Perturbative QCD:

from basic principles to current applications

I : Basic Ideas – Qualitative Descriptions

- What is Quantum Chromodynamics (QCD)?
- Why do we believe in Quarks and Gluons?
  Long-distance phys. – the Quark Model
  Short-distance phys. – the Parton Picture
- How can a Strong-interaction theory, QCD, give rise to the simple Parton Picture?
  The importance of *scaling* and *factorization*:
  Renormalization and Assyptotic Freedom;
  Infra-red Safety and *Factorization Thms.*
- Precision Tests of PQCD in e^+e^- Interactions

II : Quantitative case study – Deep Inelastic Scattering at NLO

- LO and NLO calculations
- The origin of *collinear singularities*
- The separation of long- and short- distances
- How does *Factorization* form the foundation of the QCD-Parton-Model?

III : General Formalism of PQCD

- From NLO to Higher Orders
- Universal Parton Dist. & QCD evolution
- How are Hard X-sect's actually calculated?
- Scale- & Scheme-dependencies
- Renormalization vs. Factorization – a head-to-head comparison and correspondence

IV : Survey of Hard Scattering Processes

- Q-Qbar annihilation and Drell-Yan processes: lepton-pair, W- and Z- production
- g-Q scattering and Dir. Photon Prod.
- g-g scattering and Jet Production
- Two-scale hard-scattering processes
- A 2-scale process: Heavy Quark Production

V : Global QCD Analysis

- Overview of Physical Processes and Expts.
- Overview of NLO QCD Theory Input
- Global QCD Analysis
- Parton Distribution Functions

Open Questions and Challenges

Basic Elements of Quantum Chromodynamics

–Non-abelian Gauge Field Theory with
SU(3) color Gauge Symmetry

Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$.

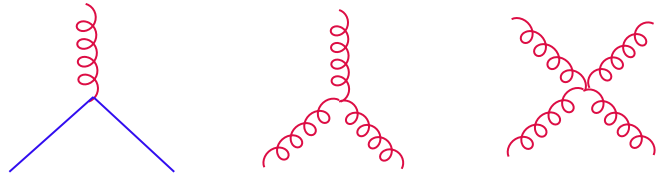
Basic Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\ \partial\!\!\!/ - g\ A\!\!\!/ \cdot t - m)\psi - \frac{1}{4} G(A\cdot T, g)\cdot G(A\cdot T, g)$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.

  Group factors: $C_F = \frac{4}{3}$; $T_F = \frac{1}{2}$; $C_A = 3$

Interaction Vertices:



Why do we believe Hadrons are made of Quarks?

– Strong Interaction at *long-distance* scale –
Hadron Spectroscope $\Rightarrow$ the *Quark Model*

Quantum #'s of Mesons given by: L=0,1,2
SU(3)$_{flv}$ Octets (nonets) of q-qbar bound states.

Addition of Charm Q.N. $\Rightarrow$ *SU(4)*

– see plots (PDG)

Combining of *SU(3)*$_{flavor}$ & *SU(2)*$_{spin}$ $\Rightarrow$ *SU(6)*

Quantum #'s of Baryons given by: L=0,1,2
SU(3)$_{flv}$ Octets & decuplets of q-q-q bound states.

– see plots (PDG)

These are the *Constituent Quarks*

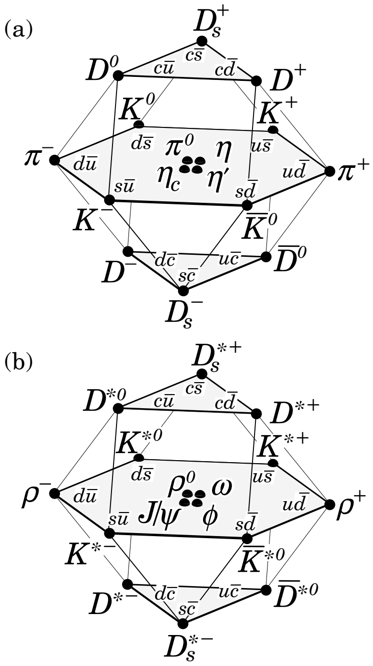
However:

Quarks have not been found in nature.

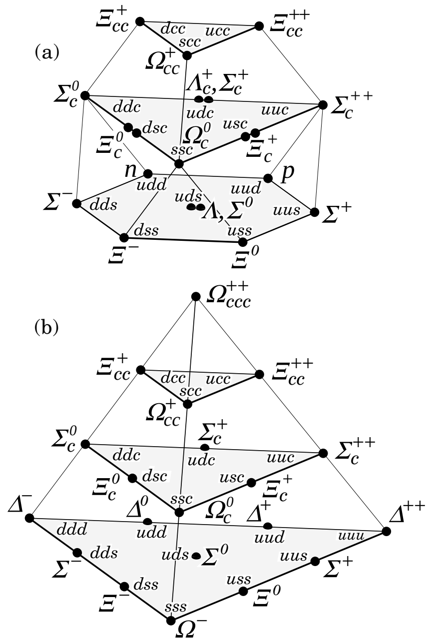
Interaction is very (infinitely?) strong at long-distances.

