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{c\bar{c}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	10680.	5146.	1	1840.			
3	2453.	989.	3	441.5	3	7749.	3558.
5	974.8	350.1	5	196.9	5	4366.	2048.
7	502.2	166.9	7	111.3	7	1622.	709.2
9	289.8	93.10	9	75.68	9	693.7	297.5
11	186.6	57.12	11	51.60	11	351.	144.0
13	126.4	37.65	13	39.07	13	188.9	78.77
15	90.91	25.96	15	27.28	15	116.3	45.67
17	68.95	19.99	17	22.55	17	75.79	27.83
19	51.44	14.43	19	18.47	19	50.16	18.82
21	41.11	11.17	21	14.14	21	30.89	12.54
23	33.29	8.965	23	13.53	23	23.02	9.024
25	27.23	7.328	25	11.02	25	18.04	6.489
27	22.28	6.031	27	9.862	27	12.32	4.547
29	18.64	4.836	29	8.612	29	10.75	3.635
31	16.10	4.203	31	6.944	31	8.112	2.609
33	13.51	3.417	33	6.359	33	5.596	2.038
35	11.55	2.961	35	5.050	35	5.217	1.719
37	9.881	2.548	37	4.683	37	4.214	1.240
39	9.078	2.212	39	4.680	39	3.500	1.039

Table 5:

[The rapidity-integrated p_T^2 distribution is given for single charm (NLO and Born) and the p_T^2 distribution in the range $|y| < 1$ is given for $c\bar{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 5.5$ TeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$c\overline{c}$ Production $\sqrt{s} = 14$ TeV

$d\sigma_c/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{c\overline{c}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{c\overline{c}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	23650.	11960.	1	4594.			
3	6067.	2473.	3	1129.	3	17250.	8046.
5	2576.	918.6	5	513.6	5	10240.	4960.
7	1368.	452.4	7	298.9	7	4119.	1840.
9	838.8	256.5	9	195.3	9	1875.	820.2
11	545.2	162.7	11	143.4	11	986.3	413.9
13	371.4	108.3	13	103.9	13	554.6	232.4
15	273.5	78.46	15	78.28	15	337.7	137.8
17	206.6	55.28	17	60.18	17	226.5	88.37
19	162.1	45.82	19	51.11	19	162.	57.77
21	130.4	33.90	21	40.63	21	107.4	41.12
23	102.5	26.90	23	34.76	23	71.90	28.14
25	84.26	22.64	25	28.13	25	59.46	21.23
27	70.85	18.27	27	24.60	27	38.62	15.25
29	60.26	15.58	29	21.12	29	30.19	12.05
31	51.43	13.08	31	17.05	31	25.45	8.619
33	45.92	11.02	33	17.66	33	22.84	6.839
35	40.26	9.718	35	16.21	35	15.55	5.642
37	33.92	7.860	37	12.86	37	13.24	4.484
39	29.80	7.281	39	10.61	39	11.64	3.454

Table 6:
[The rapidity-integrated p_T^2 distribution is given for single charm (NLO and Born) and the p_T^2 distribution in the range $|y| < 1$ is given for $c\overline{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 14$ TeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$b\bar{b}$ Production $\sqrt{s} = 200$ GeV

$d\sigma_b/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{b\bar{b}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{b\bar{b}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	0.2201	0.1123	1	0.2073			
3	0.1704	0.0883	3	0.0524			
5	0.1558	0.0680	5	0.0263			
7	0.1064	0.0541	7	0.0170			
9	0.1035	0.0577	9	0.0118	9	0.0463	0.0320
11	0.0863	0.0406	11	0.00814	11	0.4363	0.2100
13	0.0605	0.0343	13	0.00660	13	0.3184	0.1640
15	0.0478	0.0255	15	0.00441	15	0.1987	0.1050
17	0.0458	0.0264	17	0.00341	17	0.1225	0.0637
19	0.0351	0.0190	19	0.00311	19	0.0753	0.0400
21	0.0359	0.0186	21	0.00274	21	0.0492	0.0249
23	0.0300	0.0139	23	0.00237	23	0.0318	0.0160
25	0.0244	0.0122	25	0.00201	25	0.0214	0.0104
27	0.0216	0.0116	27	0.00183	27	0.0145	0.00688
29	0.0202	0.0103	29	0.00156	29	0.0091	0.00466
31	0.0171	0.0080	31	0.00147	31	0.0069	0.00321
33	0.0159	0.0083	33	0.00121	33	0.0047	0.00215
35	0.0125	0.0054	35	0.00111	35	0.0032	0.00154
37	0.0101	0.0055	37	0.00111	37	0.0022	0.00108
39	0.0097	0.0049	39	0.00086	39	0.0016	0.00075

Table 7:
[The rapidity-integrated p_T^2 distribution is given for single b quarks(NLO and Born) and the p_T^2 distribution in the range $|y| < 0.35$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 200$ GeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$b\bar{b}$ Production $\sqrt{s} = 500$ GeV

