Manual of General Physics : HW1

2.28

(a) Initial velocity($\vec{v}(0)$) is given by

$$\vec{v}(0) = 100\hat{j} = \left(\frac{dy(t)}{dt}\right)_{t=0} \hat{j} \quad in \quad Cartesian \quad coordinate(\hat{i}, \hat{j}, \hat{k}) \tag{1}$$

In this system, there is gravitational force with opposite direction of initial velocity. It can be expressed

$$\vec{F} = m\frac{d^2y(t)}{dt^2}\hat{j} = -mg\hat{j} \tag{2}$$

where g means gravitational accerelation constant. Therefore, we can calculate speed and position of this arrow

$$\frac{d^2y(t)}{dt^2} = -g \Rightarrow \frac{dy(t)}{dt} = -gt + \frac{dy(t)}{dt}|_{t=0} \tag{3}$$

As we calculate integration of eq.(3) one more time, we can get $y(t)$

$$y(t) = -\frac{1}{2}gt^2 + 100.t + y(0) \tag{4}$$

When we suppose that the initial position of arrow is zero, then $y(0) = 0$
When arrow arrive top, the velocity of y-component should be zero and then time(t_h) is

$$\frac{dy}{dt}(t) = 0 \Rightarrow t_h = \frac{100}{g} \tag{5}$$

Therefore, the arrow's the highest position with $g = 9.8m/s^2$is

$$y(t_h) = 5000/g \approx 510.2.m \tag{6}$$

(b) From eq.(4), we can calculate the time(t_r) when arrow arrive the initial position

$$y(t) = -\frac{1}{2}gt^2 + 100t = t(-\frac{1}{2}gt + 100) \tag{7}$$

We assume that the initial position is $y(0) = 0$.

$$t_r = 200/g \approx 20.4s. \tag{8}$$

3.3

(a) We already know about initial accerelation($\vec{a}$) and initial velocity($\vec{v}(0)$)

$$\vec{a} = 3.00\hat{j}(m/s^2) = \frac{d^2y(t)}{dt^2}\hat{j} \quad \vec{v}(0) = 5.00\hat{i}(m/s^2) = \frac{dx(t)}{dt}\hat{i} \tag{9}$$

From these two, we can calculate position vector

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = 3t\hat{j} + \vec{v}(0) = 5.00\hat{i} + 3t\hat{j} \tag{10}$$

$$\vec{r}(t) = 5t\hat{i} + 1.5t^2\hat{j} \tag{11}$$

(b) velocity is the same with eq.(10)
(c) when t=2.s, the position vector is

$$\vec{r}(2) = 10\hat{i} + 6\hat{j} \tag{12}$$

(d) when t=2, the speed is the magnitude of ($\vec{v}(2)$)

$$|\vec{v}(2)| = \sqrt{5^2 + 6^2} = \sqrt{61}\, m/s \tag{13}$$

3.17

There is only horrizontal direction's speed. So, the initial velocity($\vec{v}(0)$) and the acceleration(by gravitational force) are

$$\vec{v}(0) = v_0\hat{i} \quad \vec{a}(t) = -g\hat{j} \tag{14}$$

If we set the initial position($\vec{r}(0)$)=(0,40), we can calculate the position vector at certain time(t)

$$\vec{v}(t) = -gt\hat{j} + \vec{v}(0) = -gt\hat{j} + v_0\hat{i} \tag{15}$$

$$\vec{r}(t) = v_0t\hat{i} - \frac{1}{2}gt^2\hat{j} + \vec{r}(0) = v_0t\hat{i} + (40 - \frac{1}{2}gt^2)\hat{j} \tag{16}$$

Time(t_0) when ball arrive on the water is when the y component of position vector is zero

$$t_0{}^2 = \frac{80}{g} \Rightarrow t_0 \approx 2.857s \tag{17}$$

At the initial position, player heard the splash sound after 3.second. It means that, if sound travel straight, the travel distance(d_s) of sound is

$$d_s = 343(3 - t_0) = |\vec{r}(t_0) - \vec{r}(0)| \tag{18}$$

where speed of sound is $343m/s$.Then, we can take the initial speed

$$(v_0t_0)^2 = d_s{}^2 - (\frac{1}{2}gt_0{}^2)^2 \tag{19}$$

$$v_0 \approx 9.9m/s \tag{20}$$

3.21

(a) Let's suppose initial speed of ball is v_0. Then, initial velocity with angle theta(in this problem, $\theta = 53°$) is

$$\vec{v}(0) = v_0 \cos\theta \hat{i} + v_0 \sin\theta \hat{j} \tag{21}$$

In this system, there is a gravitational force with $-\hat{j}$ direction, then

$$\vec{v}(t) = \vec{v}(0) - gt\hat{j} = v_0 \cos\theta \hat{i} + (v_0 \sin\theta - gt)\hat{j} \tag{22}$$