QCD at Low Energy (long-distance) Scales
-- Confinement, Bound-states
=> the Quark Model

Mesons



Baryons



Meson States in the Quark Model

$N\ ^{2S+1}L_J$	J^{PC}	$u\bar{d}$, $u\bar{u}$, $d\bar{d}$ $I=1$	$u\bar{u}$, $d\bar{d}$, $s\bar{s}$ $I=0$	$c\bar{c}$ $I=0$	$b\bar{b}$ $I=0$	$\bar{s}u$, $\bar{s}d$ $I=1/2$	$c\bar{u}$, $c\bar{d}$ $I=1/2$	$c\bar{s}$ $I=0$	$\bar{b}u$, $\bar{b}d$ $I=1/2$	$\bar{b}s$ $I=0$	$\bar{b}c$ $I=0$
$1\ ^1S_0$	0^{-+}	π	η, η'	η_c		K	D	D_s	B	B_s	B_c
$1\ ^3S_1$	1^{--}	ρ	ω, ϕ	$J/\psi(1S)$	$\Upsilon(1S)$	$K^*(892)$	$D^*(2010)$	D_s^*	B^*	B_s^*	
$1\ ^1P_1$	1^{+-}	$b_1(1235)$	$h_1(1170)$, $h_1(1380)$	$h_c(1P)$		${K_{1B}}^\dagger$	$D_1(2420)$	$D_{s1}(2536)$			
$1\ ^3P_0$	0^{++}	$a_0(1450)^*$	$f_0(1370)^*$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$					
$1\ ^3P_1$	1^{++}	$a_1(1260)$	$f_1(1285)$, $f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	${K_{1A}}^\dagger$					
$1\ ^3P_2$	2^{++}	$a_2(1320)$	$f_2(1270)$, $f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	$D_2^*(2460)$				
$1\ ^1D_2$	2^{-+}	$\pi_2(1670)$	$\eta_2(1645)$, $\eta_2(1870)$			$K_2(1770)$					
$1\ ^3D_1$	1^{--}	$\rho(1700)$	$\omega(1600)$	$\psi(3770)$		$K^*(1680)^\ddagger$					
$1\ ^3D_2$	2^{--}					$K_2(1820)$					
$1\ ^3D_3$	3^{--}	$\rho_3(1690)$	$\omega_3(1670)$, $\phi_3(1850)$			$K_3^*(1780)$					
$1\ ^3F_4$	4^{++}	$a_4(2040)$	$f_4(2050)$, $f_4(2220)$			$K_4^*(2045)$					
$2\ ^1S_0$	0^{-+}	$\pi(1300)$	$\eta(1295)$, $\eta(1440)$	$\eta_c(2S)$		$K(1460)$					
$2\ ^3S_1$	1^{--}	$\rho(1450)$	$\omega(1420)$, $\phi(1680)$	$\psi(2S)$	$\Upsilon(2S)$	$K^*(1410)^\ddagger$					
$2\ ^3P_2$	2^{++}		$f_2(1810)$, $f_2(2010)$		$\chi_{b2}(2P)$	$K_2^*(1980)$					
$3\ ^1S_0$	0^{-+}	$\pi(1800)$	$\eta(1760)$			$K(1830)$					

Baryon States in the Quark Model

J^P	(D, L_N^P)	S	Octet members				Singlets
$1/2^+$	$(56,0_0^+)$	1/2	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$	1/2	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(?)$	
$1/2^-$	$(70,1_1^-)$	1/2	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	$(70,1_1^-)$	1/2	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$	3/2	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
$3/2^-$	$(70,1_1^-)$	3/2	$N(1700)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$	3/2	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(?)$	
$1/2^+$	$(70,0_2^+)$	1/2	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(?)$
$3/2^+$	$(56,2_2^+)$	1/2	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$	1/2	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$	1/2	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$	3/2	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$	1/2	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	
			Decuplet members				
$3/2^+$	$(56,0_0^+)$	3/2	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$	
$1/2^-$	$(70,1_1^-)$	1/2	$\Delta(1620)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$3/2^-$	$(70,1_1^-)$	1/2	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$5/2^+$	$(56,2_2^+)$	3/2	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$7/2^+$	$(56,2_2^+)$	3/2	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$	
$11/2^+$	$(56,4_4^+)$	3/2	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	

Why is Hadron Interactions at High Energies described by the QCD Parton Picture?

– Strong Interaction at *short -distance* scale –
Hard Scattering Probes $\Rightarrow$ the *Parton Model*

Evidences for the existence of Partons:

"direct": Most Hard Sc. events contain visible "jets" $\Rightarrow$ fragments of underlying partons?

Are they point-like? "Rutherford expts"

- ∗ (Bjorken) Scaling in DIS;
- ∗ annihilation into hadrons;
- ∗ Hadron-hadron scattering, ...

Properties of Partons:

2-Jet angular distributions in e^+e^- , DIS, DY proc. are the same as for scattering into leptons $\Rightarrow$ underlying partons are fermionic

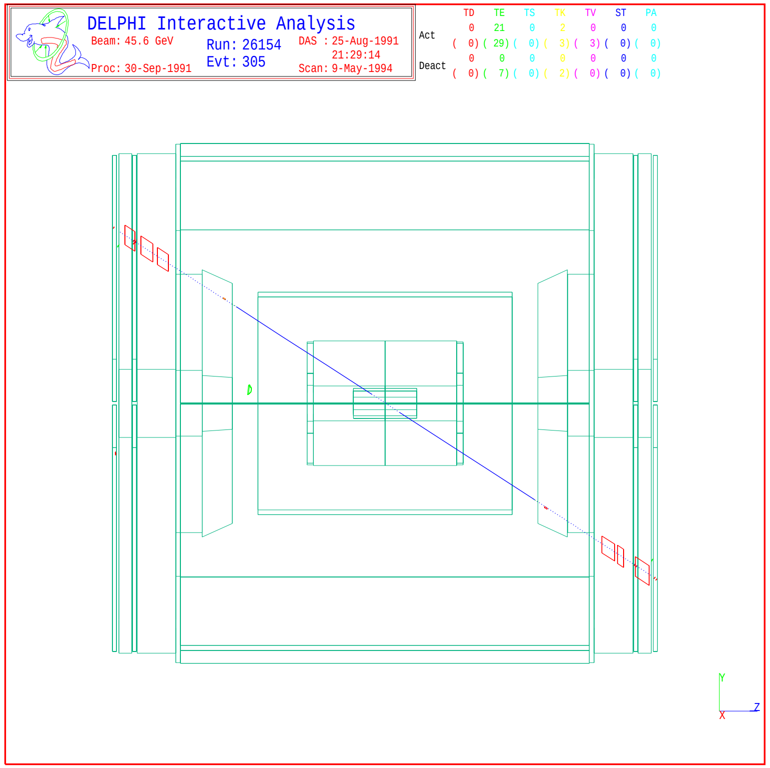
Expts. : EM & Weak Isospin couplings of partons = those of leptons $\Rightarrow$ "Current Quarks"

3-jet events and their detailed properties prove the existence, and spin of gluons

$\Rightarrow$ QCD-parton Model complete.

