

**Electric Field: analogous to gravitational field:  $F = m_1(Gm_E/r_E^2)$ ;**

**$(Gm_E/r_E^2) = g = 9.8 \text{ m/s}^2$  at surface of earth:**

**“Grav. *field* or grav. *acceleration*”**

**Important point:  $g$  depends only on earth properties ( $m_E$  and  $r_E$ ); same for all  $m_1$  vals.**

**Similarly,  $F_{\text{Electric}} = kq_1q_2/r^2 = q_1(kq_2/r^2)$ ;**

**Electric field  $E$  for charge 2  $= (kq_2/r^2)$ , or can**

**rewrite:  $F_{\text{Electric}} = kq_2q_1/r^2 = q_2(kq_1/r^2)$ ; now**

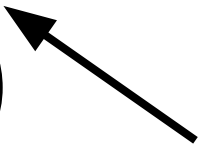


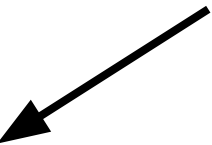
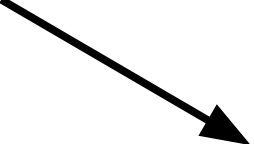
$$E_1 = (kq_1/r^2)$$

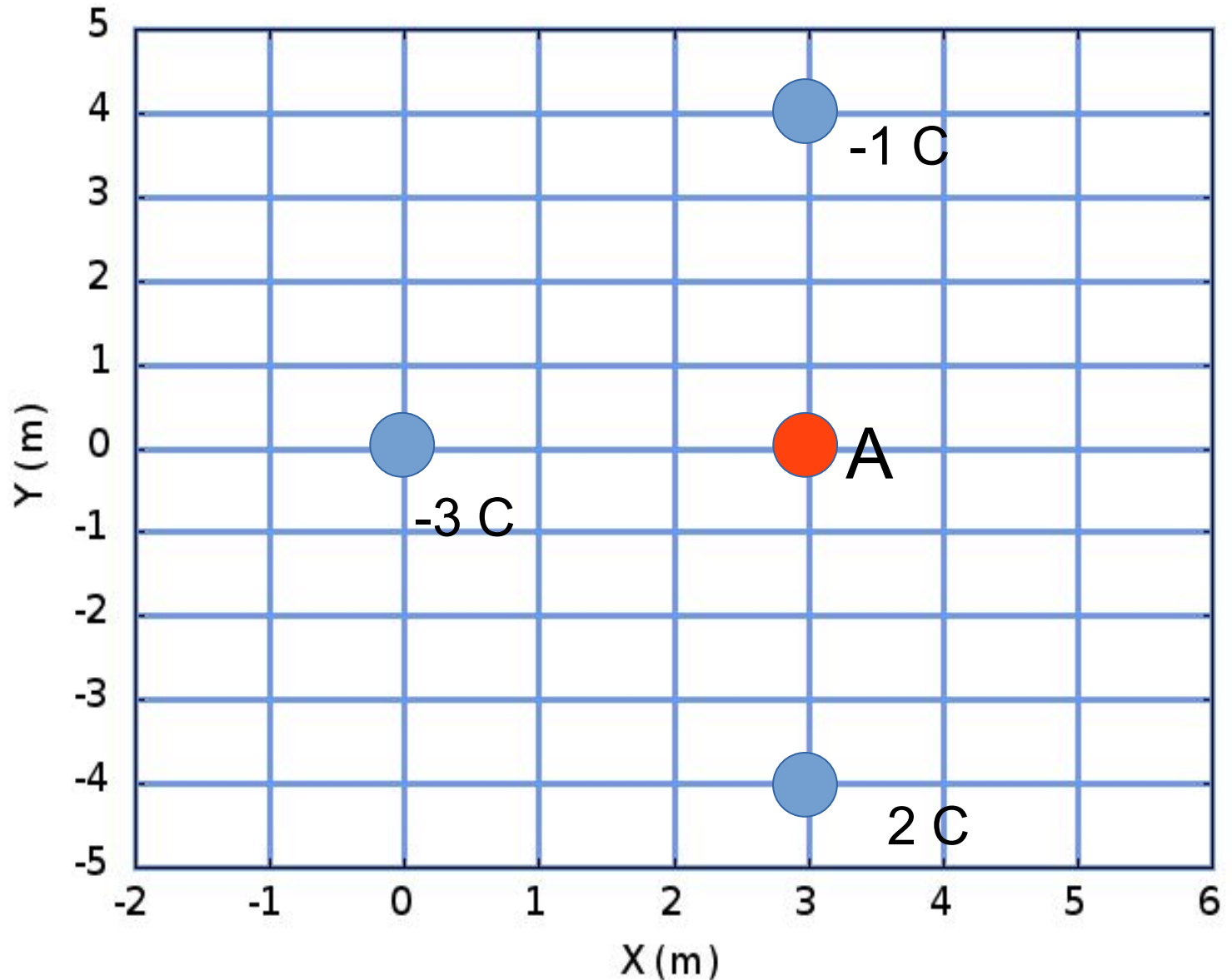
**$E$  fields point away from/towards**  
 **$+/-$  sources!**

“The Moral Arc of History is Long, But Surely it Bends Towards Justice”

- A) agree
- B) disagree
- C) who cares?

The direction of  $E_{\text{net}}$  at pt. A is closest to:

- A) 
- B) 
- C) 
- D) 
- E) 





In the previous problem,  $|E_{\text{net}}|$  at the point (A)=

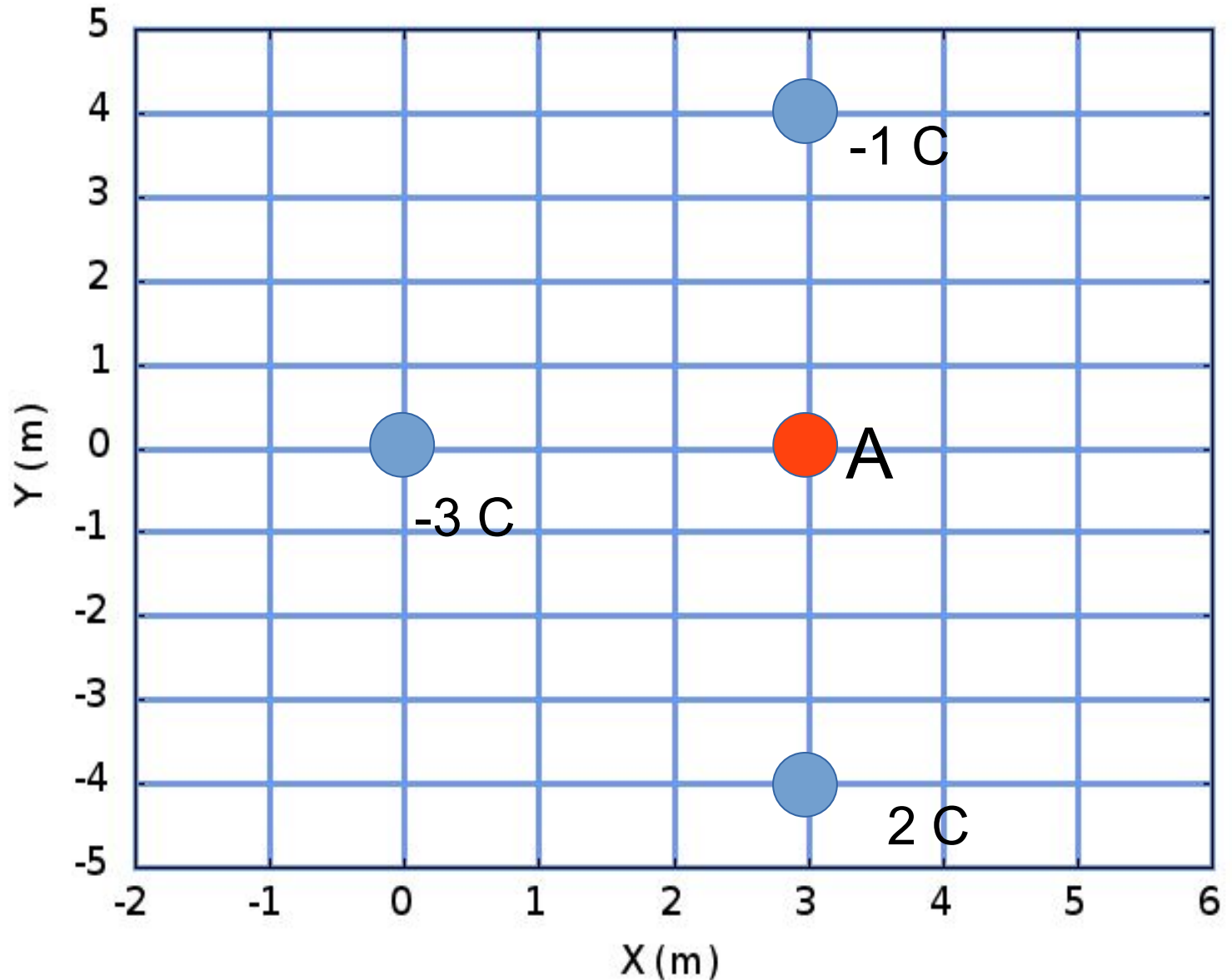
A) 0.162 k N/C

B) 0.324 k N/C

C) 0.648 k N/C

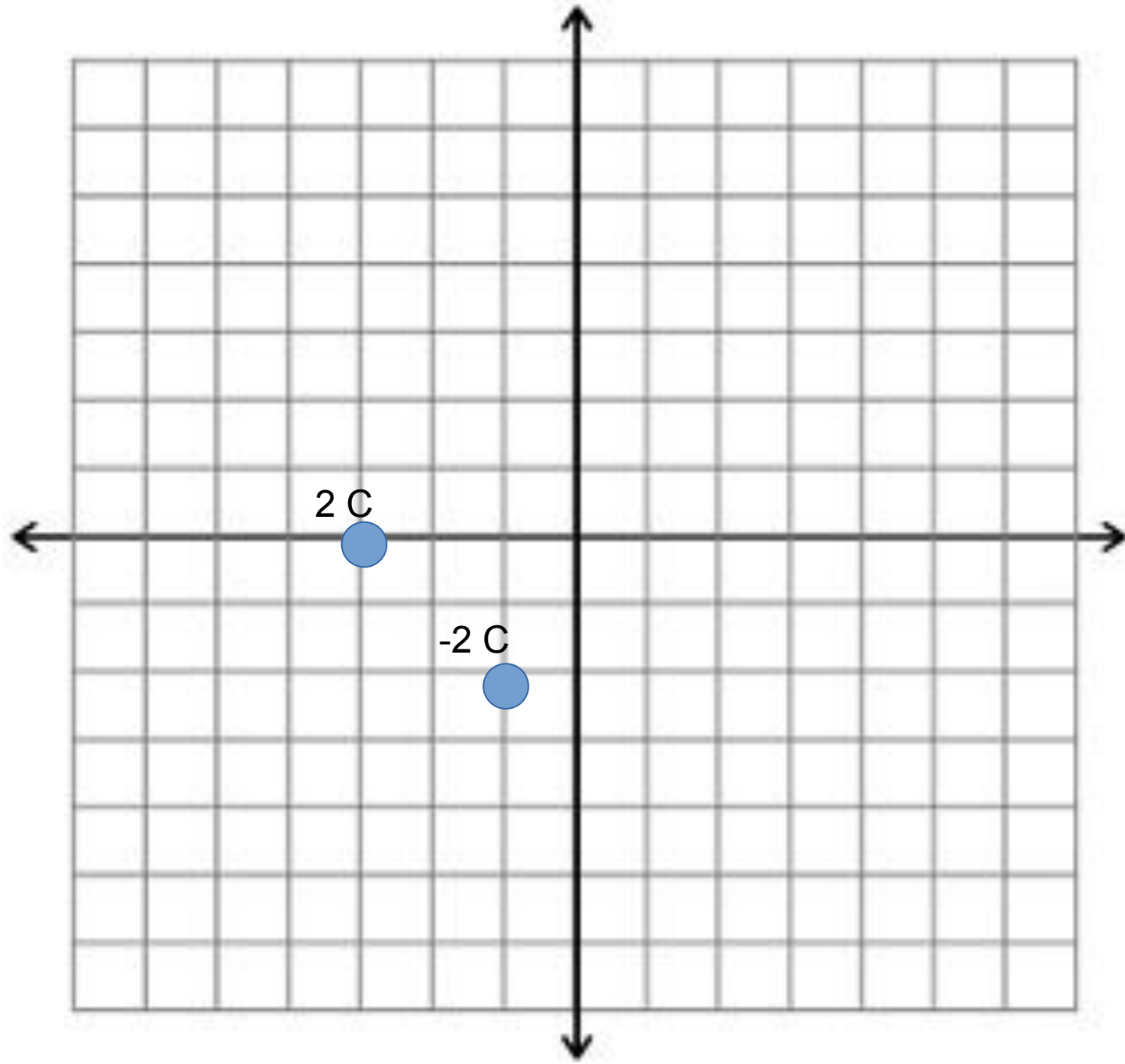
D) 0.382 k N/C

E) NOTA



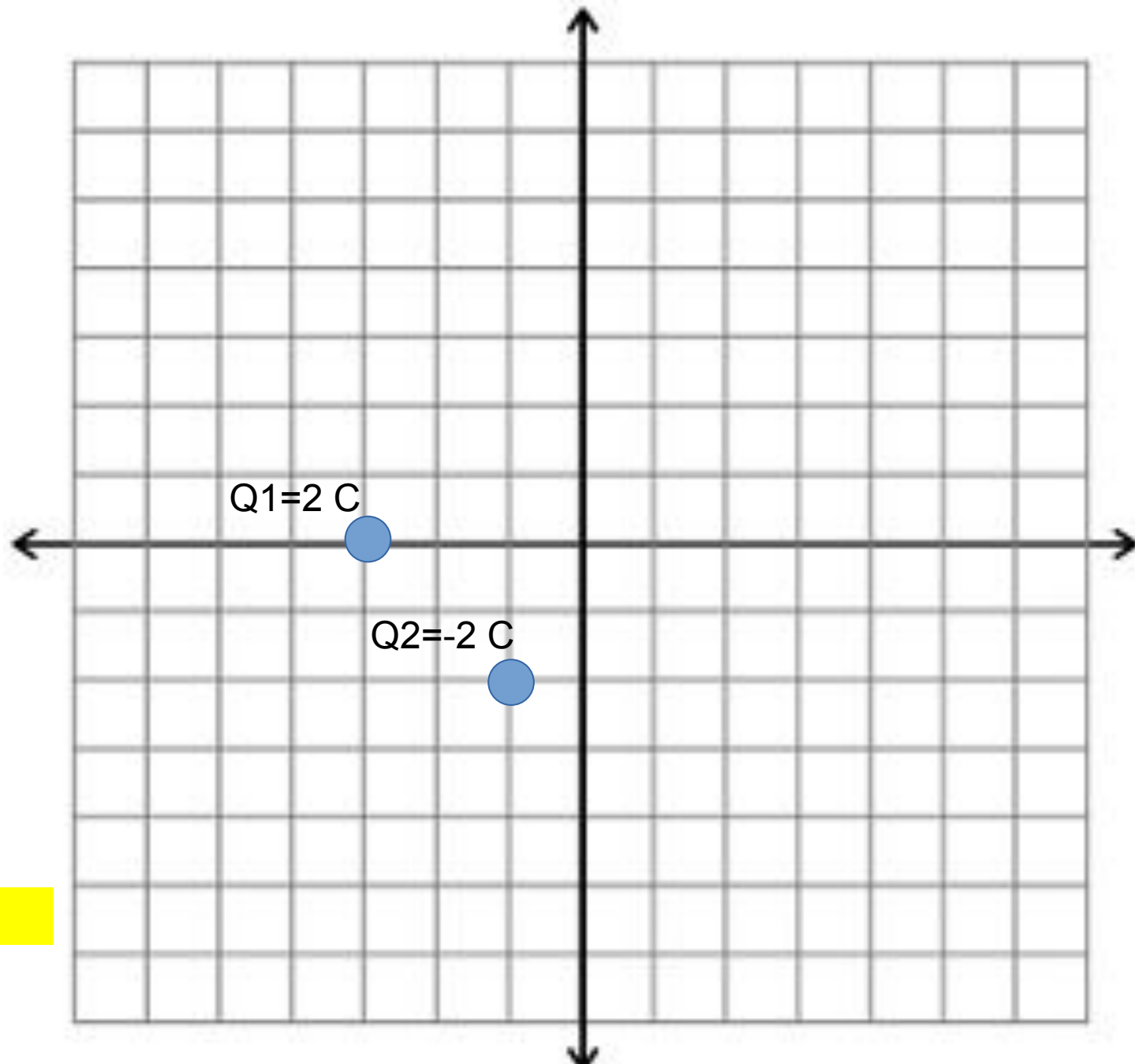
The E-field direction at (0,0) is (for CAP1, you'll need to check your graphical answer numerically!):

- A) NE
- B) NW
- C) SE
- D) SW



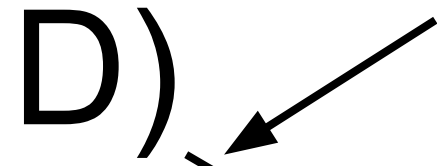
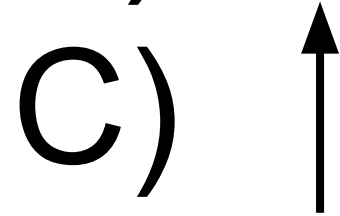
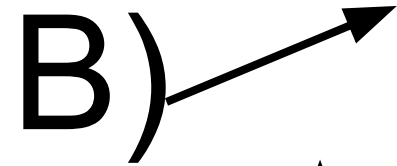
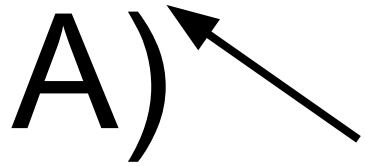
**A  $-0.1\text{ C}$  charge  $q$  is now placed at the origin, and released.  
Given that the force on  $q$  is given by  $F=qE$ , what is  $q$ 's  
acceleration direction?**

- A) NE
- B) NW
- C) SE
- D) SW



What if  $q$  was  $+0.1\text{ C}$ ?

The direction of  $E_{\text{net}}$  at point A is closest to:



3C (x,y)=(5,0)



-1C (x,y)=(0,2)



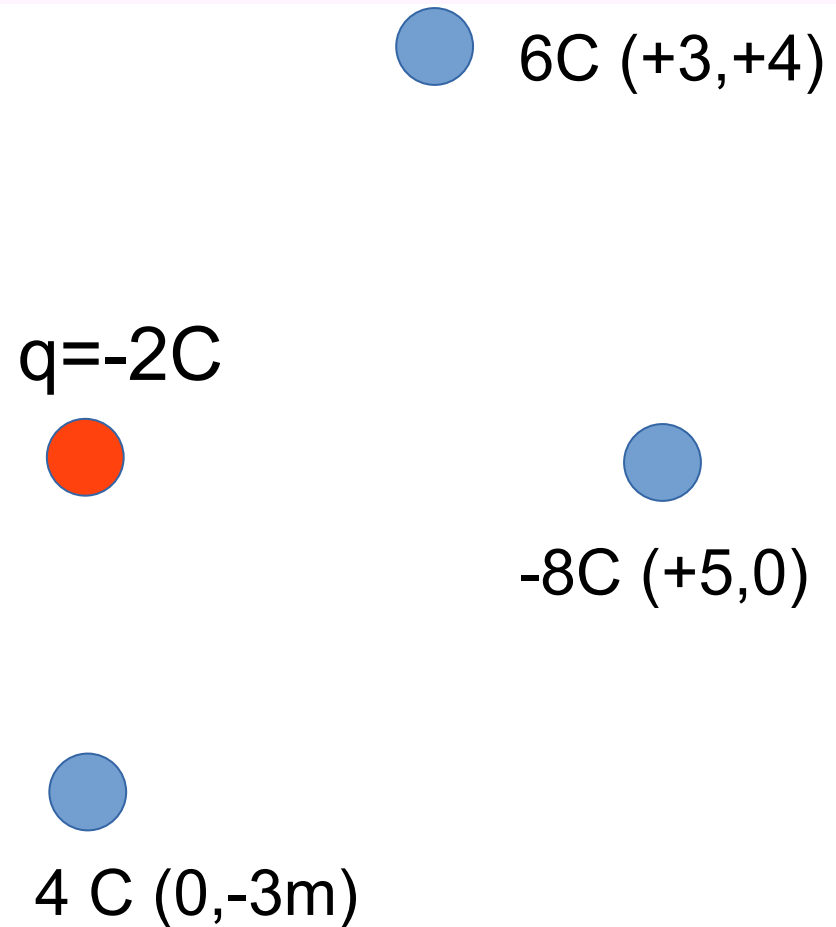
A



2 C (x,y)=(0,-4)

**What is the force direction on the  $q=-2\text{C}$  charge at  $(0,0)$ ?**

- A) NW
- B) NE
- C) SW
- D) SE
- E) NOTA



**In the previous problem, the net x-component of *force* on the -2C charge is:**

- A) 1.77k N
- B) 0.384k N
- C) -0.352k N
- D) -0.712k N
- E) NOTA

**In the same problem,  $F_y =$**

- A) 0.621k N
- B) -1.27k N
- C) 1.27k N
- D) -0.505k N
- E) NOTA

**A -1C charge and a +2C charge are placed 1 m apart. The number of points for which  $|E_{\text{tot}}|=0$  is**

- A) 0
- B) 1
- C) 2
- D) 3
- E) NOTA



**A +1C charge and a +2C charge are placed 1 m apart. The number of points for which  $|E_{\text{tot}}|=0$  is**

A) 0

B) 1

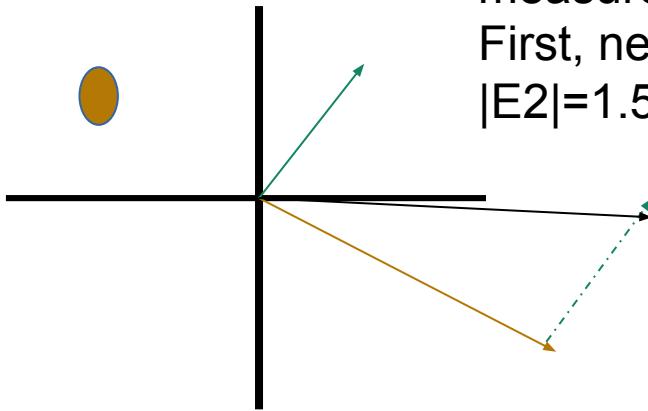
C) 2

D) 3

E) NOTA

# Source charges $Q_1 = -2\text{C}$ $(x,y) = (2,3)$ and $Q_2 = +1.5\text{C}$ $(x,y) = (-2,1)$ . $E_{\text{net}}(0,0) = ?$

- Graphically: Add two vectors;  $E_1$  points directly away from  $Q_1$ , with tail at measurement pt  $(0,0)$ ;  $E_2$  points directly towards  $Q_1$ , with tail at origin. First, need magnitudes (for numerical addition, as well):  $|E_1| = 2k/13$ ;  $|E_2| = 1.5k/5$ , so  $|E_2| \sim 2|E_1| \Rightarrow$  resultant roughly colinear with x-axis



Numerically:  $|E_1| = 2k/13$ ;  $|E_2| = 1.5k/5$ ,  
 $E_{1,x} = (2/13) * 2 / \sqrt{13}$ ;  $E_{1,x} = +0.085k$ ;  $E_{1,y} = +0.128k$ ;  
 $E_{2,x} = +0.268k$ ;  $E_{2,y} = -0.134k$ . Now add the x-components to get  $E_{x,\text{total}}$  and  $E_{y,\text{total}}$

$E_{\text{net}}$  is a vector with  $(E_x, E_y) = (0.353k, -0.006k)$ ;  
 checks out with graphical!. Net magnitude =  $0.3531 \text{ kN/C}$

NOTE: Field exists (and is calculable) at every point in space! Now we put a second charge at origin, and can calculate the force on that charge. E.g.  $Q_3 = -3\text{C}$  at origin; Force =  $3 * 0.3531 \text{ kN/C}$  in magnitude, points OPPOSITE direction of net electric field!



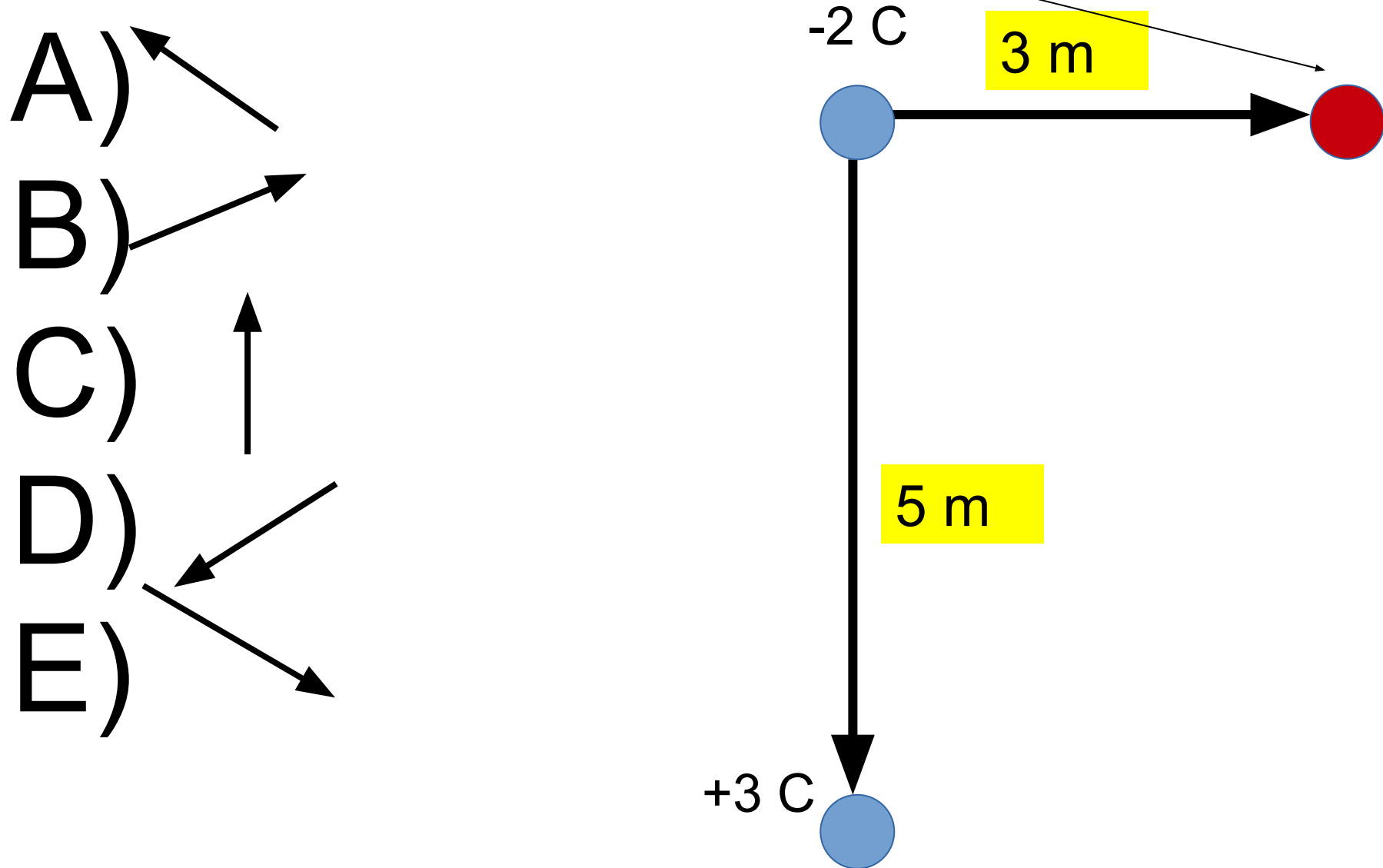
## Electric Potential (Volts):

Electric analog of 'height': defines PE, and also direction of decreasing PE.  $\Delta(\text{PE}) = mg\Delta h$ ;  $\Delta(\text{PE}) = q\Delta V$ ;  
dimensionally: Grav PE:  $GMm(r/r^2)$ ; E:  $kQq/r$

In cases where a mass or charge moves, the work done by the field (grav or electrical) is negative of  $\Delta\text{PE}$   
For source charge  $Q$ , the potential a distance  $r$  away is:  $V = kQ/r$ .

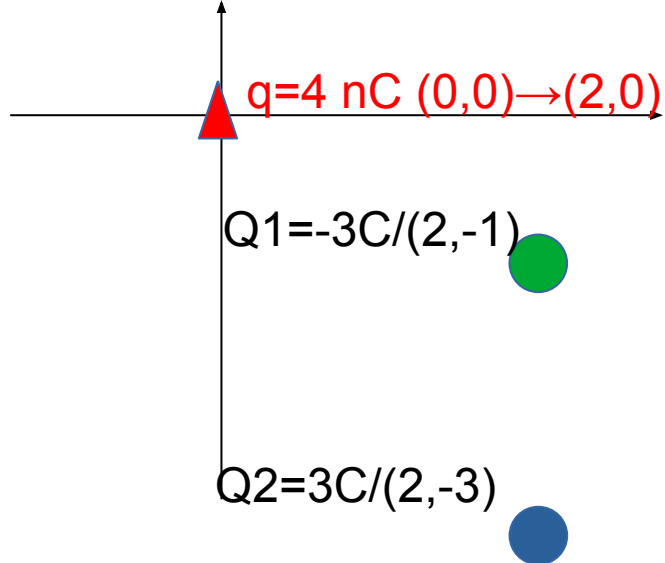
All charges spontaneously move from higher PE to lower PE. For positive charges  $+q$ :  $V_{\text{Hi}} \rightarrow V_{\text{lo}}$  (like masses move from large height to sea level);  $-q$  go from  $V_{\text{lo}} \rightarrow V_{\text{hi}}$ . Illustrate numerically by imagining how charges move relative to some  $+Q$  source

The direction of acceleration of a  $q = -0.2 \text{ C}$  charge placed at A is closest to

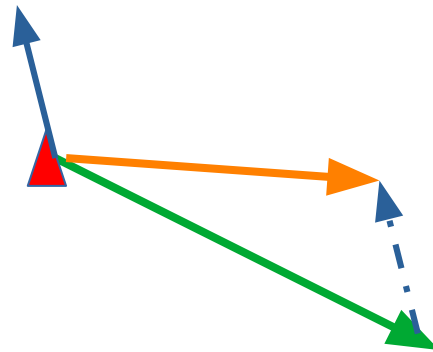


Owing to  $F_E$ , the  $-0.2\text{ C}$  charge  $q$  accelerates, eventually reaching some point B. How does the PE of  $q$  at B compare with its initial PE?

- A)  $PE(B) < PE(A)$
- B)  $PE(B) > PE(A)$
- C)  $PE(B) = PE(A)$
- D) Not enough information



1) E at origin:  $|E1| = 3k/5$ , towards  $Q1$  (neg. source charge);  $|E2| = 3k/13$ , away from  $Q2$  (pos. source charge), so vectors in ratio  $0.6:0.23 \sim 2.6$ : (resultant in orange)



Since test charge  $q$  is positive, this is also the direction along which the charge accelerates (if  $q < 0$ , moves in opposite direction)!

Now calculate change in potential ( $\Delta V = V_f - V_i$ ) and change in electrical potential energy ( $= q\Delta V$ ) in process of moving from  $(0,0) \rightarrow (0,2)$ . The change in PE is equal in magnitude, but opposite in sign, to the work done by the external electric field due to the source charges, although it is NOT necessarily the total work done on the charge (which is always given by  $W_{\text{tot}} = \Delta(KE)$ ).

$\Delta V = kQ/r_f - kQ/r_i$ ; for  $Q1$ :  $-3k \cdot (1/1 - 1/\sqrt{5})$ , for  $Q2$ :  $3k \cdot (1/3 - 1/\sqrt{13})$  – note sign of  $Q$ !  
Adding:  $\Delta V = -1.49k$  Volts.

Change in electrical PE  $= q\Delta V = 4e-9 \cdot 9e9 \cdot (-1.49) = -53.64$  Joules.

Work done by electric field  $= -\Delta PE = 53.64$  Joules

Check against intuition: change in PE should be negative! (since  $q$  is being pulled by field (mostly  $Q1$ ) to a lower potential point - similar to a mass being pulled by earth's gravity from high elevation to low elevation)

$Q1=10\text{ C @ } (0,6)$ ;  $Q2=-12\text{ C @ } (-5,0)$ ;  $A@(2,0)\rightarrow B@(0,3)$ .

$E1@A=10k/40$ ;  $E_{1x}=(10k/40)*2/\text{sqrt}(40)$ ;  $E_{1y}=(10k/40)*6/\text{sqrt}(40)*(-1)$ ;  $E2@A=12k/49 \times (-1)$ ;  $E_{2x}=E2$ ;

$E_{x,\text{net}}@A=(10./40.)*(2/\text{sqrt}(40.))-(12./49.)=-0.166\text{ k N/C}$ ;  $E_{y,\text{net}}@A=-(10/40.)*6/\text{sqrt}(40.)=-0.237\text{ k N/C}$ ; to determine angle wrt x-axis:  $\tan\theta=(E_{y,\text{net}}/E_{x,\text{net}})$ , so  $\theta=\text{atan}(-0.237\text{ k}/-0.166\text{ k})$ , or  $\theta=55\text{ degrees}$

$V1@A=10k/\text{sqrt}(40)=1.58\text{ kV}$ ;  $V2@A=-12k/7=-1.71\text{ kV}$ ;

$E1@B=10k/4=2.5\text{ k V}$ ;  $E_{1x}=0$ ;  $E_{1y}=10k/4 (-y)$ ;  $E2@B=12k/41$ ;

$V1@B=10k/2=5\text{ kV}$ ;  $V2@B=-12k/\text{sqrt}(41)=-1.87\text{ kV}$ ;

now move  $q3=-2e-9\text{ C}$  charge from  $A\rightarrow B$ ;

Work is NEGATIVE of change in PE, so:

$W1=-1*(q3\Delta V1)$ ;  $W2=-1*(q3\Delta V2)$

$W1=2e-9*k*(5-1.58)=61.56\text{ J}$ , with  $k=9e9$

$W1>0$  since

$q3$  is moving closer to  $Q1$ , consistent with attraction of  $Q1$

$W2=2*k*(-1.87-(-1.71))=-0.32\text{ kJ} * e-9=-2.88\text{ J}$

$W2$  negative

since  $q3$  is moving closer!

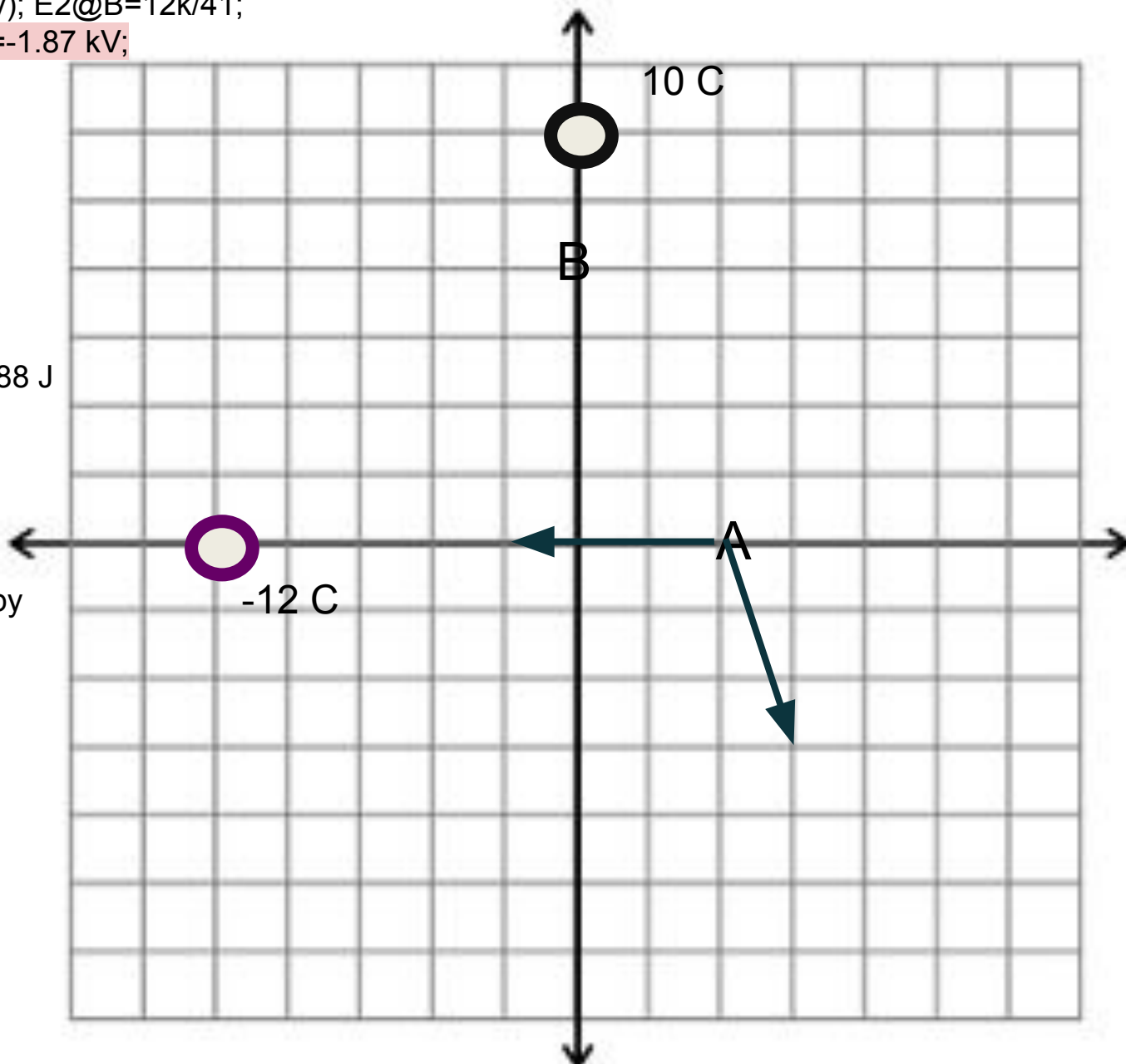
(from 7 m away to  $\text{sqrt}(40)\text{ m}$  away)

Now insert values for KE: Take  $m=4\text{ kg}$ ,

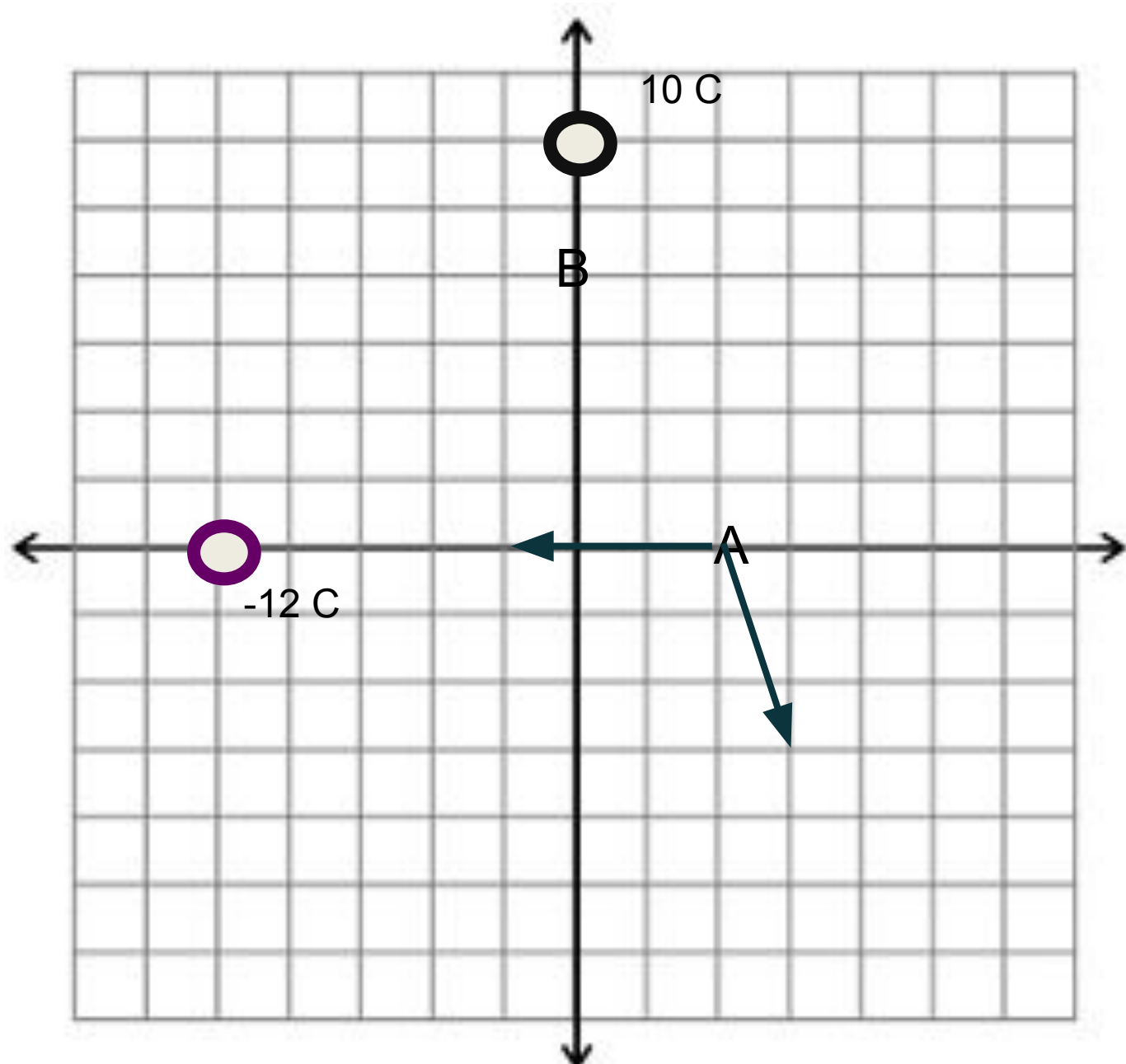
$v(\text{initial})=4\text{ m/s}$  and  $v(\text{final})=0\text{ m/s}$ ,

so  $W_{\text{total}}=-0.5*4*4*4=-32\text{ J}$ , so work done by

Agent Orange= $-32-61.56-2.88 = -96.44\text{ J}$

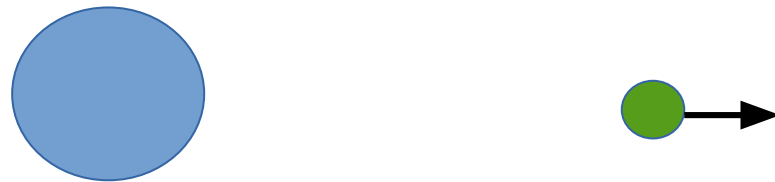






**The electric force from a heavy -10 C charge at (0,0) “pushes” a -2 C charge from (10,0)->(11,0).  
Using  $W=|F||d|\cos\theta$ , what is the approximate work done on the -2C charge?**

- A) k Joules
- B) 0.5k Joules
- C) -0.2k Joules
- D) 0.2k Joules



**A heavy  $Q=-10\text{ C}$  at the origin “pushes” a  $q=-2\text{ C}$  charge from  $(10,0)$  to  $(11,0)$ . Using  $\Delta(\text{PE})=q\Delta V$ , re-calculate the (exact) work done by the  $-10\text{C}$  charge on the  $-2\text{C}$  charge**

- A)  $0.182\text{ kJ}$
- B)  $-0.556\text{ kJ}$
- C)  $0.845\text{ kJ}$
- D)  $-0.182\text{ kJ}$
- E) NOTA



**If the mass of the  $-2\text{C}$  test charge is  $4\text{ kg}$ , what is its velocity as it passes through the point  $(11,0)$ ?**

- A)  $28.6\text{ km/s}$
- B)  $12.88\text{ km/s}$
- C)  $155.2\text{ m/s}$
- D)  $33.9\text{ m/s}$
- E) NOTA



**Bad Trip Records: Sisyphus pushes a 100 kg boulder up Mt. Oread (80 m along the base and making an angle of  $36.9^\circ$  wrt horizontal). When he reaches the top, the boulder has  $v=10$  m/s. What is  $W_{\text{net}}$  done on the boulder, assuming it starts from rest?**

- A) 5000 J
- B) -58800 J
- C) 63800 J
- D) -63800 J
- E) NOTA



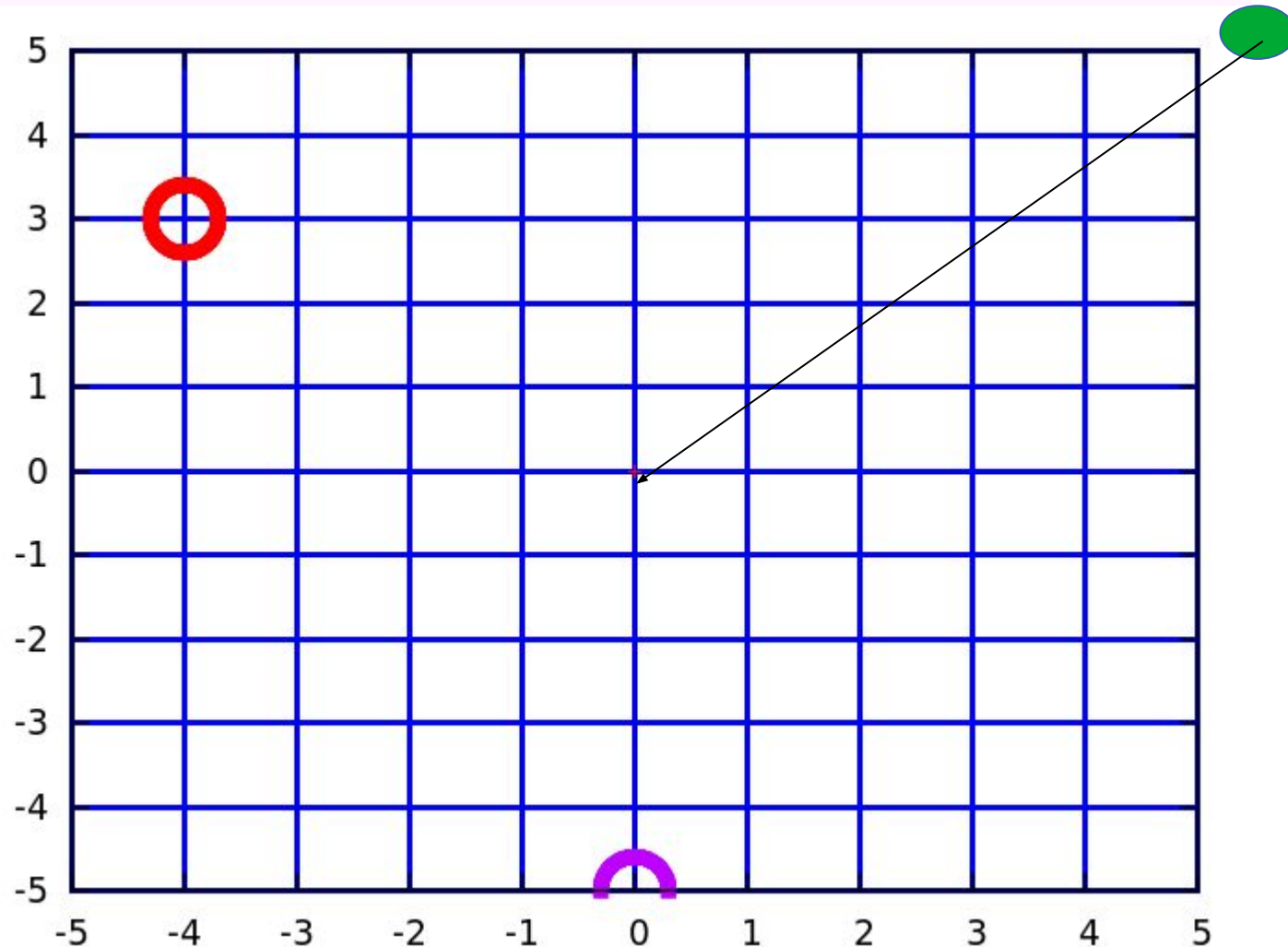
**Sisyphus pushes a 100 kg boulder up Mt. Oread (80 m along the base and making an angle of  $36.9^\circ$  wrt horizontal). When he reaches the top, the boulder has  $v=10$  m/s. What is  $W_{\text{Sisyphus}}$ ?**

- A) 5000 J
- B) -58800 J
- C) 63800 J
- D) -63800 J
- E) NOTA



Given a **-4 C source charge** at  **$(-4,3)$**  and a **2<sup>nd</sup> source +2 C charge** at  **$(0,-5)$** ; what is the work done by someone who pushes  **$q=-2$  C** from 'far away' to  **$(0,0)$** , assuming it starts and ends with  **$v=0$** ?