Also, we can set initial position with the origin. Position vector with time can be written as

$$\vec{r}(t) = v_0 \cos\theta\, t\hat{i} + (v_0 \sin\theta\, t - \frac{1}{2}gt^2)\hat{j} \tag{23}$$

At t_1=2.2s, the ball pass the wall where is (24,0). So, initial speed is

$$v_0 = \frac{24}{2.2\cos 53°} \approx 18.13m/s \tag{24}$$

(b) The height of ball(H_b) when it pass the wall can be calculated from eq.(23)'s y-component at t_1=2.2s

$$H_b = v_0 \sin\theta\, t_1 - \frac{1}{2}g{t_1}^2 \approx 8.14m \tag{25}$$

(c) As the height of roof is 6.m, the time(t_2), when ball arrive the roof, can be calculated from the root of below equation.

$$v_0 \sin\theta\, t_2 - \frac{1}{2}g{t_2}^2 = 6 \Rightarrow t_2 \approx 2.46s \tag{26}$$

Then, the horizontal distance(D) from the origin is

$$D = v_0 \cos\theta\, t_2 \approx 26.84m \tag{27}$$

3.27

Newton's 2nd law with constant mass(m) said

$$\vec{F(t)} = m\vec{a(t)} \equiv m\frac{d^2\vec{r}(t)}{dt^2} \tag{28}$$

In circular motion, if the radius(r) and angular velocity($w = w\frac{2\pi}{T}$ where T is period) are constant,

$$m\frac{d^2\vec{r}(t)}{dt^2} = mrw^2\hat{r} \tag{29}$$

When a satellite travel around the earth at 600.Km(R_s) from the ground, this satellite feels the gravitational force(mg with $g = 8.21m/s^2$ at 600.Km) as the centripetal force. Then,

$$mrw^2 = mg \Rightarrow w = \sqrt{\frac{g}{r}} \Rightarrow T = 2\pi\sqrt{\frac{r}{g}} \tag{30}$$

The radius of satellite is $r = R_e + R_s = 7000.km$.

$$T \approx 1.61hr. \tag{31}$$

4.4

From each components, we can make vector position

$$\vec{r}(t) = (5t^2 - 1)\hat{\imath} + (3t^3 + 2)\hat{\jmath} \tag{32}$$

By definition of accerelation,

$$\vec{a}(t) \equiv \frac{d^2\vec{r}(t)}{dt^2} = 10\hat{\imath} + 18t\hat{\jmath} \tag{33}$$

The magnitude of accerelation at t=2.s is

$$|\vec{a}(t)| = \sqrt{10^2 + (36)^2} = \sqrt{1396} \tag{34}$$

Then, the magnitude of force exerted on the 3.kg is

$$\left|\vec{F}(2)\right| = 3 * \sqrt{1396} = 112.N \tag{35}$$

4.6

(a) There are three forces

$$\begin{cases} \vec{F_1} = -2.00\hat{\imath} + 2.00\hat{\jmath} \\ \vec{F_2} = 5.00\hat{\imath} - 3.00\hat{\jmath} \\ \vec{F_3} = -45.0\hat{\imath} \end{cases} \tag{36}$$

The net force of these forces($\vec{F_t}$) is

$$\vec{F_t} = -42.00\hat{\imath} - 1.00\hat{\jmath} \tag{37}$$

(b) By Newton's 2nd law with constant mass, when the magnitude of accerelation is $3.75m/s^2$, the mass is

$$m = \left|\vec{F}\right|/|\vec{a}| = \sqrt{42^2 + 1^2}/3.75 \approx 11.20kg \tag{38}$$

(c) As magnitude of accerelation is 3.75 m/s^2,

$$\frac{d^2s(t)}{dt^2} = 3.75 \Rightarrow \frac{ds(t)}{dt} = 3.75t + C \tag{39}$$

if, at initial state, it is the rest, $C = 0$. Therefore, at t=10.s, the speed is

$$\frac{ds(10)}{dt} = 37.5m/s \tag{40}$$

(d) By definition and Newton's 2nd law

$$\vec{a}(t) \equiv \frac{d^2\vec{r}(t)}{dt^2} = \frac{\vec{F}_t}{m} \tag{41}$$

Therefore,

$$\frac{d^2\vec{r}(t)}{dt^2} = -3.75\hat{\imath} - 0.089\hat{\jmath} \Rightarrow \frac{d\vec{r}(t)}{dt} = -3.75t\hat{\imath} - 0.089t\hat{\jmath} \tag{42}$$