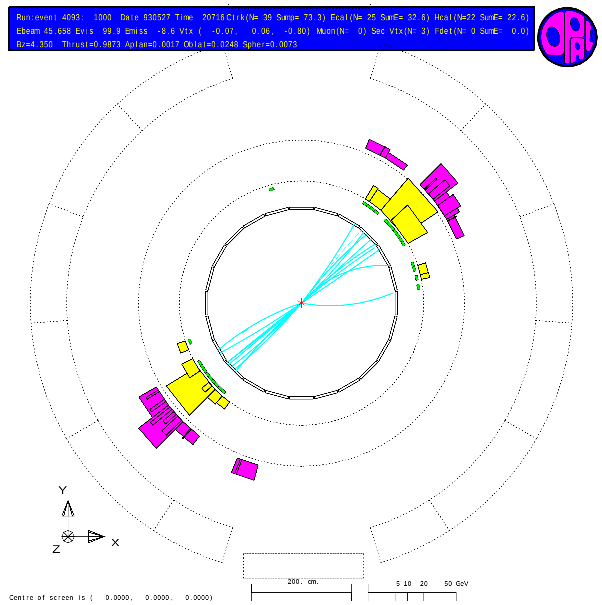
Big Question .....

An elementary particle event
– $e^+ \ e^- \longrightarrow \mu^+ \ \mu^-$

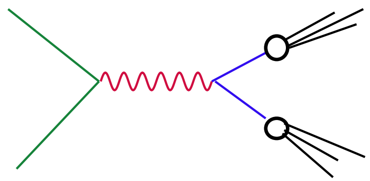


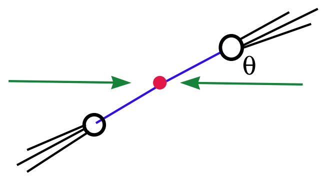
Two-jet Events:
Quark – anti-quark Pair Production

Typical $e^+ \ e^-$ event with hadron final states:



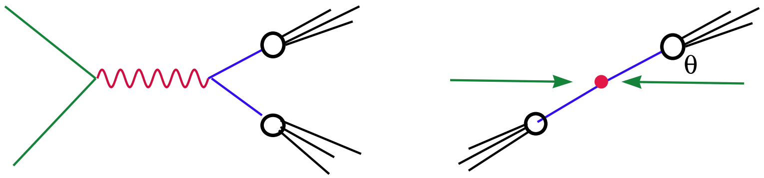
Parton process underlying 2-jet events



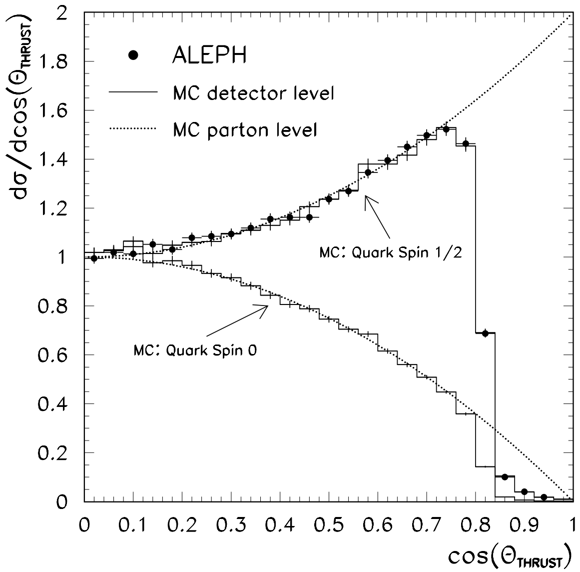


Measuring the Spin of the Quark Parton

Use the angular distribution of 2-jet events

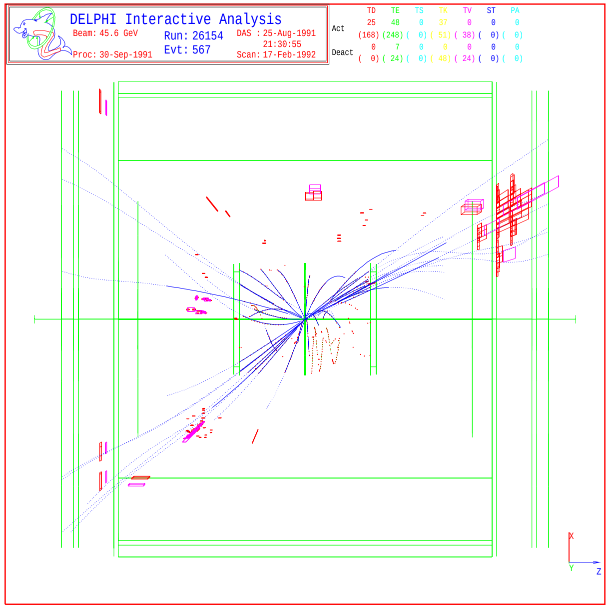


Experimental result (ALEPH):