$d\sigma_b/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{b\bar{b}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{b\bar{b}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	0.9809	0.4798	1	0.3427			
3	0.7911	0.4024	3	0.2503			
5	0.6490	0.3362	5	0.1260			
7	0.5492	0.2801	7	0.0818			
9	0.4528	0.2358	9	0.0558	9	0.2652	0.1199
11	0.3807	0.1987	11	0.0426	11	1.737	0.8711
13	0.3256	0.1688	13	0.0341	13	1.436	0.7552
15	0.2781	0.1433	15	0.0285	15	0.9909	0.5222
17	0.2428	0.1248	17	0.0235	17	0.6646	0.3503
19	0.2068	0.1057	19	0.0197	19	0.4547	0.2356
21	0.1824	0.0932	21	0.0169	21	0.3132	0.1612
23	0.1595	0.0811	23	0.0147	23	0.2183	0.1121
25	0.1429	0.0719	25	0.0133	25	0.1566	0.0797
27	0.1240	0.0622	27	0.0122	27	0.1126	0.0578
29	0.1108	0.0557	29	0.0109	29	0.0850	0.0419
31	0.0984	0.0492	31	0.0098	31	0.0640	0.0315
33	0.0898	0.0435	33	0.0085	33	0.0469	0.0236
35	0.0789	0.0387	35	0.0076	35	0.0367	0.0179
37	0.0716	0.0350	37	0.0071	37	0.0291	0.0138
39	0.0646	0.0319	39	0.0074	39	0.0220	0.0108

Table 8:

[The rapidity-integrated p_T^2 distribution is given for single b quarks(NLO and Born) and the p_T^2 distribution in the range $|y| < 0.35$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 500$ GeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$b\overline{b}$ **Production** $\sqrt{s} = 5.5$ **TeV**

$d\sigma_b/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{b\overline{b}}/dp_T^2dy$ (μb/2 GeV^2)		$d\sigma_{b\overline{b}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	23.59	11.22	1	-2.366			
3	19.38	9.650	3	12.80			
5	16.25	8.253	5	6.634			
7	13.84	7.028	7	4.424			
9	11.83	6.065	9	3.303	9	6.102	2.498
11	10.14	5.148	11	2.496	11	42.57	19.58
13	8.916	4.469	13	1.946	13	37.41	18.51
15	7.776	3.890	15	1.726	15	27.66	13.89
17	6.883	3.424	17	1.439	17	20.00	9.930
19	6.132	3.004	19	1.199	19	14.41	7.187
21	5.436	2.650	21	1.073	21	10.53	5.190
23	4.825	2.296	23	0.9512	23	8.007	3.863
25	4.357	2.098	25	0.8151	25	6.028	2.911
27	3.959	1.875	27	0.7535	27	4.583	2.202
29	3.545	1.666	29	0.6718	29	3.577	1.721
31	3.208	1.526	31	0.5796	31	2.879	1.342
33	2.950	1.367	33	0.5276	33	2.248	1.078
35	2.683	1.207	35	0.5491	35	1.813	0.8730
37	2.468	1.131	37	0.4692	37	1.507	0.7100
39	2.255	1.034	39	0.4334	39	1.261	0.5682

Table 9:
[The rapidity-integrated p_T^2 distribution is given for single b quarks(NLO and Born) and the p_T^2 distribution in the range $|y| < 1$ is given for $b\overline{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 5.5$ TeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

$b\overline{b}$ Production $\sqrt{s} = 14$ TeV

$d\sigma_b/dp_T^2$ (μb/2 GeV^2)			$d\sigma_{b\overline{b}}/dp_T^2 dy$ (μb/2 GeV^2)		$d\sigma_{b\overline{b}}/dM$ (μb/2 GeV)		
p_T^2 (GeV^2)	NLO	LO	p_T^2 (GeV^2)	NLO	M (GeV)	NLO	LO
1	68.43	32.54	1	-13.36			
3	56.73	28.24	3	34.99			
5	47.74	24.25	5	17.94			
7	41.32	20.92	7	11.83			
9	35.45	18.10	9	8.519	9	17.57	6.876
11	30.61	15.55	11	6.833	11	124.0	55.90
13	27.07	13.60	13	5.537	13	112.4	54.74
15	23.97	11.93	15	4.665	15	85.11	42.17
17	21.22	10.41	17	3.813	17	62.92	30.97
19	18.86	9.192	19	3.392	19	46.41	22.58
21	16.84	8.225	21	3.125	21	34.27	16.62
23	15.20	7.227	23	2.618	23	26.12	12.44
25	13.71	6.477	25	2.328	25	19.89	9.457
27	12.61	5.878	27	2.112	27	15.51	7.304
29	11.20	5.215	29	1.772	29	11.93	5.673
31	10.43	4.710	31	1.811	31	9.610	4.538
33	9.520	4.368	33	1.588	33	7.908	3.587
35	8.651	3.962	35	1.409	35	6.267	2.966
37	7.795	3.492	37	1.349	37	5.132	2.402
39	7.272	3.245	39	1.279	39	4.323	2.017

Table 10:

[The rapidity-integrated p_T^2 distribution is given for single b quarks(NLO and Born) and the p_T^2 distribution in the range $|y| < 1$ is given for $b\overline{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 14$ TeV and calculated with MRS D-$'$ parton distributions. Note the 2 GeV bin width for the distributions.]