- A)  $-0.8\text{ kJ}$
- B)  $0.8\text{ kJ}$
- C)  $0.4\text{ kJ}$
- D)  $-0.4\text{ kJ}$
- E) NOTA





Given a 1 mC source charge at the origin; a 4 kg.  $-2\mu\text{C}$  charge is pushed by Strelka from  $(3,0) \rightarrow (2,0)$ , after which it has a velocity of 4 m/s. In so doing,  $W_{\text{Strelka}}$  is:

- A) 29 J
- B) 35 J
- C) 32 J
- D) 3 J
- E) NOTA







### CAP3:

$Q1 = -2C$   $(-1,3)$ ;  $Q2 = +3C$   $(0,2)$ .  $q = 4 \times 10^{-9}C$   $(-4,0) \rightarrow (-2,0)$ ;  $m = 2$  kg.  $v_i = 2$  m/s;  $v_f = 4$  m/s

What we know:

a)  $W_{\text{tot}} > 0$ , since  $v_f > v_i$ :  $W_{\text{tot}} = 0.5m(16-4) = 12J$ ,

b) To get Work done by electric field, we'll calculate the change in PE of the charge, then use  $W_{\text{field}} = -\Delta PE$ , with  $\Delta PE = q\Delta V$ .

To calculate  $\Delta V$ , taking into account both  $Q1$  and  $Q2$ :

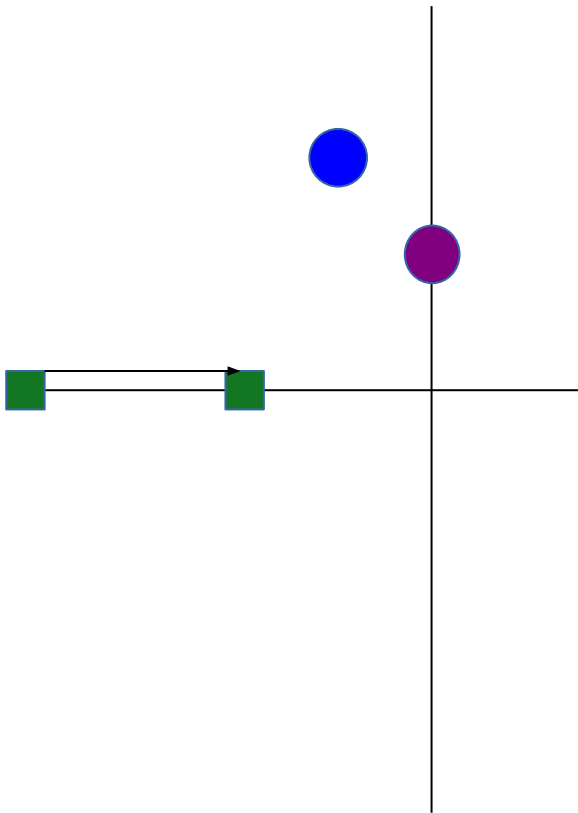
$$\Delta V = (V_{f,1} - V_{i,1}) + (V_{f,2} - V_{i,2}); V = kQ/r,$$

So:  $\Delta V = k(-2C) * (1/\sqrt{10} - 1/\sqrt{18}) + k(+3C)(1/\sqrt{8} - 1/\sqrt{20}) = +0.23k$  Volts, so  $\Delta PE = 8.23$  J,

so work done by field is *negative* 8.23 J (positive charge moves to higher PE)!

So work done by person is 20.23 J, since

$$12J = W_{\text{agentOrange}} + W_{\text{field}}$$



Connection between electrical potential and electric field;  
how to predict the path of a charge  $q_3$

## Calculation of electric potential and meaning of electric field maps

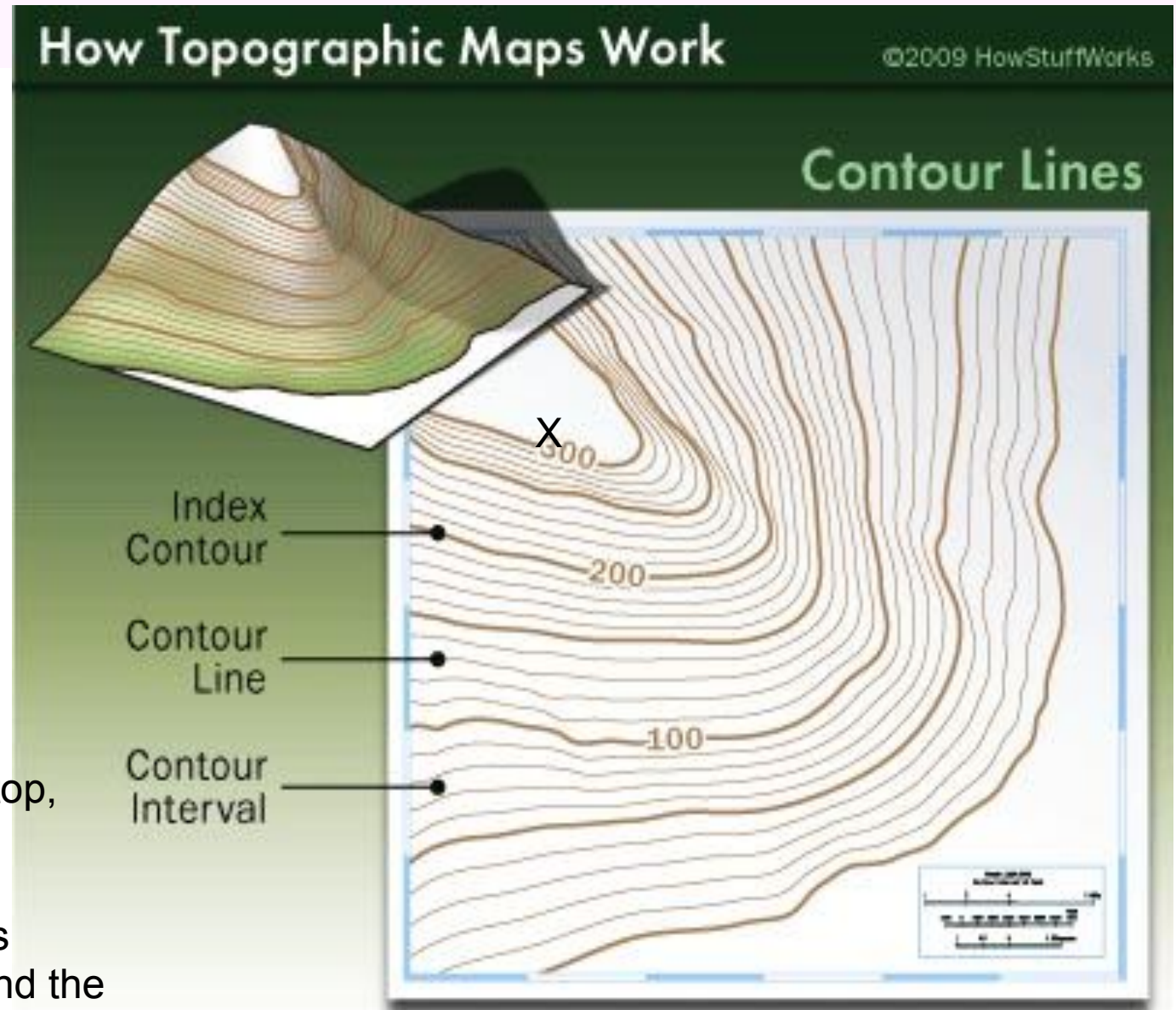
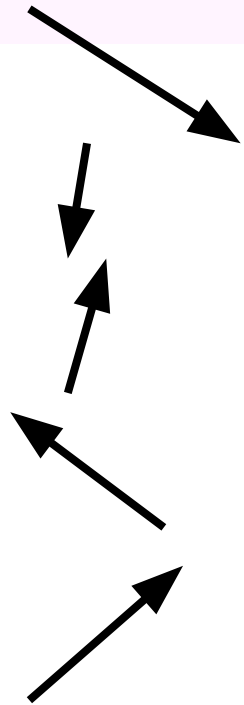
a) construct a problem with 3 non-linear charges such that the potential at the origin is zero. Numerically find  $E(0,0)$

b) construct a problem with 3 charges (not in a line) such that one component of the electric field ( $E_x$  or  $E_y$ ) is zero at the origin. Calculate the potential at the origin. c) problem following example XXX in text.

To make a potential/elevation map, we first probe the force/field, then draw contours perpendicular to force/field lines.

**A ball, starting at point X, would most likely roll**

- A)
- B)
- C)
- D)
- E)



N.B:a) for spherical mountaintop, contours are symmetric!

b) analogy not exact since this gravitational example is 3-d and the fields we've considered are 2-d



Atlantic Ocean

Indian Ocean

SP

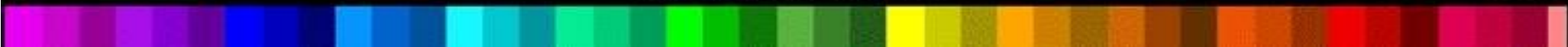
Pacific Ocean

At the South Pole, ice tends to flow in which direction?

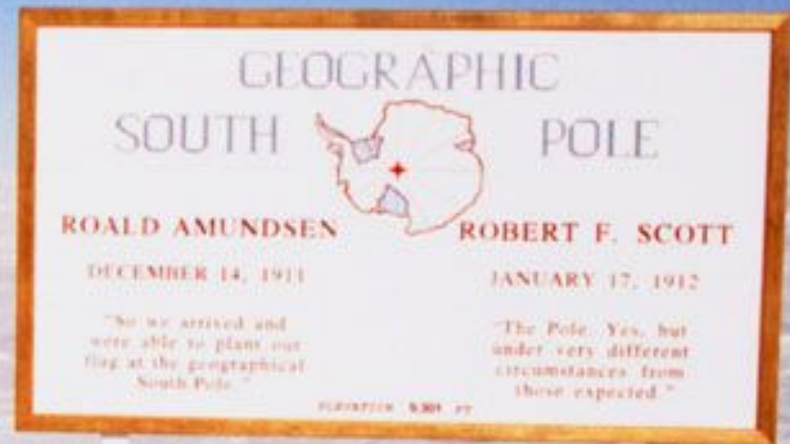
A) NE B) NW C) SE D) SW

Elevation in meters

4000









Mapcarta



Standard



Legend



Hotels



Exit

+Q source charges ↔ mountaintops:

-Q source charges ↔ depressions

+q spontaneously move from high to low potential (mass moves from mountaintop → depression)

Biggest acceleration where equipotentials 'densest'



Google

Besson Spur is a slope within Antarctica and is nearby to Papitashvili Valley and Hernandez Valley.

Map data ©2017 Imagery ©2017 DigitalGlobe, Landsat / Copernicus | Terms of Use



Enter your starting location



Enter your destination



Maps & Driving  
Directions

Start Now





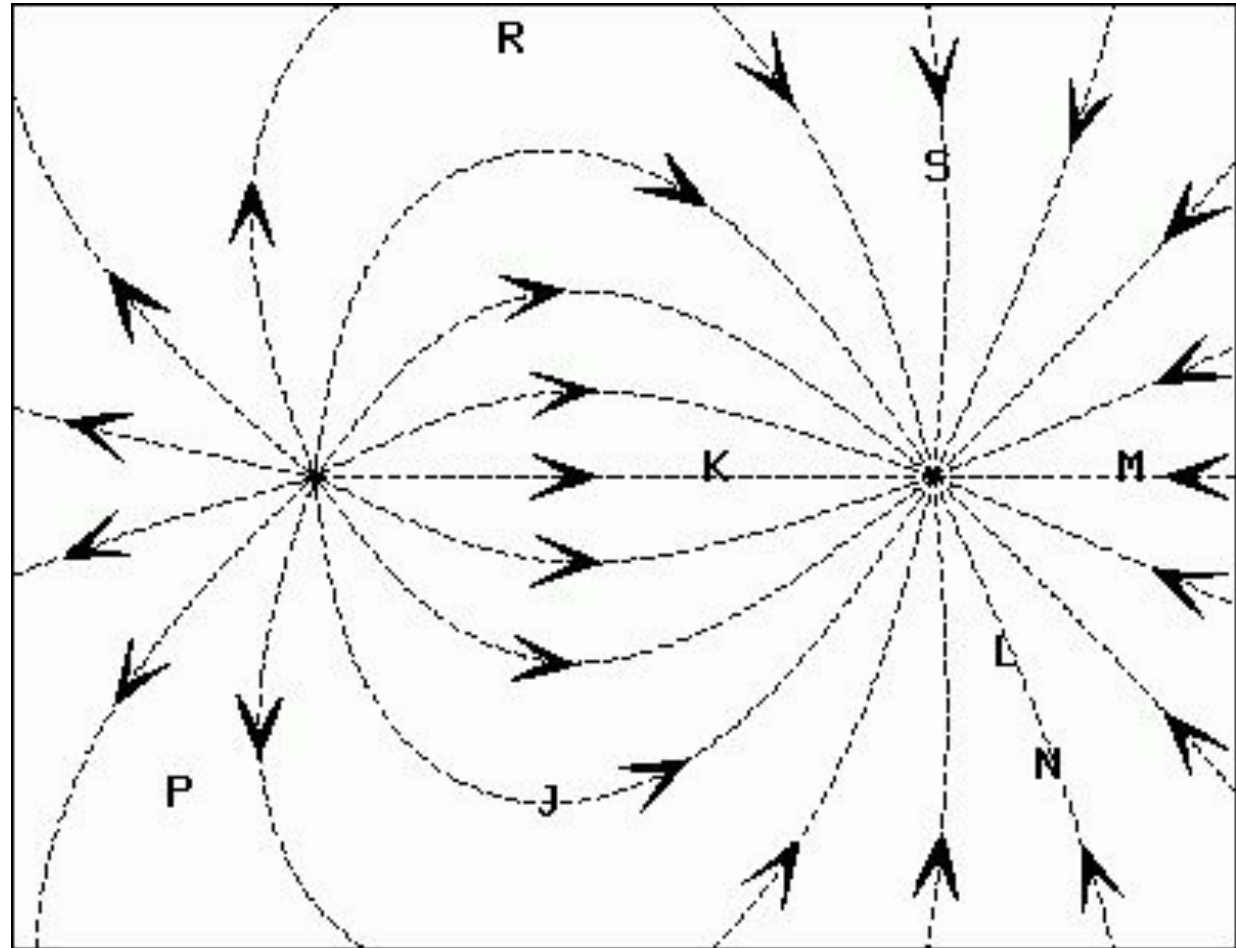
**Analogy b/w voltage and height,  $E$  and gravitational fields.**

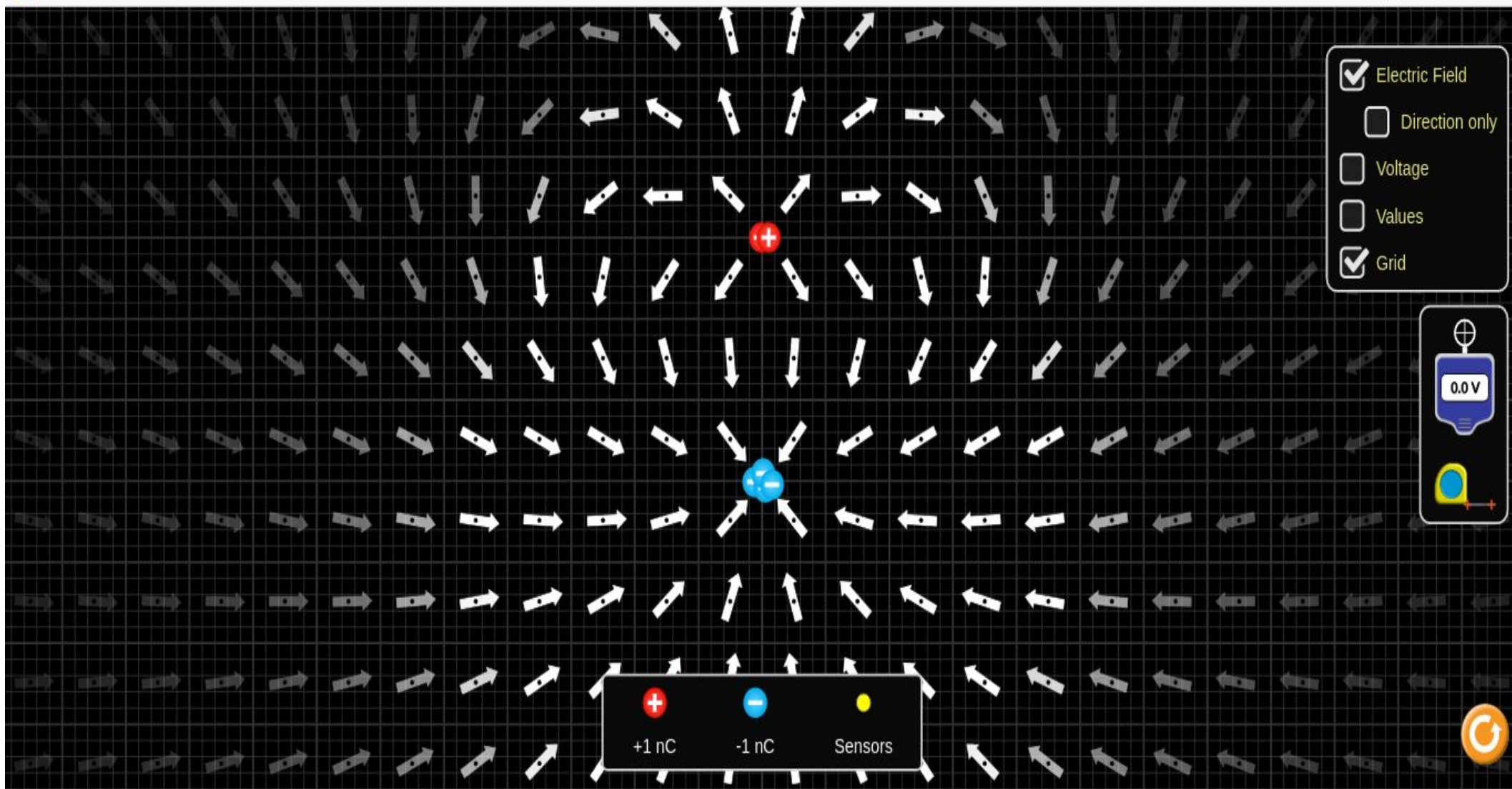
**Superimpose an equipotential map on graph paper ( $x,y$  coordinate space), labeling the lines with their voltage values. a) at some point in the map, determine the direction of the  $E$ -field and estimate the magnitude of the  $E$ -field, b) introduce a charge at some point A and specify its initial mass and velocity, now move the charge to point B and also specify its final velocity. What is the work done by the  $E$ -field corresponding to your map? How much external work is done on the charge?**

Given that E-field strength is indicated by the 'density' of field lines a given distance from a source, what value of charges (left/right) are consistent with the Electric field map shown?

- A)  $-1\text{ C}, +2\text{ C}$
- B)  $+1\text{ C}, -2\text{ C}$
- C)  $-1\text{ C}, +1\text{ C}$
- D)  $+1\text{ C}, +1\text{ C}$
- E)  $-2\text{ C}, -2\text{ C}$

Sketch the 'equipotentials'

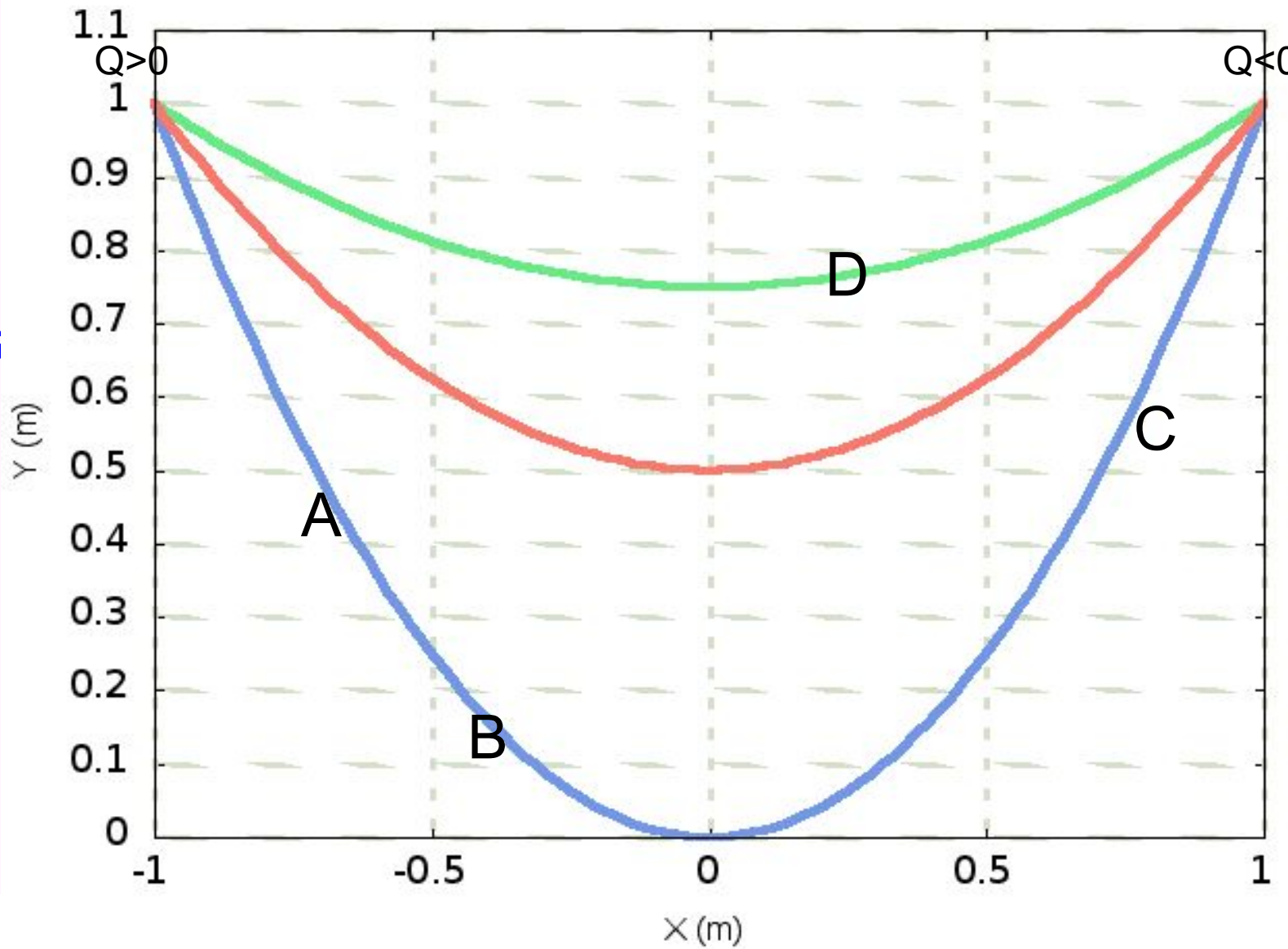




**From your knowledge of topographical maps, and the analogy between gravity and electricity ( $E \sim g$  and  $V \sim h$ ), which statement below is true?**

- A) The electric field vector is tangent to the equipotentials.
- B) The electric field vector points perpendicular to the equipotentials, in the direction of increasing potential.
- C) The electric field vector points perpendicular to the equipotentials, in the direction of decreasing potential.
- D) NOTA

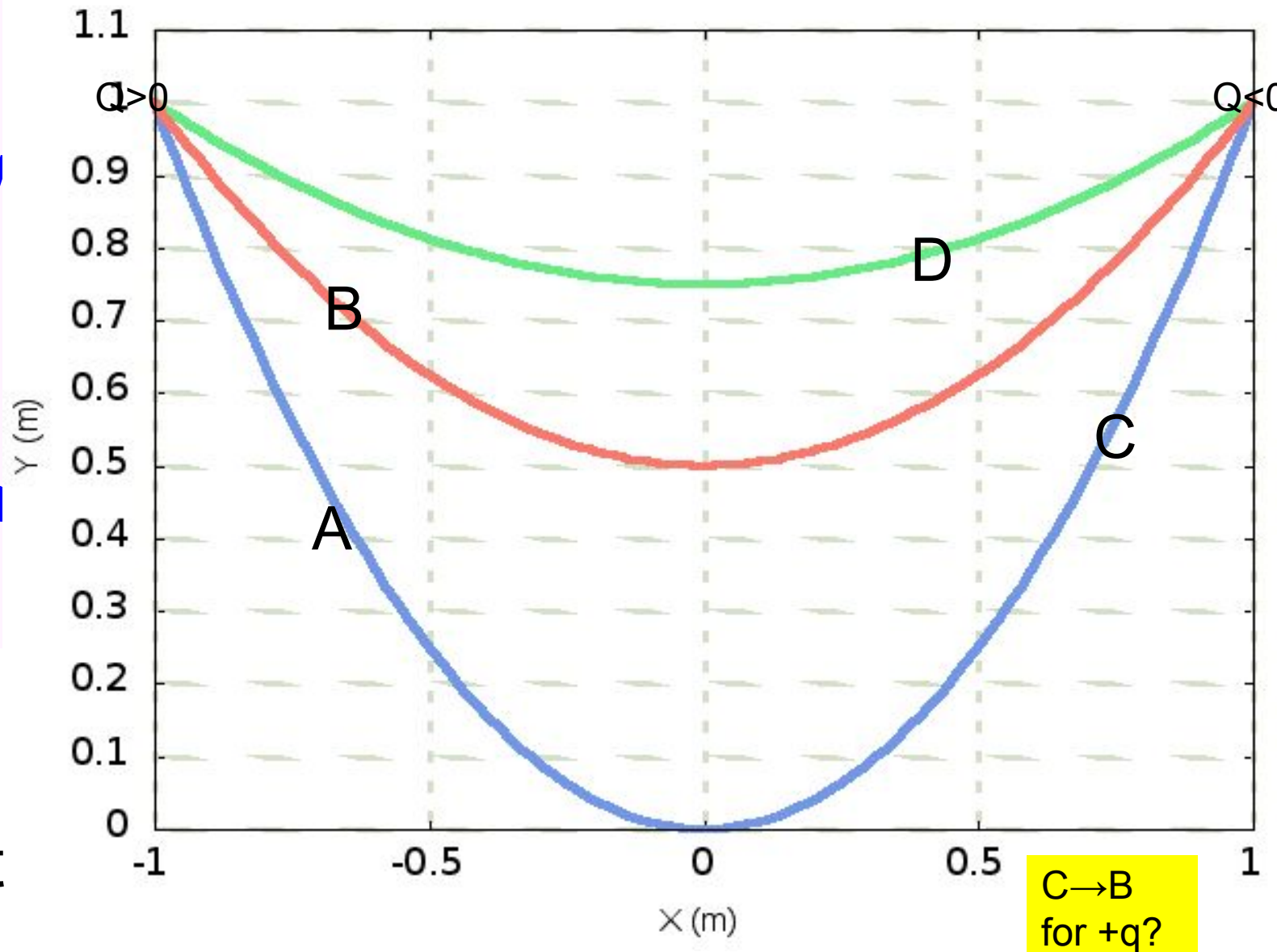
Given the  
E-field map  
shown, draw  
the  
equipotentials  
starting at the  
points A, B  
and C, if  
 $Q_{\text{left}} > 0$  and  
 $Q_{\text{right}} < 0$





In moving from  $A \rightarrow D$ , the PE of a negative charge (how does  $-q$  'want' to move and thereby gain KE?)

- A) Increases
- B) Decreases
- C) is constant

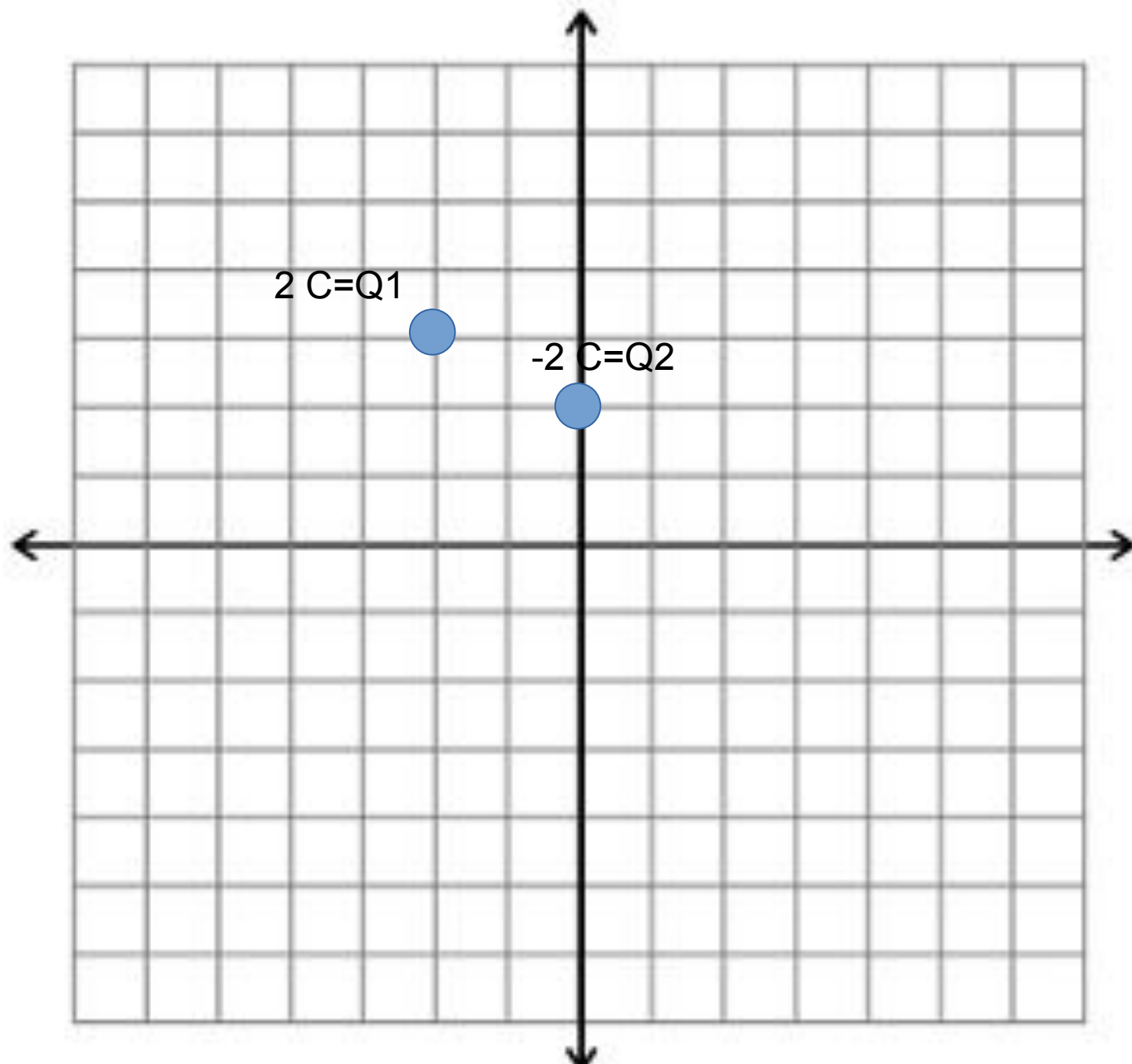


**A  $-6\text{C}$  source charge is at the origin. The change in potential, and  $\Delta\text{PE}$ , when a  $-2\text{ C}$  charge is moved from  $(x,y)=(2\text{m},0)$  to  $(0,-3\text{m})$  is:**

- A)  $3\text{k V}$ ,  $3\text{k J}$
- B)  $6\text{k V}$ ,  $\text{k J}$
- C)  $-\text{k V}$ ,  $-2\text{k J}$
- D)  $\text{k V}$ ,  $-2\text{k J}$
- E) NOTA

A charge  $q = -0.1 \text{ C}$  is placed at the origin and released. The work done on  $q = -0.1 \text{ C}$  subsequent to its release is:

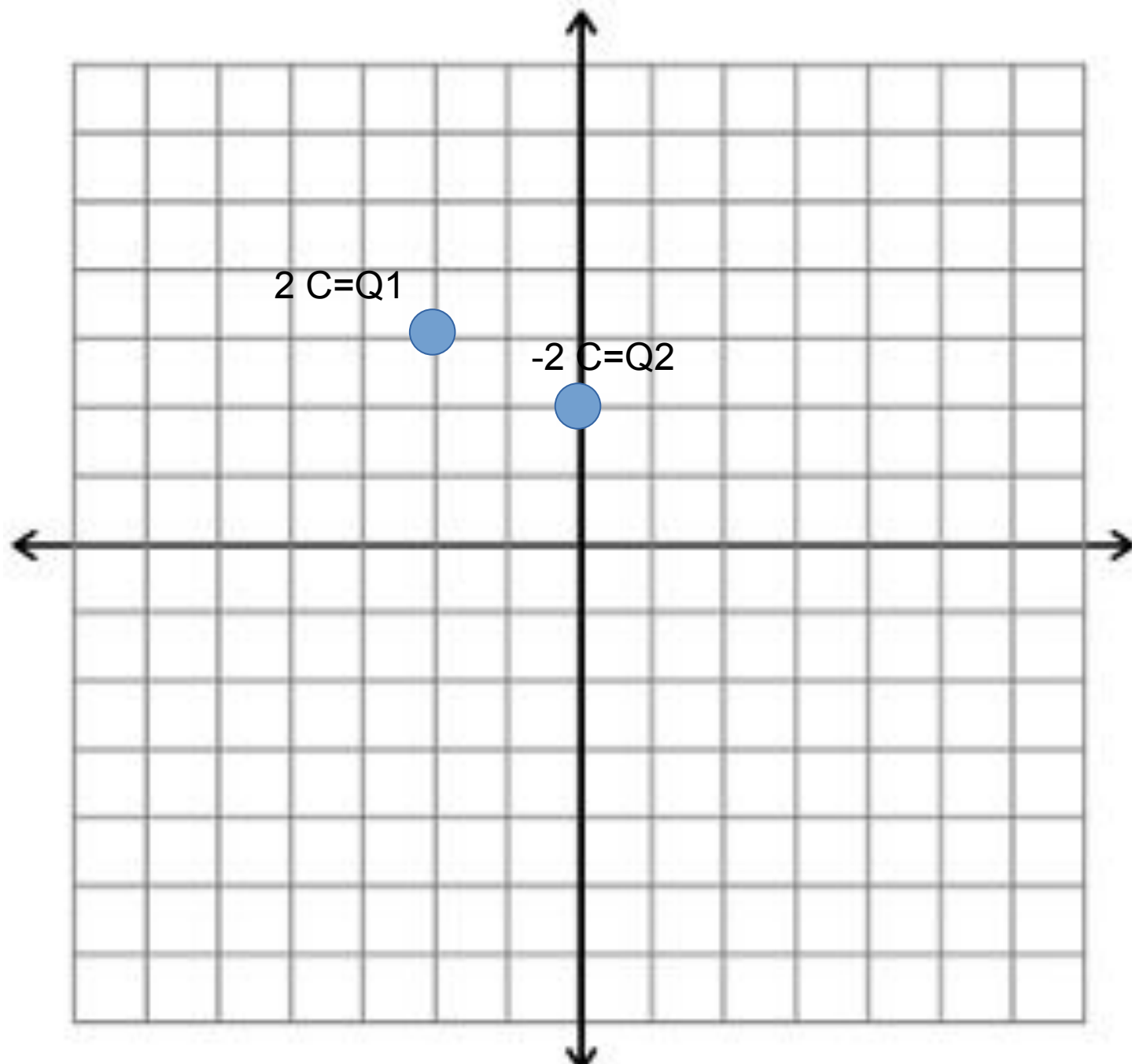
- A)  $W_1 > 0$ ,  $W_2 < 0$
- B)  $W_1 > 0$ ,  $W_2 > 0$
- C)  $W_1 < 0$ ,  $W_2 < 0$
- D)  $W_1 < 0$ ,  $W_2 > 0$
- E) NOTA





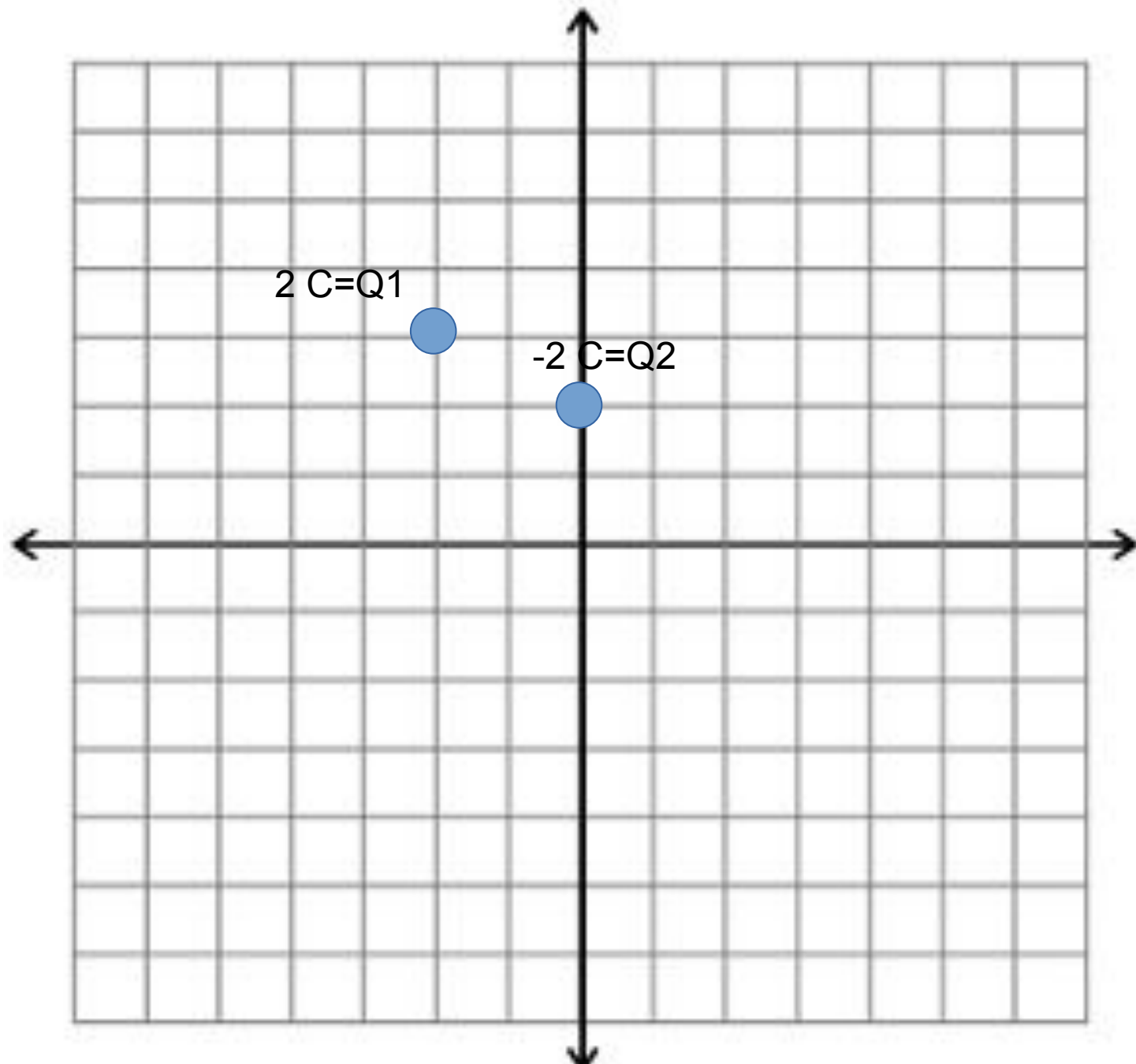
realizing that positive test charges  $q$  tend to move from higher to lower potential (while negative test charges  $q$  tend to move from lower to higher potential), what is the relative potential at  $(x,y)=(0,0)$  vs.  $(x,y)=(-1,-2)$ ? Check your answer numerically

- A)  $V(0,0) > V(-1,-2)$
- B)  $V(0,0) < V(-1,-2)$
- C)  $V(0,0) = V(-1,-2)$



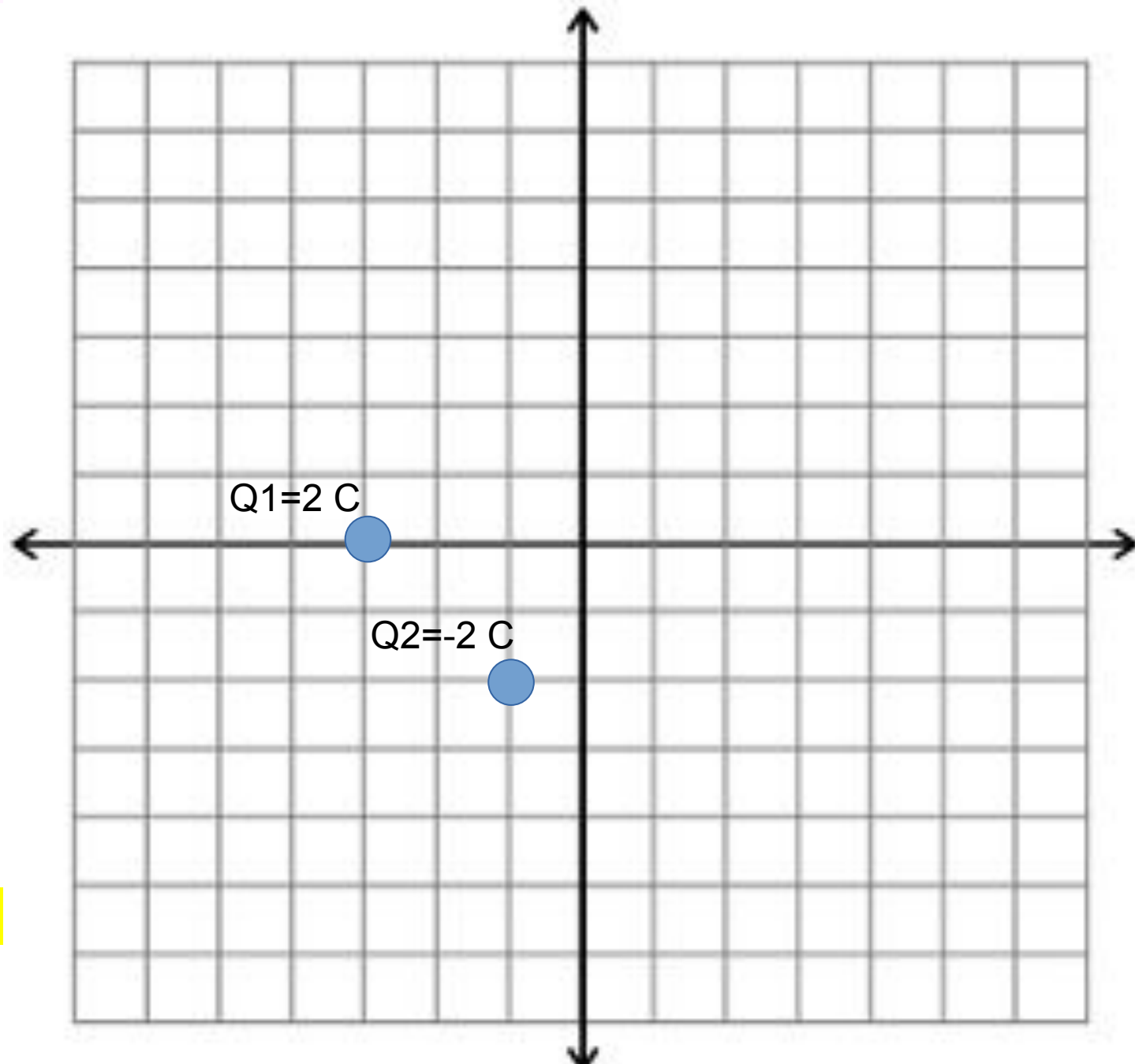
The E-field direction at (0,0) is (check your graphical answer numerically!):

- A) NE
- B) NW
- C) SE
- D) SW



**A  $-0.1\text{ C}$  charge  $q$  is now placed at the origin, and released. Given that the force on  $q$  is given by  $F=qE$ , what is the direction of acceleration of  $q$ ?**

- A) NE
- B) NW
- C) SE
- D) SW



What if  $q$  was  $+0.1\text{ C}$ ?

**Given the two source charges shown.  
The number of points along x- for which  
 $|E_{\text{tot}}|=0$  and also  $V_{\text{tot}}=0$  is, respectively:**

A) 1,2

B) 1,1

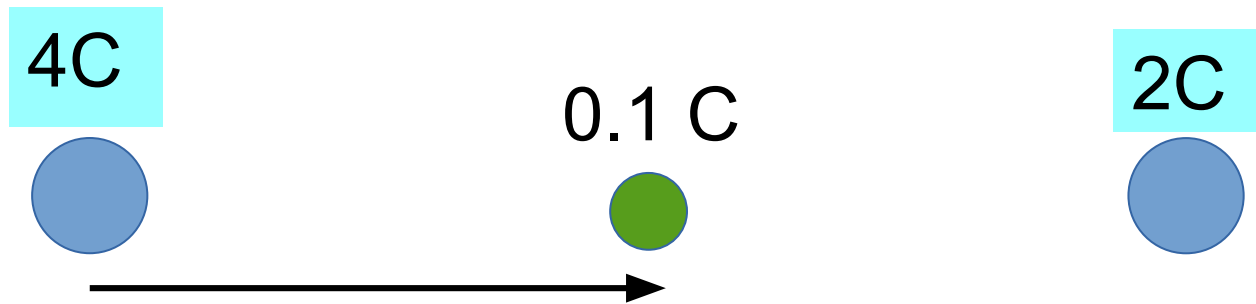
C) 2,1

D) 2,2

E) NOTA



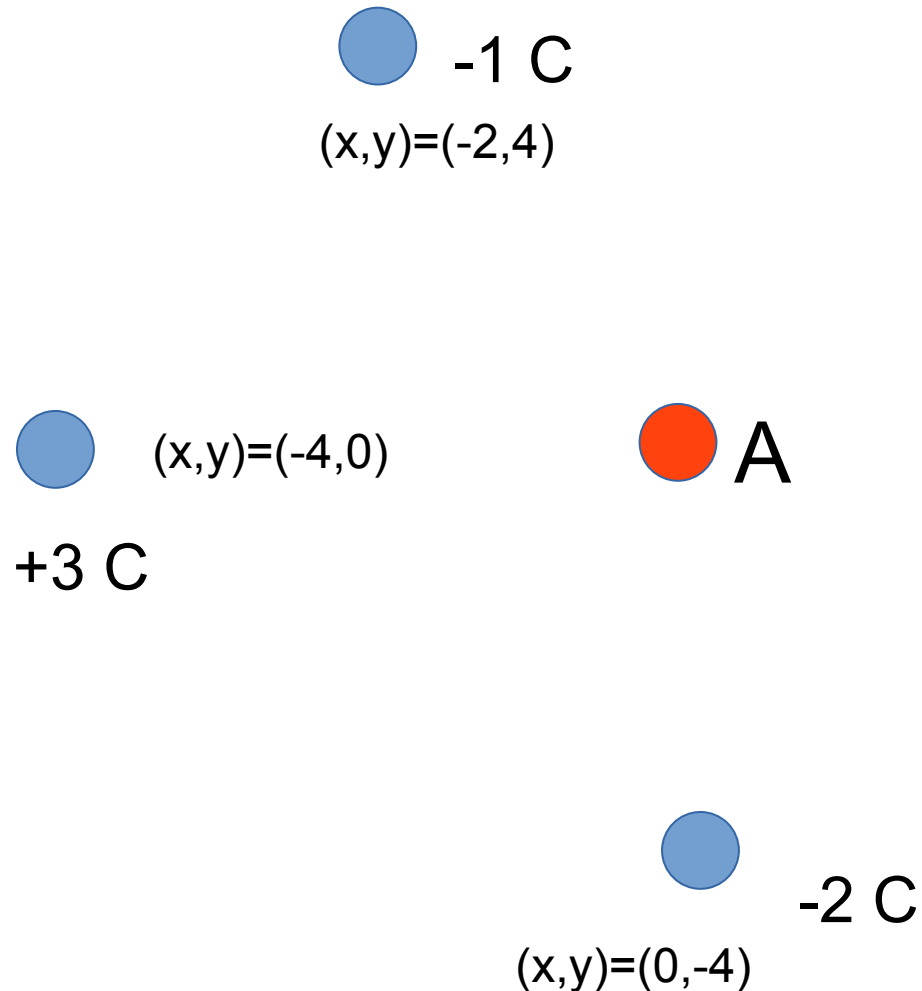
**A second  $+2\text{C}$  charge is placed  $2\text{ m}$  to the right of the  $0.1\text{C}$  charge, and the  $0.1\text{C}$  charge released from  $x=0$ . At what point does the charge change direction?**



- A)  $X=1\text{ m}$
- B)  $X=0.345\text{ m}$
- C) Not finished yet....