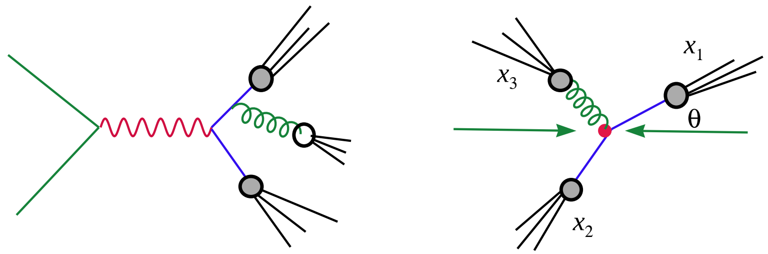


3 Jet Events and the Gluon Parton

A typical 3-jet event ($\sim$ 10% prob.):

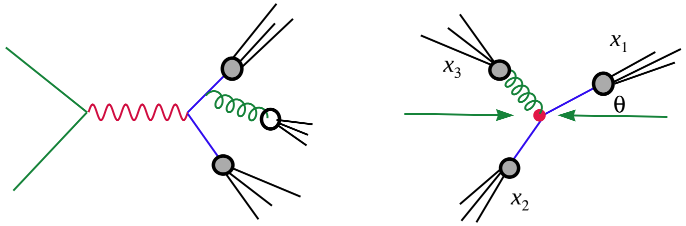


Parton process underlying 3-jet events

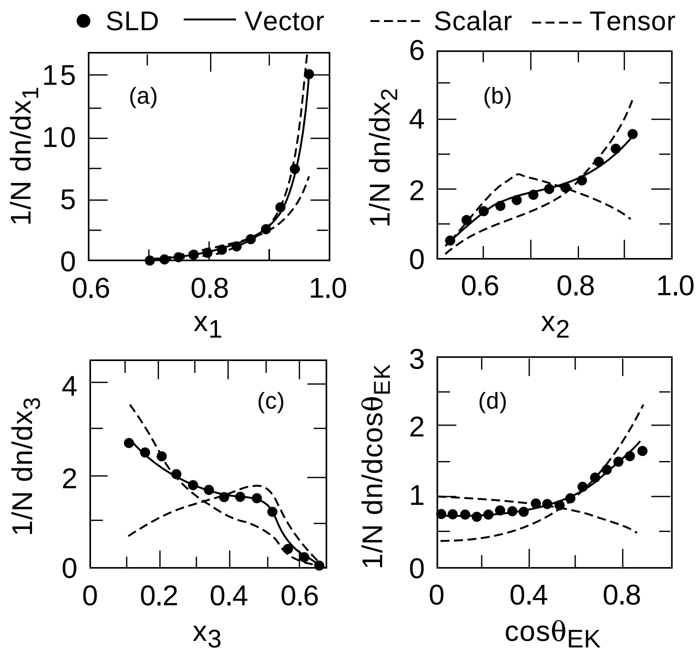


Measuring the Spin of the Gluon

Use the angular distributions of 3-jet events



Experimental result from SLD:



Question:

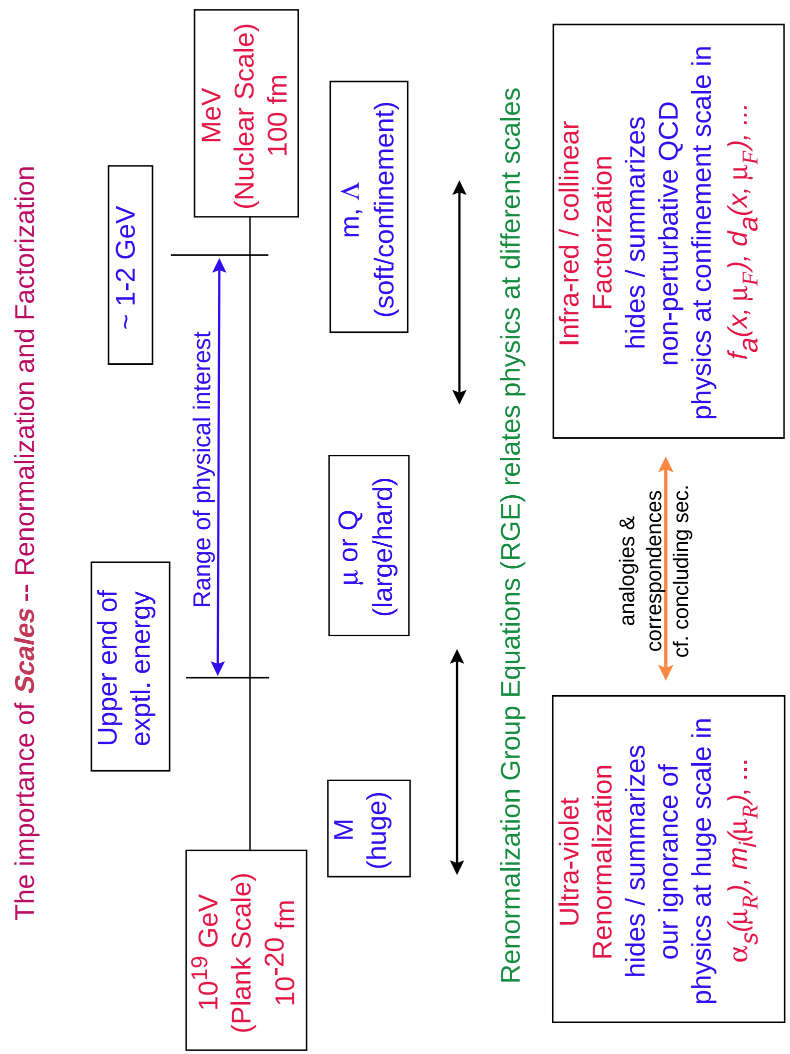
How could the *simple parton picture* possibly hold in a strongly interacting gauge field theory such as Quantum Chromodynamics?