Figure Captions

1. (a) Gluon distributions from GRV HO (solid), MRS D0′ (dashed), MRS D-′ (dot-dashed) at $Q = 2.4$ GeV and GRV HO (dotted) at $Q = 1.2$ GeV. (b) The running of the coupling constant with scale.

2. Investigation of uncertainties in the total cross section as a function of scale. Variation of the $c\overline{c}$ production cross sections with scale at (a) RHIC and (b) LHC. Variation of the $b\overline{b}$ production cross sections with scale at (c) RHIC and (d) LHC. Variation of the $c\overline{c}$ production cross sections at $\sqrt{s}$ at 200 GeV with μ_R at fixed μ_F (e) and with μ_F at fixed μ_R (b). In each case, the circles represent the NLO calculation, the crosses, the Born calculation.

3. Total charm production cross sections from pp and pA measurements [18, 19, 20, 21, 22] compared to calculations. The curves are: MRS D-′ $m_c = 1.2$ GeV, $\mu = 2m_c$ (solid); MRS D0′ $m_c = 1.2$ GeV, $\mu = 2m_c$ (dashed); GRV HO $m_c = 1.3$ GeV, $\mu = m_c$ (dot-dashed); GRV HO $m_c = 1.5$ GeV, $\mu = m_c$ (dotted).

4. (a) Total charm production cross sections from $\pi^- p$ measurements [18, 25, 26, 27, 28] compared to calculations. The curves are: GRV HO $m_c = 1.3$ GeV, $\mu = m_c$ on a nucleon (solid) and proton target (dashed); MRS D-′ $m_c = 1.2$ GeV, $\mu = 2m_c$ with SMRS P2 (dot-dashed) and SMRS P1 (dotted) on a proton target. (b) The $b\overline{b}$ production cross section from $\pi^- p$ interactions [29, 30, 31, 32]. The calculations use $m_b = 4.75$ GeV and $\mu = m_b$. The curves use GRV HO (solid) and MRS D-′ with SMRS P1 (dashed).

5. Comparison with D meson (a) p_T^2 and (b) x_F distributions at 800 GeV [19]. The NLO calculations are with MRS D-′ (solid) and GRV HO (dashed) parton distributions.

6. Comparison with $D\overline{D}$ production for (a) p_T^2 and (b) M and (c) ϕ at 800 GeV [36]. The NLO calculations are with MRS D-′ (solid) and GRV HO (dashed) parton distributions.

7. Comparison with b quark production cross sections at (a) UA1 [37] and (b) CDF [38]. The NLO calculations are with MRS D-′ (solid) and GRV HO (dashed) parton distributions.

8. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 200$ GeV with MRS D-′ distributions. The c quark p_T distributions at NLO (solid) are shown in (a) and the rapidity distributions at LO (dashed) and NLO (solid) are shown in (b). The $c\overline{c}$ pair distributions are shown in (c)-(f). The LO (dashed) distributions are shown only for mass and rapidity. Additionally, the p_T and p_{T_p} distributions are shown with a central cut in rapidity. (The rapidity bin widths are removed.) The corresponding distributions

from HIJING are also shown, again with the rapidity bin width divided out.

9. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 200$ GeV with GRV HO distributions. The corresponding distributions from HIJING are also shown.

10. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 500$ GeV with MRS D-$'$ distributions.

11. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 500$ GeV with GRV HO distributions.

12. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 5.5$ TeV with MRS D-$'$ distributions. The corresponding distributions from HIJING are also shown.

13. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 5.5$ TeV with GRV HO distributions. The corresponding distributions from HIJING are also shown.

14. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 14$ TeV with MRS D-$'$ distributions.

15. Predictions for c and $c\overline{c}$ production at $\sqrt{s} = 14$ TeV with GRV HO distributions.

16. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 200$ GeV with MRS D-$'$ distributions.

17. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 200$ GeV with GRV HO distributions.

18. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 500$ GeV with MRS D-$'$ distributions.

19. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 500$ GeV with GRV HO distributions.

20. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 5.5$ TeV with MRS D-$'$ distributions.

21. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 5.5$ TeV with GRV HO distributions.

22. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 14$ TeV with MRS D-$'$ distributions.

23. Predictions for b and $b\overline{b}$ production at $\sqrt{s} = 14$ TeV with GRV HO distributions.

24. Dilepton (a) mass, (b) rapidity, and (c) p_T distributions at $\sqrt{s} = 200$ GeV from $c\overline{c}$ decays calculated using MRS D-$'$ distributions are shown.

25. Dilepton distributions at $\sqrt{s} = 200$ GeV from $b\overline{b}$ decays.

26. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from correlated $c\overline{c}$ decays.

27. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from uncorrelated $c\overline{c}$ decays.

28. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from $b\overline{b}$ decays.