**The voltage (= `potential') at point A (0,0) is:**

- A) 0k V
- B) 0.337k V
- C) 0.0143k V
- D) 0.0264k V
- E) NOTA

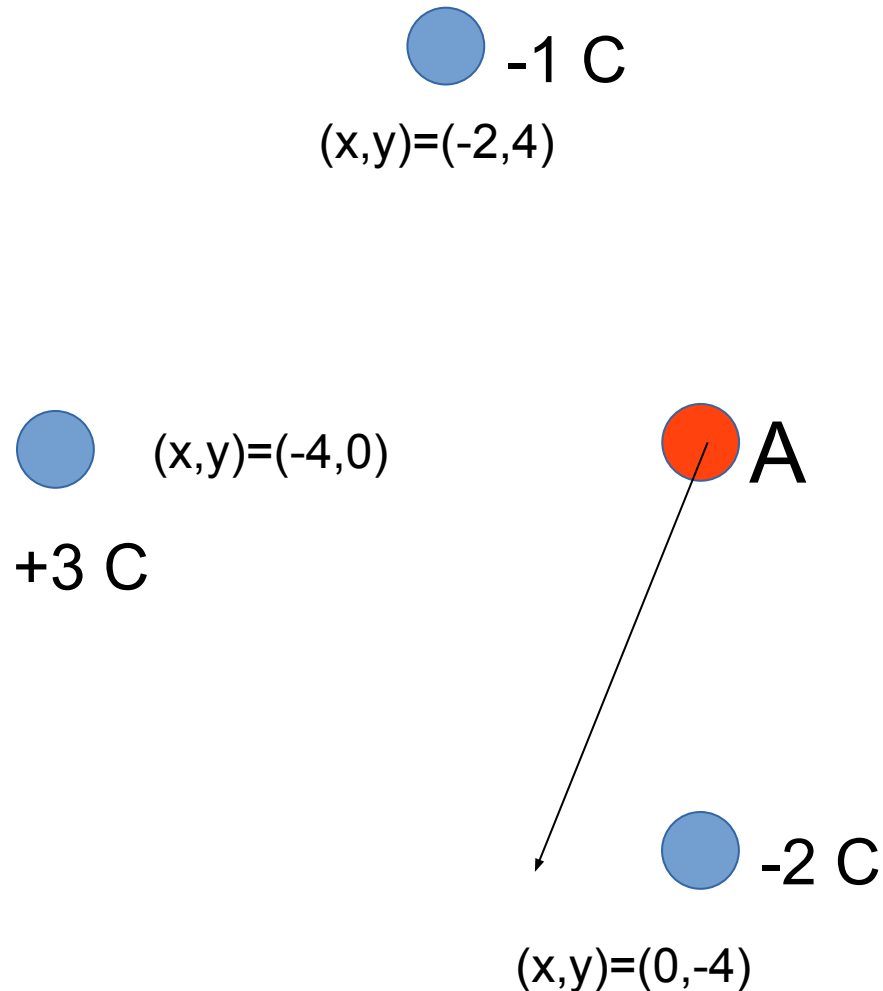


**In the previous problem, consider 3 pts  $P_0$ ,  $P_1$ , and  $P_2$ , which are at  $y=-1$  m,  $-2$  m, and  $-3$  m, respectively. Rank the potentials at the 3 points from most negative to most positive (low to high)**

- A)  $V_0 > V_1 > V_2$
- B)  $V_0 < V_1 < V_2$
- C)  $V_1 > V_2 > V_0$
- D)  $V_1 > V_0 > V_2$
- E) NOTA

**A  $-2\text{C}$  charge  $q$  is placed at A and moved to  $(-2,-4)$ . The  $\Delta\text{PE}$  of  $q$  is:**

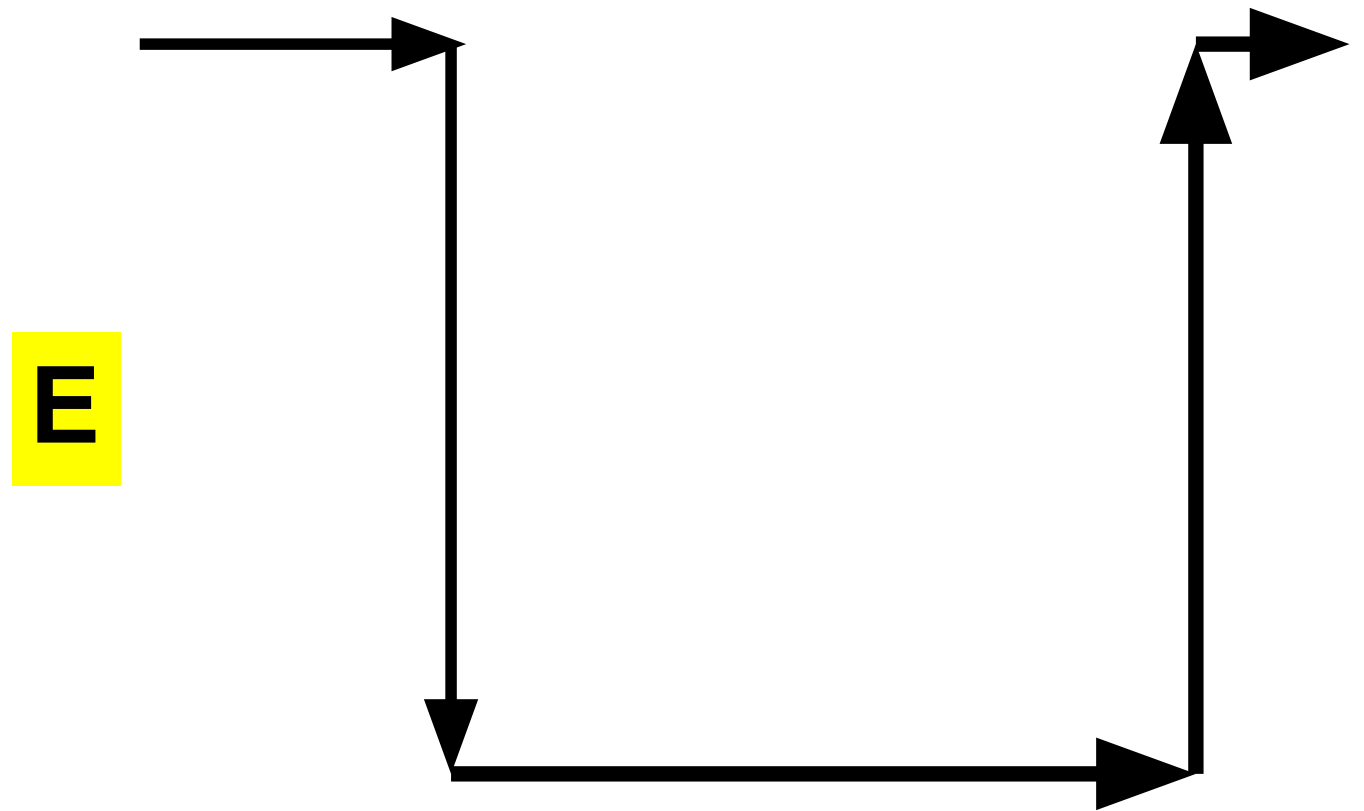
- A)  $0\text{k J}$
- B)  $0.337\text{k J}$
- C)  $0.961\text{k J}$
- D)  $0.126\text{k J}$
- E) NOTA





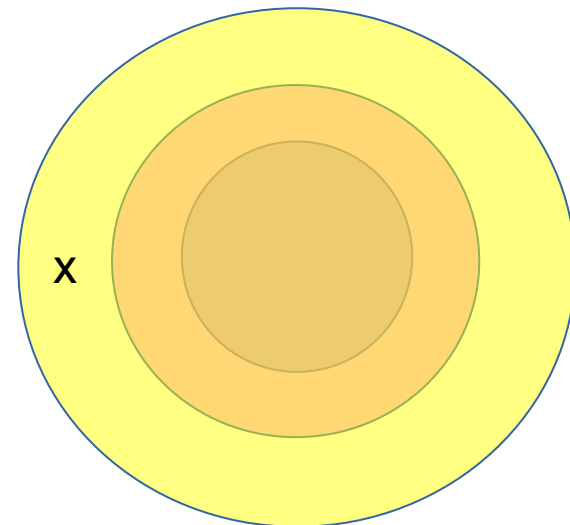
**Given the  $E(x)$  map; what is the final velocity of a 0.48 kg, -2 C charge that starts at  $x=-2$  m and ends at  $x=6$  m?**

- A) 0 m/s
- B) 2 m/s
- C) 8 m/s
- D) 10 m/s
- E) NOTA



**Three concentric metallic spheres containing charges  $-2C$ ,  $+3C$ , and  $-C$  (inner to outer sphere) have diameter  $2m$ ,  $4m$  and  $6m$ , respectively. At the point  $x$ , the electric field has magnitude and direction**

- A)  $k/25$ ;  $+y$
- B)  $k/25$ ;  $+x$
- C)  $5k/25$ ;  $+y$
- D)  $3k/25$ ;  $+x$
- E) NOTA



Given the graph of potential as a function of position  $x$ ; the  $E$  field values at  $x=-1\text{m}$ ,  $x=1.5\text{m}$ , and  $x=4.5\text{m}$ , are closest to:

A)  $2\text{ V/m}$ ,  $-2\text{ V/m}$ ,  
 $3\text{ V/m}$

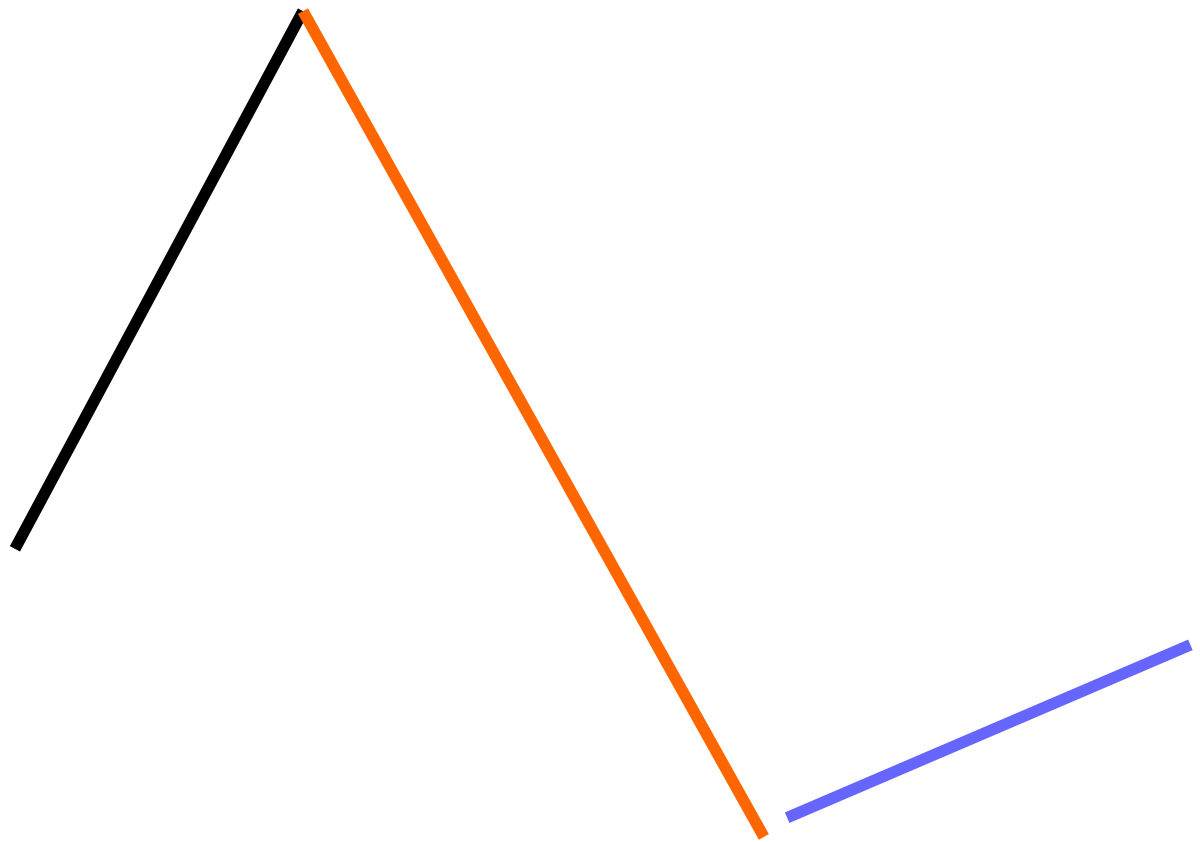
B)  $-2\text{ V/m}$ ,  $4\text{ V/m}$ ,  
 $-5\text{ V/m}$

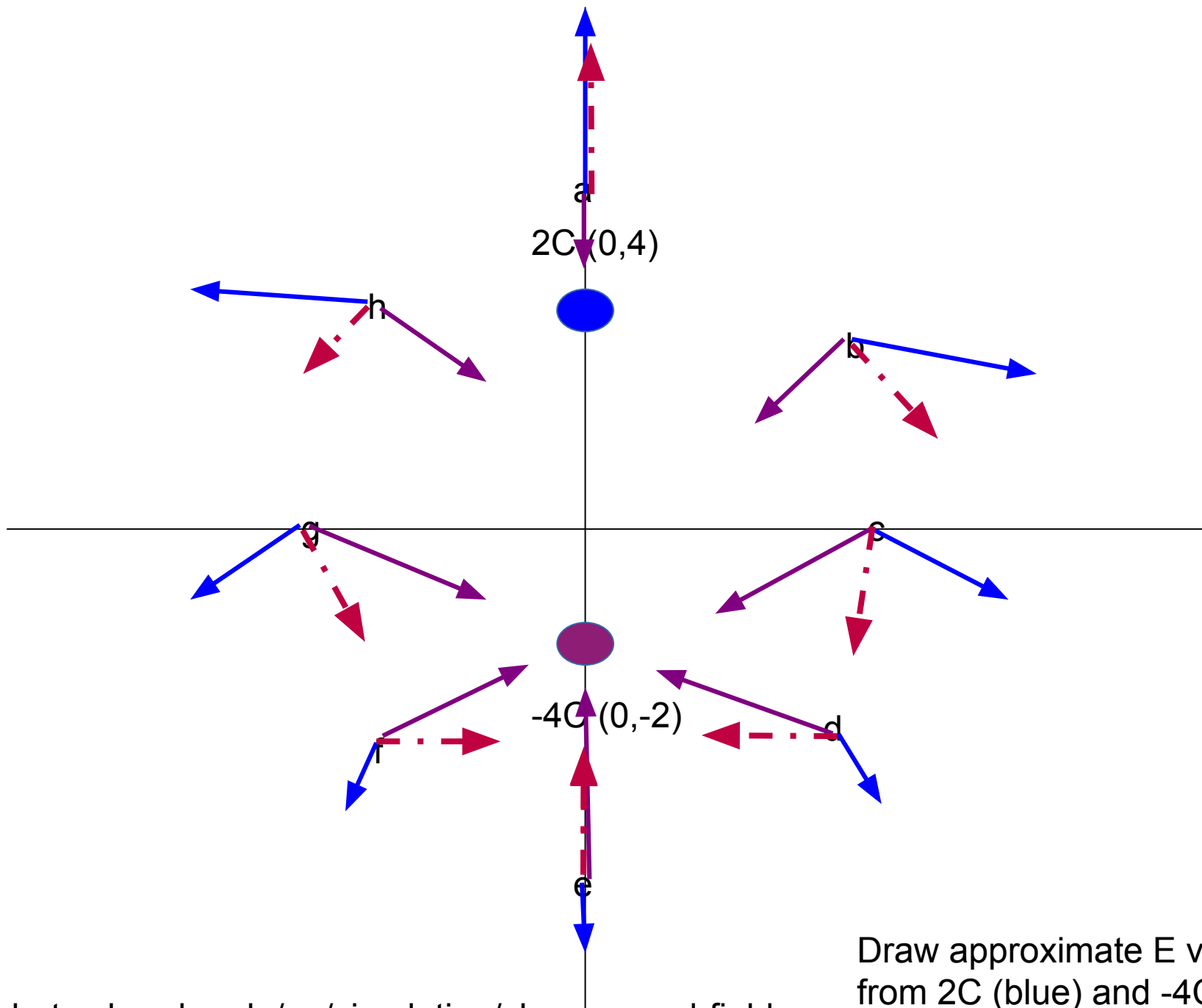
C)  $4\text{ V/m}$ ,  $-5\text{ V/m}$ ,  
 $-3\text{ V/m}$

D)  $-3\text{ V/m}$ ,  $3\text{ V/m}$ ,  
 $-1.5\text{ V/m}$

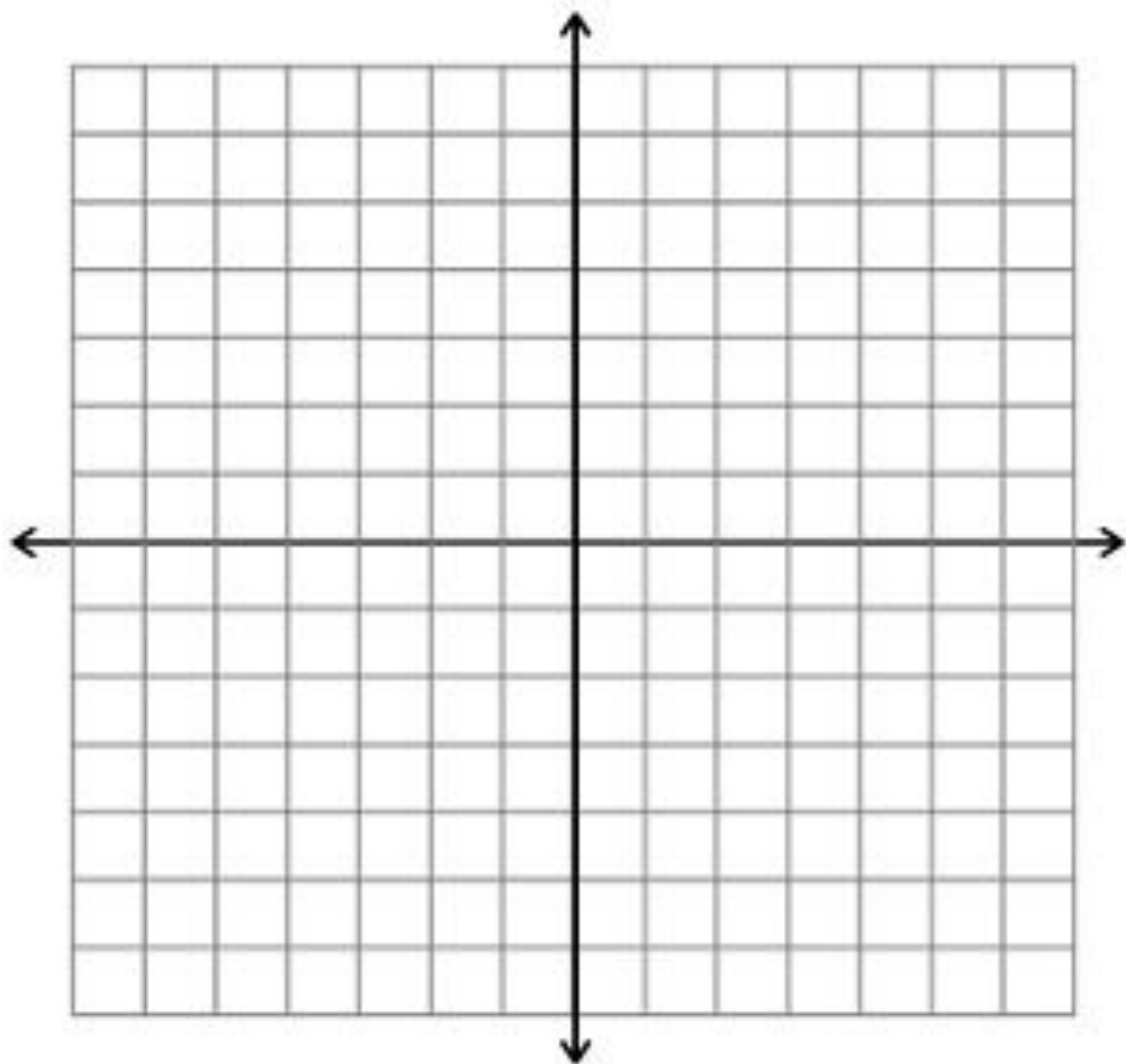
E) NOTA

**V**



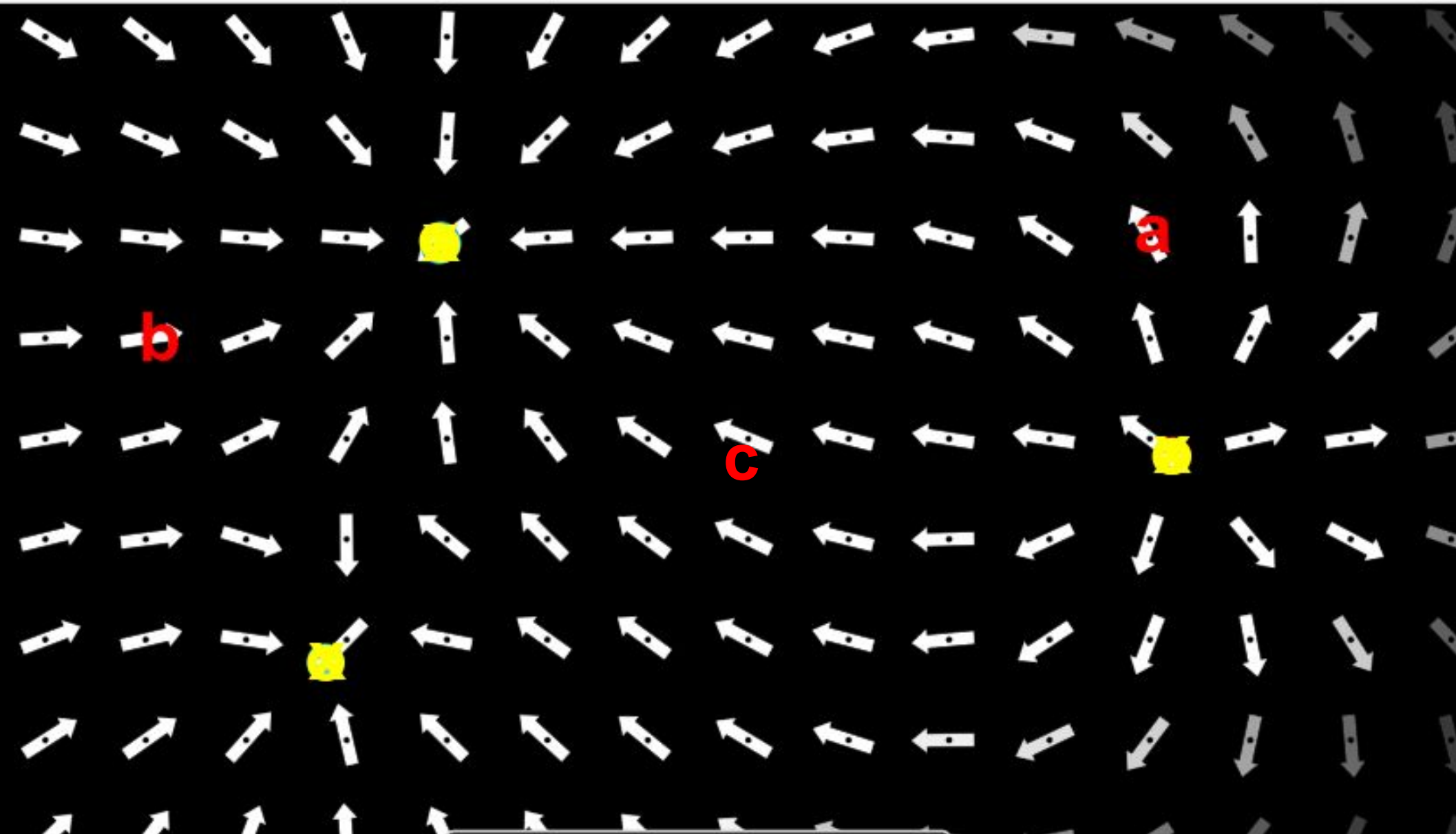


Draw approximate E vectors  
from  $2C$  (blue) and  $-4C$  (purple)  
and add graphically (red dash)

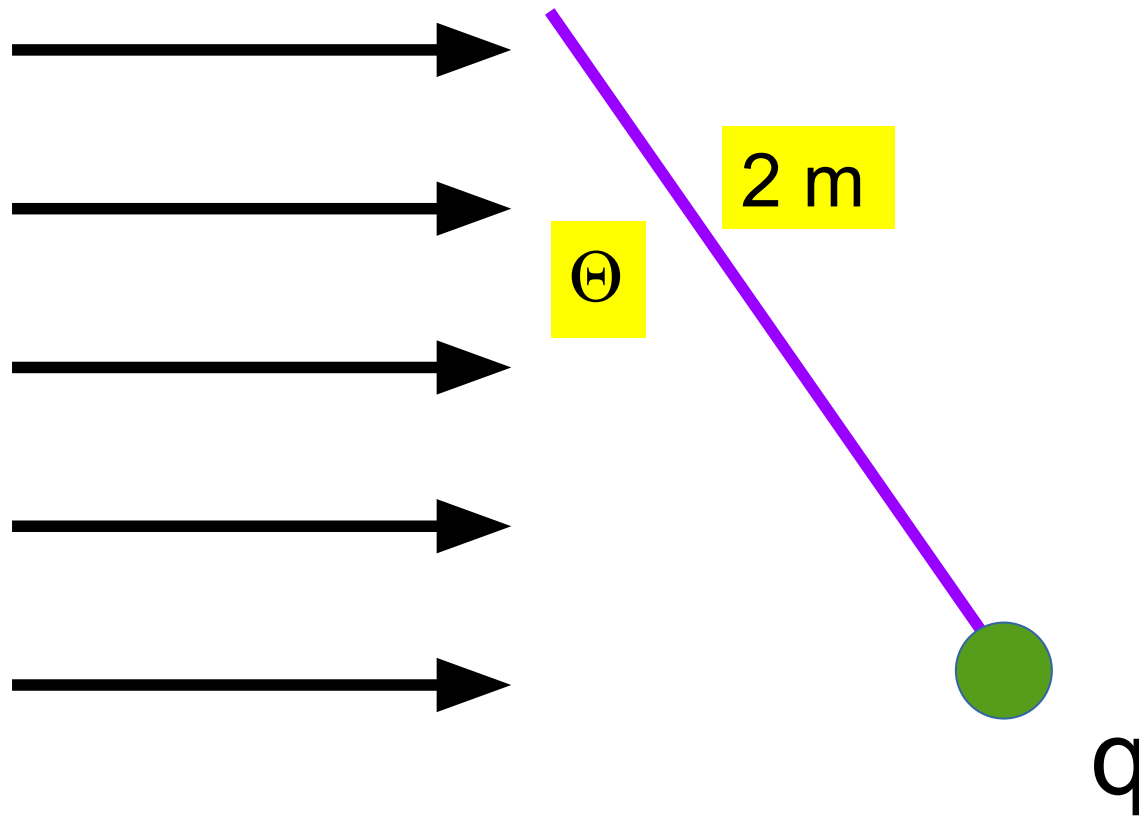


Using the mountain/depression analogy, the PE of a +q charge follows:

A)  $PE(a) > PE(b) > PE(c)$ ; B)  $PE(a) < PE(b) < PE(c)$ ; C)  $PE(a) > PE(b) > PE(c)$ ; D)  $PE(c) > PE(b) > PE(a)$   
Check your answer by: i) identifying the signs of the source charges, and ii) make a rough estimate of  $\Sigma kQ/r$ , taking into account those signs

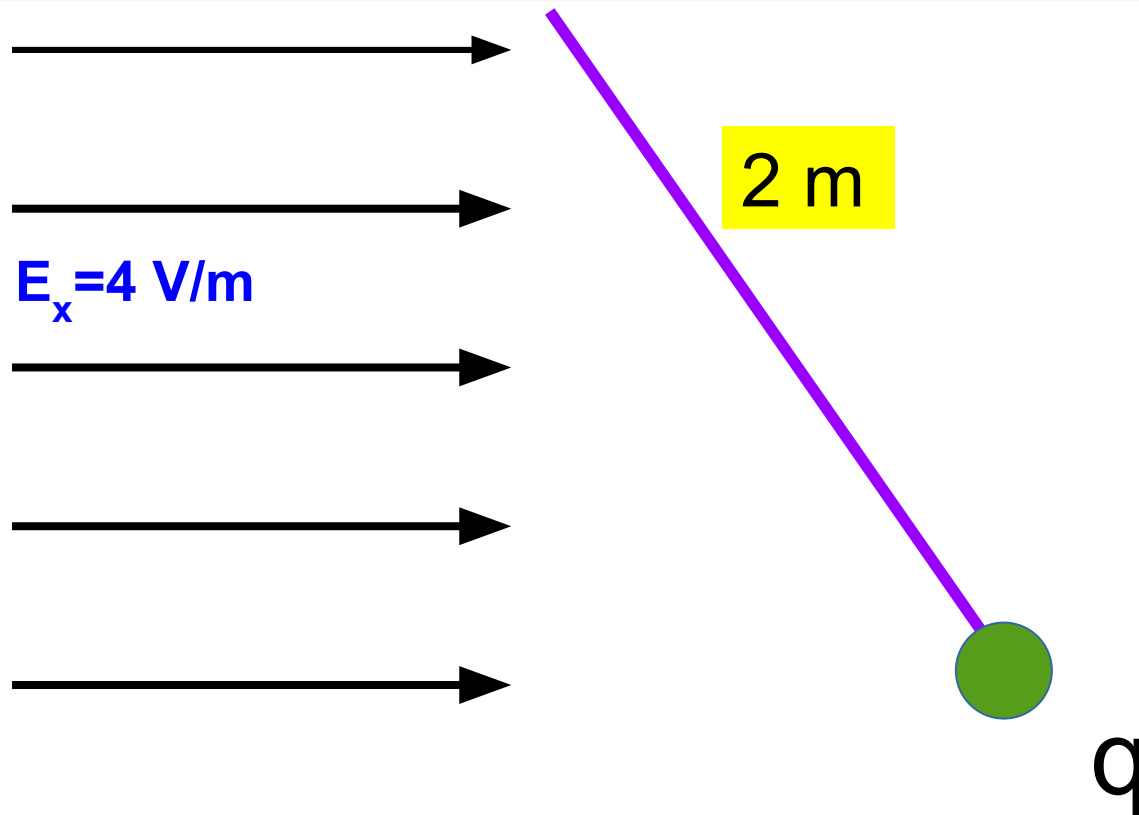


A 1.02 kg mass, with  $|q|=2$  C, on a 2 m rope in a field with  $E_x=4$  V/m makes an angle  $\theta$ . The total # of forces acting on  $q$ , and the sign of  $q$  is:



- A) 1, +
- B) 2, -
- C) 3, +
- D) 4, -
- E) NOTA

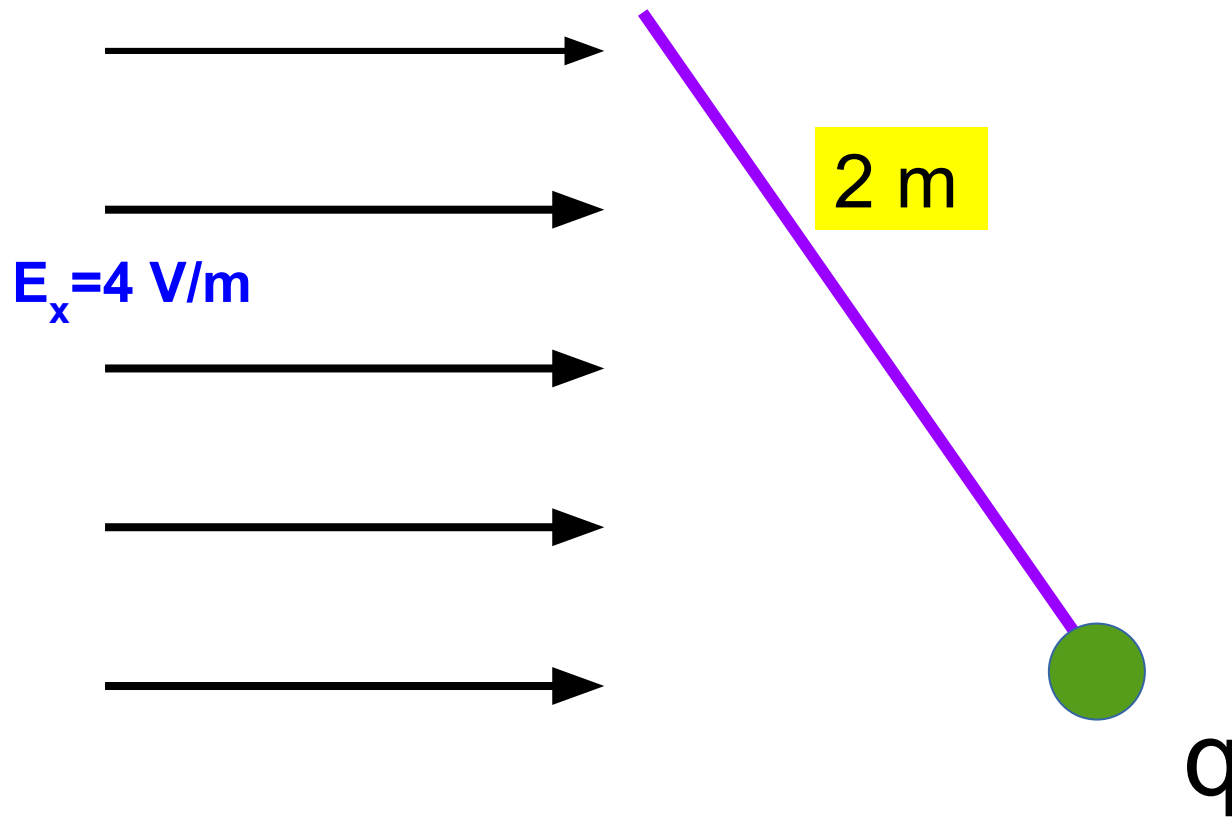
**The tension in the string is:**



- A) 8 N
- B) 10 N
- C) 12.8 N
- D) 15 N
- E) NOTA

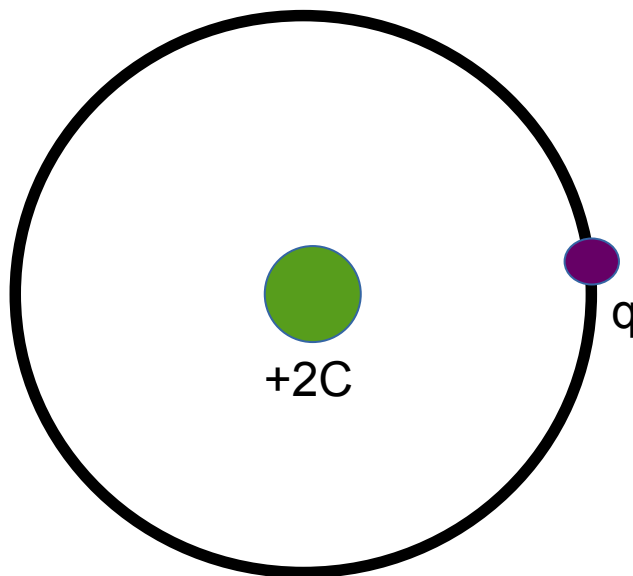


For  $m=1.02$  kg, the angle  $\theta$  is:



- A)  $38.6^\circ$
- B)  $51.3^\circ$
- C)  $45^\circ$
- D)  $30^\circ$
- E) NOTA

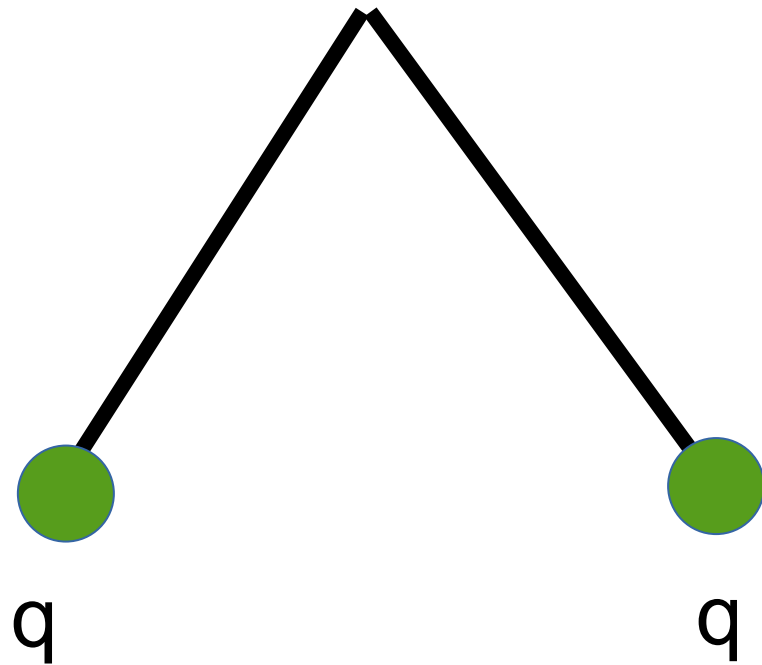
**Charge  $q$ , with mass 1 kg orbits (radius 10 m) around  $+2\text{C}$  source with  $v=100\text{ m/s}$ . The value of the charge  $q$  is:**



- A)  $-10^{-6}\text{ C}$
- B)  $-5.55 \times 10^{-6}\text{ C}$
- C)  $-10^{-4}\text{ C}$
- D)  $-2\text{ C}$

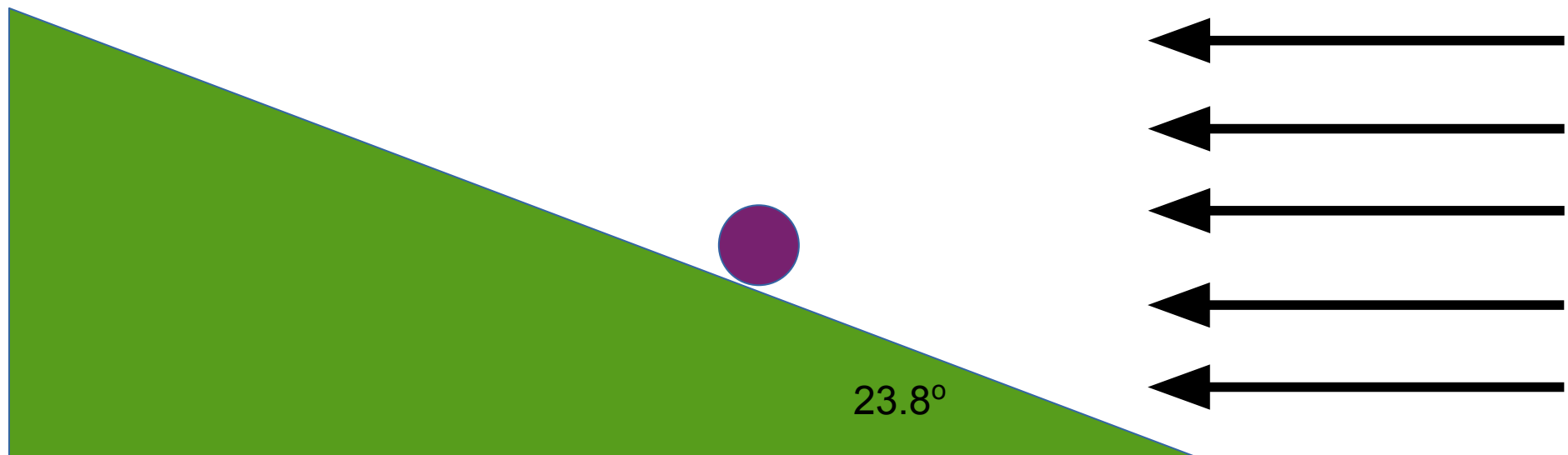
**Two identical electric charges, each with charge  $q$  and mass 4 kg, hang at the ends of two 2m ropes, which are separated by a total angle of  $60^\circ$ .  $q =$**

- A)  $6.4 \times 10^{-5} \text{ C}$
- B)  $3.2 \times 10^{-5} \text{ C}$
- C)  $1.0 \times 10^{-4} \text{ C}$
- D)  $9.33 \times 10^{-5} \text{ C}$
- E) NOTA



**A charged 2.04 kg mass is stationary on a frictionless incline with angle  $23.8^\circ$  wrt. horizontal, in a field  $E_x = -4\text{ N/C}$ ;  $q = ?$**

- A) -2.2 C
- B) -1.1 C
- C) +2.2 C
- D) +1.1 C
- E) NOTA



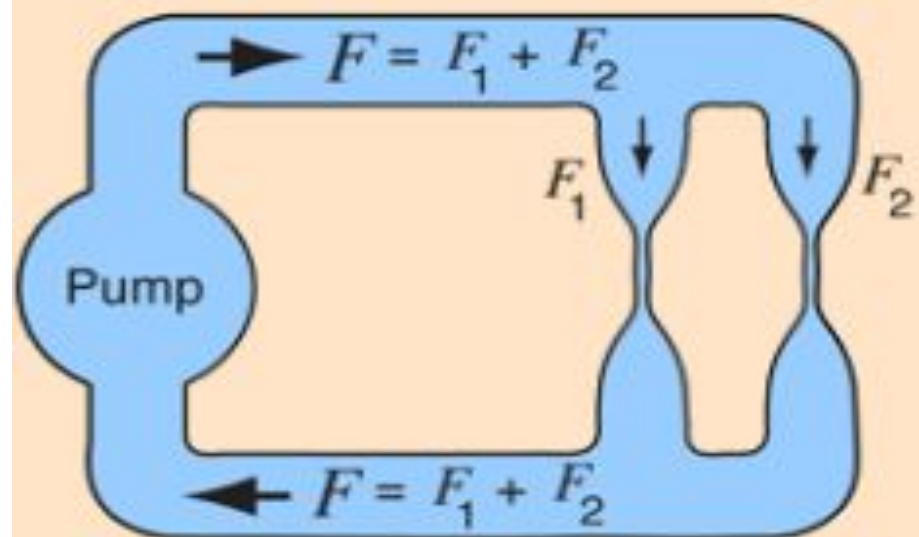
## Capacitance and Batteries.

For gravity, we can use gravitational potential energy as an energy source (hydroelectric, e.g.). The KE of masses stored at high PE is recovered when water flows over the Kaw river; water tower is another example.

A capacitor is an electrical “water tower”, which stores charge. A battery is analogous to the water pump that gets water up to the top of the water tower. Principle: Series: sum of voltage drops through height  $h$  = voltage supplied; Parallel: all voltage drops same. When caps added in series, they must have the same charge and  $\text{Sum}(V) = V_{\text{ext}}$  => add as inverses. Added in parallel, each cap has same  $V$  (add linearly). Charge  $Q = CV$

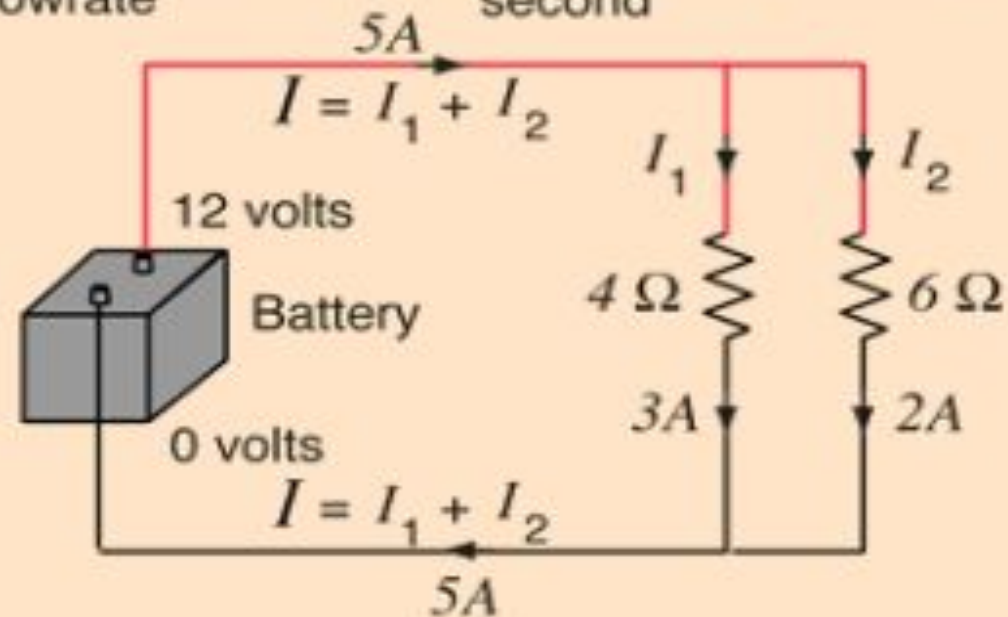
### Conservation of liquid

volume  
flowrate e.g.,  $\text{cm}^3/\text{sec}$



### Conservation of charge

charge  
flowrate = current =  $\frac{\text{coulombs}}{\text{second}} = \text{amperes}$



## **Procedure for solving circuit problems:**

### **1) Battery+capacitors/resistors:**

**Reduce circuit to battery+series C or R**

### **2) Capacitors: Reduce circuit to equivalent series capacitors**

### **3) Resistors: Determine I for equivalent series resistors**

### **4) Trace voltage drops through circuit, then use $Q=CV$ or $I=V/R$ to determine Q or I for circuit elements**

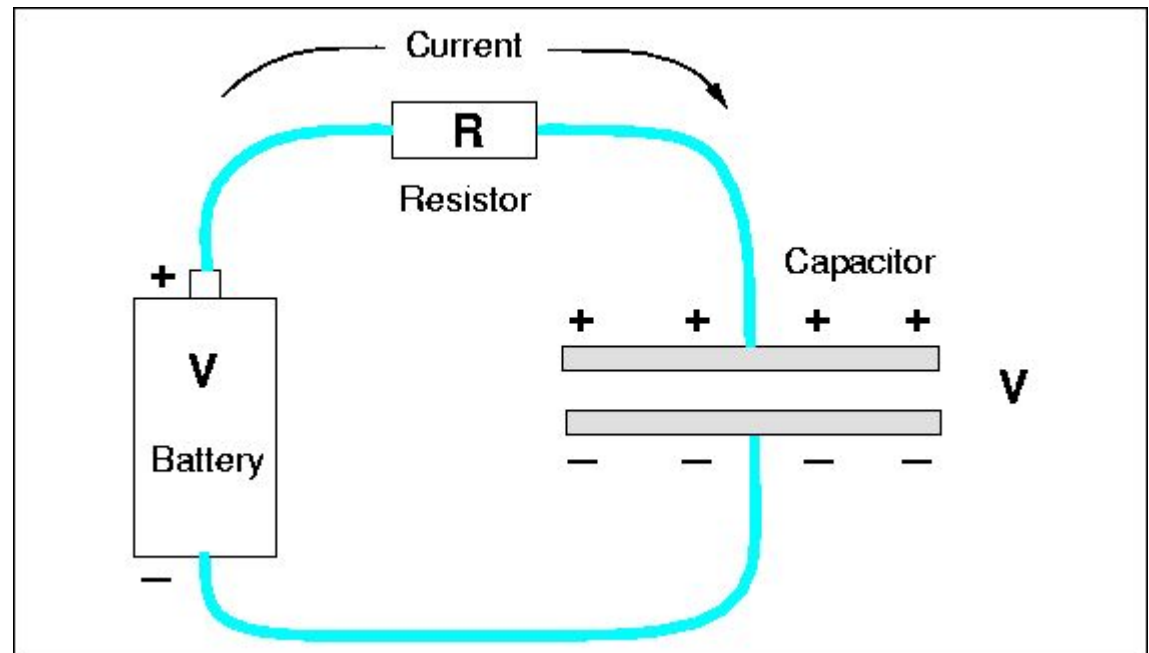
**In the water analogy, a capacitor is most like a:**

- A) Water pump
- B) Water tower
- C) Plumbing pipe
- D) Hydroelectric generator



**Which of the following formulas would you expect to be correct for the total charge  $Q$  stored on a ppc with capacitance  $C$  attached to a battery of voltage  $V$ ?**

- A)  $Q = C/V$
- B)  $Q = V/C$
- C)  $Q = CV$
- D)  $Q = 1/(CV)$
- E) NOTA

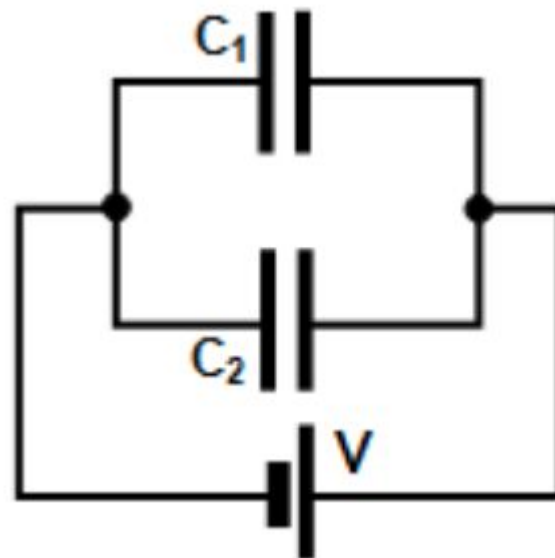


**A battery charges up a ppc (parallel plate capacitor). As time goes on, which of the following increase?**

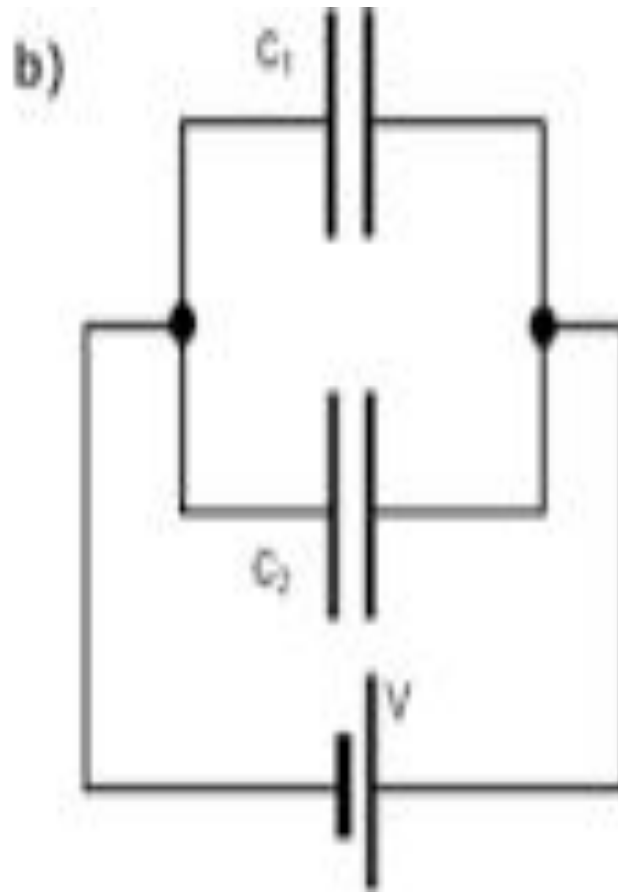
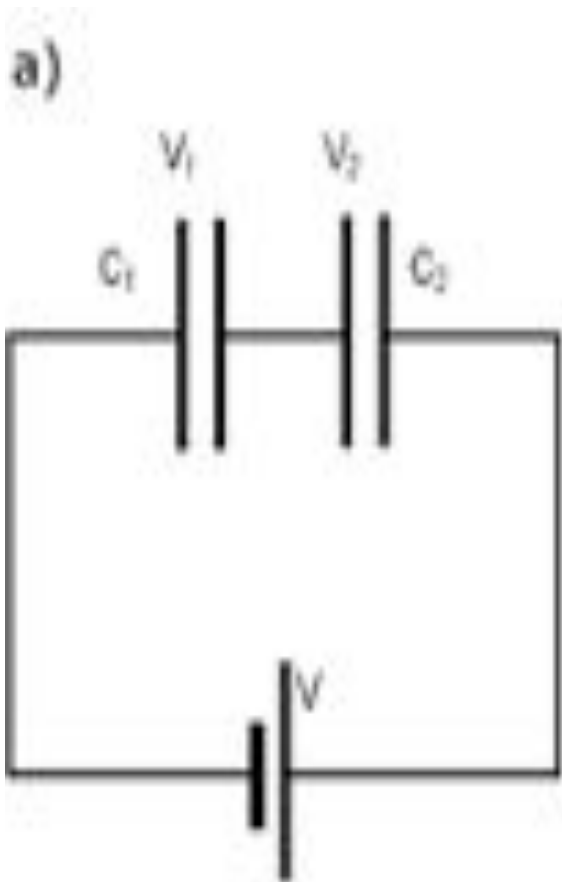
- A) The rate at which charge flows onto the ppc
- B) The total electrical energy stored in the ppc
- C) The voltage across the ppc
- D) Two of the above
- E) All of the above

An 18V battery is attached to a  $3\text{F}=\text{C}_1$  and  $6\text{F}=\text{C}_2$  capacitor, in parallel. After charging up, the ppc's have charge

- A) 12C, 6C
- B) 54C, 108C
- C) 36C, 36C
- D) 12C, 24C
- E) NOTA

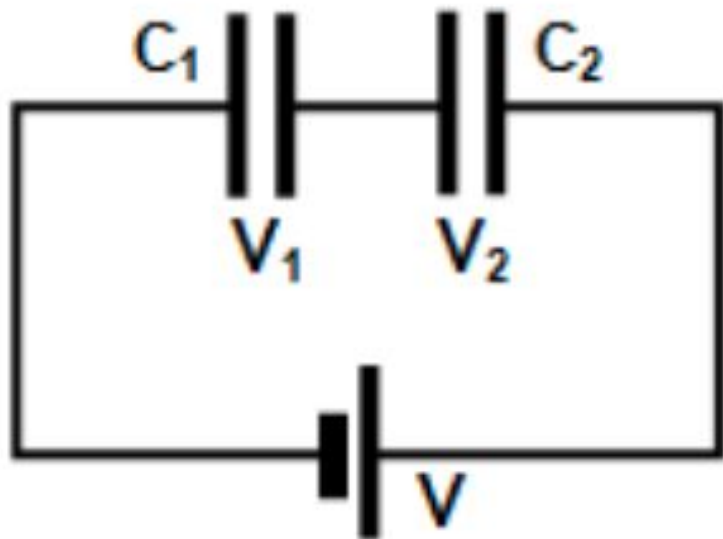


An 18V battery is attached to a 3F ( $C_1$ ) and 6F ( $C_2$ ) capacitor, in series. After charging up, which would you expect to be true?