Answers:

1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.
2. Infra-red Safety:
   There are classes of "infra-red safe" quantities which are independent of long-distance physics, hence are calculable in PQCD.
3. Factorization:
   There are an even wider class of physical quantities which can be *factorized* into long-distance pieces (not calculable, but *universal*) and short-distance pieces (process-dependent, but infra-red safe, hence *calculable*).

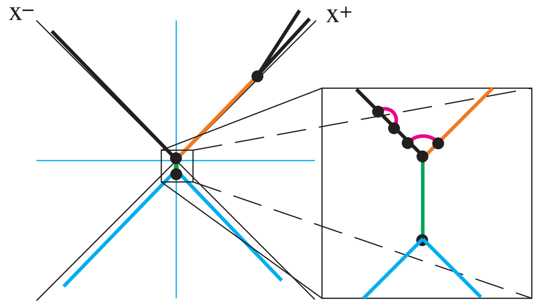
"Bottom Line"

PQCD does not give all the answers; but it does cover quite a lot of ground!



The smallest time (shortest distance) scales and *renormalization*

What does renormalization do?



⋆ Say, $\overline{\text{MS}}$ renormalization with *ren. scale* μ_R.
in principle, μ ($\equiv \mu_R$) is arbitrary;
in practice, μ is chosen $\approx$ a phys. scale Q.†

⋆ Physics of scales $|t| \ll 1/\mu$ removed from perturbative calculation; renormalization hides:

- the ugly: ultra-violet divergences; and
- the beautiful: short-distance physics $< \frac{1}{\sqrt{s}}$
  (*New Physics*: Q. Gravity, GUT, Super-xx, ...)

⋆ Effects of small time physics are absorbed into the *running coupling* (function) $\alpha_s(\mu)$ (also running masses $m_i(\mu)$, field operators, ...).

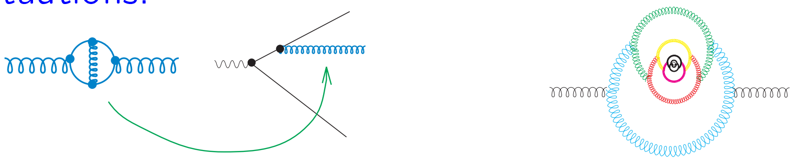
⋆ For QCD, $\alpha_s(\mu)$ decreases as μ increases — *Asymptotic freedom*.

Running Coupling & Asymptotic Freedom

- The *running coupling* results from renormalization, accounts for physics in the RG range:

(long-time)

It sums the leading effects of short-time fluctuations:



- The μ dependence of $\alpha_s(\mu)$ is given by the *renormalization group equation* :

$$\frac{d\ \alpha_s(\mu)}{\pi\, d\ln(\mu^2)} = -\beta(\alpha_s(\mu)) = -\beta_0 \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 - \beta_1 \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + \cdots.$$

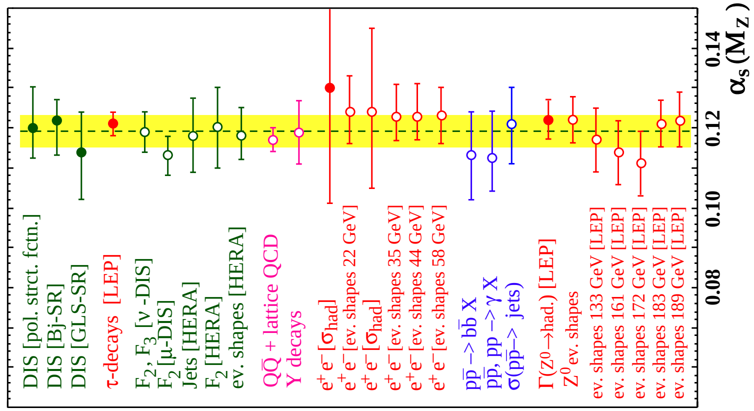
Solution (to 1-loop order) sums *leading* quantum fluctuation effects to all orders:

$$\alpha_s(\mu) \approx \alpha_s(M) - \ln(\frac{\mu^2}{M^2})\ \alpha_s^2(M) + (\frac{\beta_0}{\pi})^2 \ln(\frac{\mu^2}{M^2})\alpha_s^3(M)$$

$$= \frac{\alpha_s(M)}{1+\frac{\beta_0}{\pi}\alpha_s(M)\ln(\frac{\mu^2}{M^2})} = \frac{\pi}{\beta_0 \ln(\frac{\mu^2}{\Lambda^2})}.$$

$\alpha_s(M)$, or Λ, is a parameter in the solution.

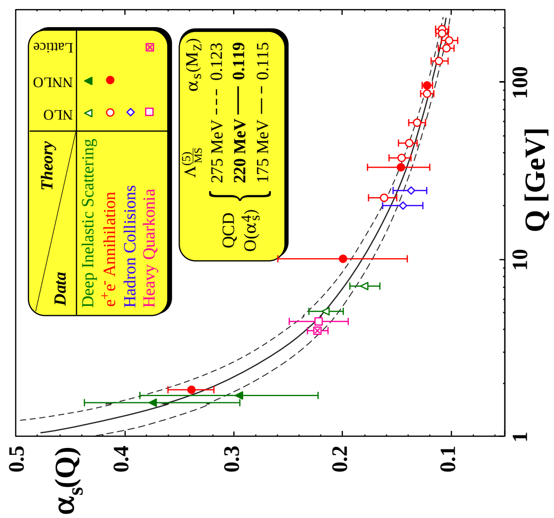
- $\beta > 0 \Rightarrow \alpha_s(\mu)$ decreases as μ increases — *Asymptotic freedom*.



Asymptotic Freedom

Universal (running) coupling:

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)}(1+\ldots)$$

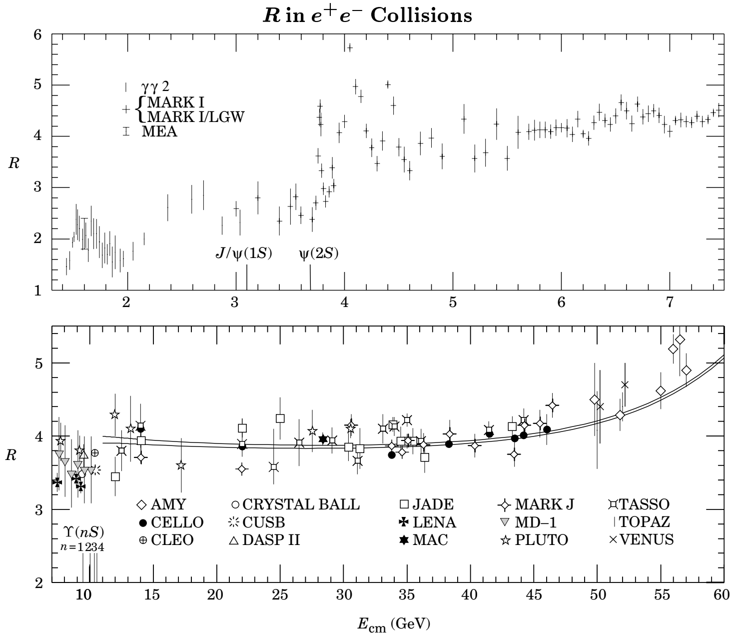


(S. Bethke)

Asymptotic Freedom and Total Cross-section for $e^+e^- \to$ hadrons

$$R = \frac{\sigma_T^{e^+e^- \to \text{hadrons}}}{\sigma^T_{e^+e^- \to \mu^+\mu^-}}(E) = \left(\sum_f Q_f^2\right)[1 + \Delta]$$

(Δ = Order α_s QCD correction term.)



- Energy-indep. $\Rightarrow$ point-like partons.
- Value of R measures sum of quark charges.
- Rise for $E > 45$ GeV due to Z-tail.

Factorization of short- and long-distance physics in e^+e^- Annihilation

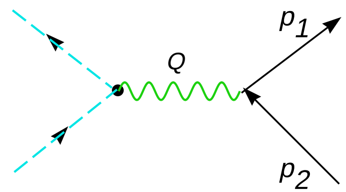
Example: One particle/jet inclusive cross-section

(Zero-mass quarks)

Leading Order:

Back-to-back jets:

$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2\theta)$$



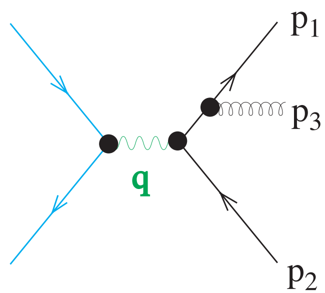
Next-to-Leading Order:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2\,q \cdot p_i}{s}$$

$i = 1, 2, 3$

$$\sum x_i = \frac{2\,q \cdot \sum p_i}{s} = 2$$

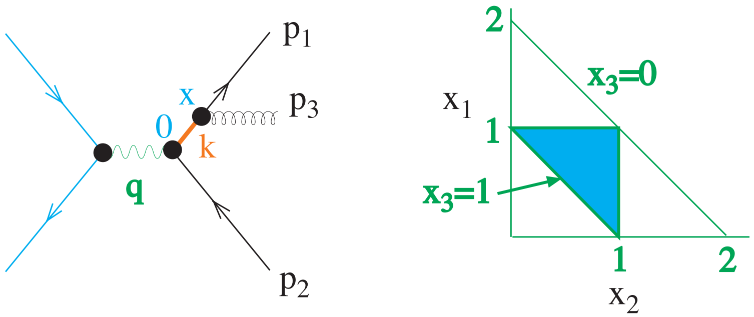
$$2(1 - x_1) = x_2 x_3 (1 - \cos\theta_{23}), \quad cycl.$$

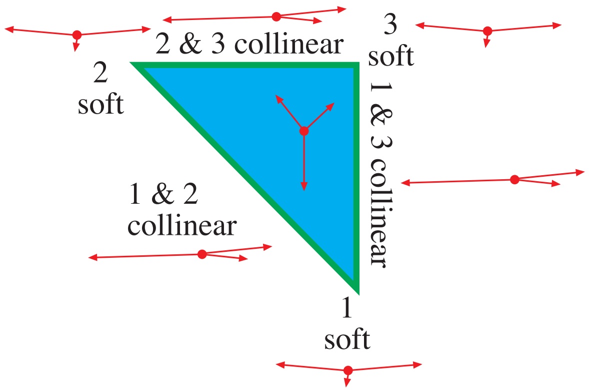


$$\frac{d\sigma}{\sigma_0\, dx_1\, dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_1^2}{(1 - x_1)(1 - x_2)}$$

Singularities in the perturbative cross-section formula

$$\frac{d\sigma}{\sigma_0\, dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_1^2}{(1-x_1)(1-x_2)}$$





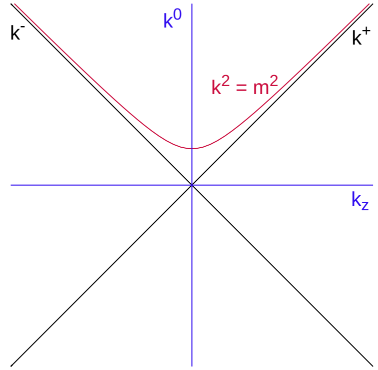
Moral: Singularities occur at boundaries of phase space (colinear/soft) where 2-->3 kinematics collapses to 2-->2 and *the 4-mom. k of the internal line goes on-mass shell.*

(These singularities correspond to solutions to the Landau equations for pinch surfaces of the Feynman diagrams.)