- A)  $Q_{-,1} = Q_{+,2}$
- B)  $V_1 = V$
- C)  $V_2 = V$
- D)  $V_1 + V_2 = V$
- E) 2 or more are true

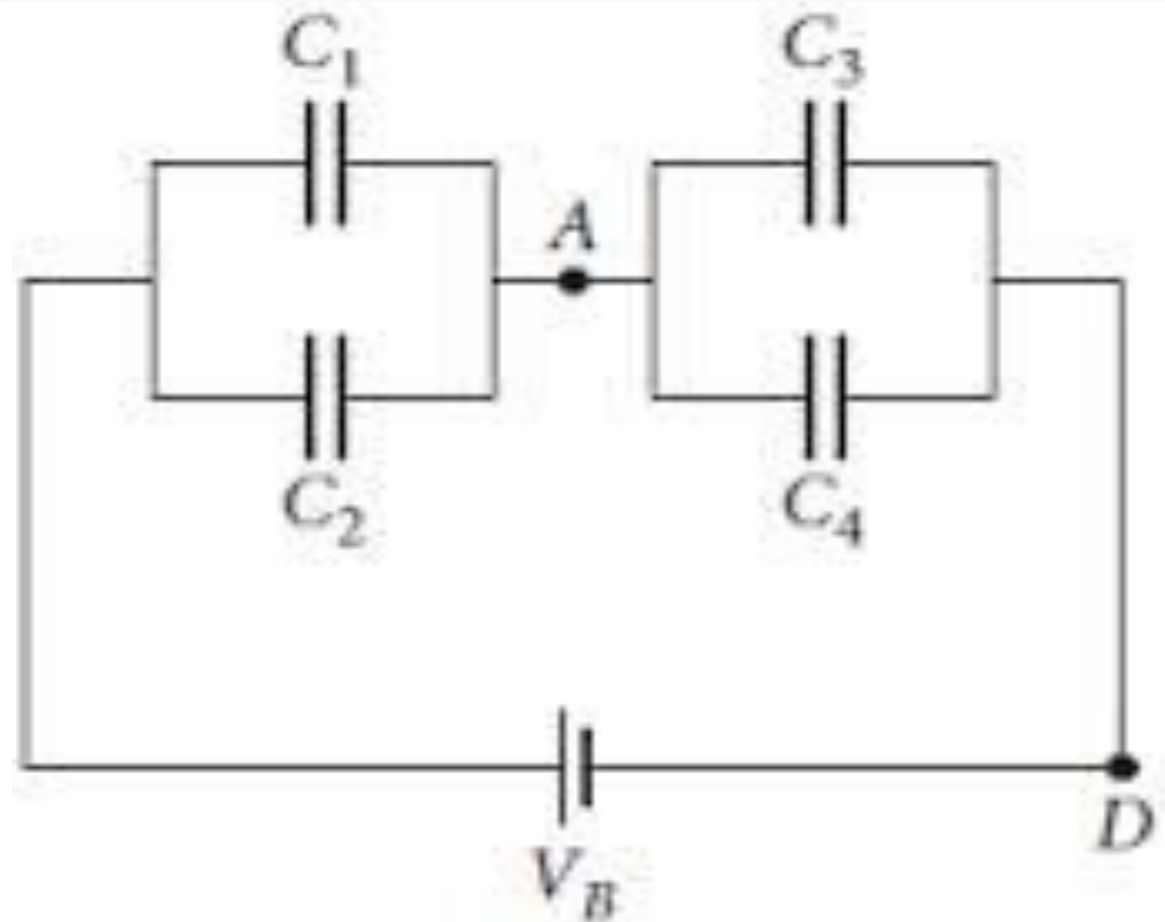
An 18V battery is attached to a 3F ( $C_1$ ) and 6F ( $C_2$ ) capacitor, in series. After charging up, the charge on the plates of the ppc's is (verify using Kirchhoff's conservation of Voltage)



- A) 6C
- B) 36C
- C) 0.111C
- D) 9C
- E) NOTA

In the circuit below,  $V_B = 6\text{ V}$ ,  $C_1 = 2\text{ F}$ ,  $C_2 = 4\text{ F}$ ,  $C_3 = C_4 = 6\text{ F}$ ; after fully charging up, the charge on capacitor  $C_1$  and  $C_3$  are, respectively:

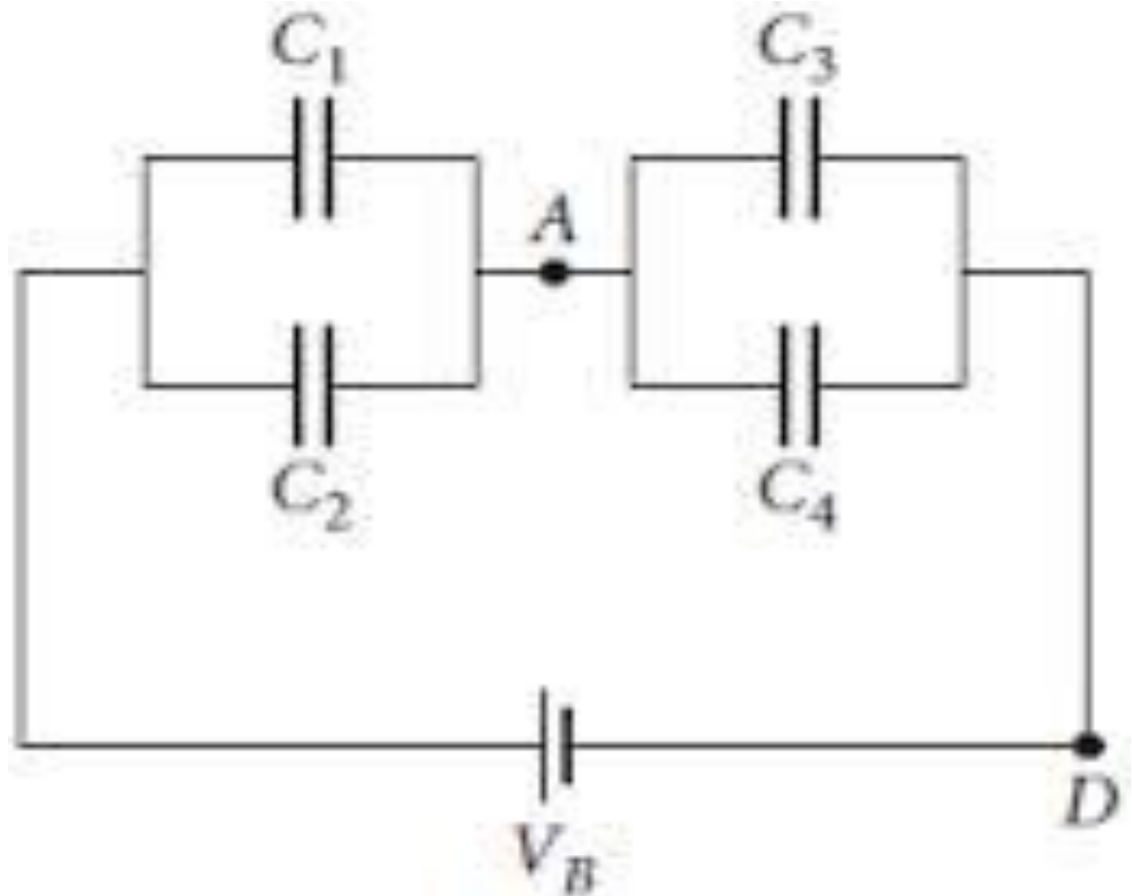
- A) 12C, 12C
- B) 16C, 8C
- C) 8C, 12C
- D) 4C, 4C
- E) NOTA



**In the same circuit, the voltage at A is:**

- A) 6V
- B) 4V
- C) 2V
- D) 0V
- E) NOTA

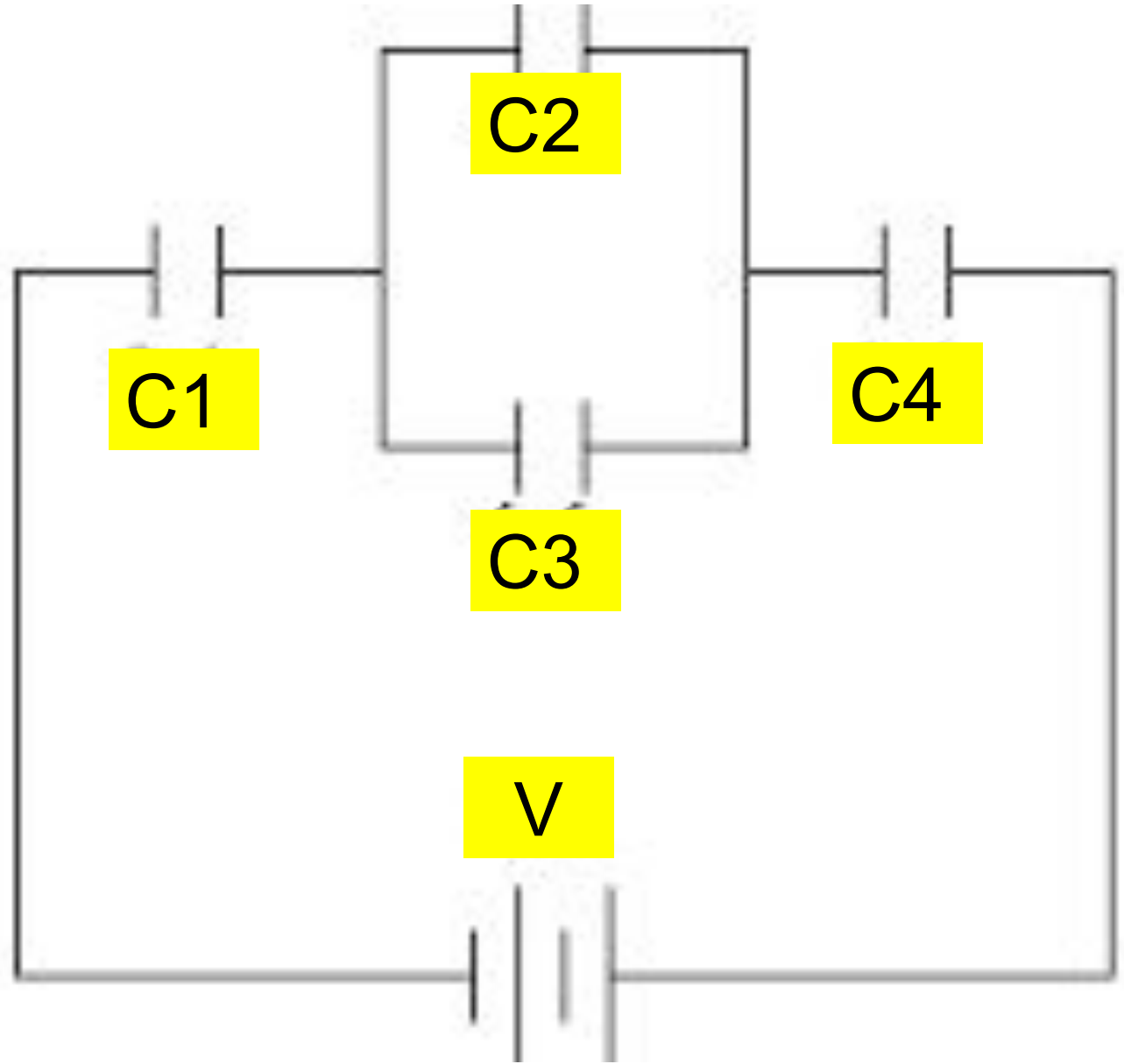
Suppose  $V_B$  was reversed?



If  $V=12\text{V}$ ,  $C_1=2\text{F}$ ,  $C_2=4\text{F}$ ,  $C_3=2\text{F}$ ,  $C_4=3\text{F}$ . The charge on  $Q_4=$

1

- A) 2C
- B) 6C
- C) 12C
- D) 18C
- E) NOTA

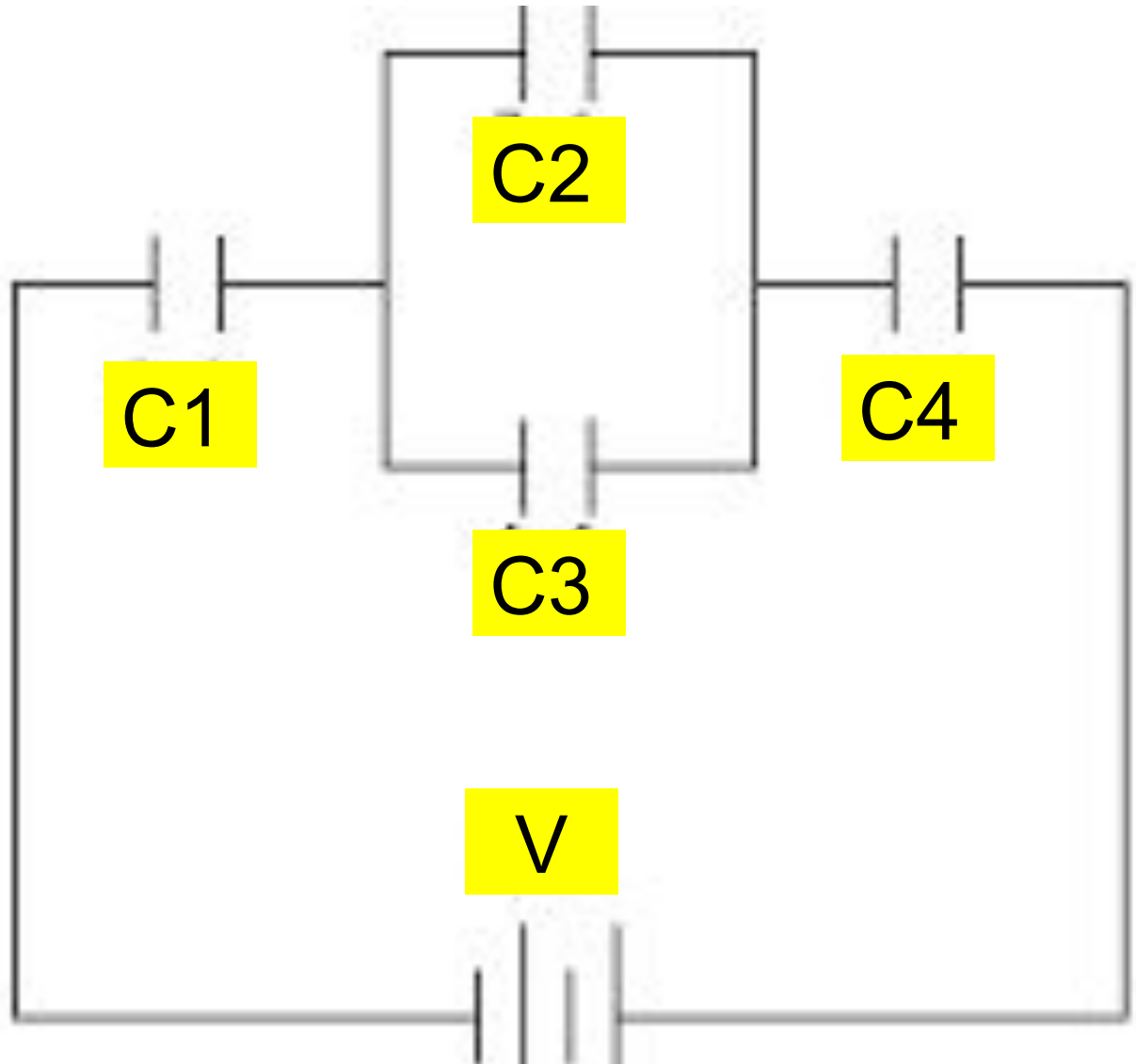




In the circuit,  $V=12\text{V}$ ,  $C_1=2\text{F}$ ,  $C_2=4\text{F}$ ,  $C_3=2\text{F}$ ,  $C_4=3\text{F}$ . The energy stored on  $C_2$  is:

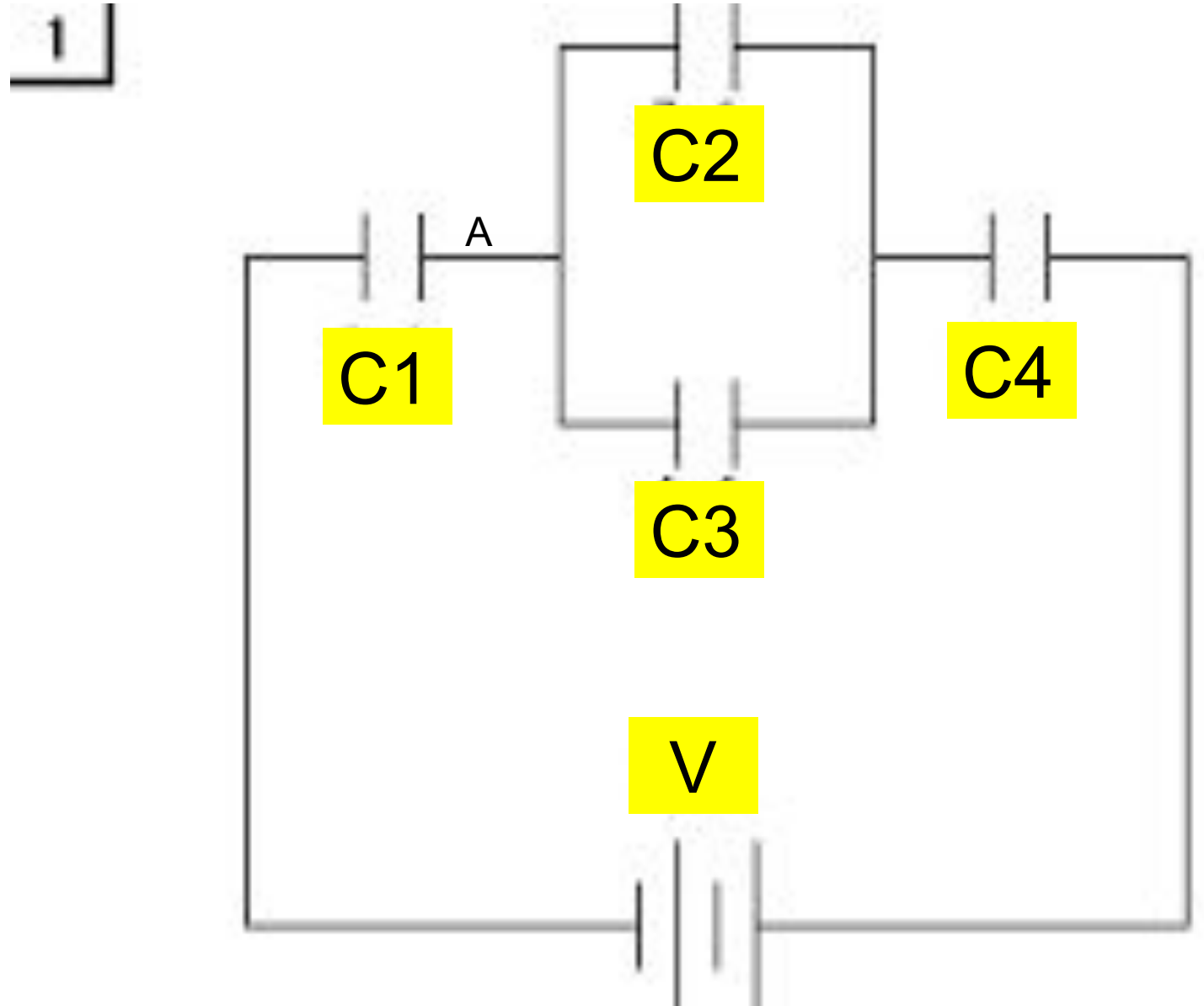
1

- A) 2J
- B) 6J
- C) 12J
- D) 8J
- E) NOTA



In the circuit,  $V=12\text{V}$ ,  $C_1=2\text{F}$ ,  $C_2=4\text{F}$ ,  $C_3=2\text{F}$ ,  $C_4=3\text{F}$ . The voltage at point A is (check by working both ways through the circuit!):

- A) 2V
- B) 6V
- C) 12V
- D) 8V
- E) NOTA

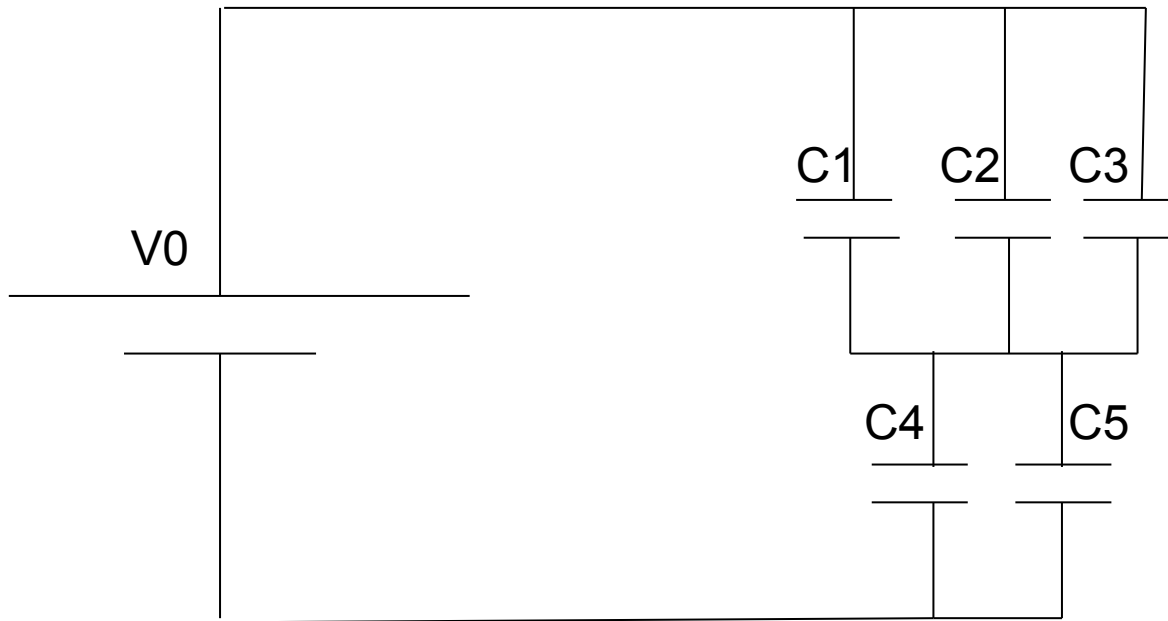


# Capacitance and Batteries

Given one battery and five capacitors. Assign unique values to the 5 capacitors and the one battery. Determine the charge on each capacitor, when fully charged up, the total charge from the battery, and the total energy required to charge up the 5 capacitors. Verify your results using Kirchhoff's Laws for one 'closed loop'.

Given  $V_0=24V$ ,  $C_1=1F$ ,  $C_2=2F$ ,  $C_3=3F$ ,  $C_4=8F$  and  $C_5=4F$ , find charges on all Caps.  
Verify your answers using Kirchoff's principle.

Soln: 1) Find equivalent circuit:  $C_{123}=1F+2F+3F=6F$ ;  $C_{45}=12F$ , so circuit is equivalent to a 24V source with a 6F and 12F Capacitor in series:



Alternately: 1. reduce circuit to series combination, then 2. use Kirchoff with Q same on series combinations & solve for Q:

$24V=Q/6+Q/12$  gives  $Q=96C$ , so  $V_{123}=16V$ , so  $Q_1=16C$ ,  $Q_2=32C$ ,  $Q_3=48C$ ,  $V_{45}=8V$ ,  $Q_4=64C$ ,  $Q_5=32C$

2) Find equivalent resistance of series combinations:

$$1/C_{12345}=1/C_{123}+1/C_{45} \Rightarrow C_{12345}=4F$$

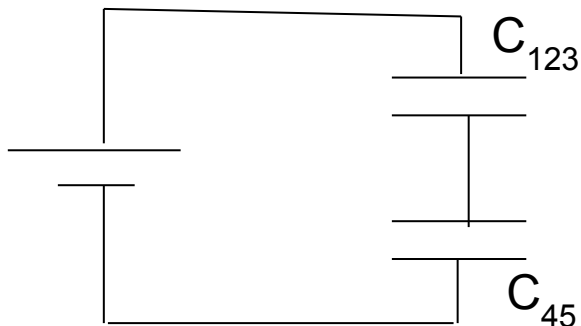
3) Solve for charge Q on  $C_{12345}=96C$ . This also is the charge on each of the series combinations, so  $96C=Q_{123}=Q_{45}$

4) Solve for voltage drops through caps:  $V_{123}=96C/6F=16V$ ;  $V_{45}=8V$  (check that sums to  $V_0$ !)

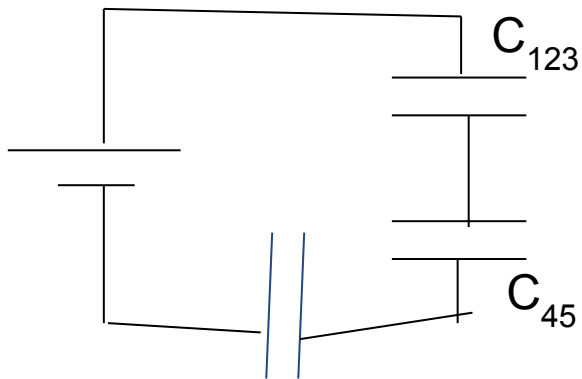
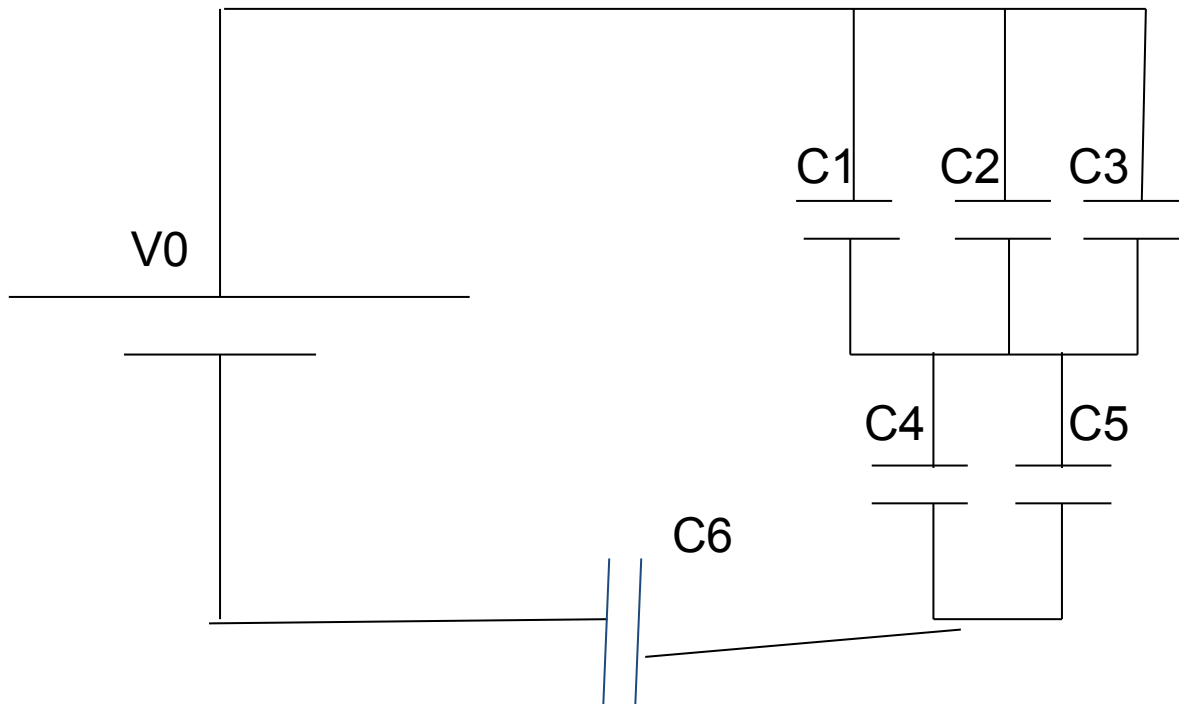
5) Now solve for individual charges on individual caps:

$$Q_1=16V \cdot 1F=16C; Q_2=32C; Q_3=48C$$

$$Q_4=8V \cdot 8F=64C; Q_5=8V \cdot 4F=32C$$



Given  $V_0=24V$ ,  $C_1=1F$ ,  $C_2=2F$ ,  $C_3=3F$ ,  $C_4=8F$ ,  $C_5=4F$  and  $C_6=4F$ , find charges on all Caps. Verify your answers using Kirchoff's principle.



# Equivalent Circuit Solution

Applicable to circuits consisting of one battery and some number of R or C

1) Add parallel caps/resistors from inside→out and reduce circuit to battery + series capacitors

2) Use  $V_{\text{Battery}} = Q/C_{1,\text{series}} + Q/C_{2,\text{series}} + Q/C_{3,\text{series}} \dots$  to find charge on each equivalent series capacitor.

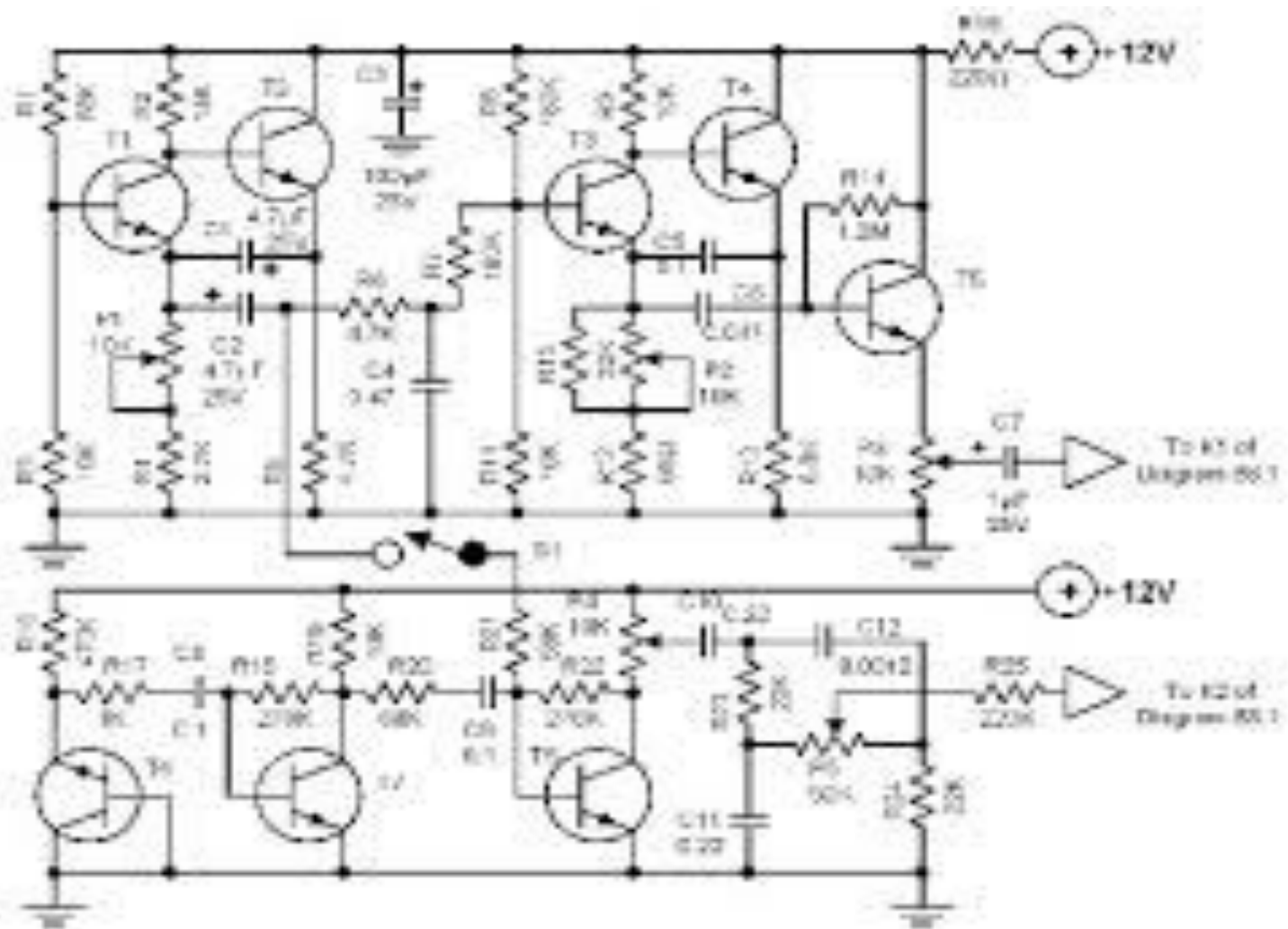
3) This now gives the voltage across each equivalent series capacitor, which can be used to find the charge on each individual capacitor.

4) PE in each capacitor is given by  $Q^2/2C$

Checks:

- a) Kirchoff's rules for each closed loop
- b) Q should equal the same value of Q when circuit is reduced to one equivalent capacitor.

**In the circuit below, what is the voltage difference between points A and B?**







# Relating charge flow to current

$[qv]$ : Coulombs-meters/second

$[IL]$ : (Coulombs/sec) x meters

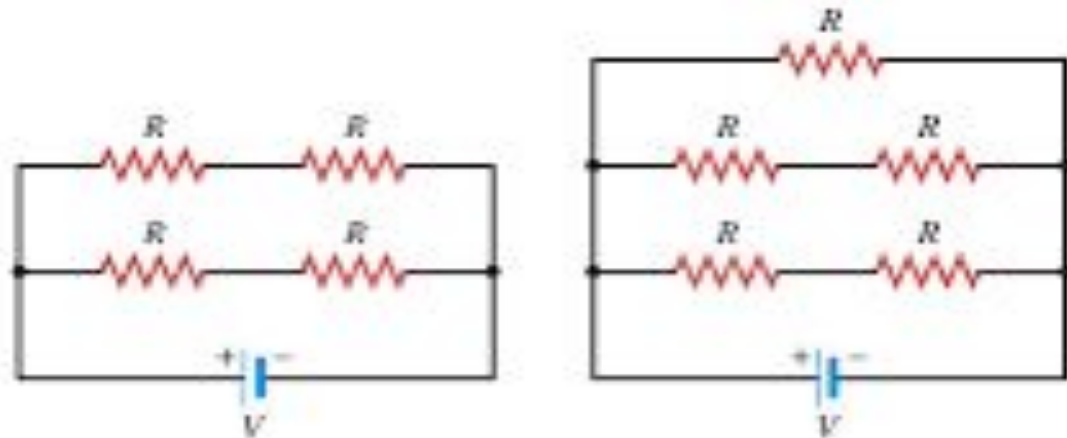
so  $[I]:[qv/L]$ , where  $L$  is distance traveled by charge in one second.

**Resistors: In the water tower analogy – pump=battery, tower=capacitor, voltage across capacitor is the water pressure. How is the water conveyed to the ground? Can have multiple tubes/pipes connecting tower, either in parallel or series... parallel => more paths for water to flow through, so reduced resistance (or think about blowing through straws, either in series or parallel)**

Kirchoff's Laws: after assigning current flow directions:

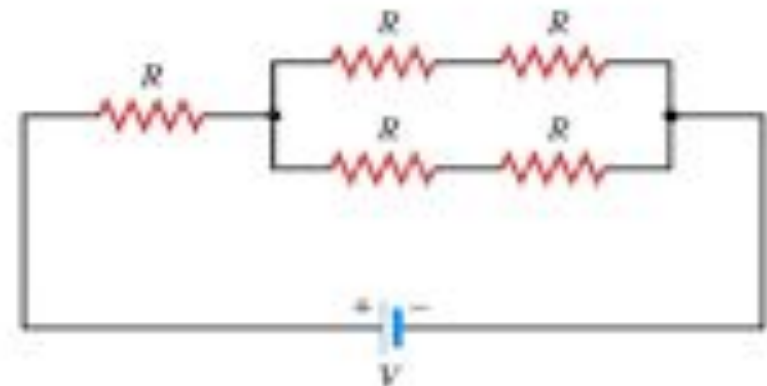
1.  $\sum V$ , through closed loop=0
2.  $\sum I$  at a junction=0, with currents flowing into junction positive and currents flowing out negative.

$I_{\text{tot}}$  is largest for which circuit?  $V=12\text{V}$ ;  $R=12\ \Omega$



Circuit A

Circuit B



Circuit C

A)

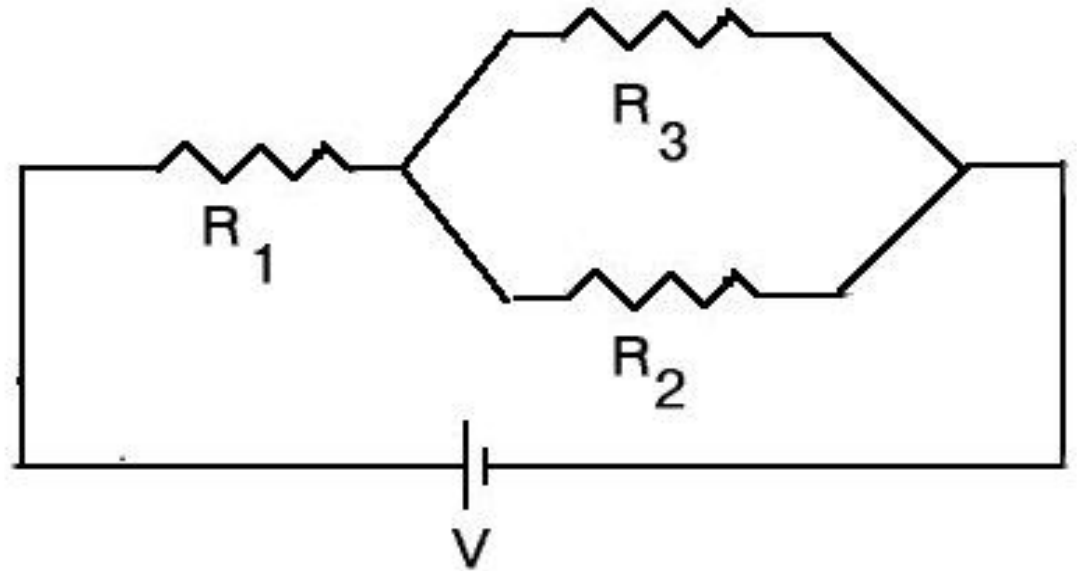
B)

C)

D) all=

**Given  $V=12\text{ V}$ ; for which values of  $R_1$ ,  $R_2$  and  $R_3$  is the total current from the battery largest?**

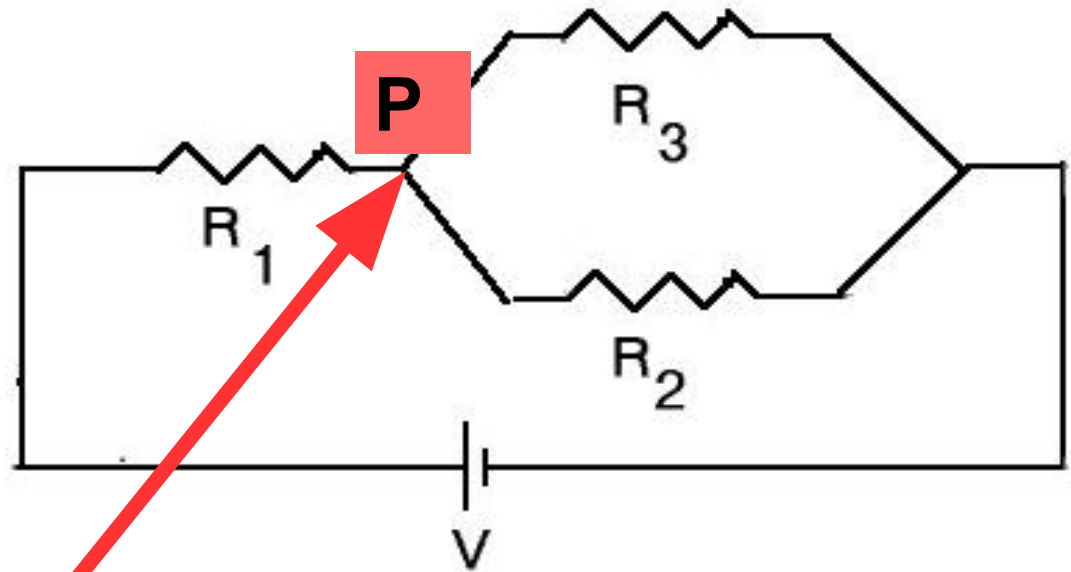
- A)  $4\Omega$ ,  $5\Omega$ ,  $6\Omega$
- B)  $6\Omega$ ,  $4\Omega$ ,  $4\Omega$
- C)  $3\Omega$ ,  $6\Omega$ ,  $6\Omega$
- D) Same for all cases



**In which of the 3 cases above is the voltage drop across  $R_1$  largest?**

Given  $V=12\text{ V}$ ;  $4\Omega=R_1$ ,  $3\Omega=R_2$  and  $6\Omega=R_3$ , the total current from the battery is:

- A)  $12/13\text{ A}$
- B)  $13/12\text{ A}$
- C)  $6\text{ A}$
- D)  $2\text{ A}$
- E) NOTA

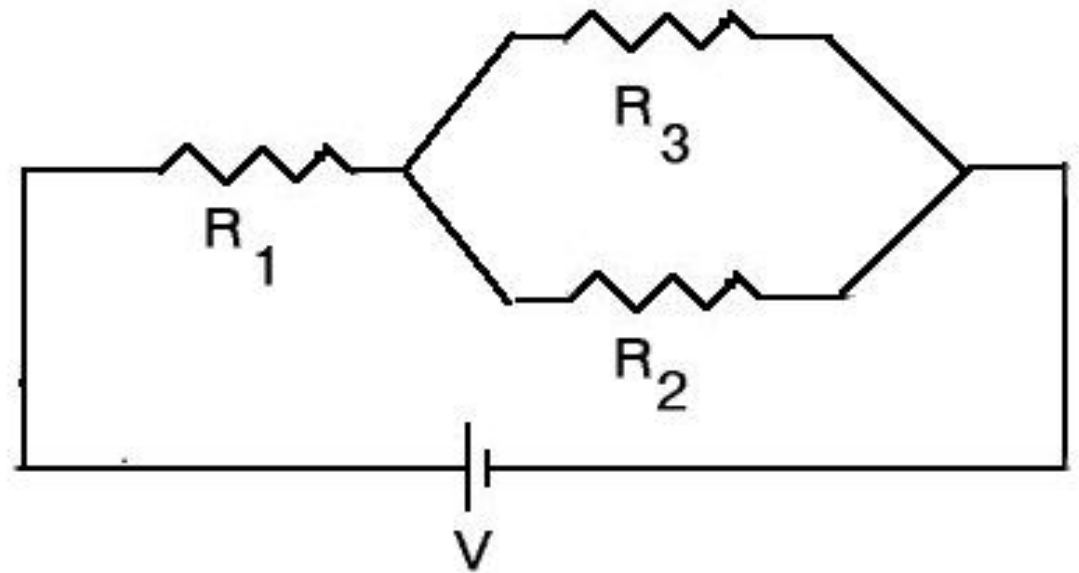


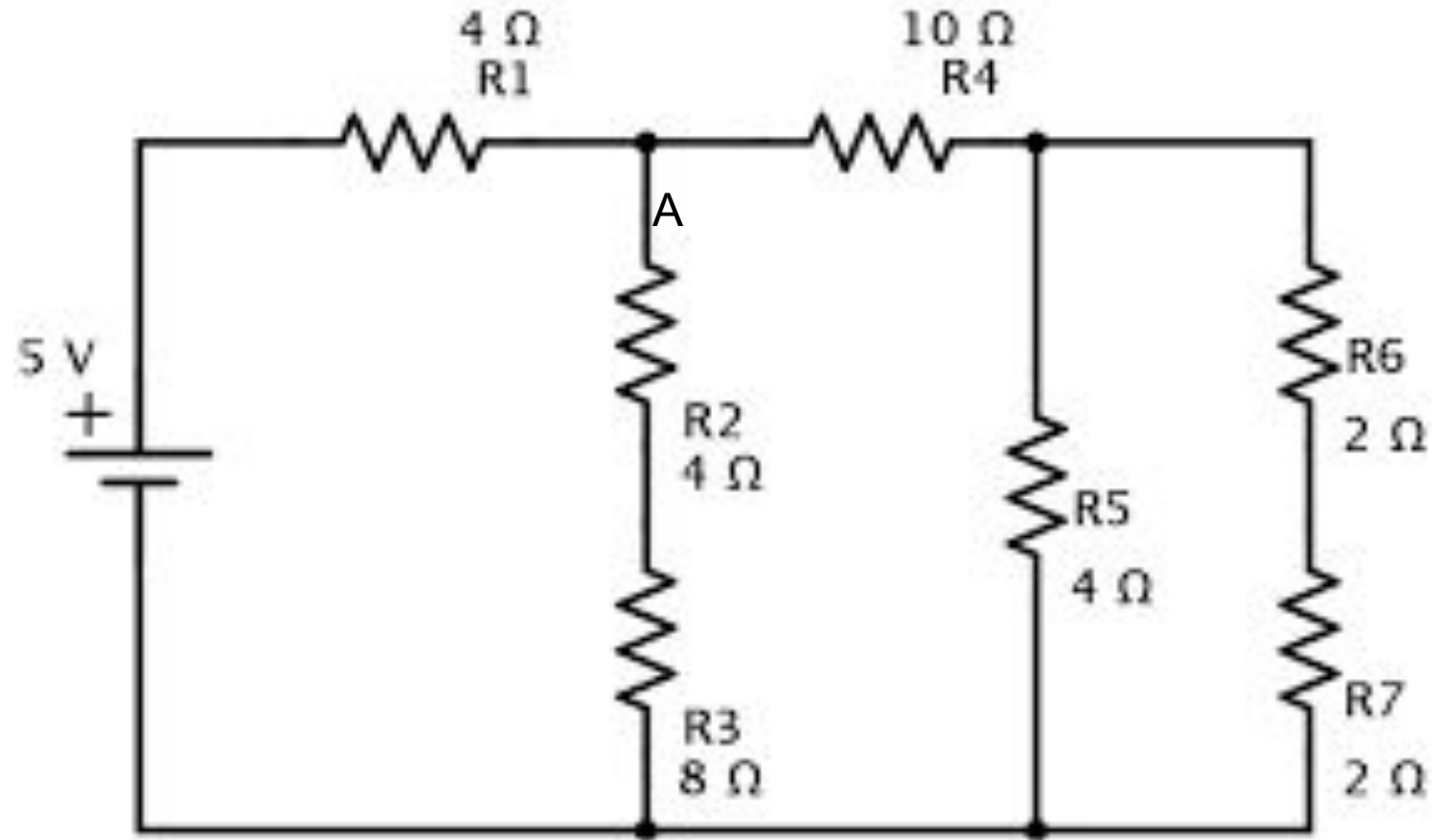
**The voltage at P is**

- A)  $12\text{ V}$ , B)  $8\text{ V}$  C)  $4\text{ V}$ , D)  $0\text{ V}$  E) NOTA