Separation of Short- and Long-distance Interactions
Space-time Picture

Null Plane coordinates:

$$k^{\pm} = \frac{k^0 \pm k^3}{\sqrt{2}} \;\; ; \;\; k^2 = 2k^+k^- - \vec{k}_T^{\,2}$$



Space-time connection:

$$\int d^4x\, e^{i\,x\cdot k} \cdots$$

$$x \cdot k = x^- k^+ + x^+ k^- - \vec{x}_T \cdot \vec{k}_T$$

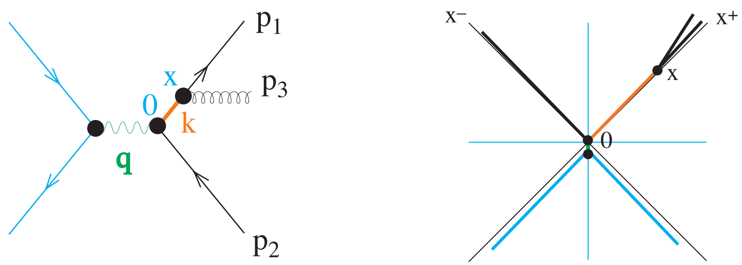
On mass shell:

$$k^- = \frac{\vec{k}_T^{\,2} + m^2}{2k^+}$$

High-energy interaction:

$$k^+ \to \infty \;\Rightarrow\; k^- \to 0 \;\Rightarrow\; x^+ \to \infty$$

Correspondence between singularities in p-space and the development of the system in space time:



Moral: Singularities associated with divergent perturbative X-sec <--> interactions a long time after the creation of the initial quark-antiquark pair.

What to do with the long-distance physics associated with these colinear/soft singularities?

Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

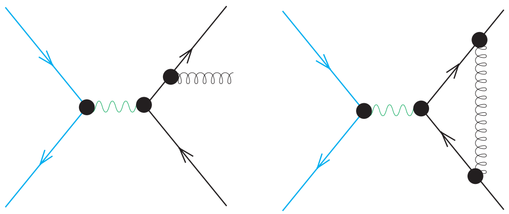
Total Hadronic Cross-section (*inclusive*):

$$\sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)\, c_1 + \ldots]$$

Block – Nordsieck Thm $\rightarrow$ $c_{1,2,\ldots}$ *are finite, i.e. IRS (unitarity)*

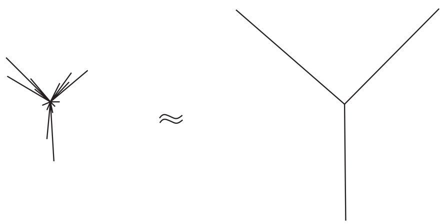
Order α_s:

Cancellation of the colinear/soft singularities between real and virtual diagrams



Essential feature of a general IRS physical quantity:

the observable must be such that it is insensitive to whether n or n+1 particles contributed -- if the n+1 particles has n-particle kinematics



e.g. a IRS "jet algorithm"

Infrared-safe Observables and Factorization Theorems

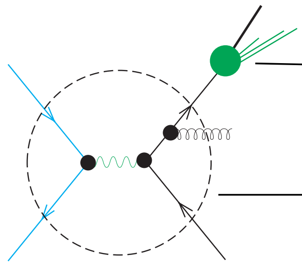
Other examples of IRS observables:

- Sterman-Weinberg jet cross-sections and their modern variations (*Jade-, Durham-, ... algorithms*);
- Jet shape observables: Thrust, ... ;
- energy-energy correlation ; ....

A different strategy: Factorization — QCD Parton model

Factorize the physical observable into a calculable IRS part and a non-calculable but universal piece.

Example: One particle inclusive cross-section



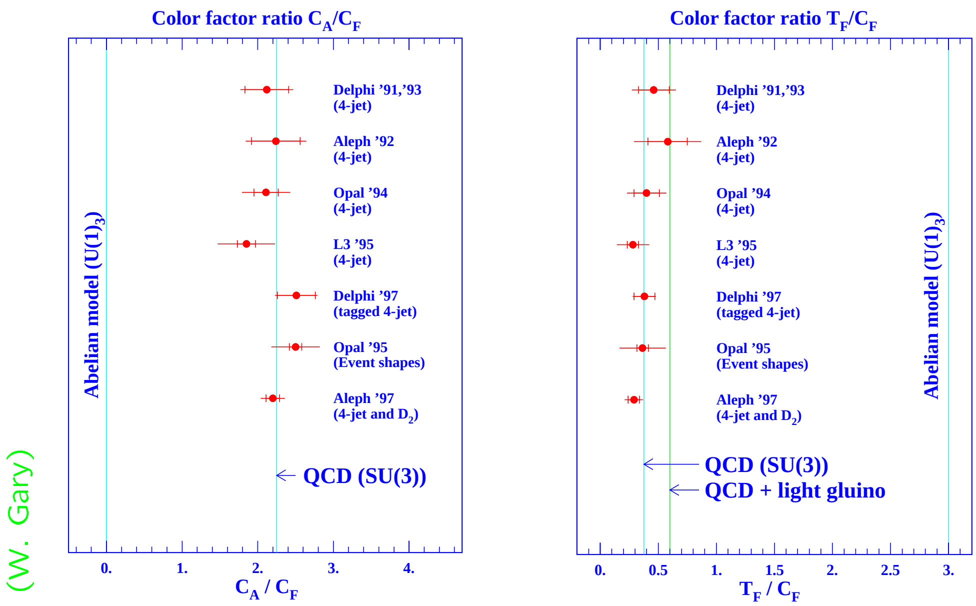
Fragmentation function: $D_a^h(z,\mu)$
Long-distance physics;
Universal.