**Given  $V=12\text{ V}$ ;  $4\Omega=R_1$ ,  $3\Omega=R_2$  and  $6\Omega=R_3$ , the total amount of power dissipated as heat in the  $3\Omega$  and  $6\Omega$  resistors is, respectively:**

- A)  $5.33\text{ W}$ ,  
 $2.66\text{W}$
- B)  $2.66\text{W}$ ,  $5.33\text{W}$
- C)  $2.66\text{W}$ ,  $2.66\text{W}$
- D)  $5.33\text{W}$ ,  $5.33\text{W}$
- E) NOTA



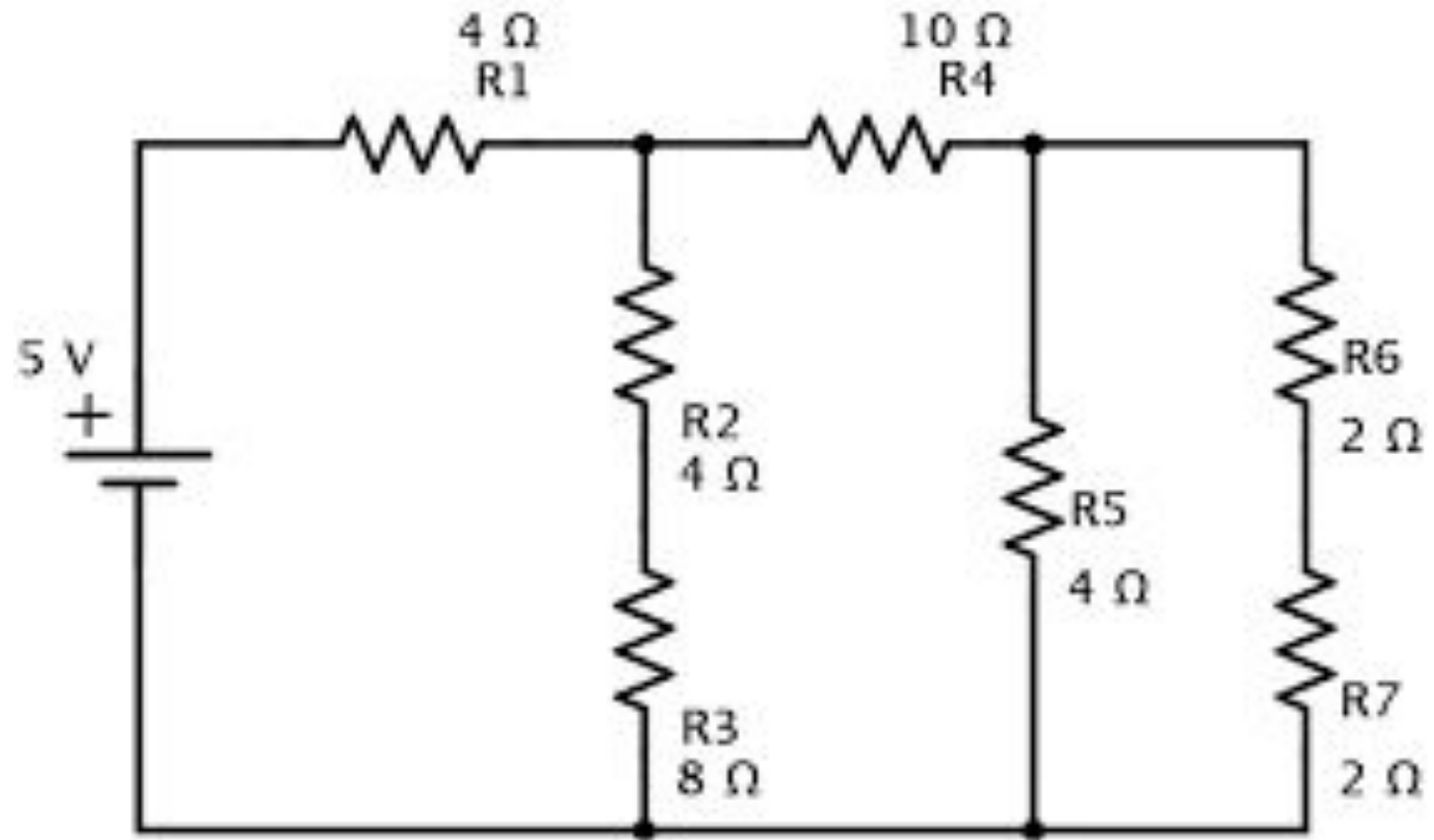


- A) 0.5 A
- B) 0.25 A
- C) 0.125 A
- D) 2 A
- E) NOTA

**The current  $I_{\text{tot}}$  is:**

**Remember to check your results for currents by verifying that the total voltage drop through a closed loop is equal to the voltage supplied by the battery!**

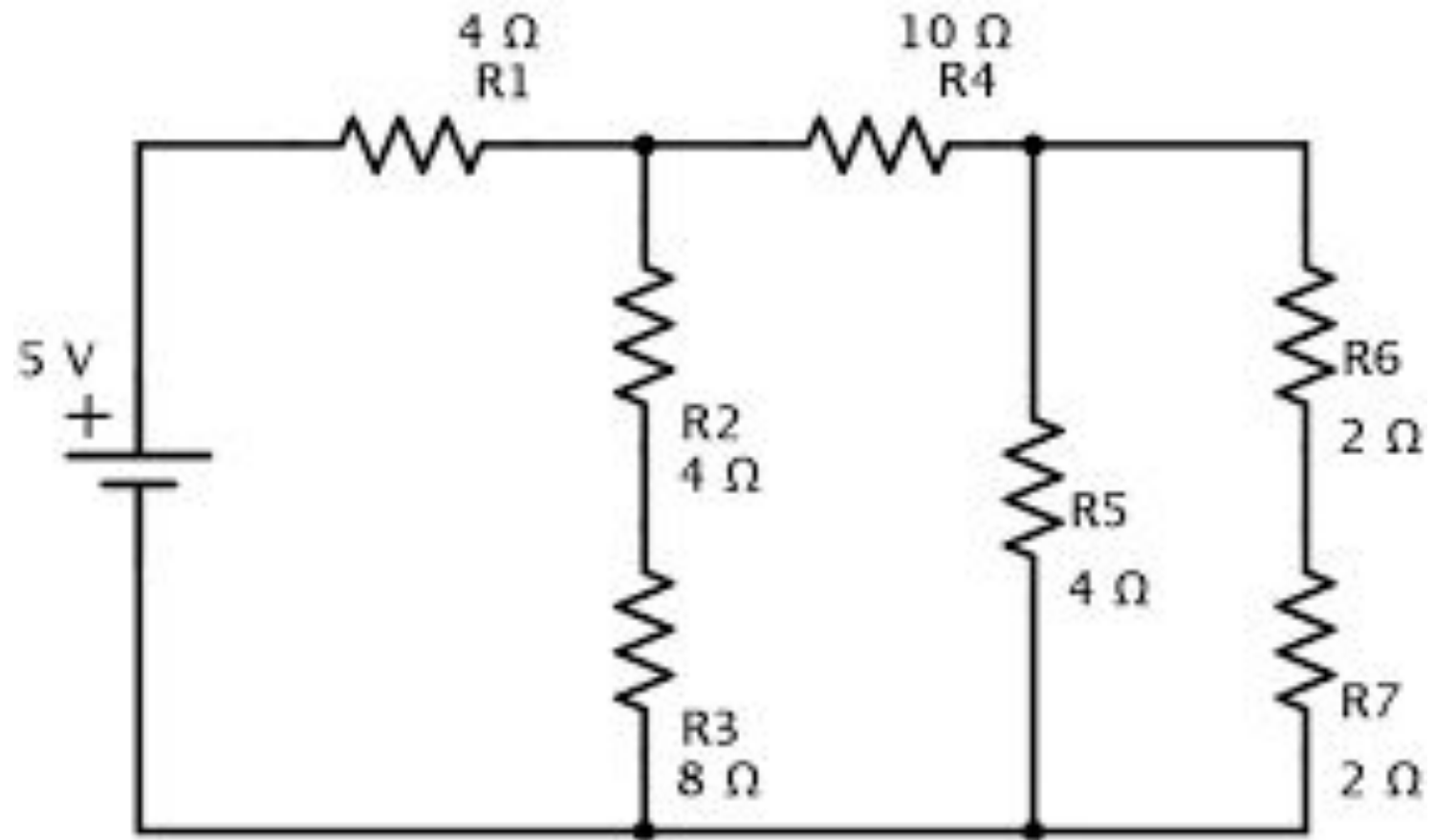
- A) 5 V
- B) 2.5 V
- C) 3 V
- D) 2 V
- E) NOTA



**The voltage drop across  $R_{23}$  is:**



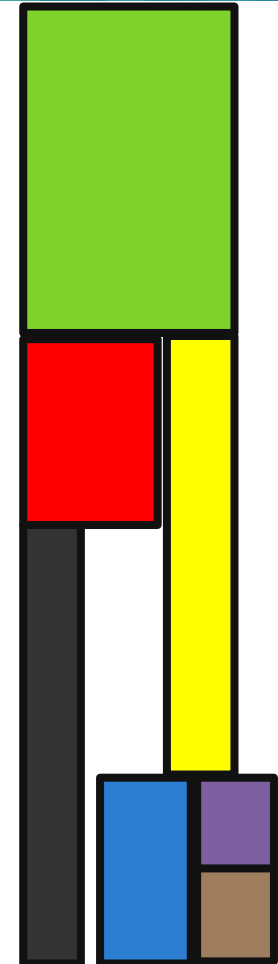
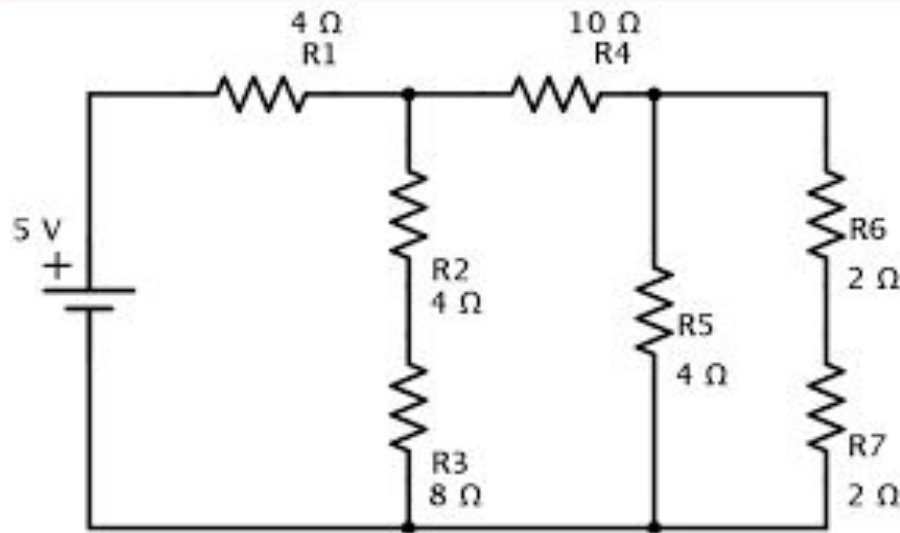
- A) 5V
- B) 0.25 V
- C) 0.125 V
- D) 2 V
- E) NOTA



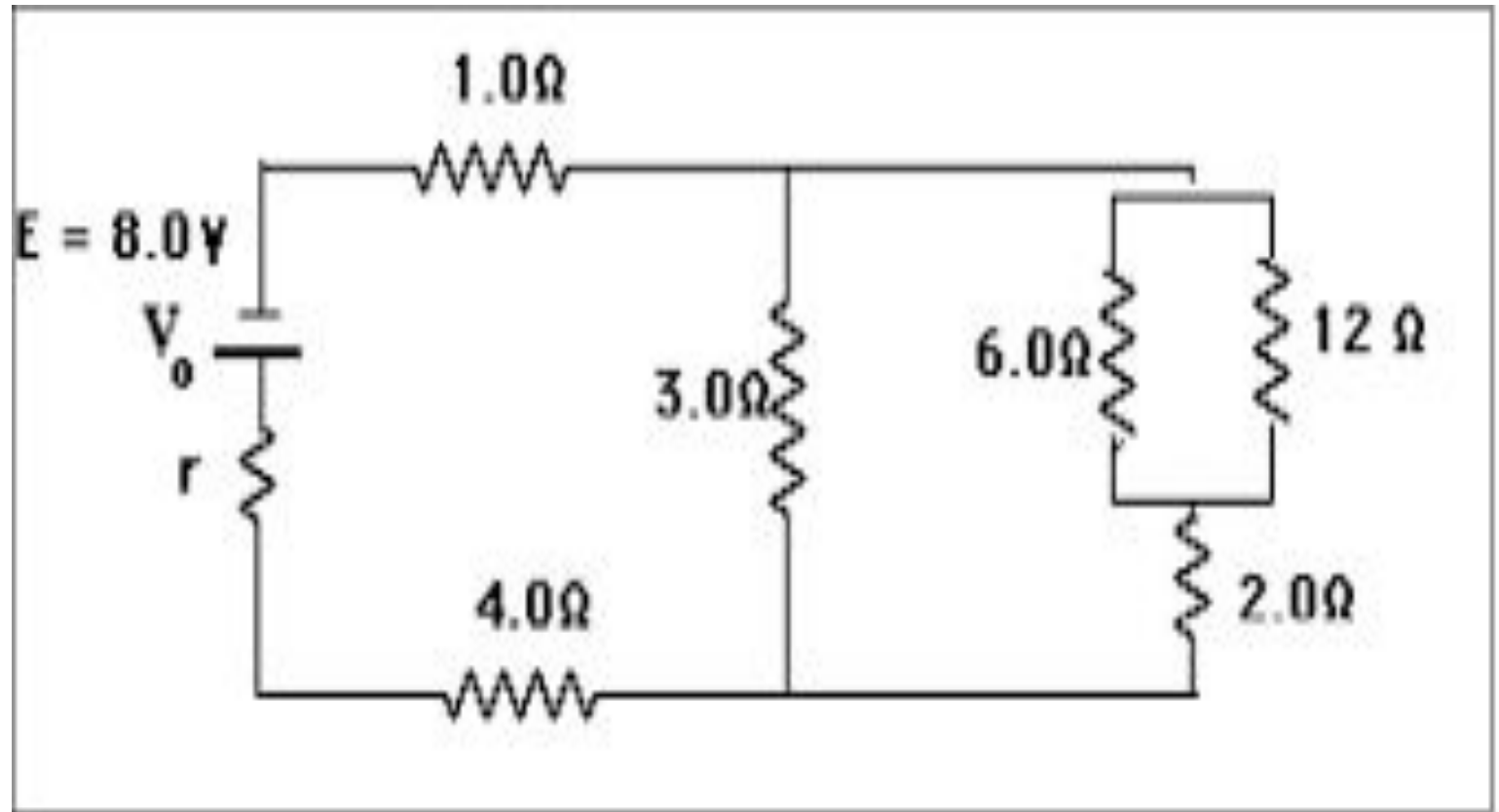
**The voltage drop across  $R_6$  is:**

What color are 'R2' and 'R5', respectively?

- A) Black, yellow
- B) Yellow, black
- C) Red, blue
- D) Green, yellow
- E) NOTA

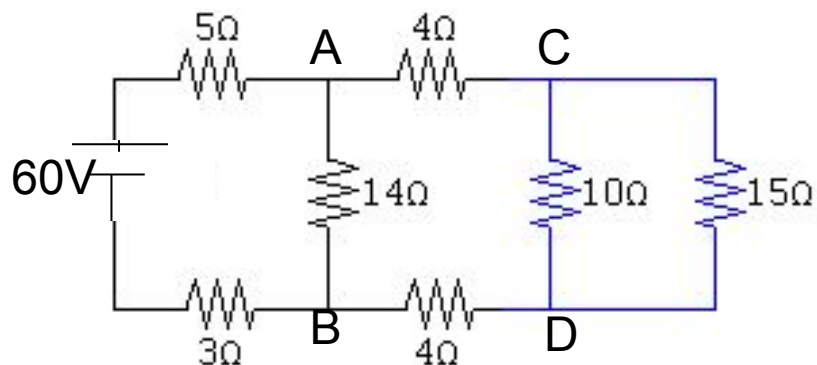


- A) 1V
- B) 2V
- C)  $\frac{2}{3}$  V
- D)  $\frac{4}{3}$  V
- E) NOTA



**If  $r=1\Omega$ ,  $\Delta V_{AB}$  across the  $3\Omega$  resistor=**

**Remember to check your results for currents by verifying that the total voltage drop through a closed loop is equal to the voltage supplied by the battery!**



$R_1=5\Omega$ ;  $R_2=3\Omega$ ;  $R_3=14\Omega$ ;  $R_4=4\Omega$ ;  $R_5=4\Omega$ ;  $R_6=10\Omega$ ;  $R_7=15\Omega$ .

$R_6 \parallel R_7 \Rightarrow R_{67}=6\Omega$ ;  $R_4, R_5, R_{67}$  all series  $\Rightarrow R_{4567}=14\Omega$ ; in  $\parallel$  w/  $R_3 \Rightarrow R_{34567}=7\Omega$ ;  $R_{\text{tot}}=15\Omega$   
 $\Rightarrow I_{\text{tot}}=60\text{V}/15\Omega=4\text{A}$

Now determine currents and voltages through all individual resistors:

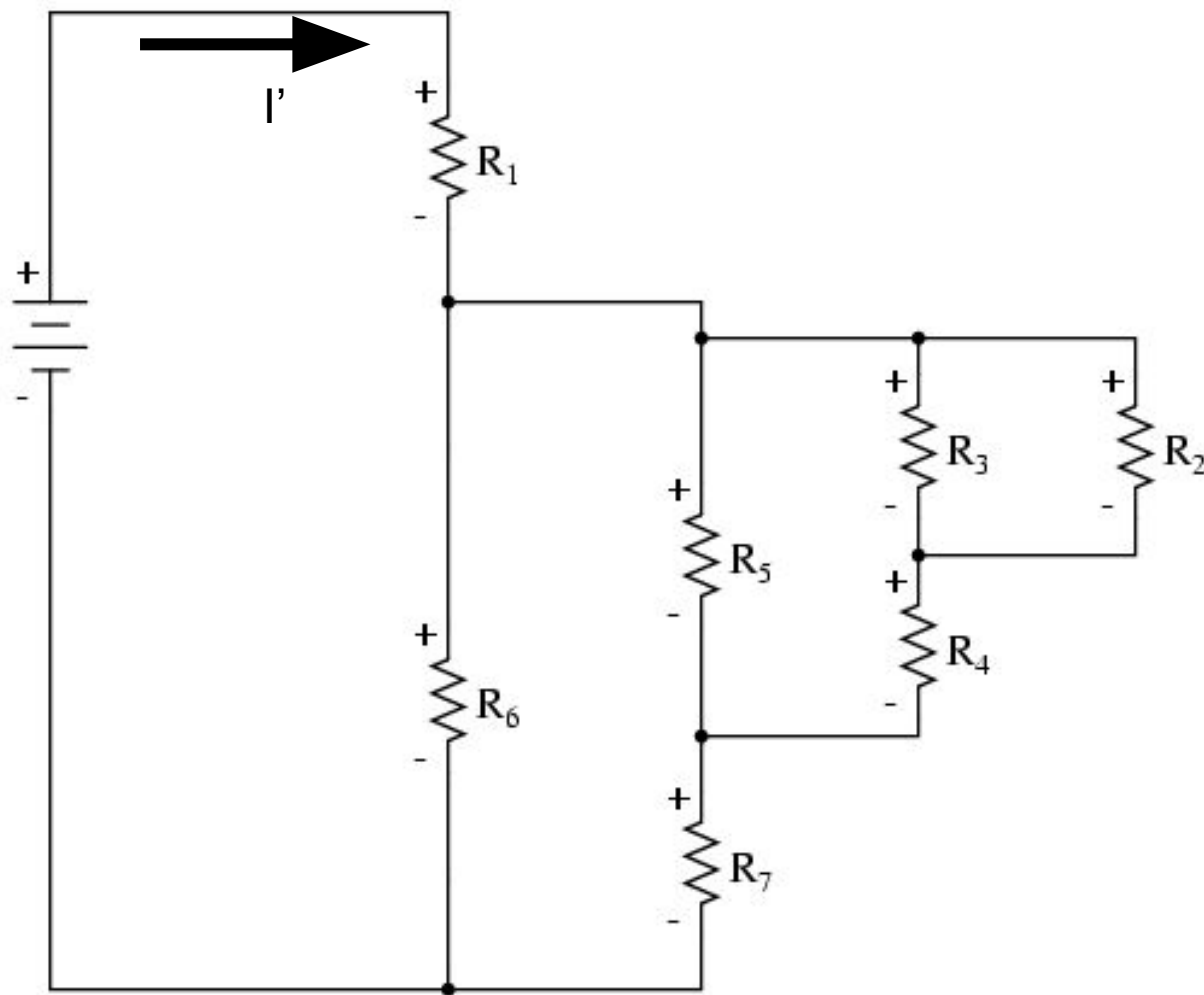
$V_1=20\text{V}$ ;  $V_2=12\text{V}$ , so  $V(\text{A})=60\text{V}-20\text{V}=40\text{V}$ ;  $V(\text{B})=0\text{V}+12\text{V}=12\text{V} \Rightarrow V_3=40\text{V}-12\text{V}=28\text{V}$   
 $\Rightarrow I_3=28\text{V}/14\Omega=2\text{A}$ .

If 4A flows into pt. A (and also out of pt. B), then current flowing  $\text{A} \rightarrow \text{C} = 4\text{A} - 2\text{A} = 2\text{A} = I_4 = I_5$   
 So  $V(\text{C}) = 40\text{V} - 2\text{A} \cdot 4\Omega = 32\text{V}$ ;  $V(\text{D}) = 12\text{V} + 8\text{V} = 20\text{V}$ , so  $V_6 = 32\text{V} - 20\text{V} = 12\text{V} = V_7$  (since  $\parallel$ ), so  $I_6 = 1.2\text{A}$ ;  
 $I_7 = 0.8\text{A}$  (check that  $I_6 + I_7 = 2.0\text{A}$ , as expected).

As mentioned in class, the current through each resistor literally makes each resistor hot, so for  $R_1$ , e.g., the power dissipated  $= I^2 R = 4^2 \cdot 5 = 80\text{ Watts}$

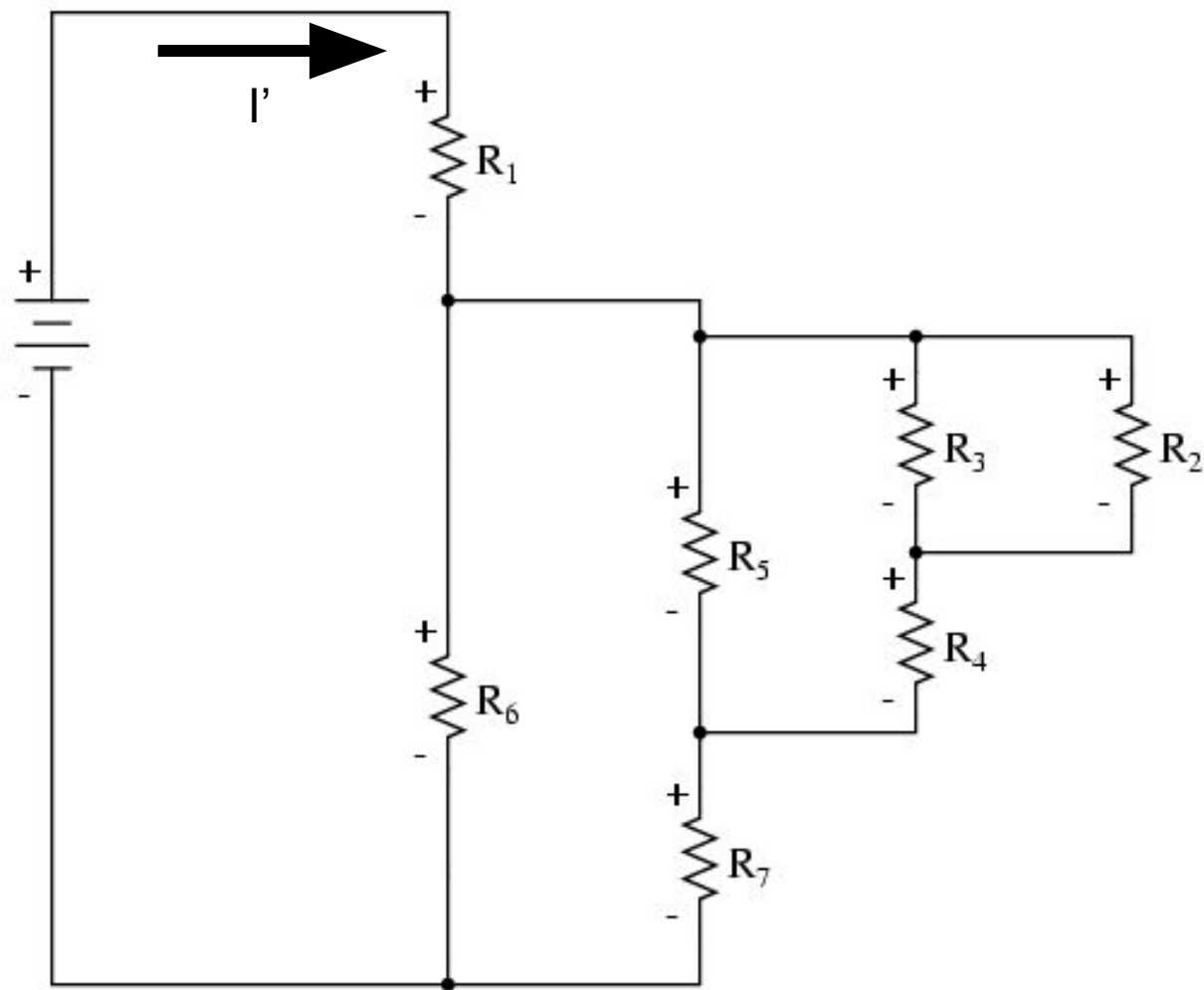
$R_2=6\Omega$ ,  $R_3=3\Omega$ ,  $R_4=4\Omega$ ,  $R_5=12\Omega$ ,  $R_7=2\Omega$ ,  $R_6=6\Omega$ ,  
 $R_1=3\Omega$ ,  $V_{\text{battery}}=24\text{V}$ .  $\Delta V_6$  and  $I'=?$

- A) 8V, 1A
- B) 12V, 2 A
- C) 16V, 2A
- D) 16V, 1A
- E) NOTA



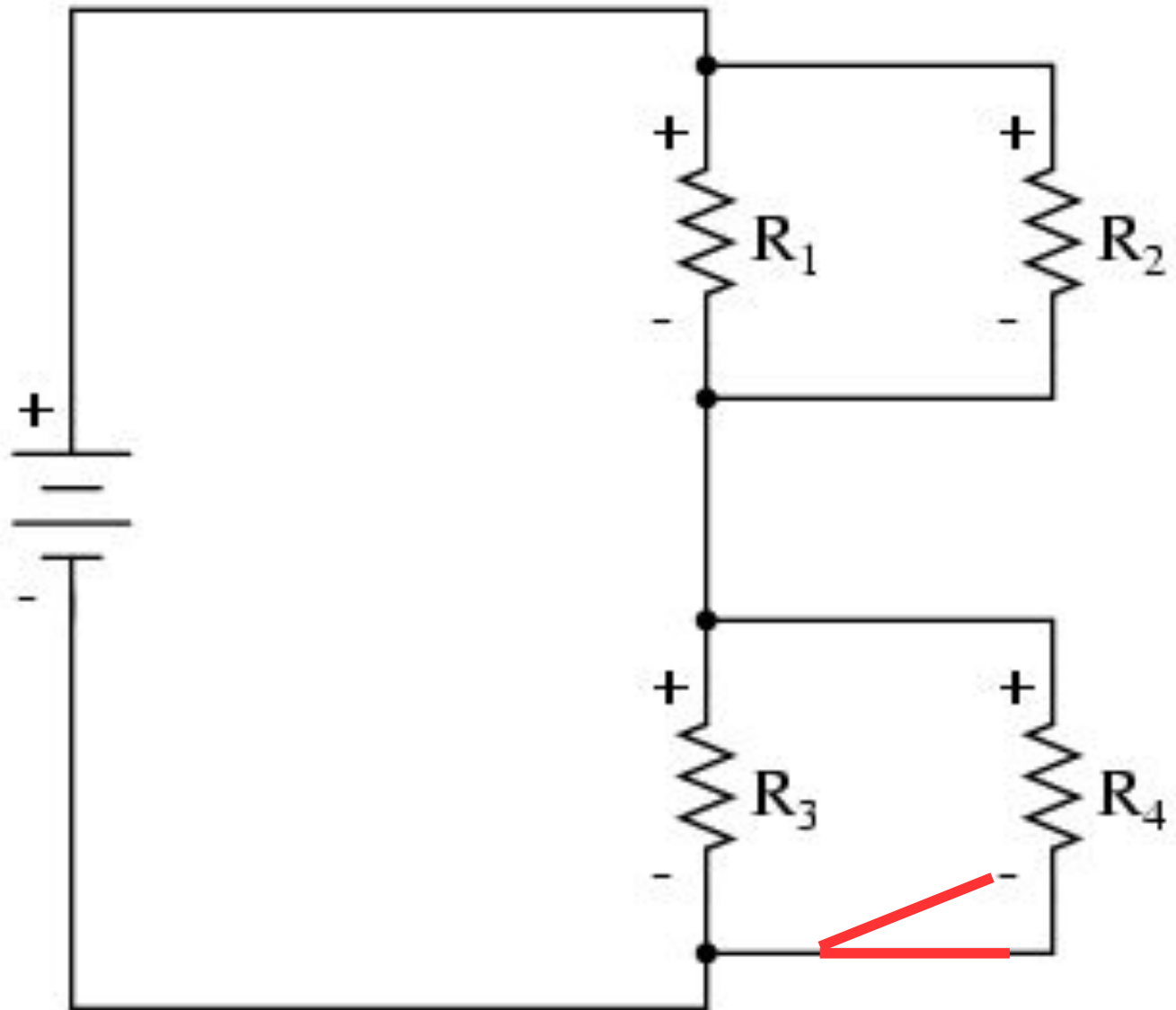
$R_2=6\Omega$ ,  $R_3=3\Omega$ ,  $R_4=4\Omega$ ,  $R_5=12\Omega$ ,  $R_7=2\Omega$ ,  $R_6=6\Omega$ ,  
 $R_1=3\Omega$ ,  $V_{\text{battery}}=24\text{V}$ .  $\Delta V_7$  and  $I_5=?$

- A) 4V, 2/3 A
- B) 8V, 2/3 A
- C) 12V, 2A
- D) 16V, 1A
- E) NOTA



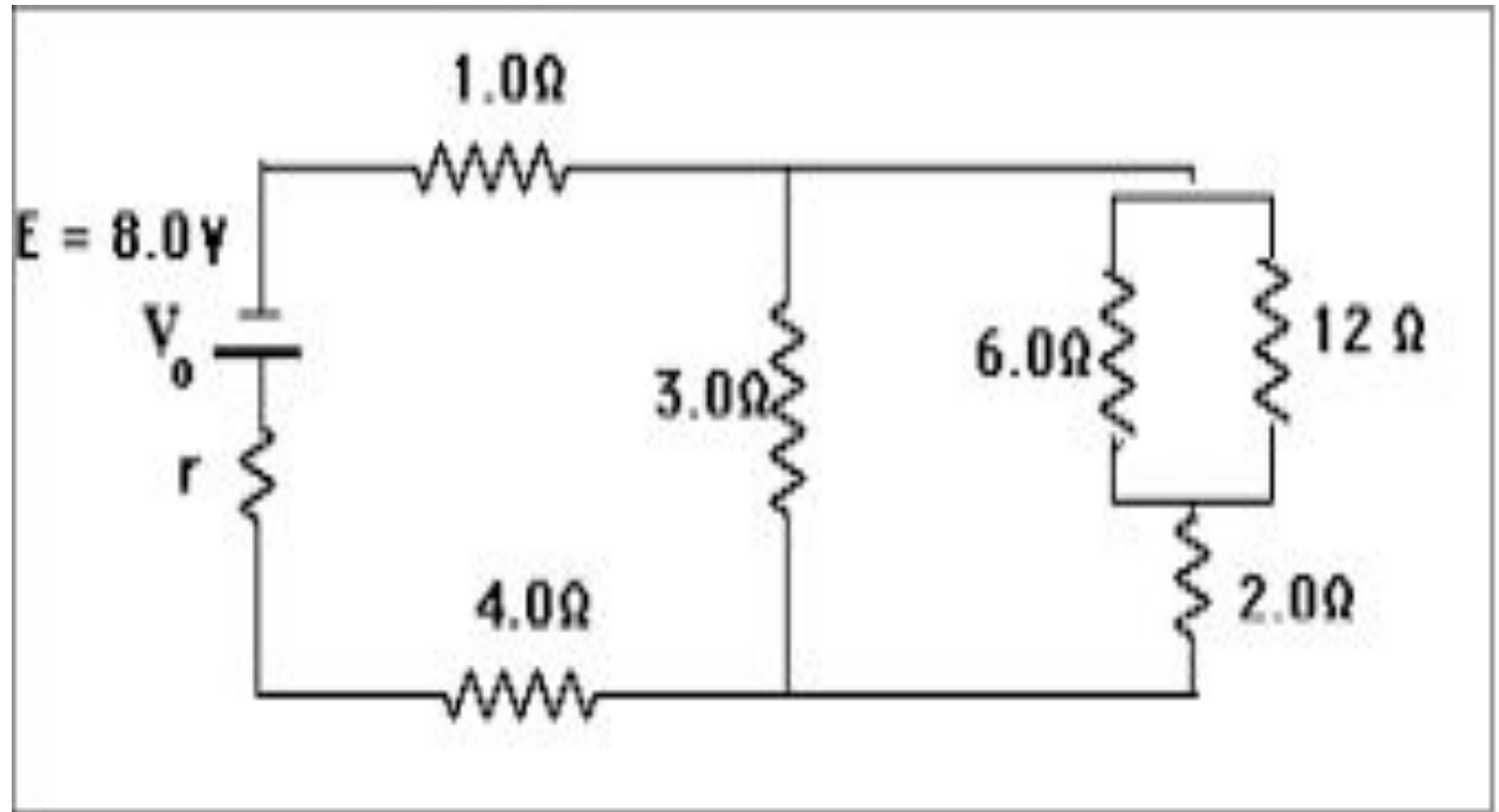
4 light bulbs each have  $R=6\Omega$ . When the switch  $S$  is opened, which bulbs burn brighter ( $V=144\text{V}$ )?

- A)  $R_1, R_2, R_3$
- B)  $R_1$  &  $R_2$  only
- C)  $R_3$  only
- D) All burn with the same brightness
- E) NOTA



**If  $r=1\Omega$ ,  $\Delta V$  across the  $2\Omega$  resistor=**

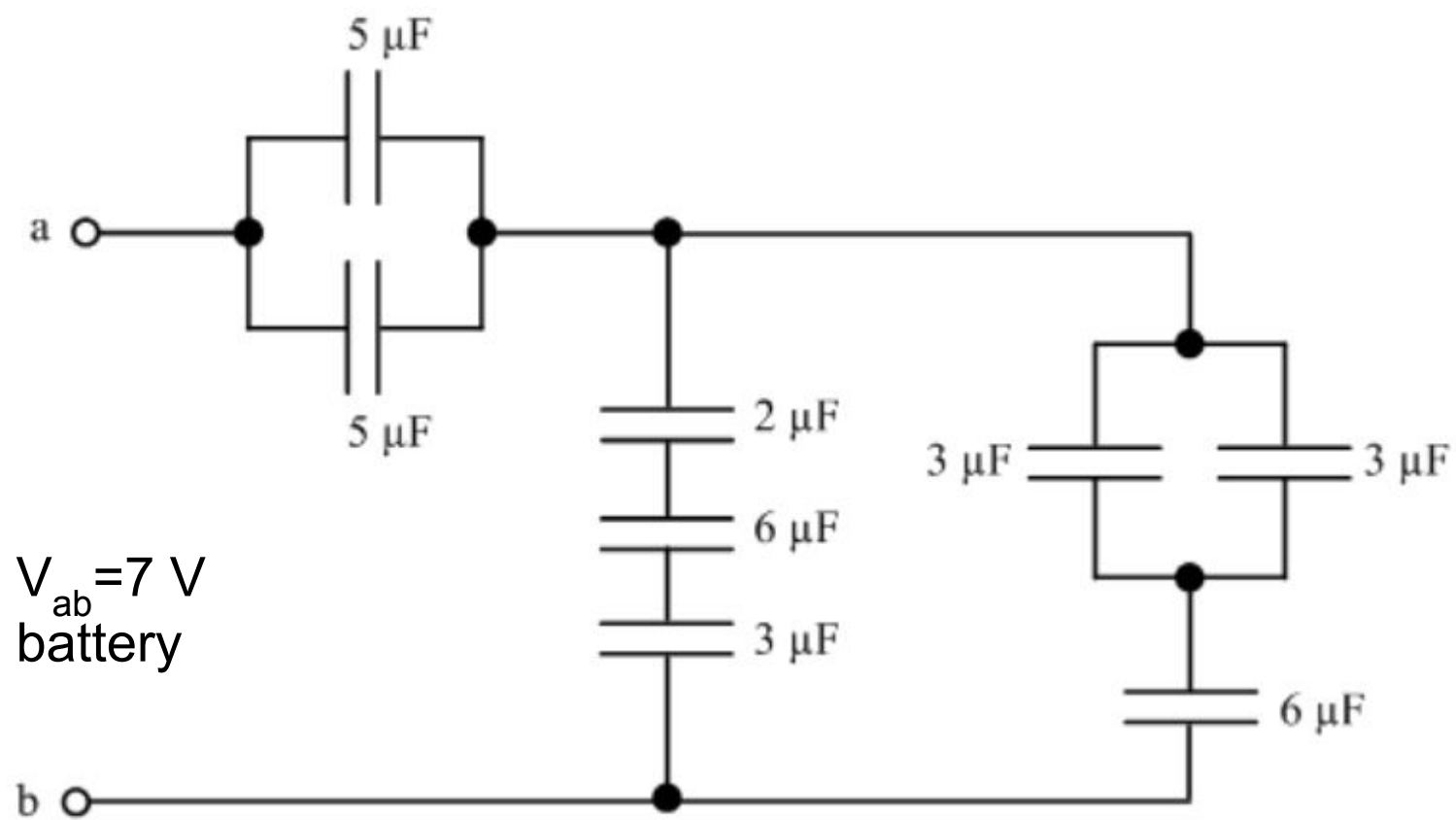
- A) 1V
- B) 2V
- C)  $\frac{2}{3}$  V
- D)  $\frac{4}{3}$  V
- E) NOTA

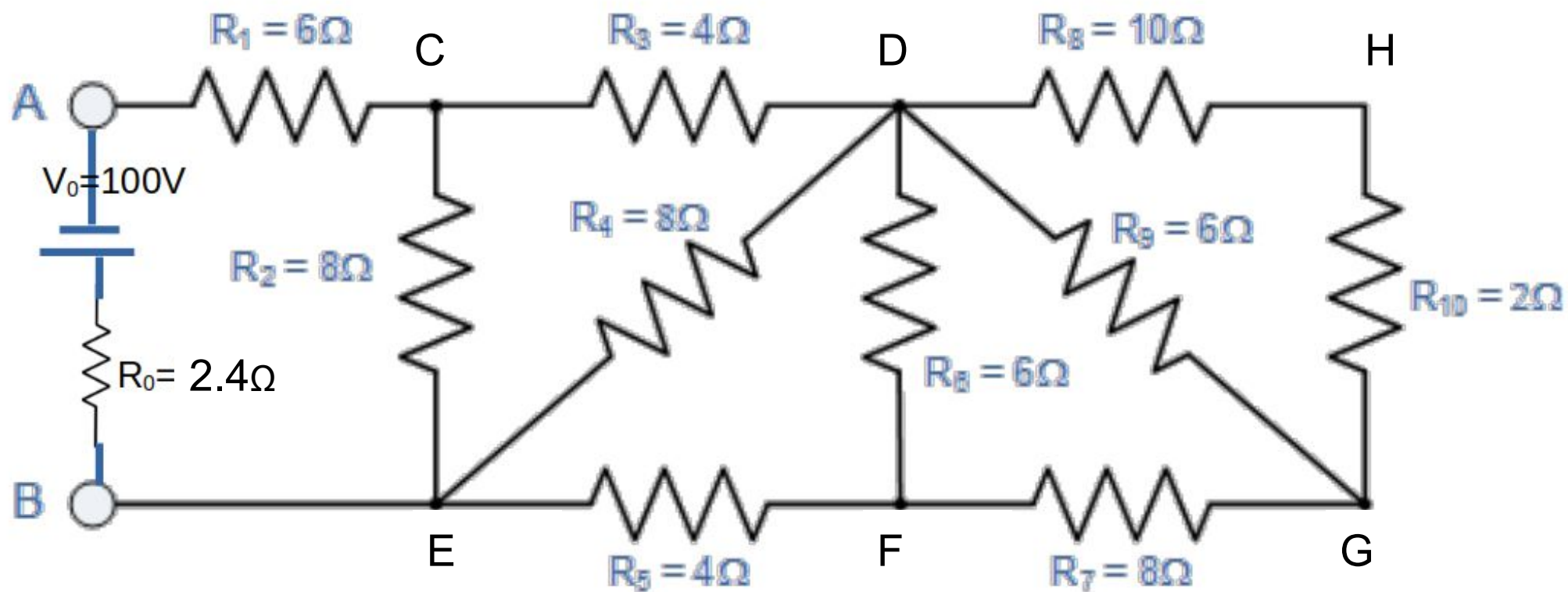


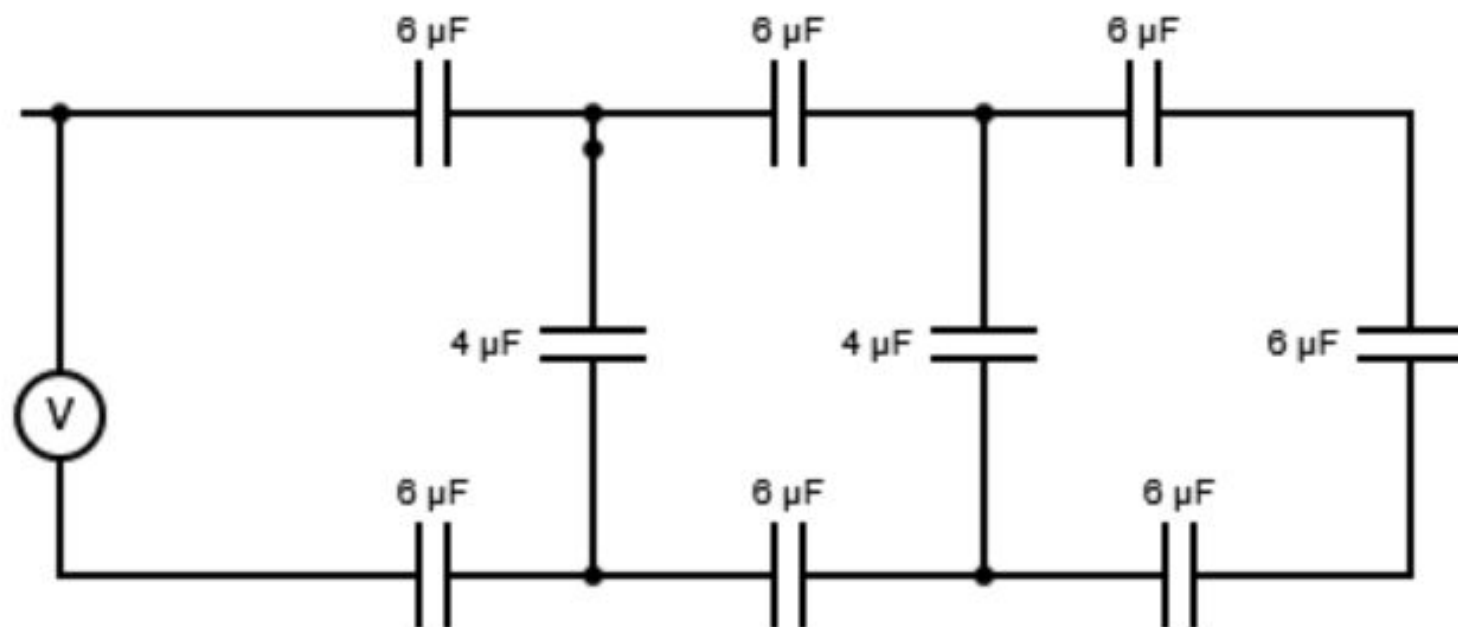
**In the circuit shown, the number of individual currents would be:**

- A) 1      B) 2      C) 3      D) 4      E) 5



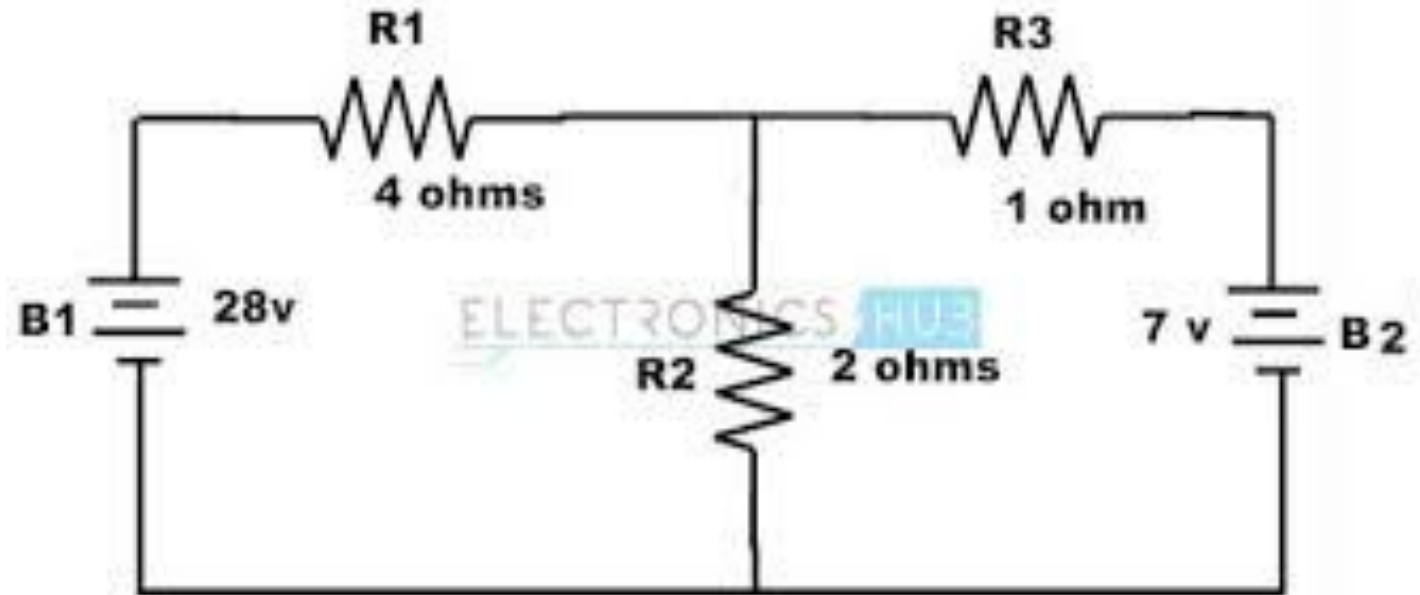


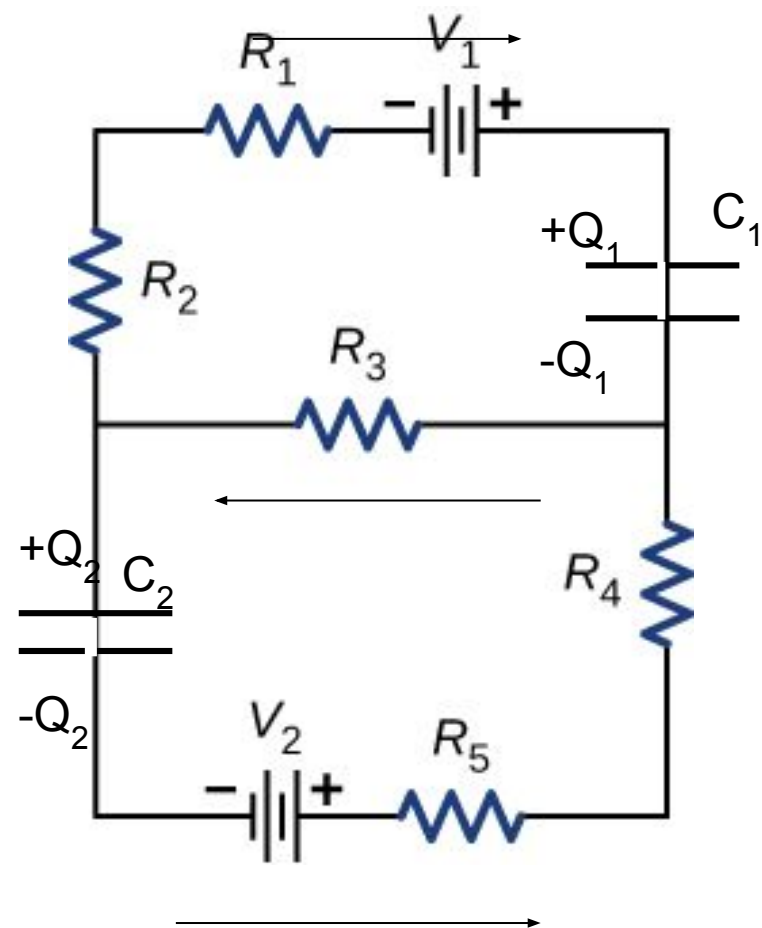




**How many currents are needed to fully describe the circuit below?**

- A) 1
- B) 2
- C) 3
- D) 4
- E) NOTA





1. Guess current flow direction in all branches

=>  $\rightarrow$  through  $R_1$  and  $R_5$ ; reverse for  $R_3$

2. Based on current flow direction, assign +/- sides for Caps  
=> top of  $C_1$  positive; bottom of  $C_2$  negative

3. Write Kirchhoff's Laws for any loop.

=> top loop, traced clockwise, starting at - terminal of  $V_1$ :

$$+V_1 - Q_1/C_1 - I_3 R_3 - I_1 R_2 - I_1 R_1 = 0$$

Bottom loop, traced counter-clockwise,

$$+V_2 - I_5 R_5 - I_3 R_3 - I_5 R_4 - Q_2/C_2 = 0$$

Outer loop, traced clockwise, starting at  $V_1$ :

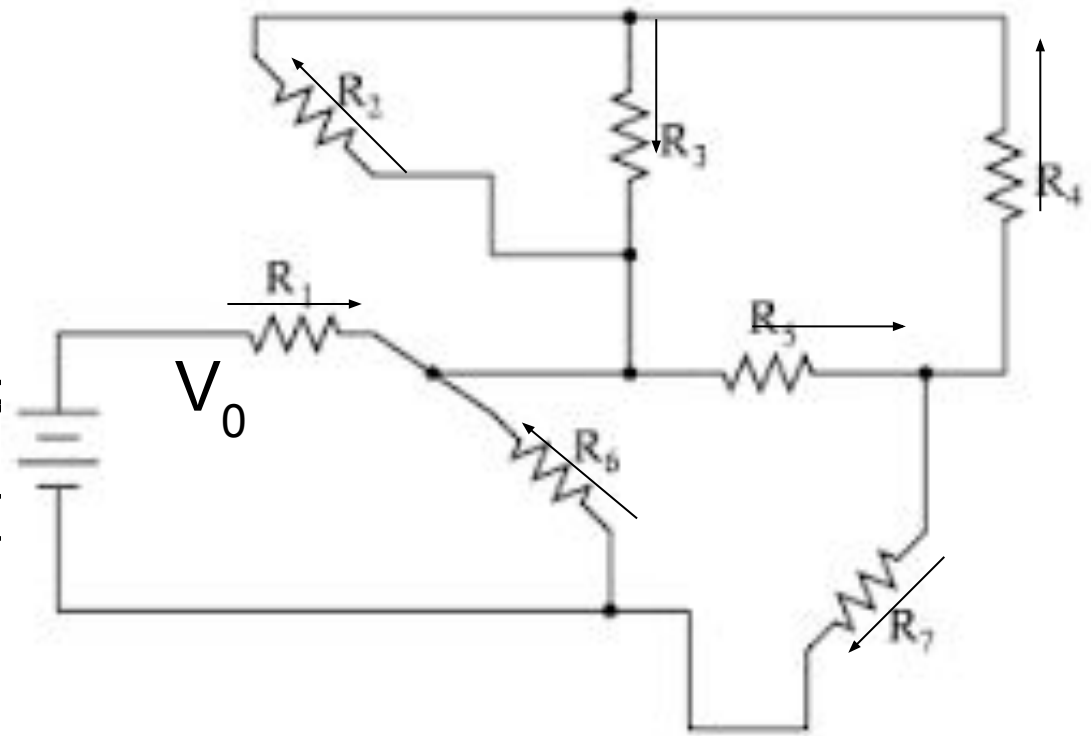
$$0 = +V_1 - Q_1/C_1 + I_5 R_4 + I_5 R_5 - V_2 + Q_2/C_2 - I_1 R_2 - I_1 R_1$$

NOTE: positive sign before  $I_5 R_4$ ,  $I_5 R_5$  and  $Q_2/C_2$  since tracing lower potential to higher potential; negative sign in front of  $V_2$  since tracing from +  $\rightarrow$  - terminal of battery.