Hard scattering: $\hat{\sigma}^a(s,z,\mu)$
Short distance physics;
IRS, perturbatively cal.

$$\sigma(s,z) = \int_z^1 \frac{d\varsigma}{\varsigma}\, \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\varsigma}, \alpha_s(\mu)\right) \cdot D_a(\varsigma,\mu)$$

Experimental Analyses of IRS Quantities: Color Factors in Quark and Gluon Couplings

Detailed fits to 2,3-jet event shapes and 4-jet distr. yield, in addition to $\alpha_s(Q)$:



Scale Dependence and Independence

Physical X-sect's know nothing about μ (μ_R); what about the PQCD theory predictions?

Example: Cross section for $e^+e^- \to$ hadrons:

$$\sigma_{\rm tot} = \frac{12\pi\alpha^2}{s}\left(\sum_f Q_f^2\right)[1+\Delta] \qquad \text{(infra-red safe)}$$

$$\Delta = \frac{\alpha_s(\mu)}{\pi} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 C_2(\frac{\mu^2}{s}) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 C_3(\frac{\mu^2}{s}) + \cdots$$

$$C_2(\frac{\mu^2}{s}) = 1.4092 + 1.9167\ \ln\left(\frac{\mu^2}{s}\right)$$

$$C_3(\frac{\mu^2}{s}) = -12.805 + 7.8186\ \ln\left(\frac{\mu^2}{s}\right) + 3.674\ \ln^2\left(\frac{\mu^2}{s}\right)$$

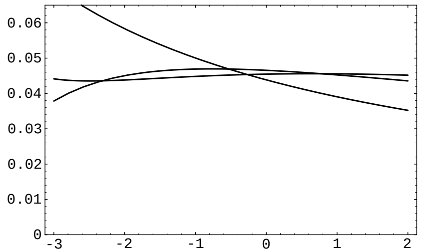
- $\alpha_s(\mu)$ mustn't be big $\Rightarrow$ choose $\mu \gg \Lambda$;
- $\log\left(\mu^2/s\right)$ not big $\Rightarrow$ must choose $\mu \sim \sqrt{s}$;
- Consistency of theory $\Rightarrow$ μ-dep. of $\alpha_s(\mu)$ and $C_n(\frac{\mu^2}{s})$ must compensate each other: $\frac{d}{d\ln\mu^2}\Delta = \frac{d}{d\ln\mu^2}\sum_{n=1}^{\infty} C_n(\mu)\ \alpha_s(\mu)^n = 0$
- For the truncated perturbative series, one can prove:

$$\frac{d}{d\ln\mu}\sum_{n=1}^{N} C_n(\mu)\ \alpha_s(\mu)^n \sim \mathcal{O}(\alpha_s(\mu)^{N+1})$$

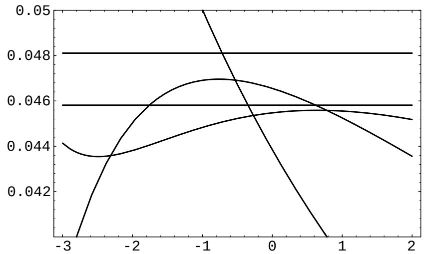
The science and art of choosing μ

Take $\alpha_s(M_Z) \approx 0.117$, $\sqrt{s} = 34$ GeV, 5 flavors.
Plot $\Delta(\mu)$ versus p defined by

$$\mu = 2^p \sqrt{s} \qquad \text{i.e.} \qquad p = log_2(\mu/\sqrt{s})$$



μ-dep.—ground zero.



Expanded view of μ-dep.

LO: $\Delta_1(\mu) = \alpha_s(\mu)/\pi$ (monotonic μ-dep.)

NLO: $\Delta_2(\mu)$ up to α_s^2 (note improvement).

NNLO: $\Delta_3(\mu)$ up to α_s^3 (tiny residual μ-dep.)

Error band:

Central value: $\hat{\mu}$ based on $\left[\frac{d\Delta_2(\mu)}{dln\mu}\right]|_{\mu=\hat{\mu}} = 0$
Error size: range of $\Delta(\mu)$ for $2\hat{\mu} > \mu > 0.5\hat{\mu}$,

What does $\Delta_3(\mu)$ say about the error estimate based on $\Delta_{1,2}(\mu)$?

Note: The IRS σ_{tot} only depends on the *renormalization scale*, μ_R. The *factorization scale* μ_f will appear in hadronic X-sect.'s with factorized long- and short- distance pieces.

Summary : Salient Features of Perturbative QCD

Strong interaction, yet perturbative method is applicable;
Confined quarks and gluons, yet calculations based on partons can explain large classes of observed hadronic processes.

Keys to resolve the apparent dilemma:

Infra-red safe observables:
Long-distance interaction (non-calculable) does not change the short-distance perturbative results.

Factorization Theorems:
Large classes of inclusive cross-sections can be factorized into an IRS (calculable) hard X-sec. and a set of universal parton distribution/fragmentation functions which characterize long-distance hadron structure.

Some important features and consequences:

Ultra-violet Renormalization → Asymptotic freedom
Running coupling $\alpha(\mu_r)$ and masses $m_i(\mu_r)$;

Factorization of mass- (colinear-) singularities →
Both *parton-distribution/fragmentation functions* and *hard cross-sections* are factorization (and renormalization) scheme- and scale(μ_f)- dependent.
"Scaling violation"