Current equations:  $I_1 = I_2$  (series) and  $I_4 = I_5$  (series);  $I_1 + I_4 = I_3$

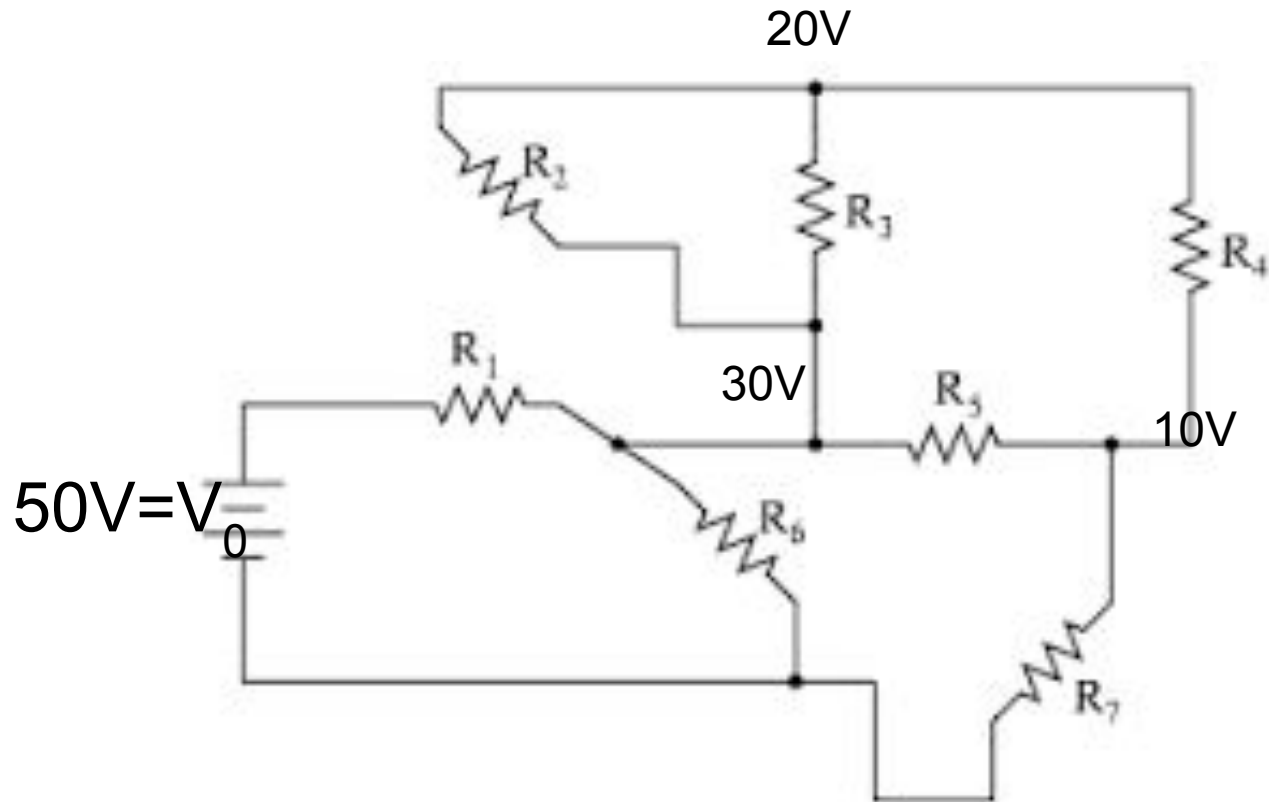
**Assuming currents shown, which correctly states conservation of voltage?**

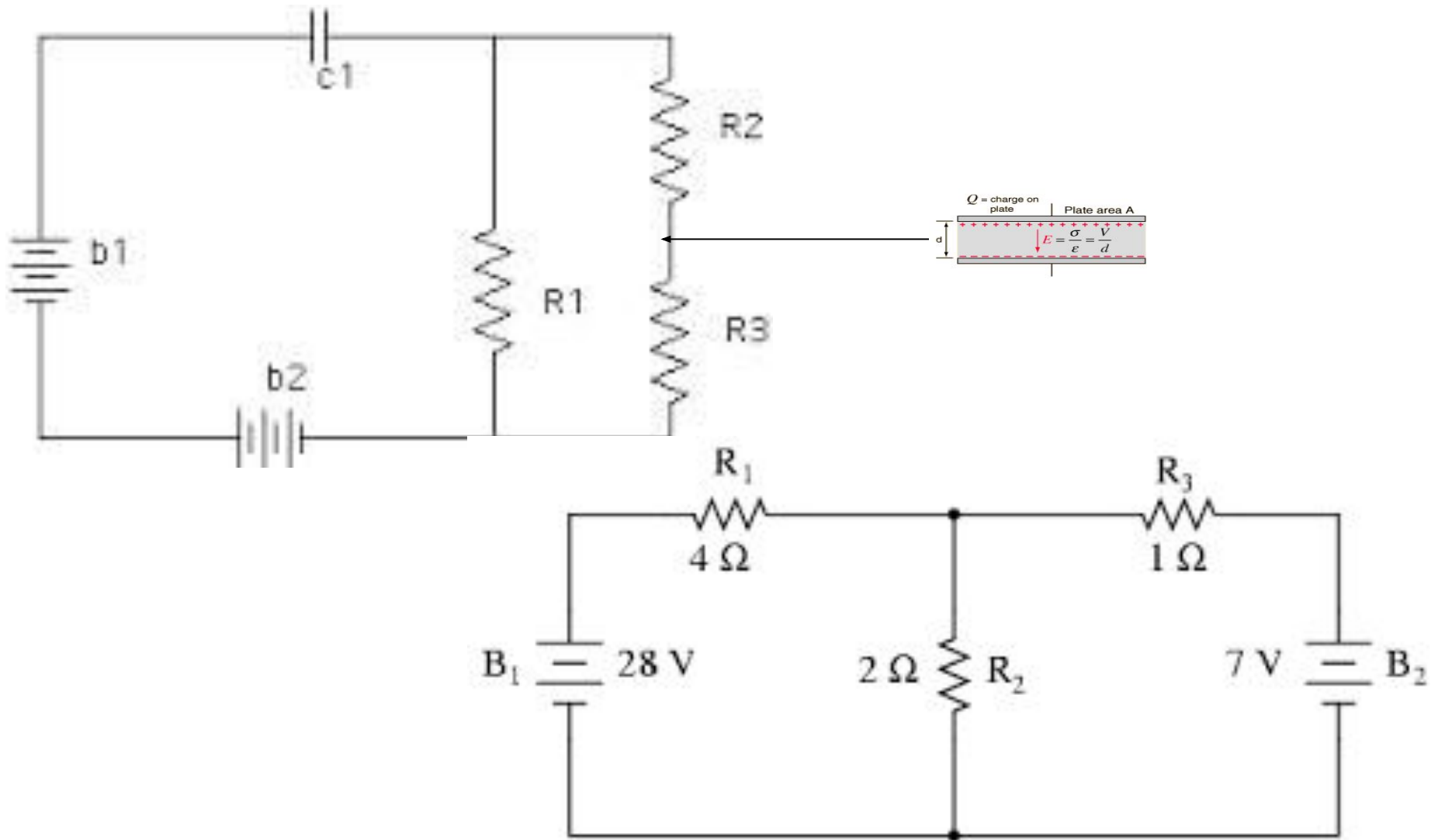
- A)  $V_0 - I_1 R_1 - I_6 R_6 = 0$
- B)  $V_0 - I_1 R_1 + I_3 R_3 + I_4 R_4 + I_7 R_7 = 0$
- C)  $V_0 + I_1 R_1 + I_3 R_3 + I_4 R_4 + I_7 R_7 - I_5 F = 0$
- D)  $-V_0 + I_7 R_7 - I_4 R_4 - I_3 R_3 + I_1 R_1 = 0$
- E) NOTA



**You measure the voltages at the indicated points, from which you can guess:**

- A)  $I_4$  flows in the +y direction
- B)  $I_7$  flows SW (↙)
- C)  $I_5$  flows in the +x direction
- D) Two of the above
- E) NOTA



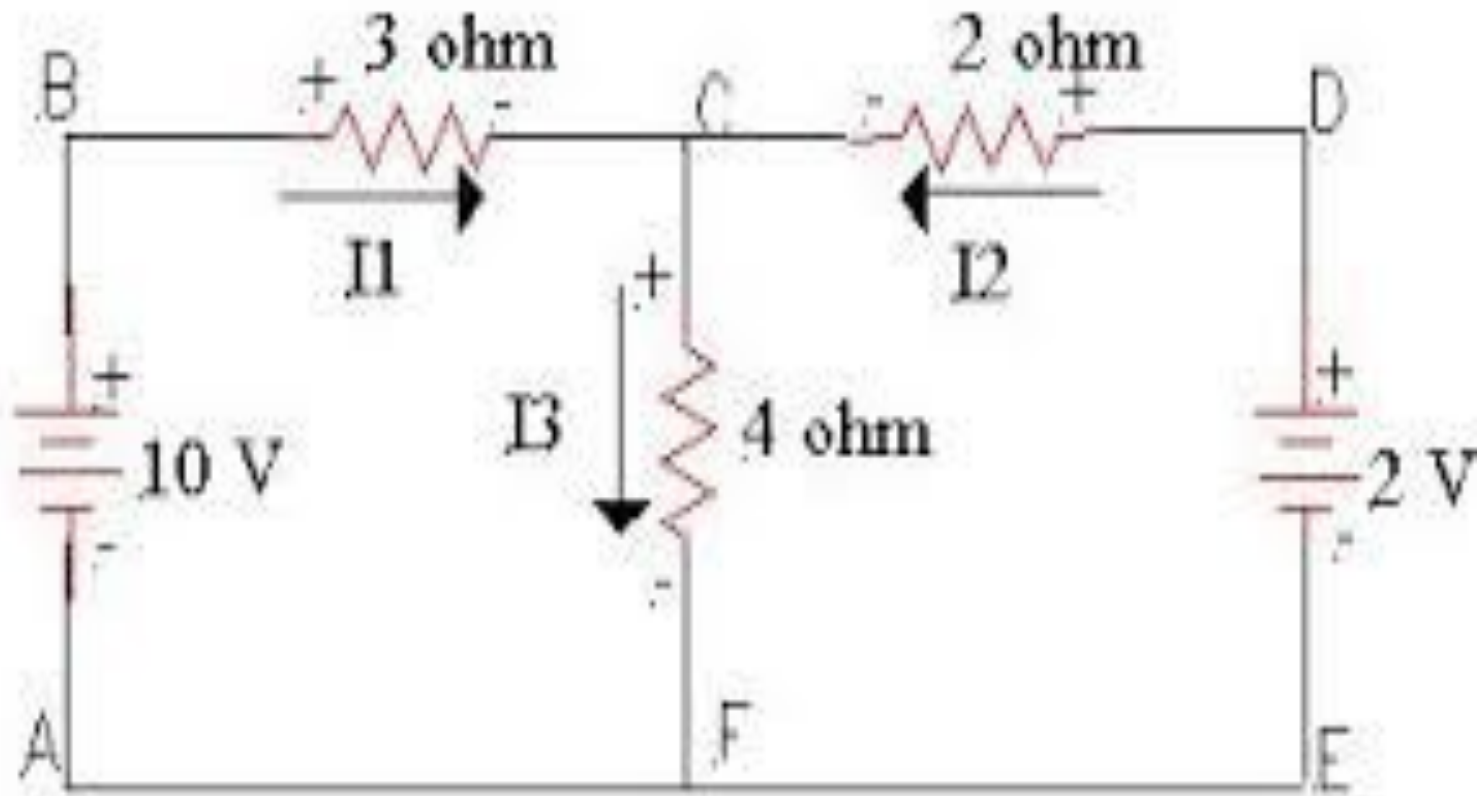


# Multi-battery/capacitor circuits



**Solve for  $I_1$  using conservation of current/voltage;  $I_1 =$**

- A) 1A
- B) 2A
- C) 3A
- D) 4A
- E) NOTA



**The number of currents you would write to define this circuit is:**

- A) 1
- B) 2
- C) 3
- D) 4
- E) NOTA

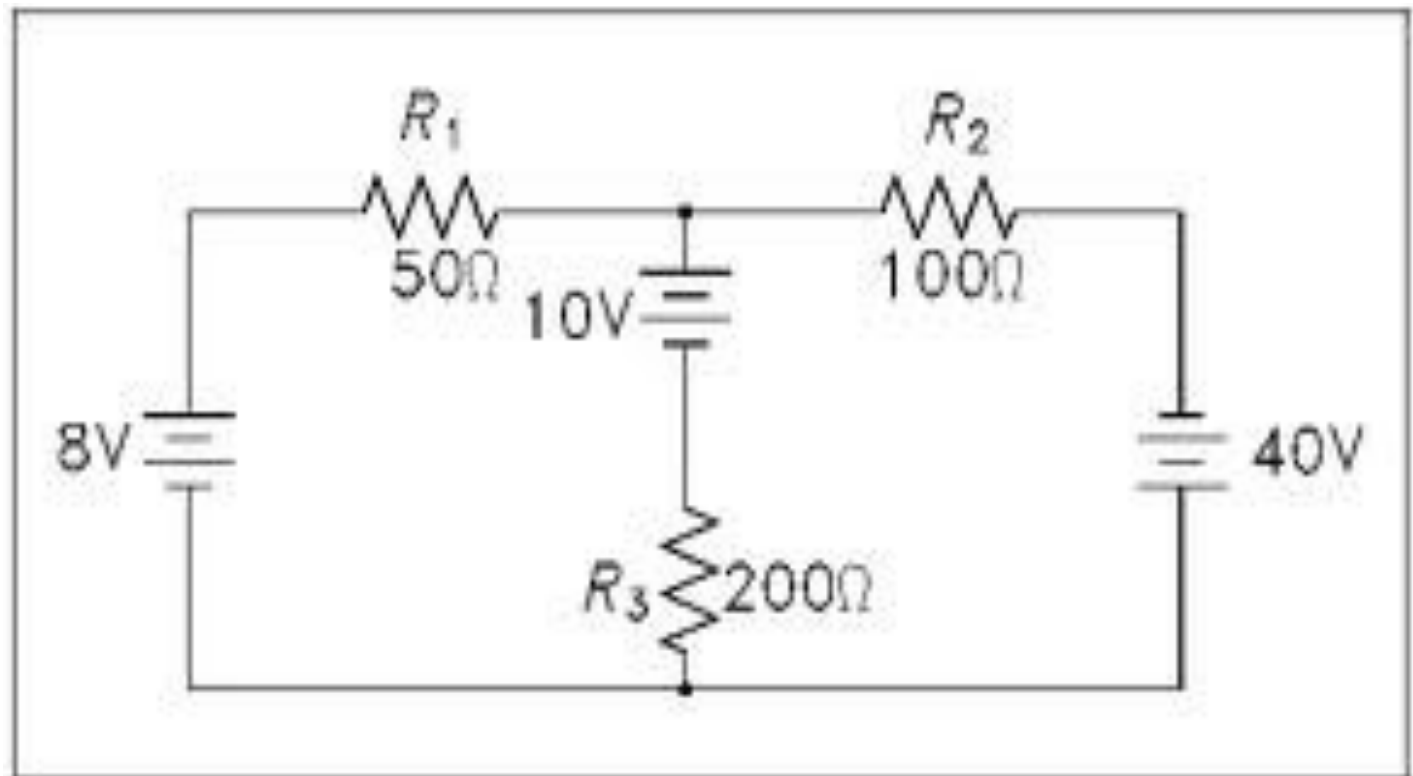
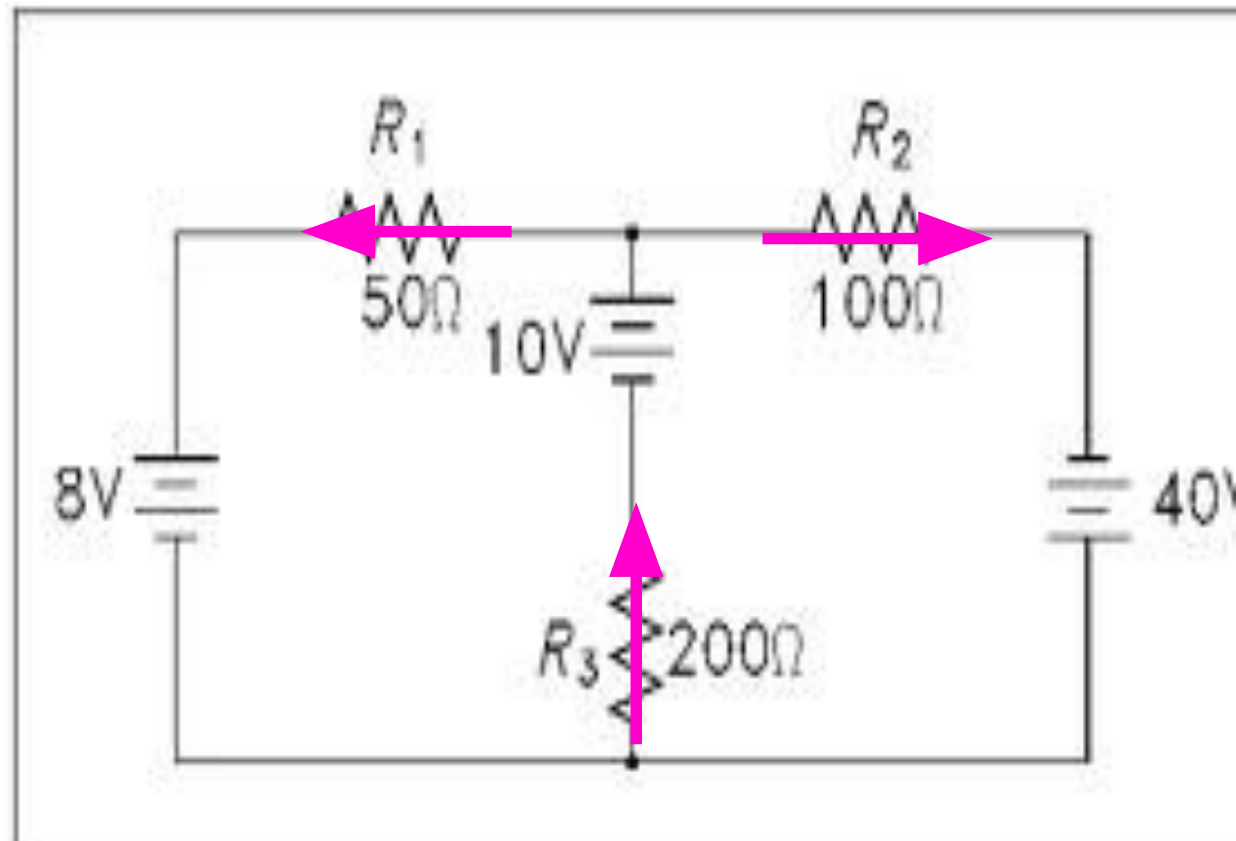


Figure 37 Example Circuit for Loop Equations

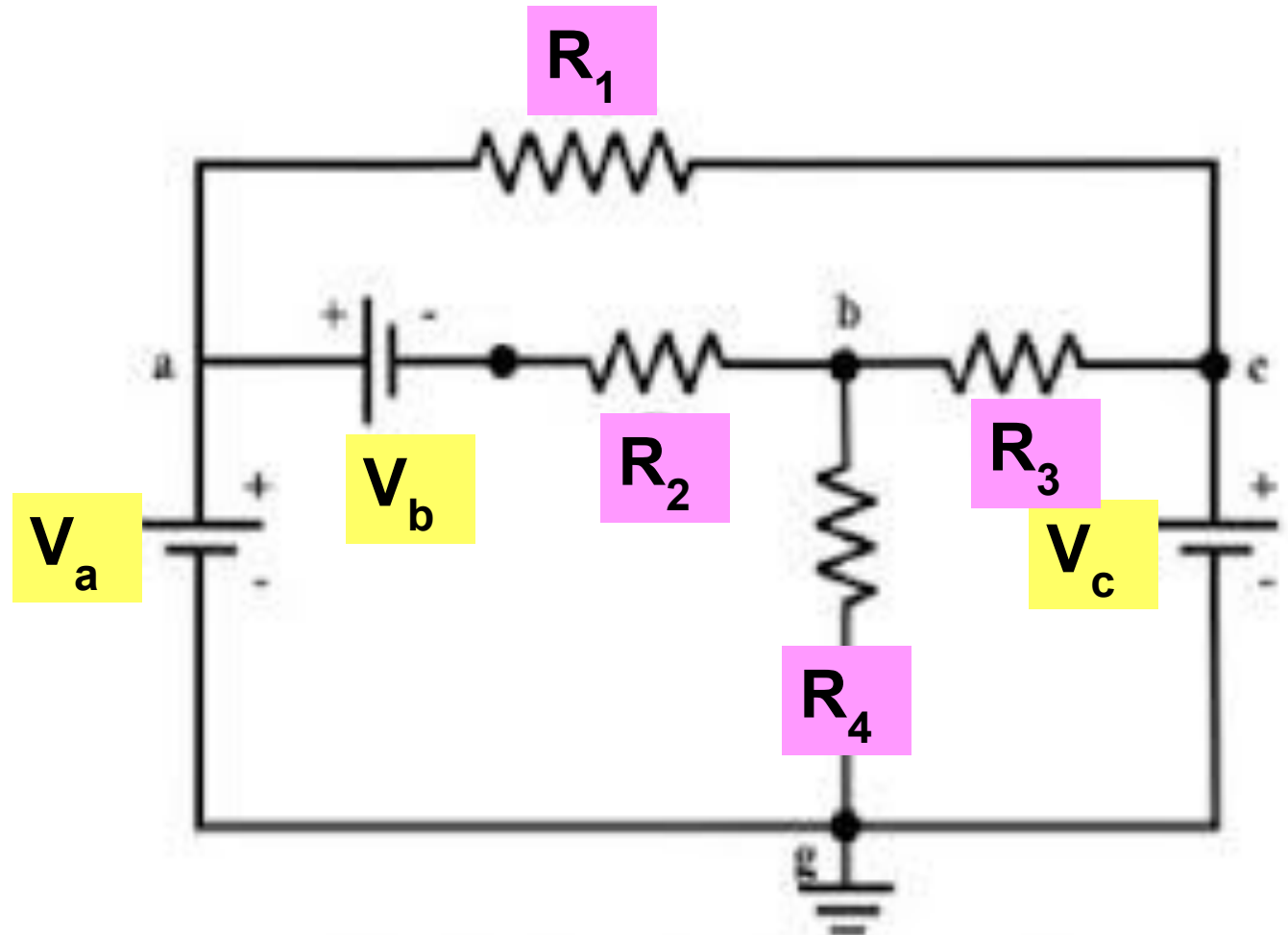
- A)  $200I_3 + 10V - 50I_1 - 8V = 0$
- B)  $-40V + 100I_2 - 10V + 200I_3 = 0$
- C)  $-8V - 200I_3 - 10V - 50I_1 = 0$
- D)  $40V + 8V + 50I_1 + 100I_2 = 0$
- E) NOTA

Which eqn. above expresses voltage conservation?

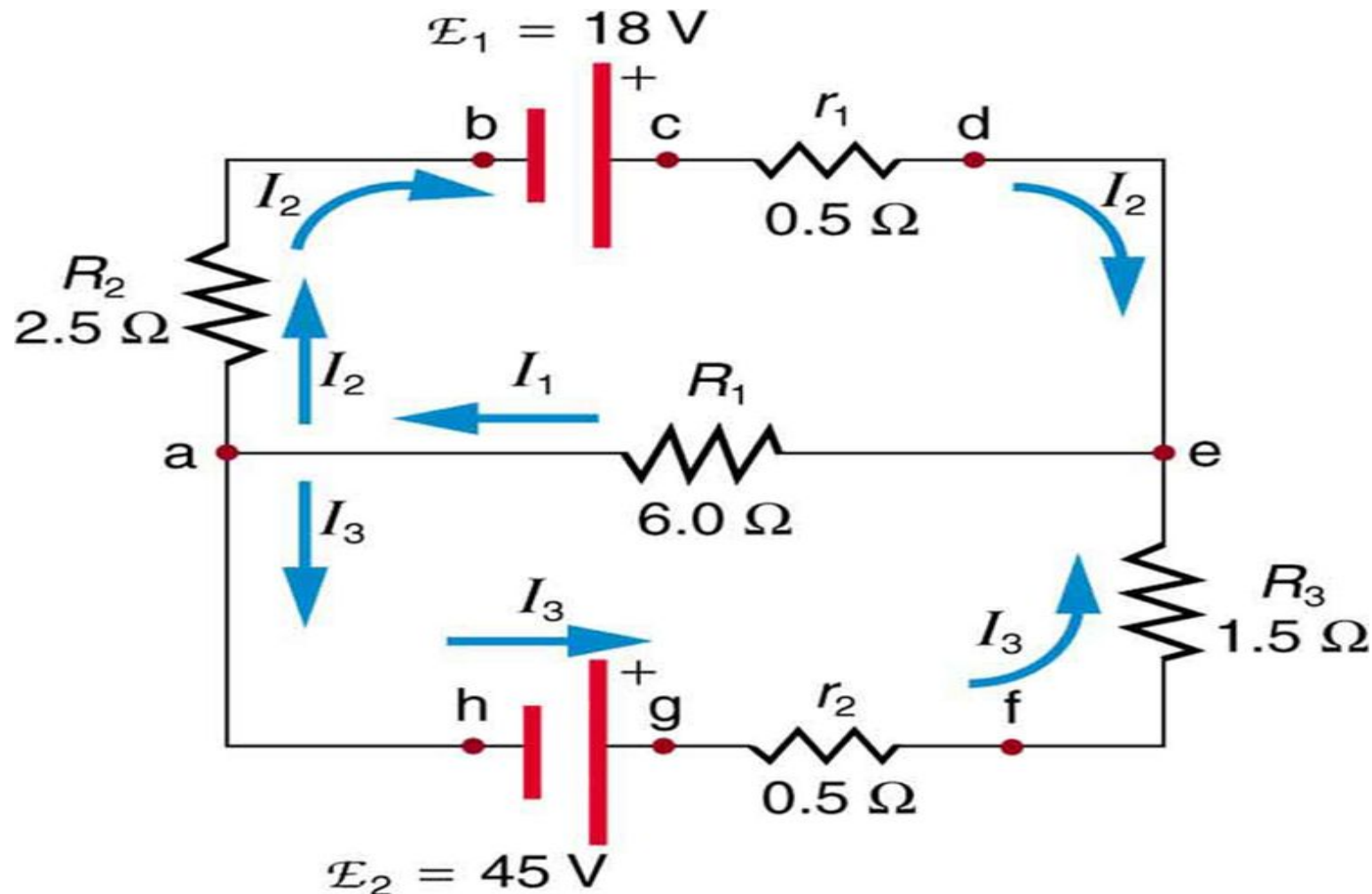


The total number of loops for which you can write conservation of voltage is:

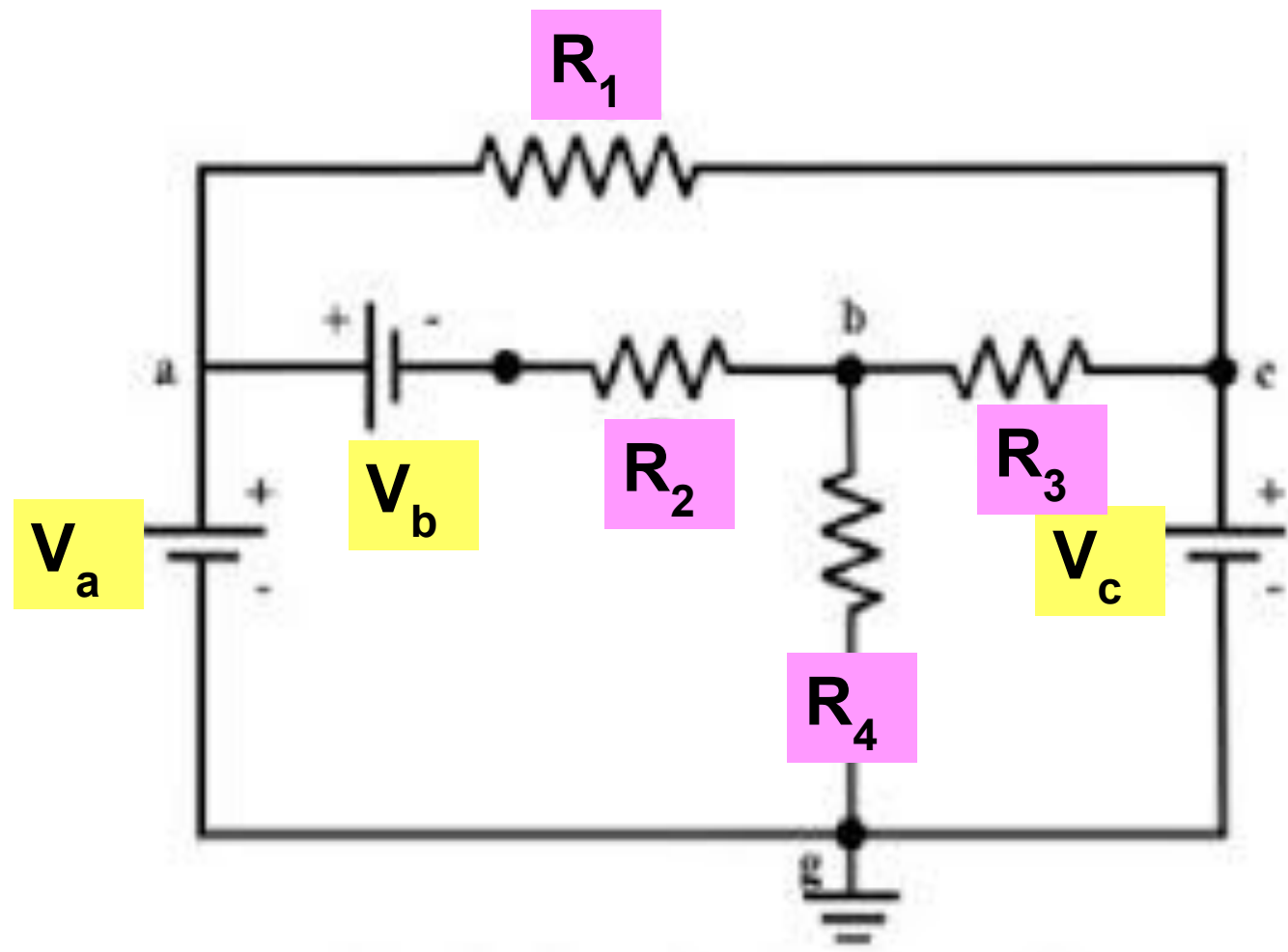
- A) 3
- B) 5
- C) 6
- D) 7
- E) NOTA



**Write Kirchhoff's conservation of voltage and current laws for loop abcdefgh & junctions a and e**

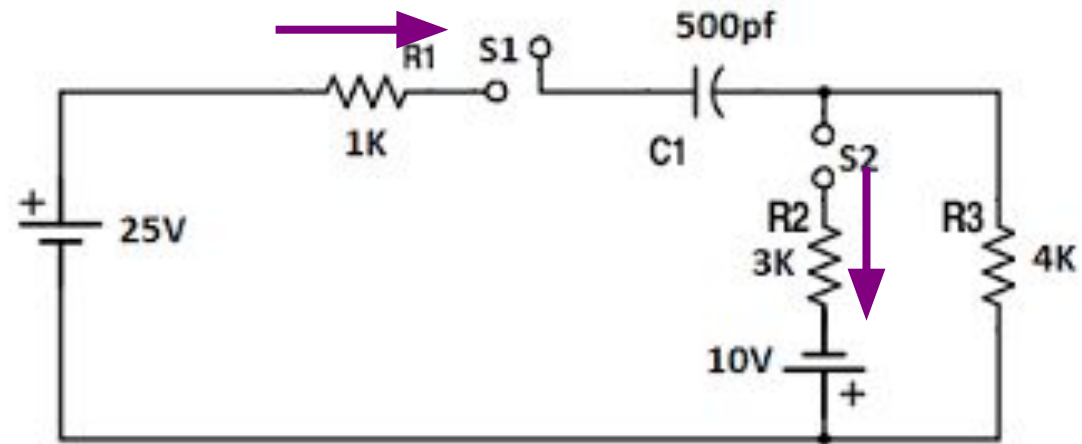


**Circuits with resistors and capacitors: two limits:  $t=0 \Rightarrow$  branch containing the capacitor is a 'straight wire';  $t \rightarrow \infty \Rightarrow$  a) no current flows into branch containing capacitor, b) the capacitor charges to the point where the voltage across the capacitor is equal and opposite to the voltage being 'pushed' from the battery into that branch (think of what happens with water flowing through parallel pipes if one of the pipes clogs). In that limit, we can ignore the branch containing the capacitor in calculating  $R_{\text{tot}}$ .**



**Assuming currents shown, which statement expresses conservation of voltage after the switches are closed?**

- A)  $25V - I_1 R_1 - Q_1 / C_1 - I_2 R_2 - 10V = 0$
- B)  $-25V - 10V + I_2 R_2 + Q_1 / C_1 + I_1 R_1 =$
- C)  $25V - I_1 R_1 + Q_1 / C_1 - I_2 R_2 + 10V = 0$
- D)  $25V + I_1 R_1 - Q_1 / C_1 - I_2 R_2 - 10V = 0$
- E) NOTA



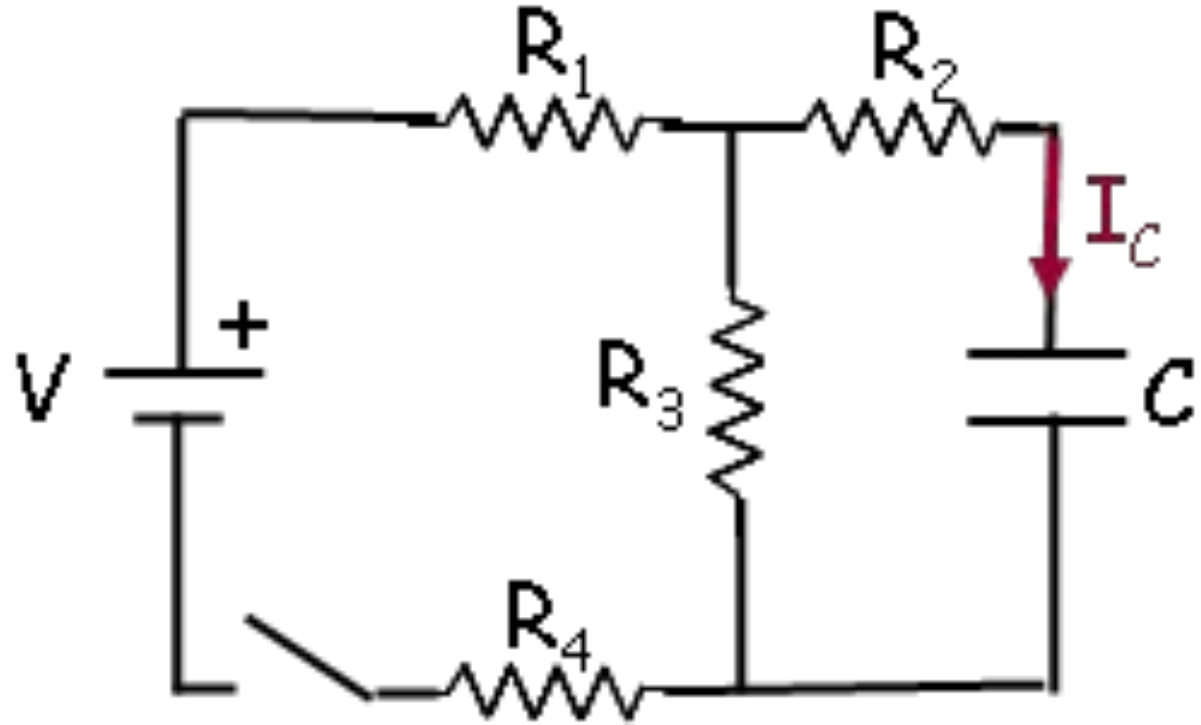
If C1 were replaced by a simple wire, which would result in greater current through R3?

- A) Opening S1
- B) Opening S2
- C) current would be the same for both A) and B) above



$R_1=6\Omega$ ,  $R_2=12\Omega$ ,  $R_3=6\Omega$ ,  $R_4=4\Omega$ ,  $V=84V$ ;  $C=2F$ . At  $t=0$ ,  $I_3=$

- A) 5.25 A
- B) 6A
- C) 4A
- D) 12A
- E) NOTA



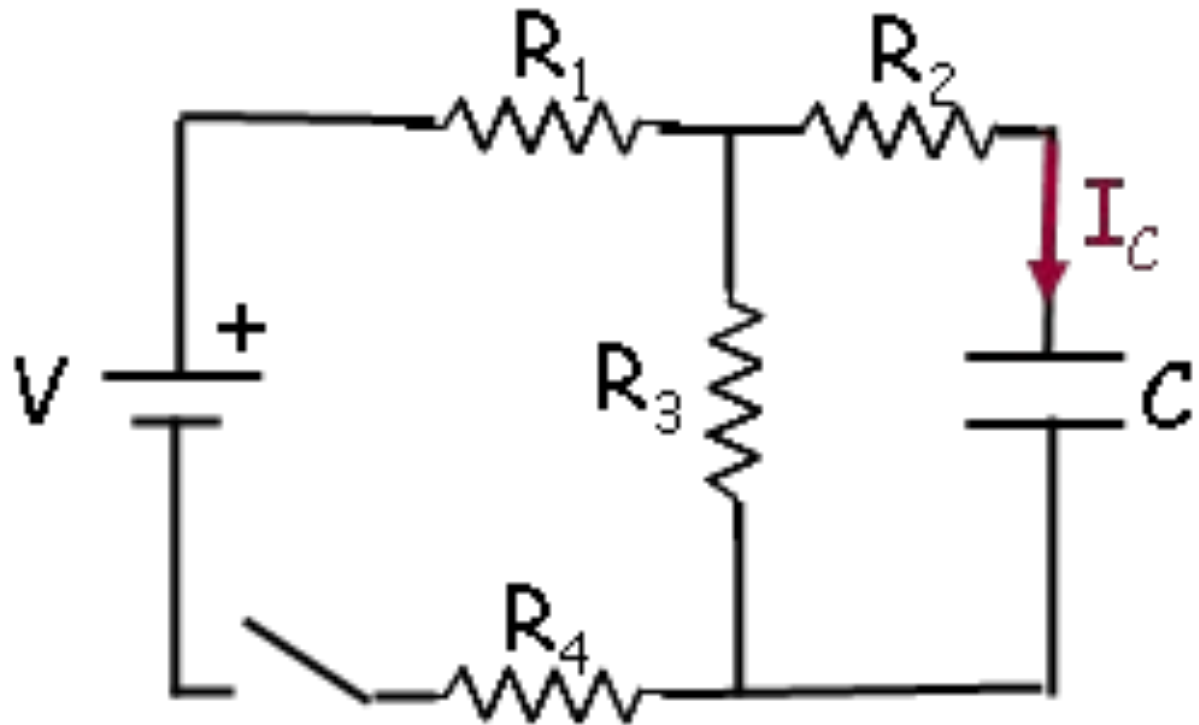
as  $t \rightarrow \infty$ ,  $I_3=$

as  $t \rightarrow \infty$ ,  $Q=$  A) 164C B) 63C C) 31.5C E)

NOTA

**$R_1=6\Omega$ ,  $R_2=12\Omega$ ,  $R_3=6\Omega$ ,  $R_4=4\Omega$ ,  
 $V=96V$ ;  $C=2F$ . At  $t=\infty$ ,  $I_3=$**

- A) 4.57A
- B) 6A
- C) 9.14A
- D) 12A
- E) NOTA

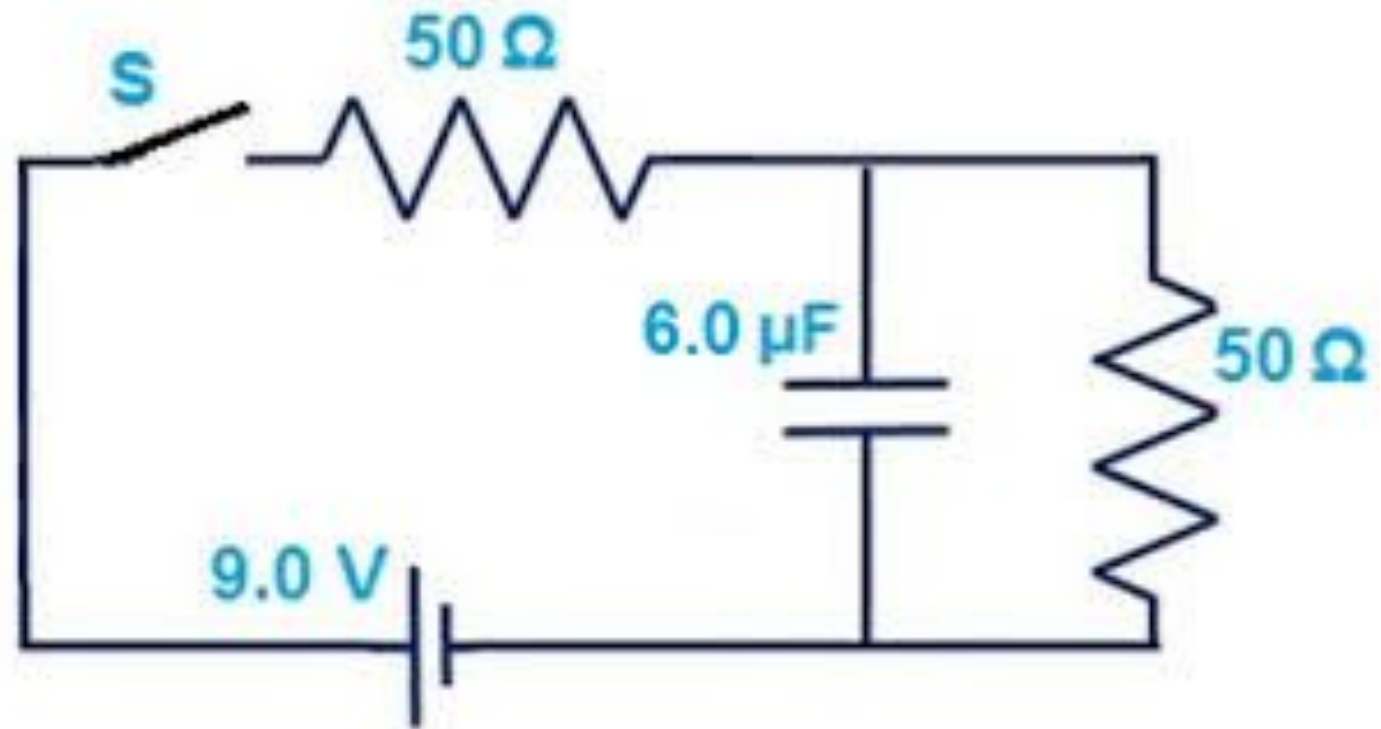


The charge on  $C$  as  $t \rightarrow \infty$  is:

- A) 72C B) 24C C) 48C D) 96C E) NOTA

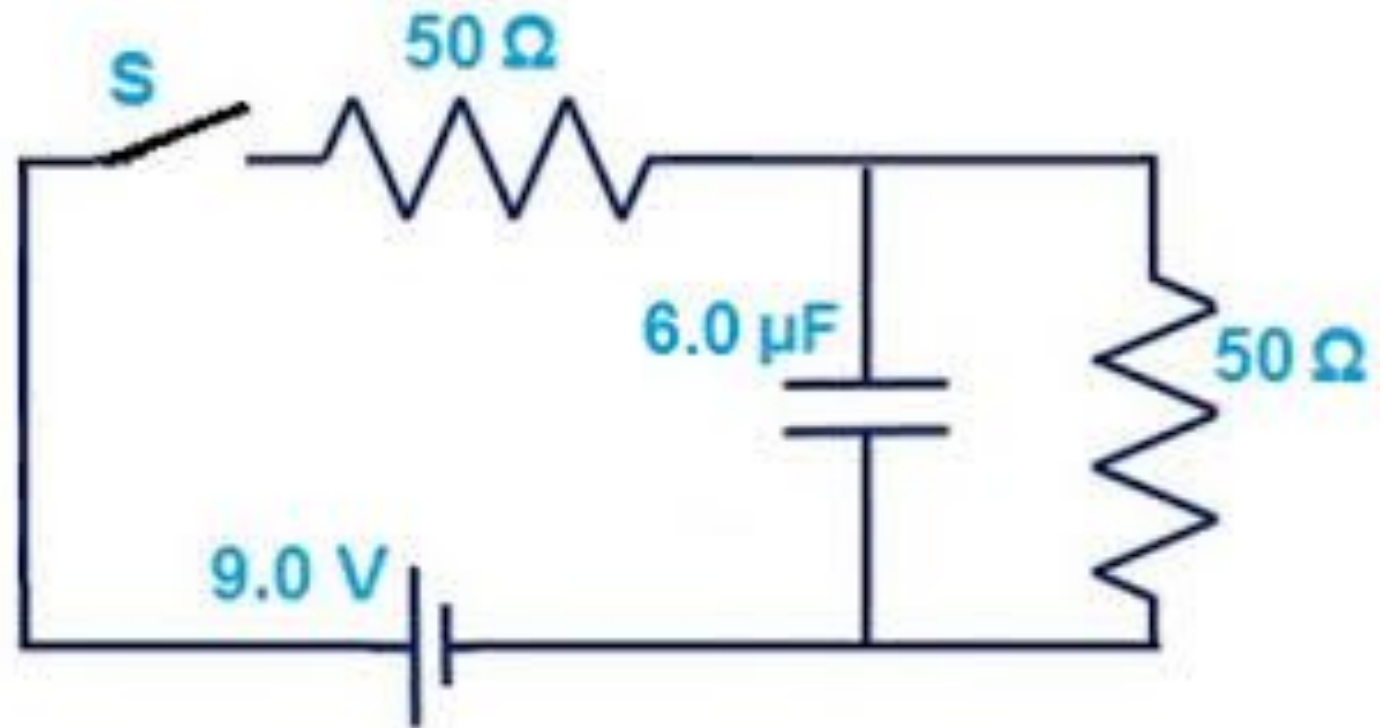
**What is the charge on the capacitor as  $t \rightarrow \infty$ ?**

- A)  $24 \mu\text{C}$
- B)  $27 \mu\text{C}$
- C)  $30 \mu\text{C}$
- D)  $9 \mu\text{C}$
- E) NOTA

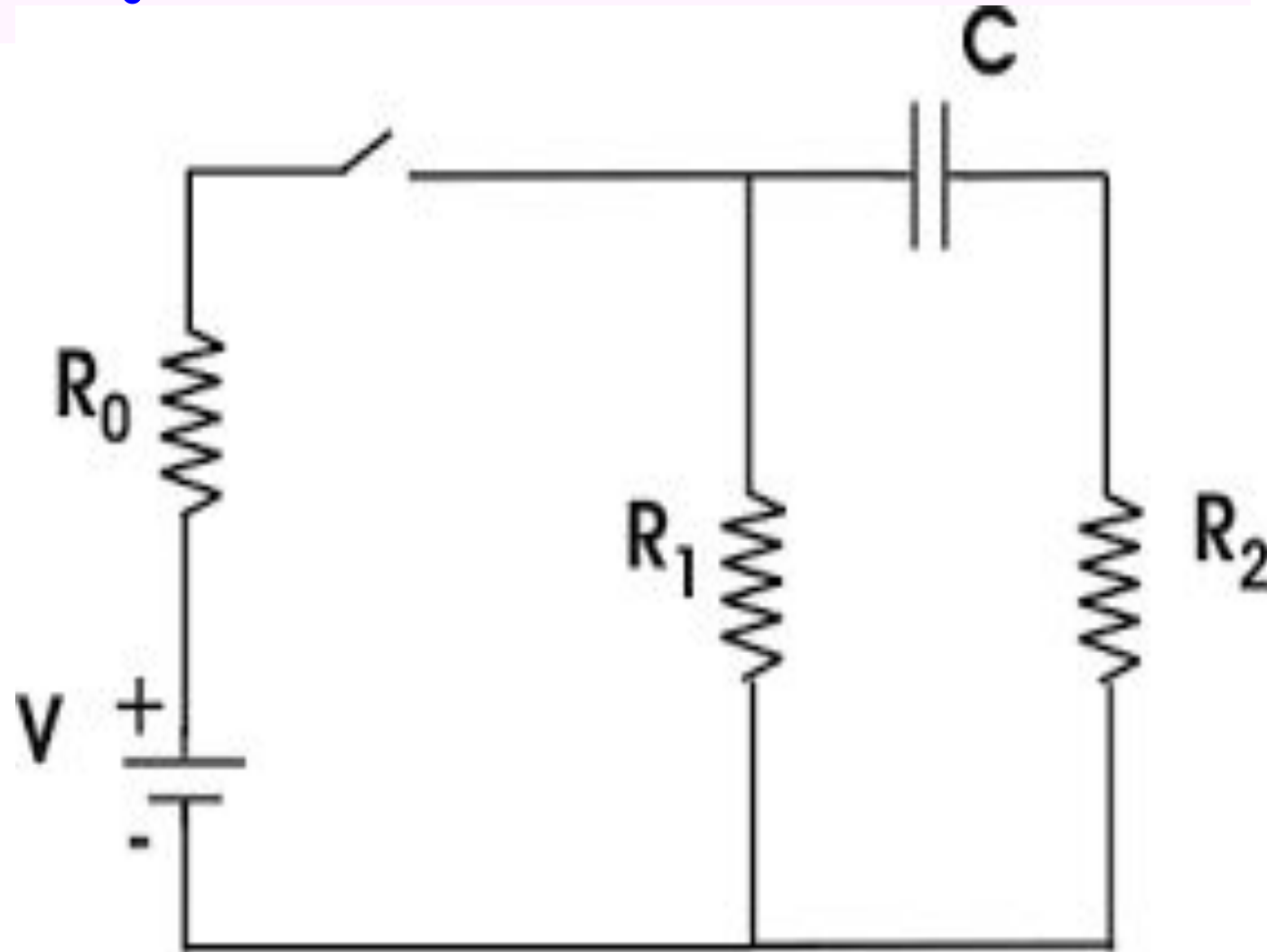


**What is the ratio of the current through the top resistor at  $t=0$  vs.  $t \rightarrow \text{infinity}$ ?**

- A) 0.5
- B) 1.0
- C) 1.5
- D) 2.0
- E) NOTA



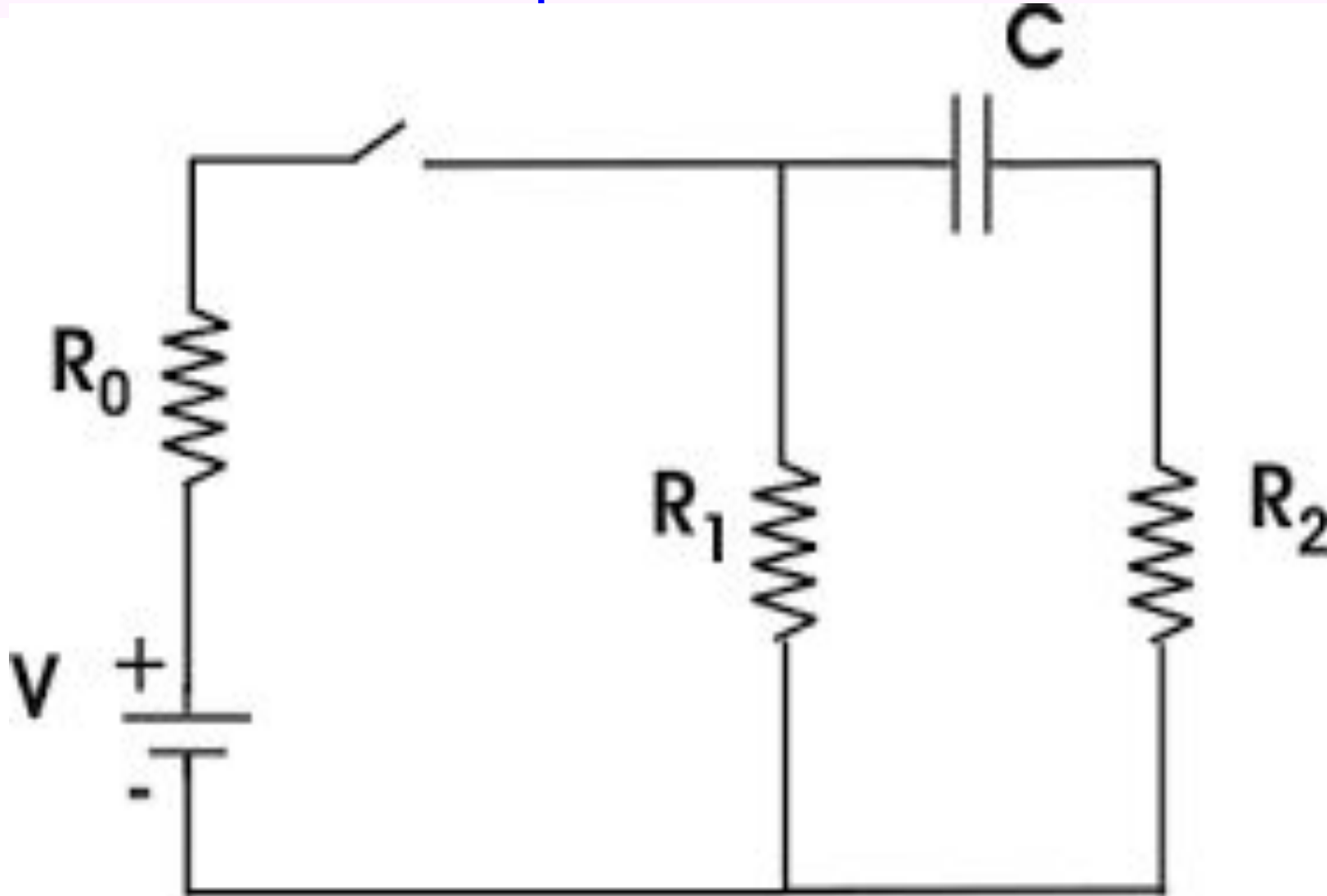
For  $V=12\text{ V}$ ,  $C=2\text{ F}$ ,  $R_0=6\Omega$ , and  $R_1=R_2=12\Omega$ , how does the current through  $R_0$  change for  $t=0$  vs.  $t \rightarrow \infty$ ?



- A) increase
- B) decrease
- C) unchanged

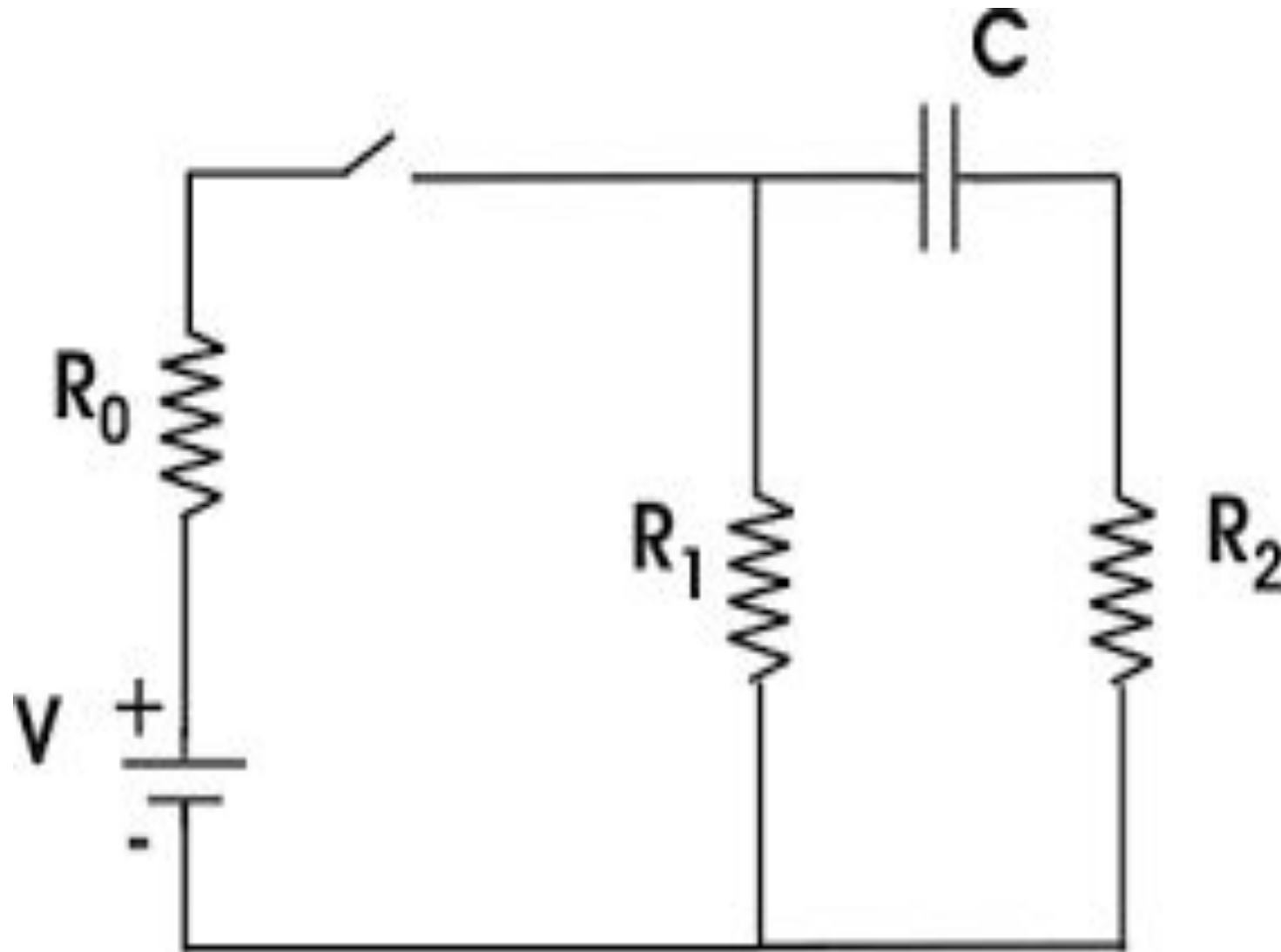
For  $V=12\text{ V}$ ,  $C=2\text{F}$ ,  $R_0=6\Omega$ , and  $R_1=R_2=12\Omega$ , what is the ratio of voltage across  $R_1$  at  $t=0$  vs.  $t \rightarrow \text{infinity}$ ?

- A) 1
- B)  $2/3$
- C)  $3/4$
- D)  $4/3$
- E) NOTA



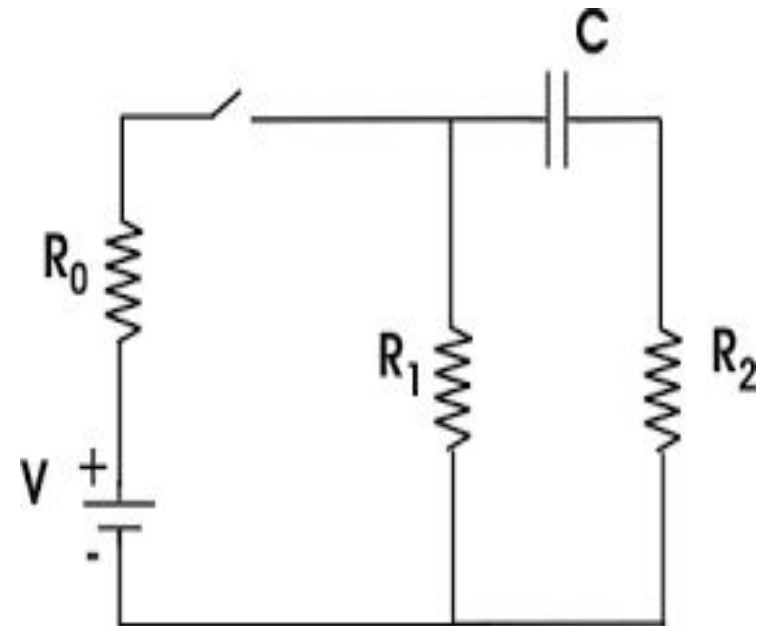
**What is the charge across the capacitor as  $t \rightarrow \infty$ ?**

- A)  $4C$
- B)  $8C$
- C)  $16C$
- D)  $32C$
- E) NOTA



**If  $R_0=R_1=R_2=6\Omega$  and  $V=24V$ , as  $t \rightarrow \infty$ ,**

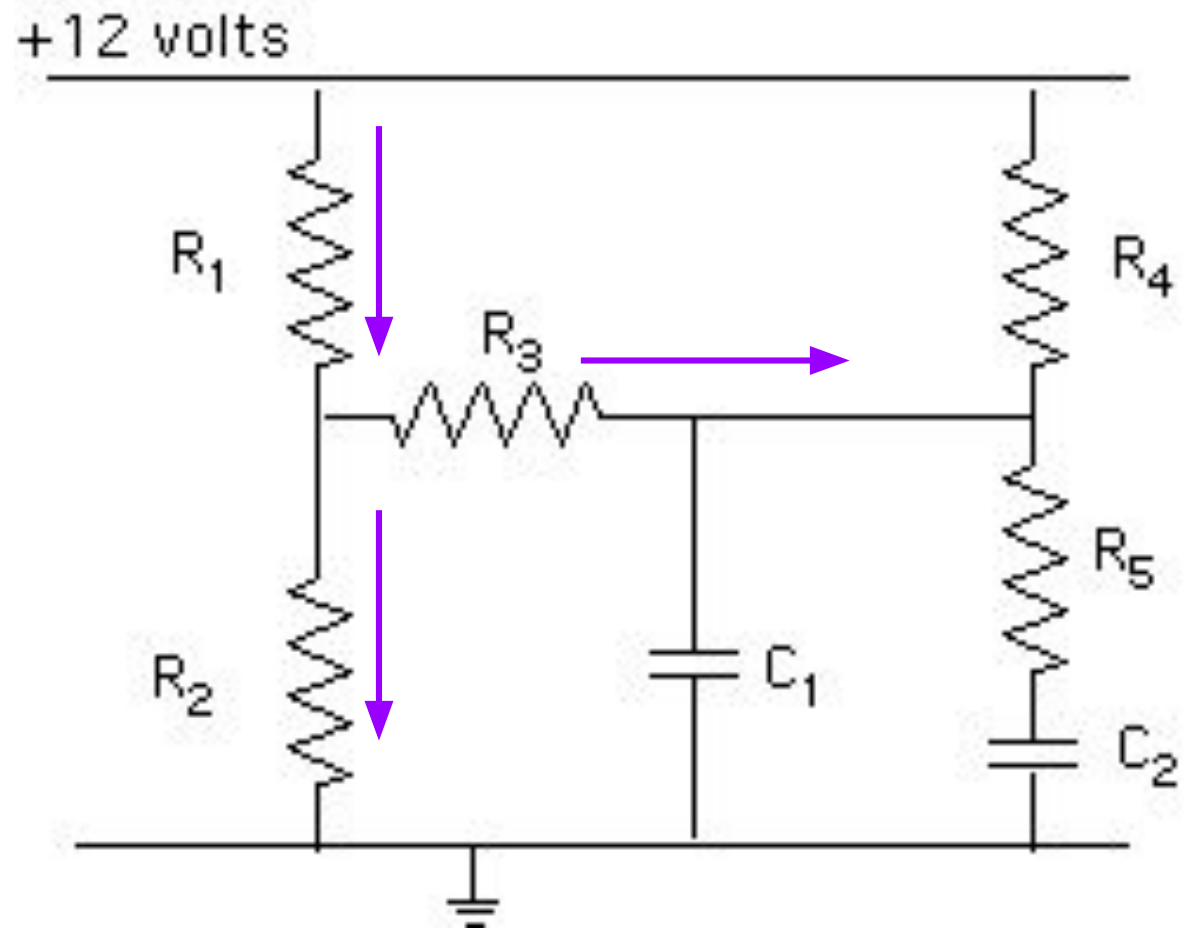
- A) The voltage drop through  $R_2$  is the same as for  $R_1$
- B) The voltage drop through C is the same as for  $R_0$
- C) The voltage drop through C is the same as for  $R_1$
- D) There is no longer any current thru resistor  $R_2$
- E) At least two of the above are true





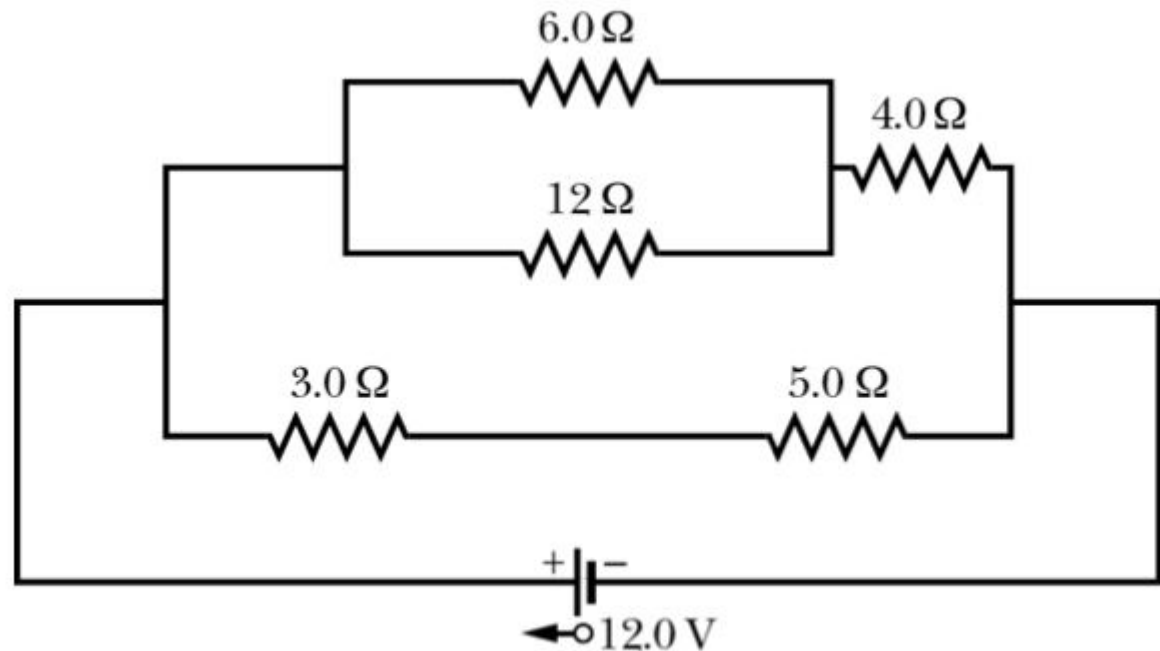
If the current direction is shown at  $t=0$ , and if  $R_1=R_2$ , then, at that time:

- A)  $V_2=V_1$
- B)  $V_2>V_1$
- C)  $V_2<V_1$
- D) Need more info



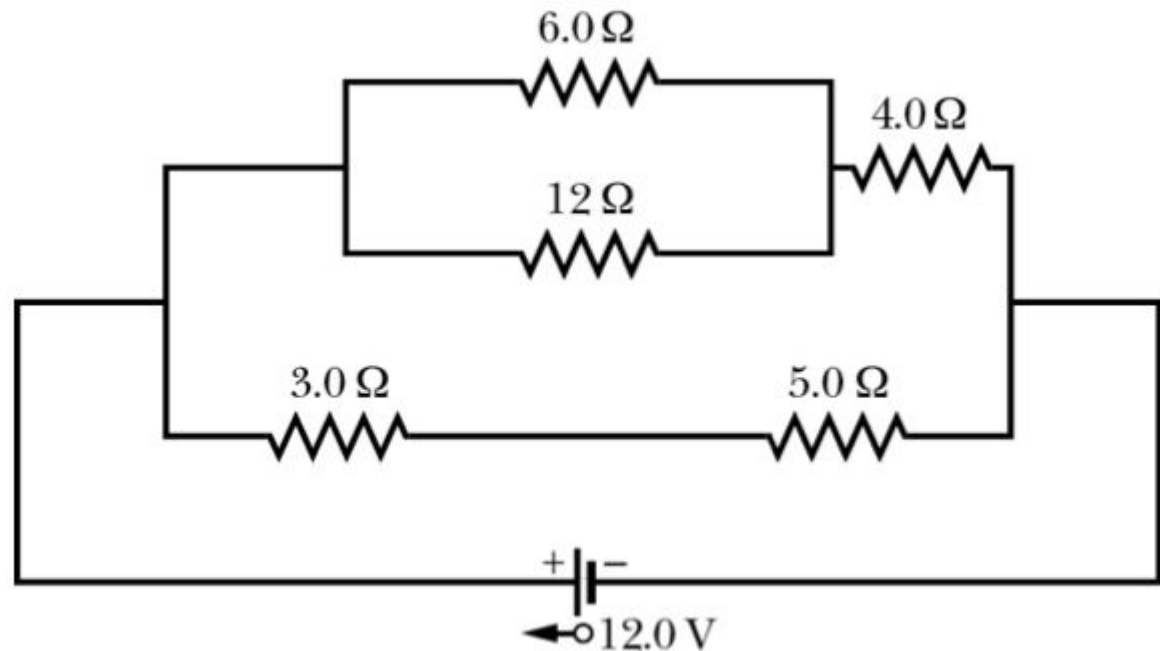
**A 3F capacitor is inserted in series with the 6 Ohm resistor.  $I_{\text{tot}}(t=0)$  and  $I_{\text{tot}}(t \rightarrow \text{infinity})$  are:**

- A) 1A, 0.675A
- B) 1A, 1A
- C) 2A, 0.8A
- D) 3A, 2.25A
- E) NOTA



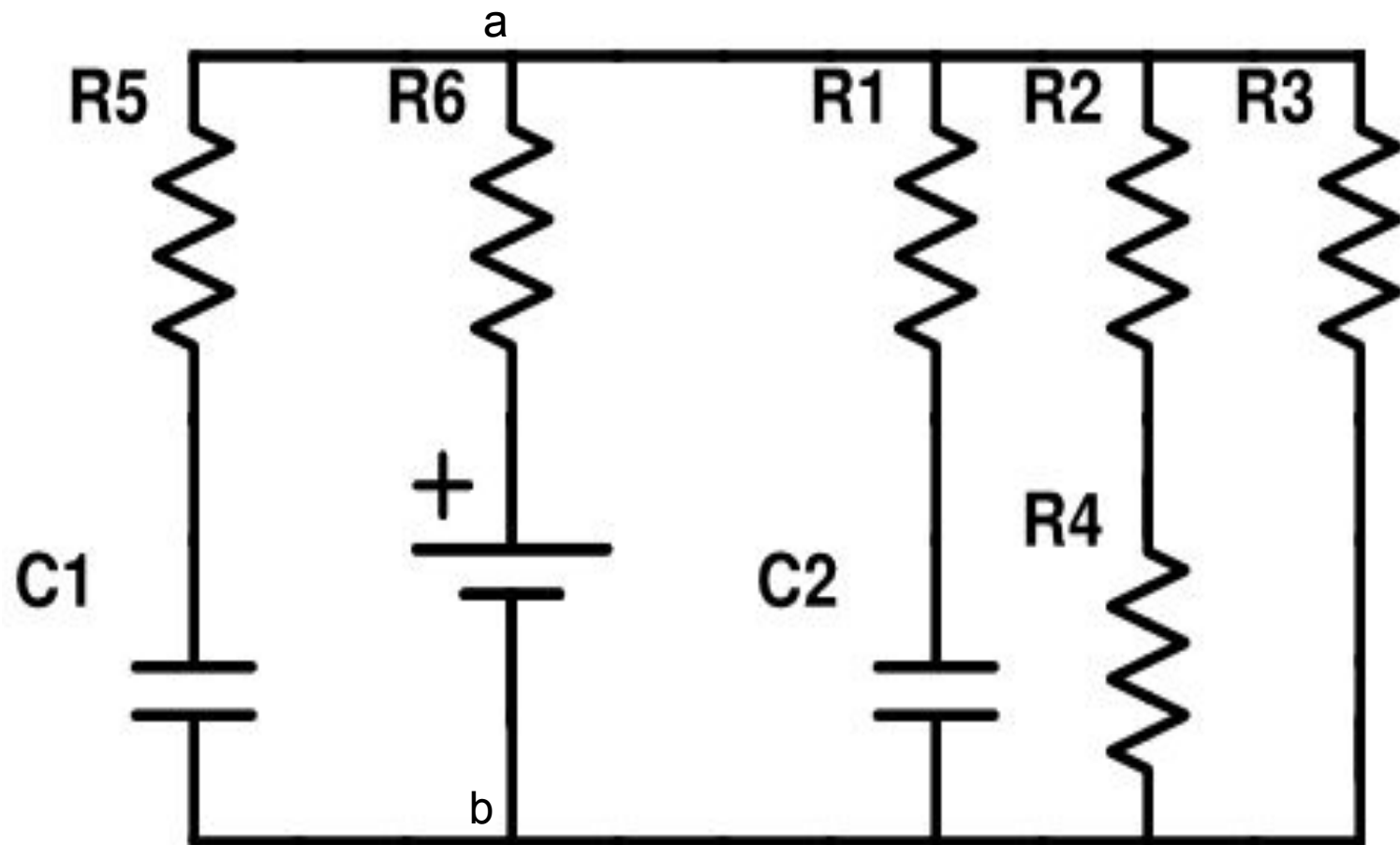
**A 3F capacitor is inserted in series with the 6 Ohm resistor. The current through the 3Ω resistor at  $t=0$  and  $t \rightarrow \infty$  is:**

- A) 1.5A, 3A
- B) 1.5A, 1.5A
- C) 3A, 3A
- D) 3A, 1.5 A
- E) NOTA



$R_1=3\Omega$ ;  $R_2=6\Omega$ ;  $R_3=24\Omega$ ;  $R_4=2\Omega$ ;  $R_5=2\Omega$ ;  
 $R_6=5\Omega$ ;  $C_1=2F$ ;  $C_2=4F$ ;  $V=96V$ .  $V_4$  @  $t=0$ ,  
 $t \rightarrow \infty =$

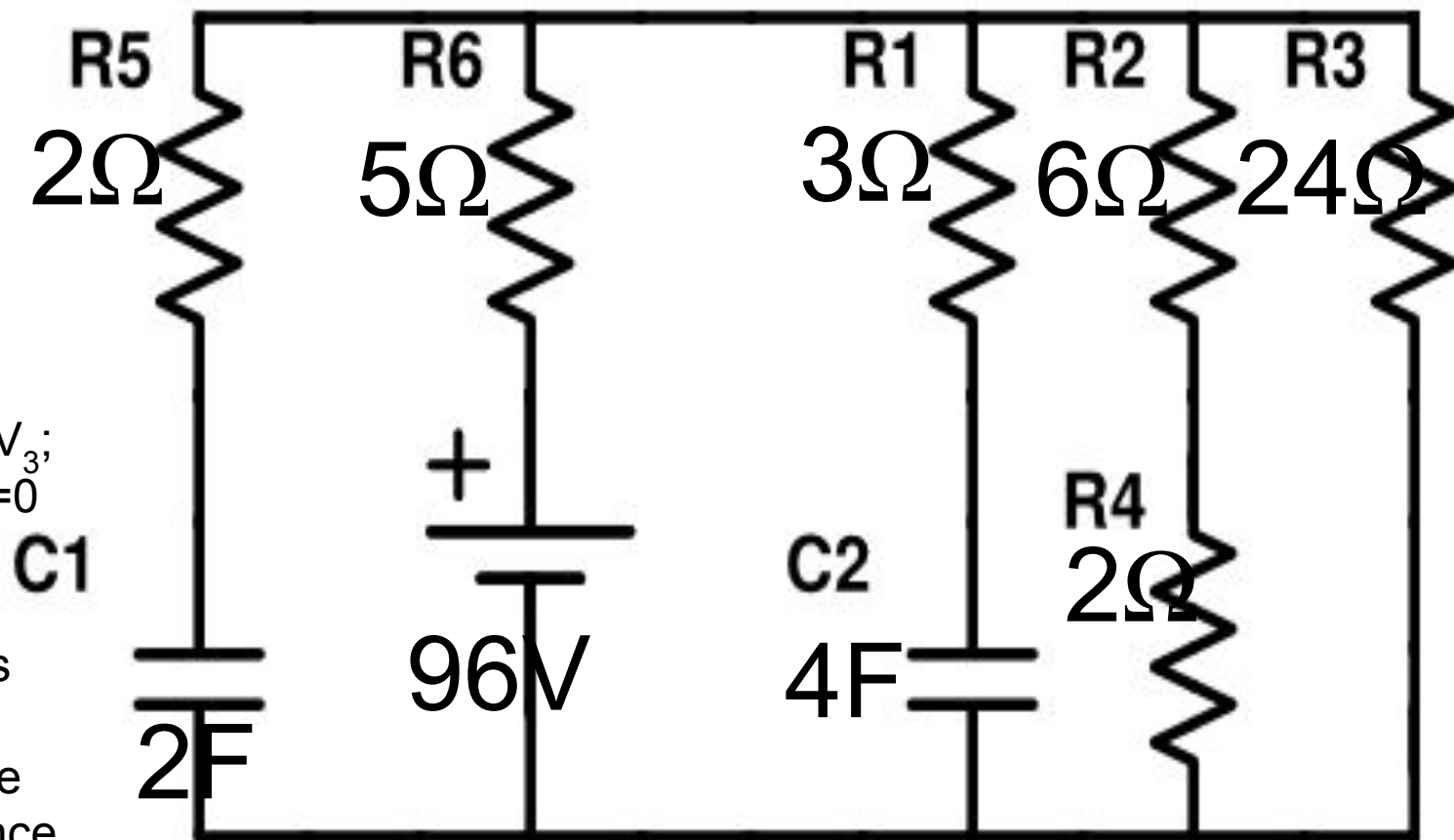
- A) 2V, 8V
- B) 4V, 13.09V
- C) 8V, 4V
- D) 6V, 6V
- E) NOTA

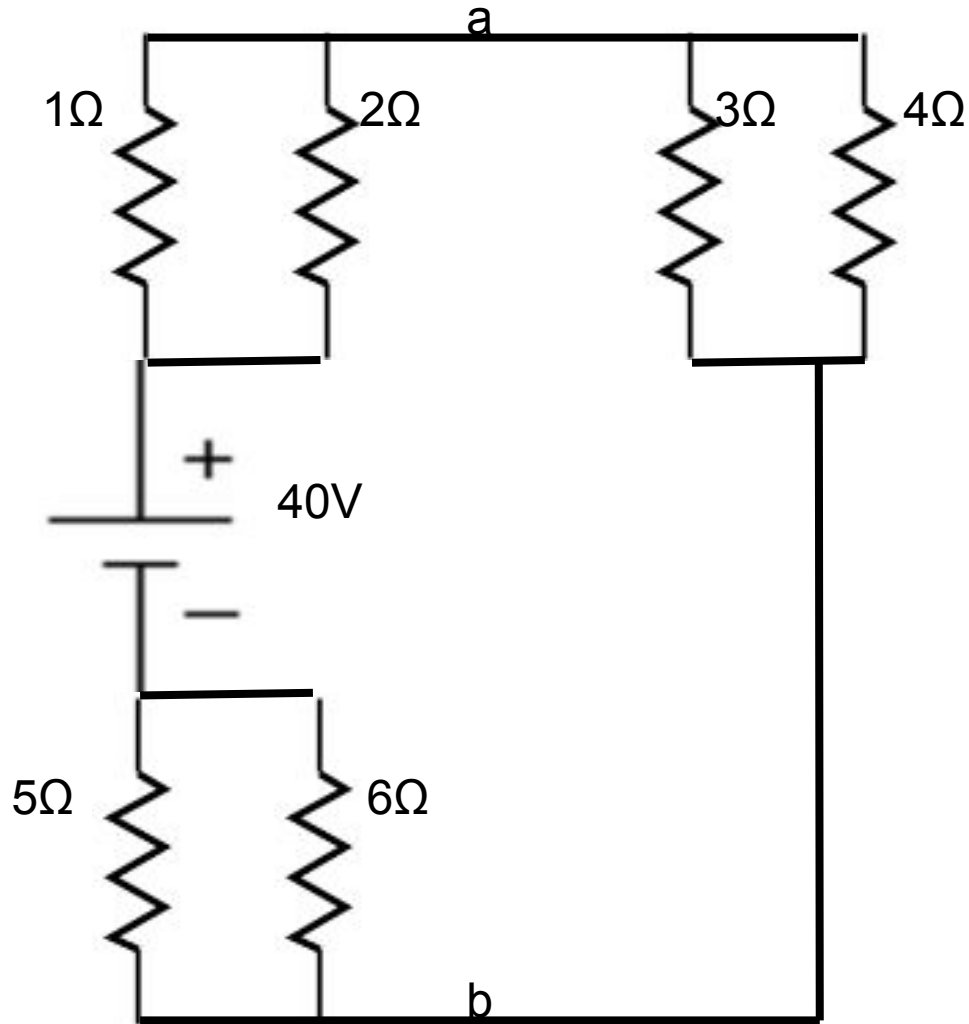


$Q_1(t \rightarrow \infty) =$  A) 209.4 C B) 104.7 C C) 2C D) 192C E) NOTA

$T=0: R_{24}=8\Omega;$   
 $R_1 \parallel R_{24} \parallel R_5 \parallel R_3$   
 So:  $1/R_{12345} =$   
 $1/2 + 1/3 + 1/8 + 1/24 \Rightarrow$   
 $R_{12345}=1\Omega$ , so  $R_{tot}=6\Omega$ , and  
 $I_{tot}=96/6\Omega=16\text{ A}$ , so:  
 $V_6=5\Omega \cdot 16\text{A}=80\text{ V}$ , so  
 $V_5=96-80\text{V}=16\text{V}=V_1=V_{24}=V_3;$   
 voltage drop through caps=0

$t \rightarrow \infty$ : capacitor blocks  
 current flow in all segments  
 containing that resistor, so  
 each of those segments are  
 (effectively) infinite resistance  
 branches, so to calculate  $R_{tot}$ ,  
 only include  $R_6$ ,  $R_{24}$  and  $R_3$ ,  
 $R_{24} \parallel R_3$ ;  $R_{234}$  in series with  $R_6$ ,  
 so  $R_{234}=6\Omega$ ,  $R_{tot}=11\Omega$ ,  
 $I_{tot}=8.73\text{ A};$   
 $V_6=8.73\text{A} \cdot 5\Omega=43.63\text{ V}$ , so  
 $V_{24}=V_3=96\text{V}-43.63\text{V}=52.37\text{V};$   
 This is also the voltage drop  
 across both  $C_1$  and  $C_2$ , so  
 $Q_1=104.74\text{C}$  and  $Q_2=209.48\text{C}$





Find voltages at a and b, as well as voltage drops through all resistors in circuit.

Repeat exercise for case where resistors are replaced by equivalent capacitors (e.g.,  $1\Omega \rightarrow 1\text{F}$  capacitor, etc. In that case, again find the voltage at points a and b, and also the charges on all the capacitors in the circuit.

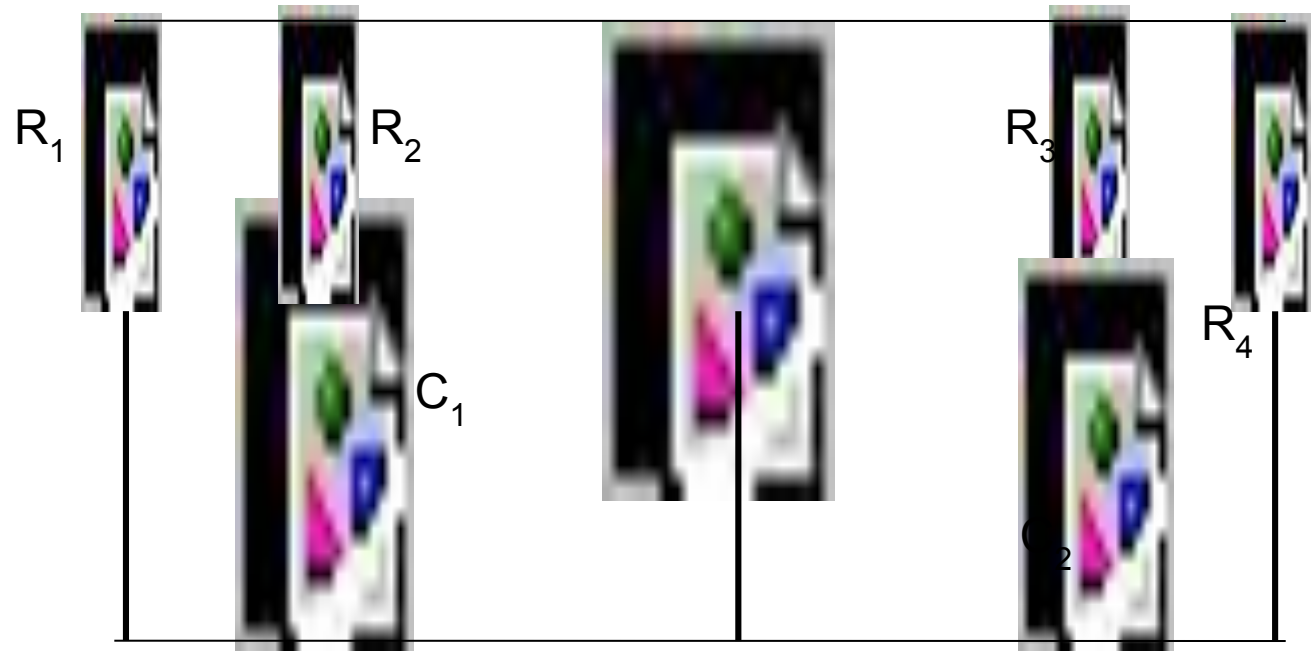
In the circuit below, determine  $I_{\text{tot}}$  and also  $I_2$  and  $Q_1$  at  $t=0$  and as  $t \rightarrow \infty$ .  $R_1=4\Omega$ ,  $R_2=12\Omega$ ,  $R_3=9\Omega$ ,  $R_4=18\Omega$ ,  $C_1=2F$ ,  $V=36V$ ,  $C_2=1F$

$T=0: (I_{\text{tot}}, I_2)=$

- A) 18A, 18A
- B) 3A, 3A
- C) 18A, 3A
- D) 3A, 18A
- E) NOTA

$T \rightarrow \infty: (I_{\text{tot}}, C_1)=$

- A) 11A, 36C
- B) 11A, 72C
- C) 18A, 36C
- D) 18A, 72C
- E) NOTA



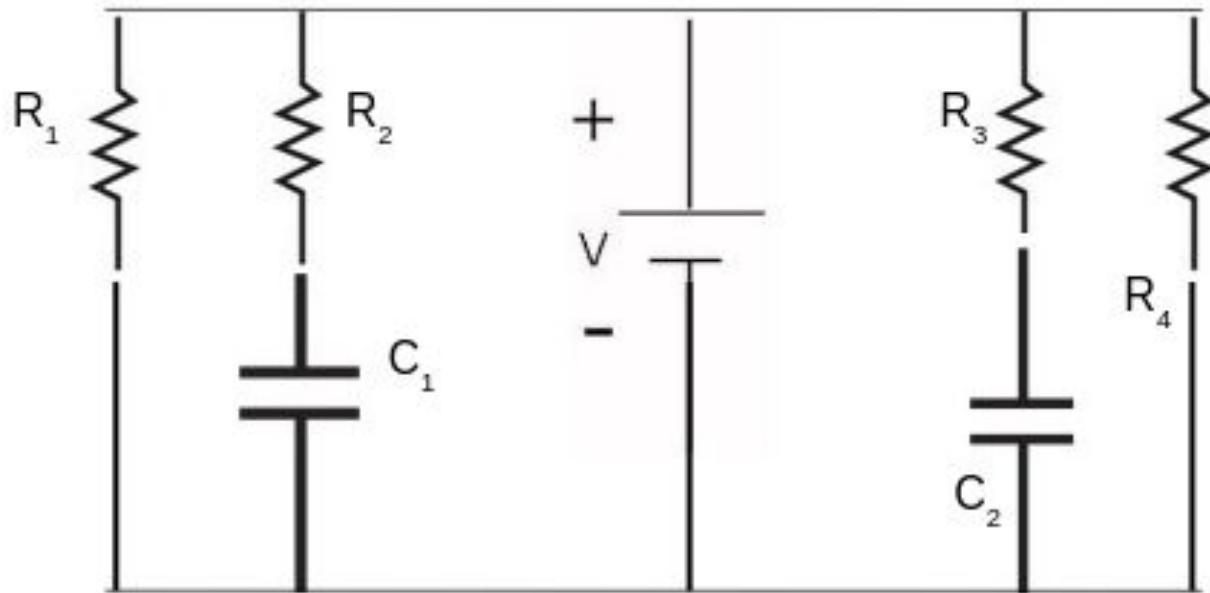
In the circuit below, determine  $I_{\text{tot}}$  and also  $I_2$  and  $Q_1$  at  $t=0$  and as  $t \rightarrow \infty$ .  $R_1=4\Omega$ ,  $R_2=12\Omega$ ,  $R_3=9\Omega$ ,  $R_4=18\Omega$ ,  $C_1=2\text{F}$ ,  $V=36\text{V}$ ,  $C_2=1\text{F}$

$T=0$ : ( $I_{\text{tot}}, I_2$ )=

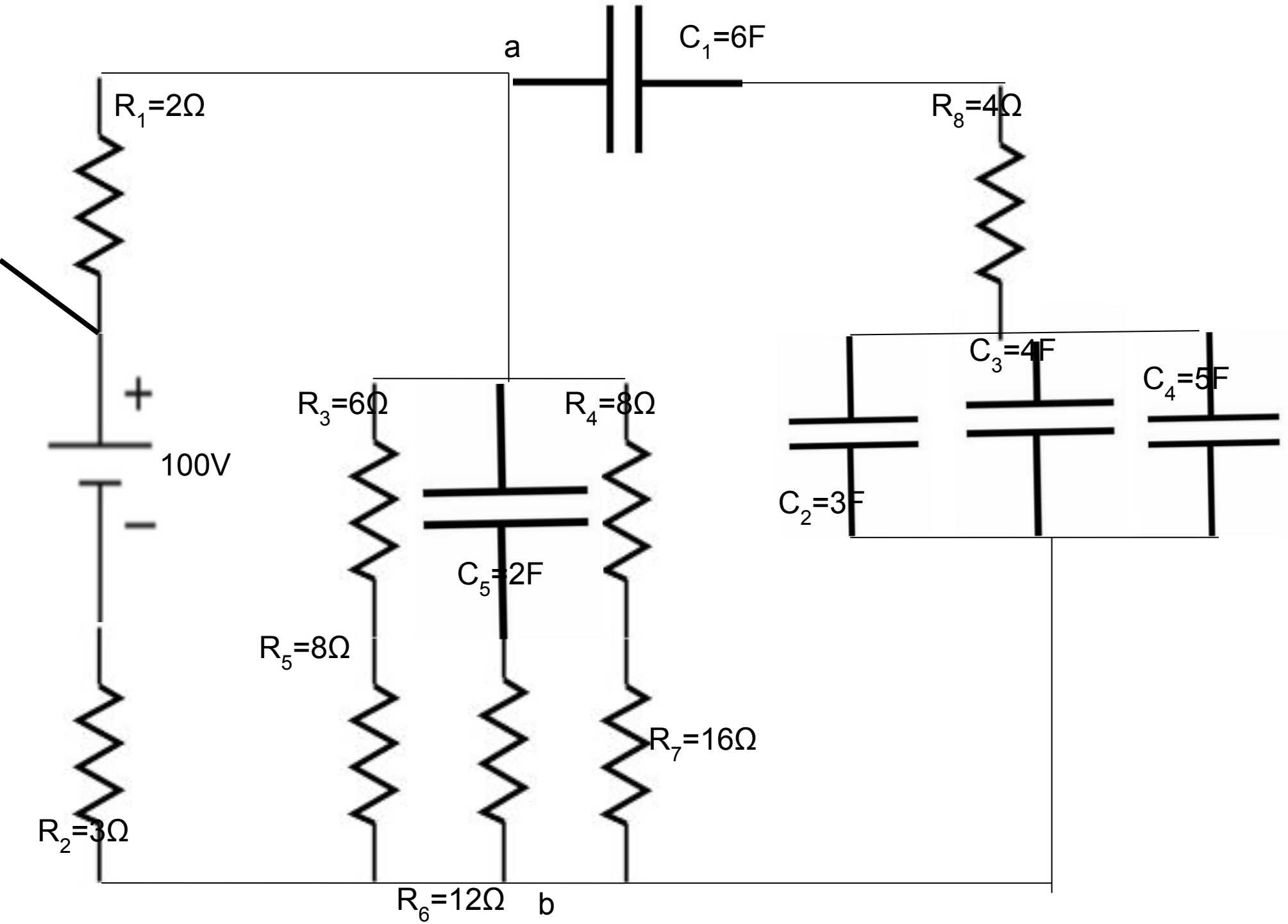
- A) 18A, 18A
- B) 3A, 3A
- C) 18A, 3A
- D) 3A, 18A
- E) NOTA

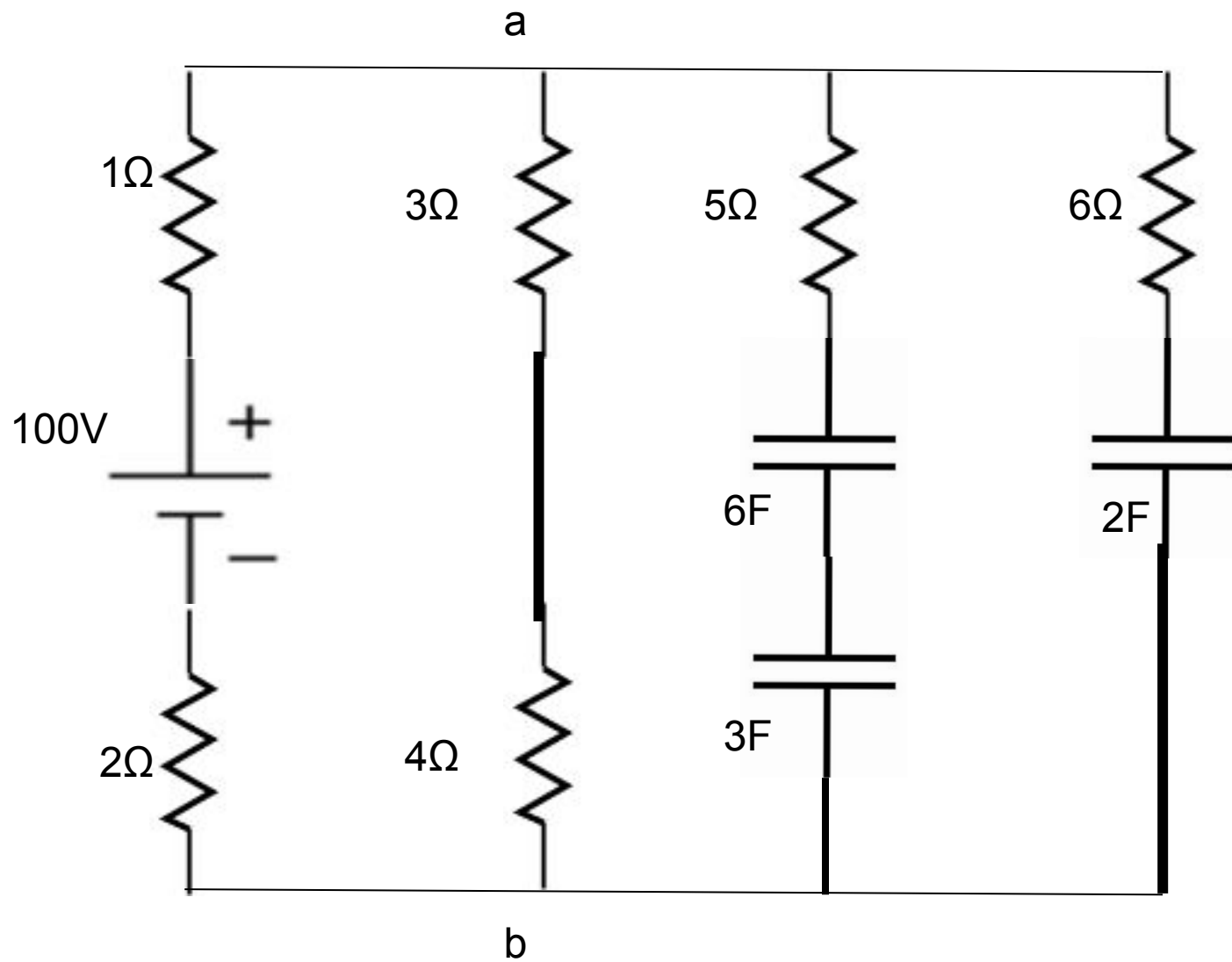
$T \rightarrow \infty$ : ( $I_{\text{tot}}, C_1$ )=

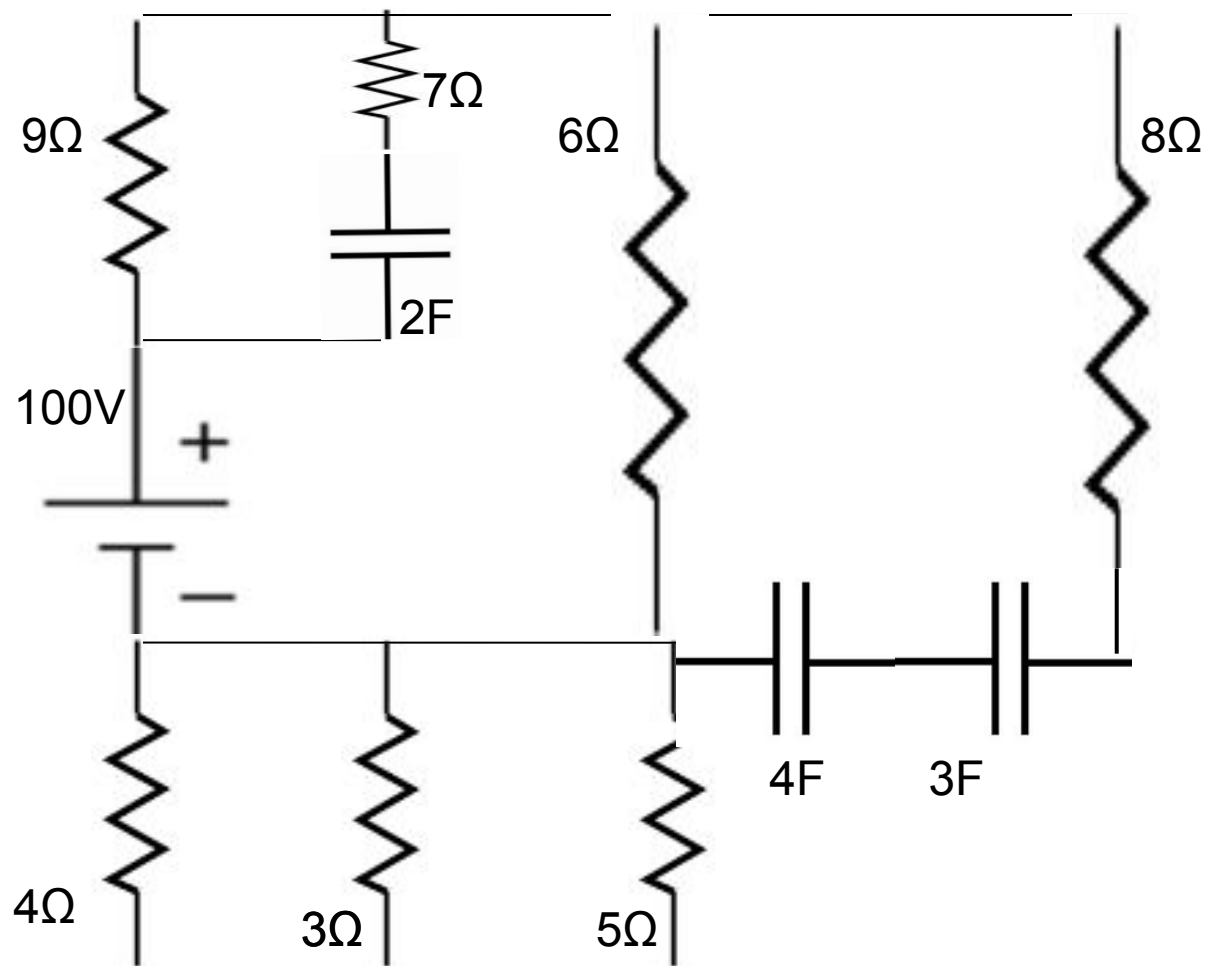
- A) 11A, 36C
- B) 11A, 72C
- C) 18A, 36C
- D) 18A, 72C
- E) NOTA





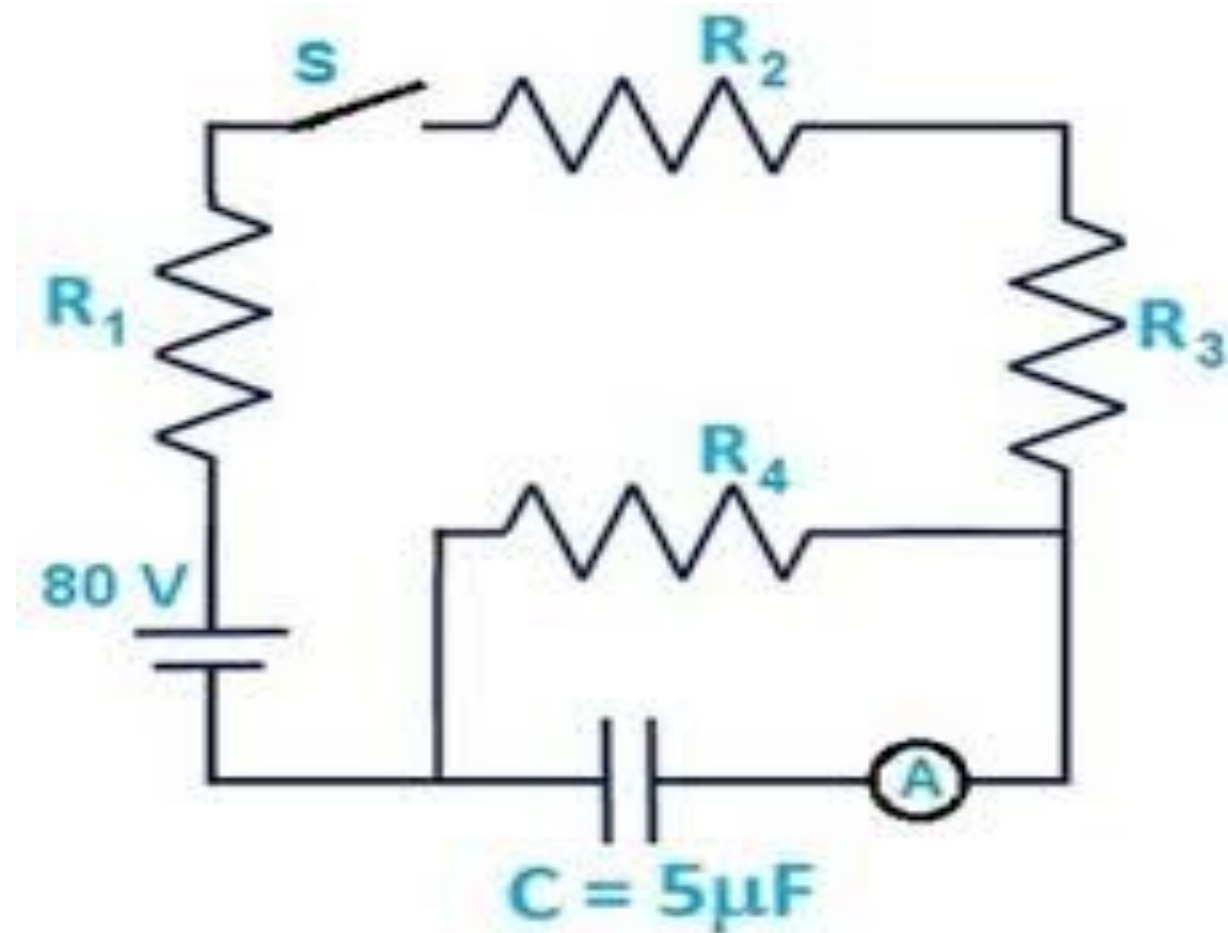






**$R_1=R_2=R_3=R_4=10\Omega$ ; what is the charge on the capacitor as  $t\rightarrow\infty$ ?**

- A)  $20\ \mu\text{C}$
- B)  $40\ \mu\text{C}$
- C)  $80\ \mu\text{C}$
- D)  $100\ \mu\text{C}$
- E) NOTA



Power dissipated in resistors and energy stored on capacitors=...

Power stored in electric field in capacitors= $Q^2/2C$   
Heat lost as current flows through resistor= $I^2R$

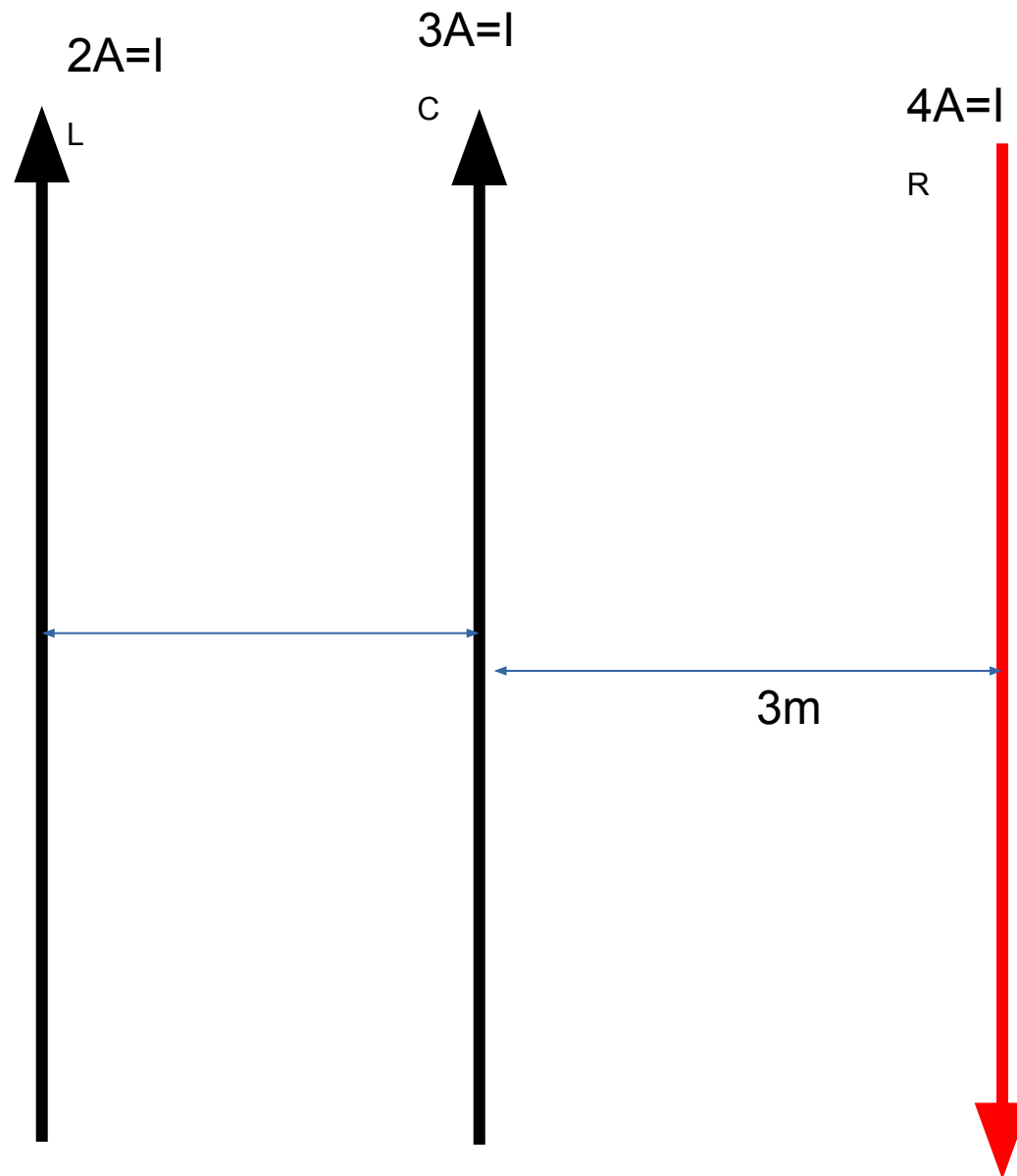
Not for all the tea in china...

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = qvB\sin\theta = qvB_{\text{perpendicular}} \quad (\text{cf } \mathbf{F}_E = E_{\parallel})$$

$B_{\text{wire}} = \mu_0 I / (2\pi r)$  (Ampere's Law);  $\mu_0 = 4\pi \times 10^{-7}$ ,  $\Rightarrow B = k'I/r$ ,  
with  $k' = 2 \times 10^{-7}$  T-m/A; two right-hand rules: (2) for  
direction of field (*magnetic field lines are tangents to  
circles centered on given current; i.e., B is at 90° to line  
joining current and measurement point P*), and (1) for  
direction of Force.

$$B_{\text{loop}} = \mu_0 I / (2r) = \pi k' I / r$$

$$\text{EMF} = -\Delta\Phi / dt$$



$I_1 = 3\text{A}$



$I_2 = 5\text{A}$



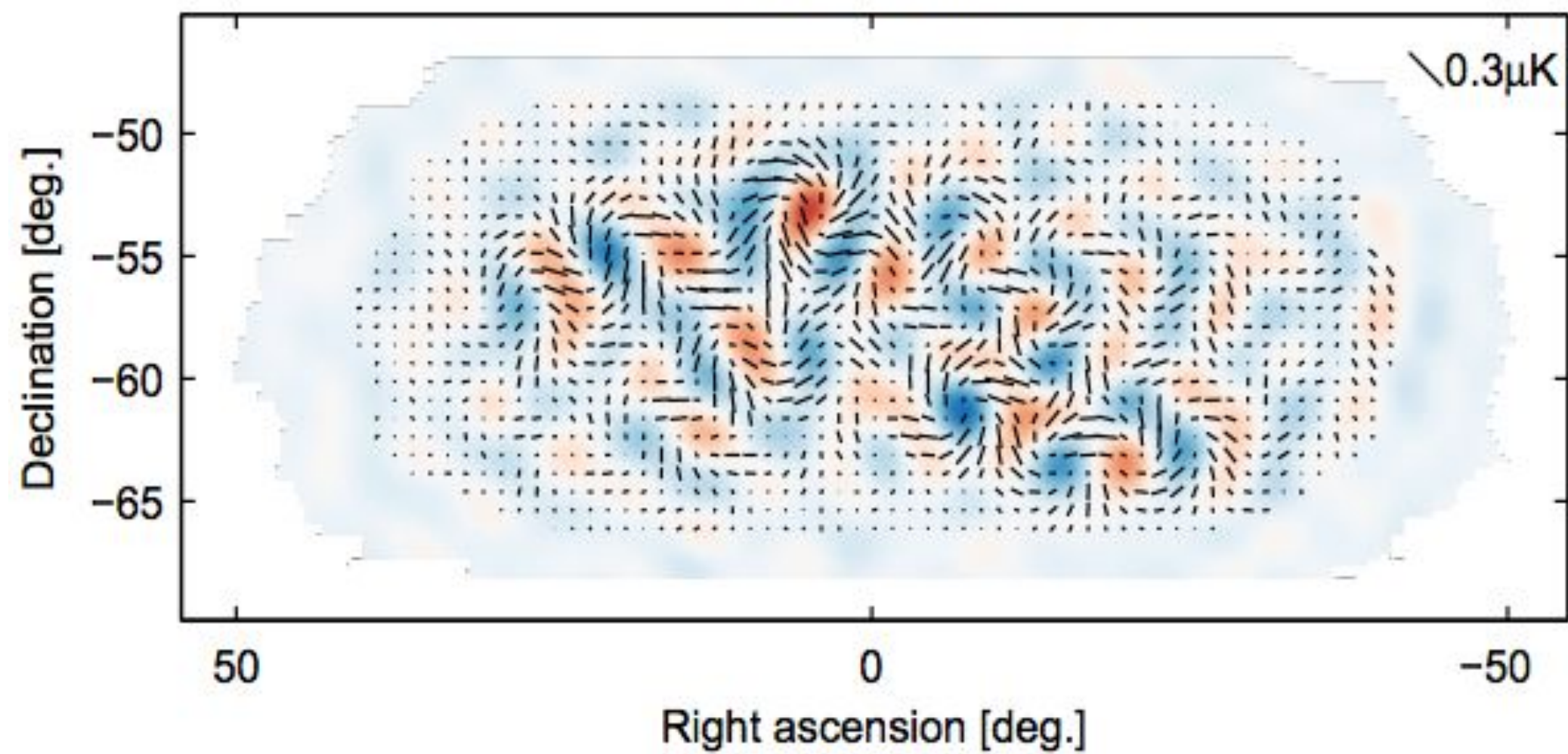
$I_3 = 4\text{A}$



1.5m



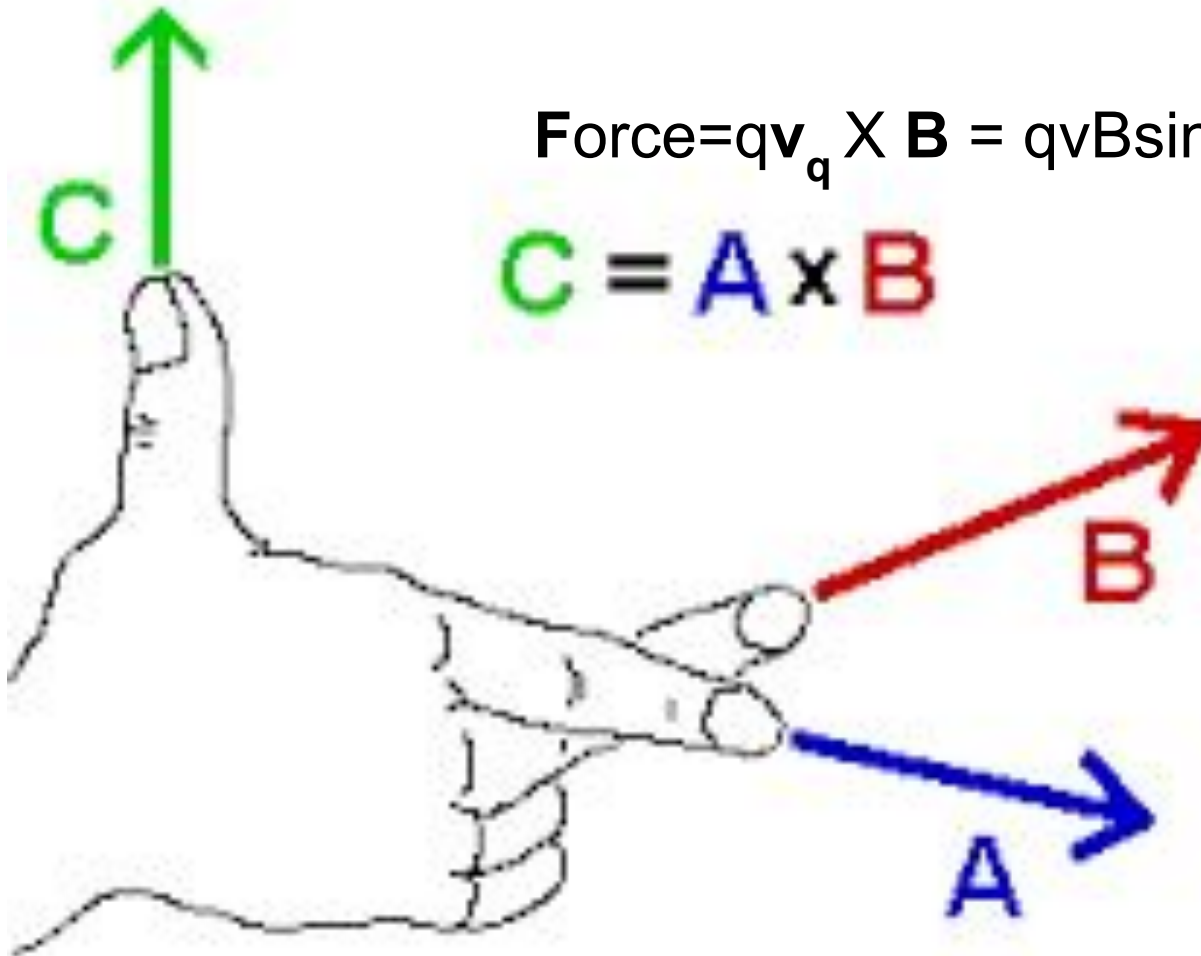
# BICEP2: B signal





# “Force” RHR(1)

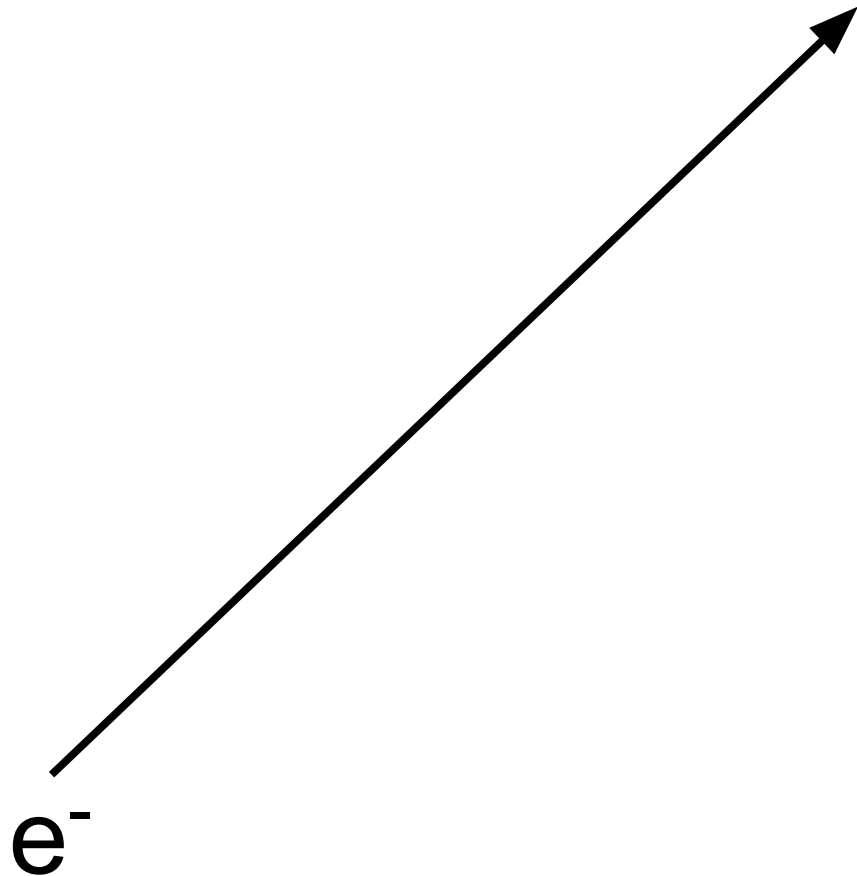
$$\text{Force} = q\mathbf{v}_q \times \mathbf{B} = qvB\sin\theta_{v,B}$$

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

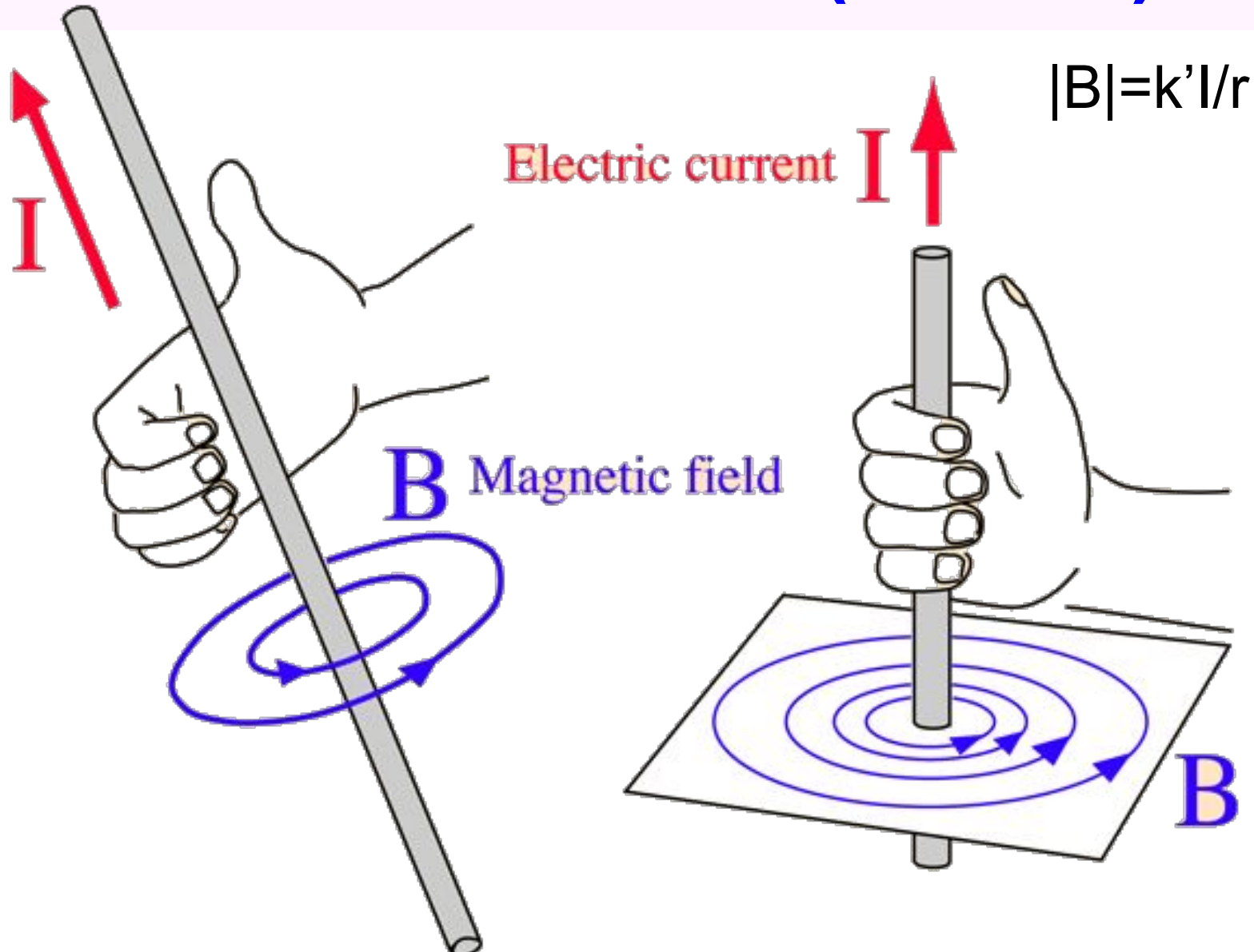


**A magnetic field  $B_z = +1$  T (out-of-page) fills the region below. The direction of deflection of an electron moving as shown would be:**

- A) +z
- B) -z
- C) 
- D) 
- E) NOTA

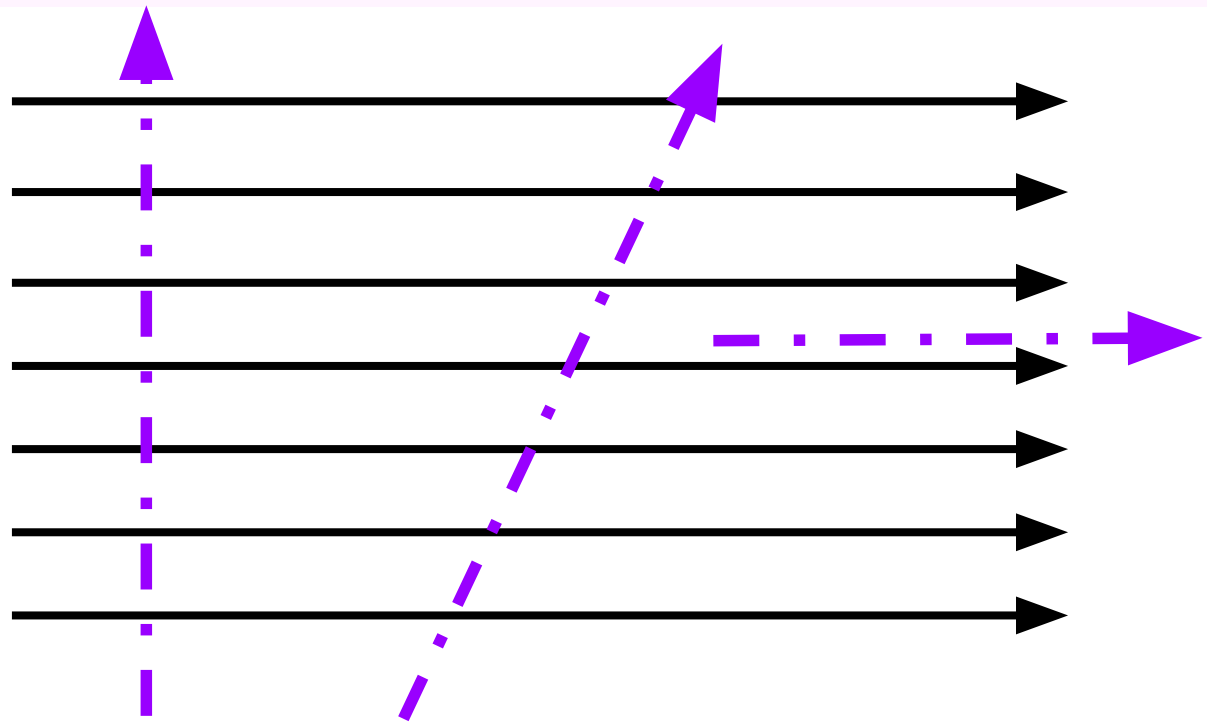


## B-field direction (RHR,2)



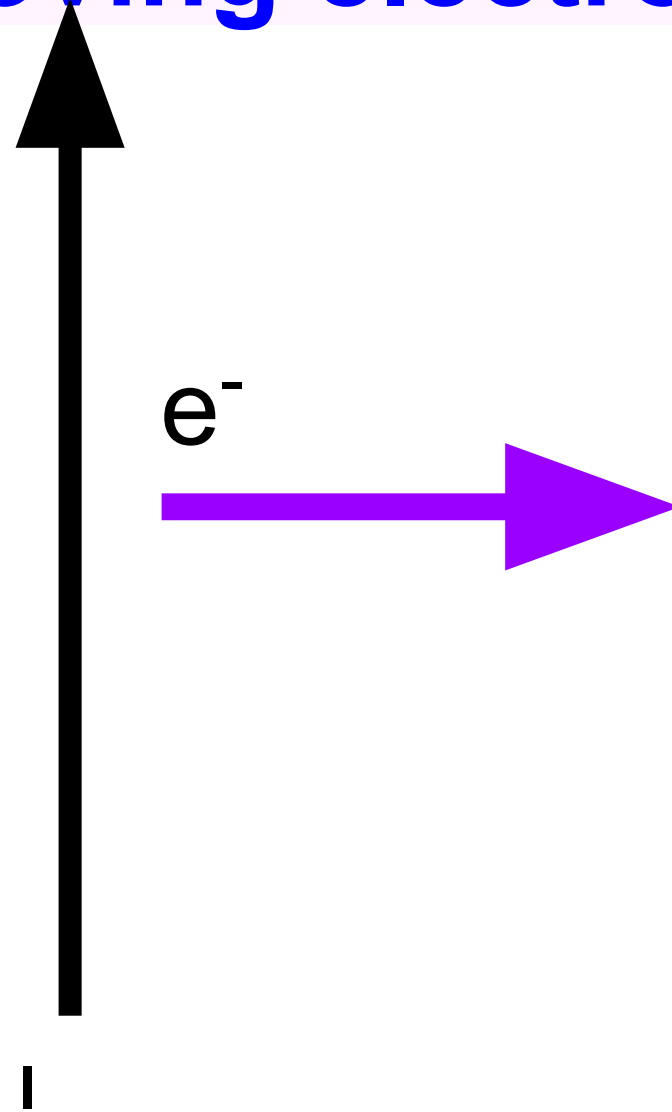
Three protons move with  $v=1$  m/s,  $1.2$  m/s (at an angle of  $30^\circ$  wrt vertical) and  $2$  m/s (left-to-right). Which proton experiences the biggest deflection?

- A) (left)
- B) (center)
- C) (right)



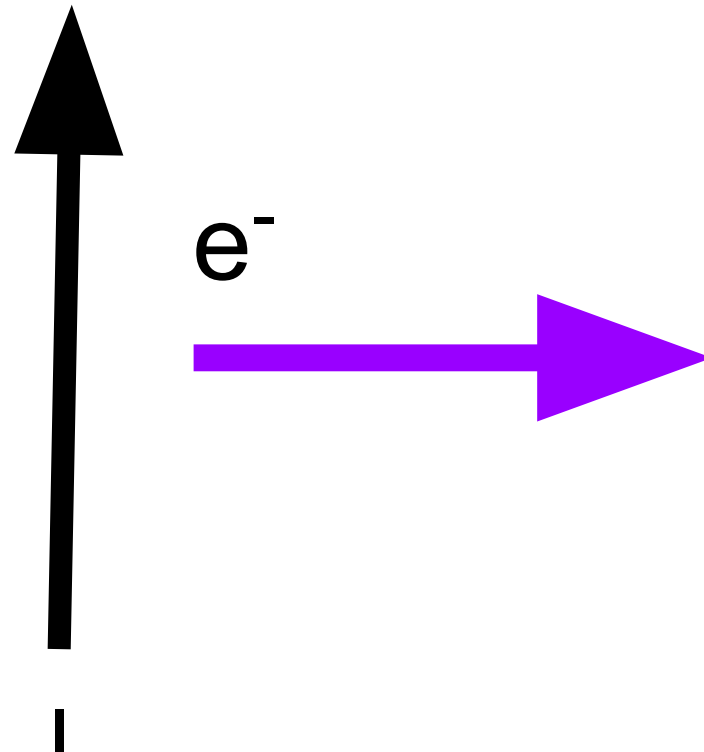
**A current  $I$  flows as shown. The deflection of the moving electron is:**

- A) Up
- B) Down
- C)  $+z$  (out-of-page)
- D)  $-z$  (into page)
- E) NOTA



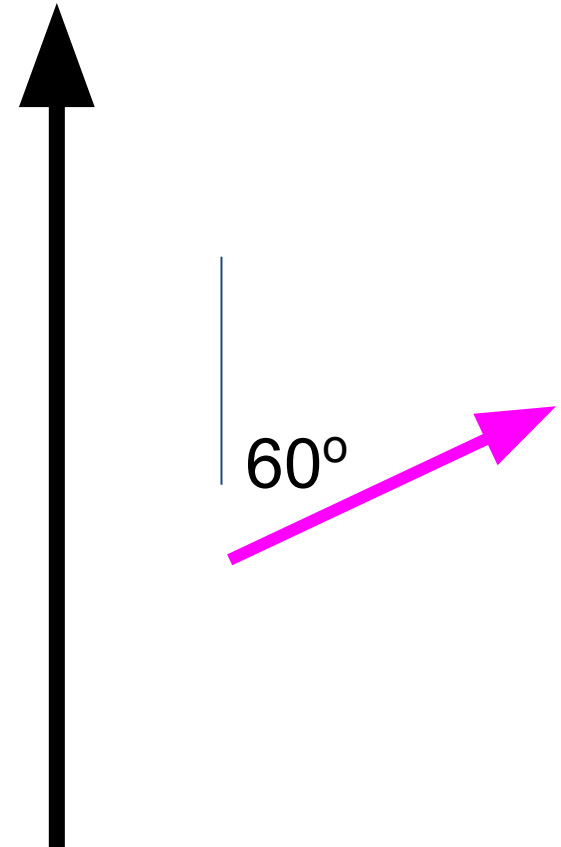
**As the electron moves, it must (recall the definition of  $W = |F||d|\cos\theta_{F,d}$ )**

- A) move faster
- B) move slower
- C) Move with constant velocity



A current of 4 amps flows through a wire in the +y-direction. A particle with charge -2 Coulombs and mass 4 kg at the point  $\mathbf{x}=2 \text{ m}$  moves with velocity  $\mathbf{v}$  making an angle of  $60^\circ$  relative to the wire, with a magnitude of  $3 \times 10^6 \text{ m/s}$ . What is the magnetic field vector at the location of the charge?

- A)  $4 \times 10^{-7} \text{ T}$ , -z (into page)
- B)  $4 \times 10^{-7} \text{ T}$ , +z (out of page)
- C)  $10^{-7} \text{ T}$ , -z (into page)
- D)  $10^{-7} \text{ T}$ , +z (into page)
- E) NOTA





A current of 4 amps flows through a wire in the +y-direction. A particle with charge -2 Coulombs and mass 4 kg at the point  $x=2$  m moves with velocity  $v$  making an angle of  $60^\circ$  relative to the wire, with a magnitude of  $3 \times 10^6$  m/s. What is the Force at the location of the charge?

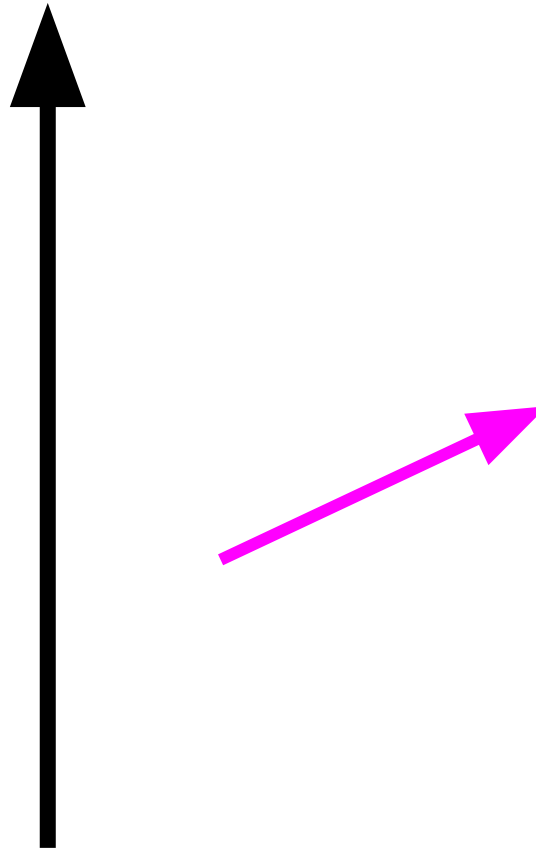
A) 2.4 N,  $+120^\circ$   
wrt +x

B) 2.4 N,  $-60^\circ$   
wrt +x

C) 1.2 N, +z

D) 1.2 N, -z

E) NOTA



A)  $9 \times 10^{-5} \text{ A}$

B) 9 milliAmps

C) 4.5 Amps

D) 4.5  
milliAmps

E) NOTA

Stationary 4C source  
charge; initially 2  
meters away

-0.1 C



**After 0.2 ns, the current constituted by the moving 2 kg, -0.1 C charge is about:**

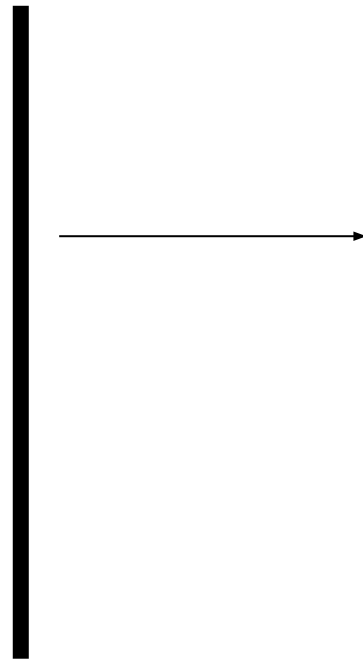
**A 2 kg, -3C charge moving at  $v_y = -100$  m/s enters a magnetic field region with  $B_x = +4$  T. The magnitude of the acceleration of the mass is:**

- A) Approximately equal to  $g$
- B) About 60X greater than  $g$
- C) About 60X less than  $g$
- D) About 6000x less than  $g$



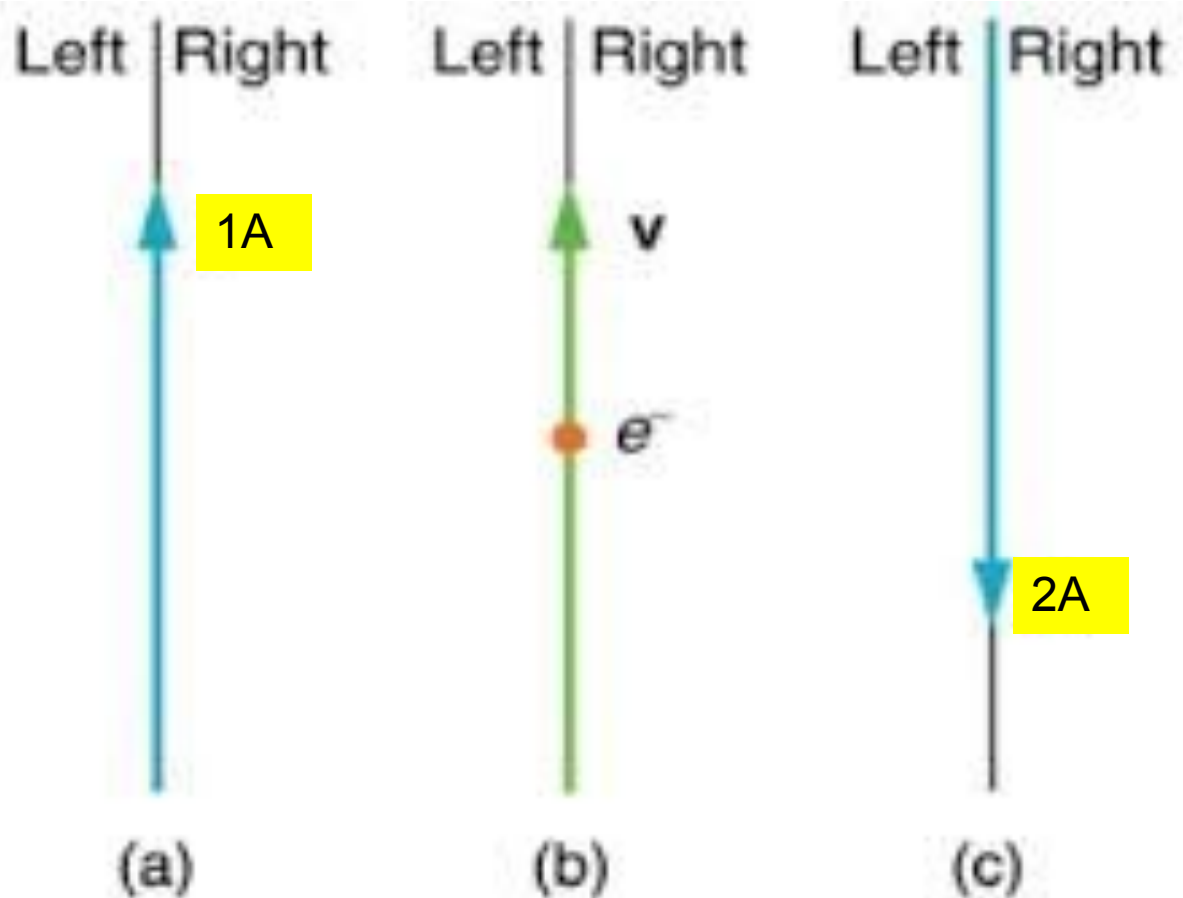
**An uncharged wire, containing free electrons, moves in the  $+x$ -direction, in a region with a magnetic field in the  $+z$ -direction. In which direction do the electrons move?**

- A) Up
- B) Down
- C) Left
- D) Right
- E) NOTA

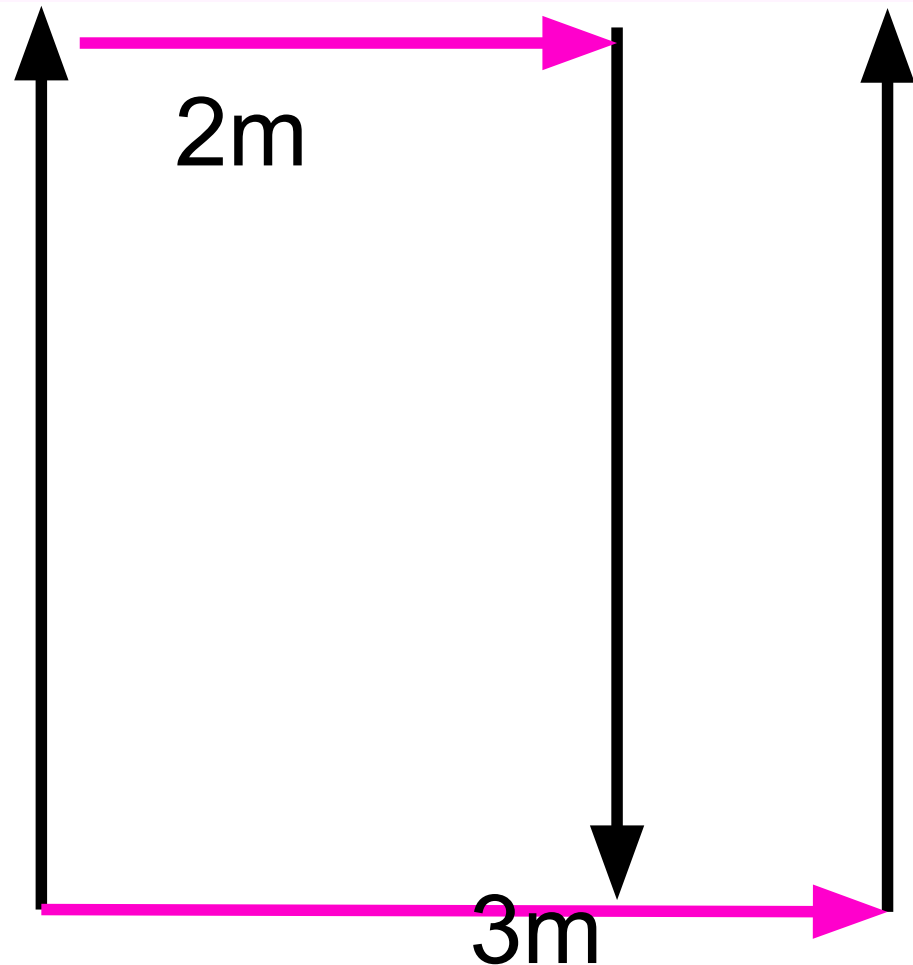


**In which direction is the center  $e^-$  deflected?**

- A) Left
- B) Right
- C) +z
- D) -z
- E) NOTA



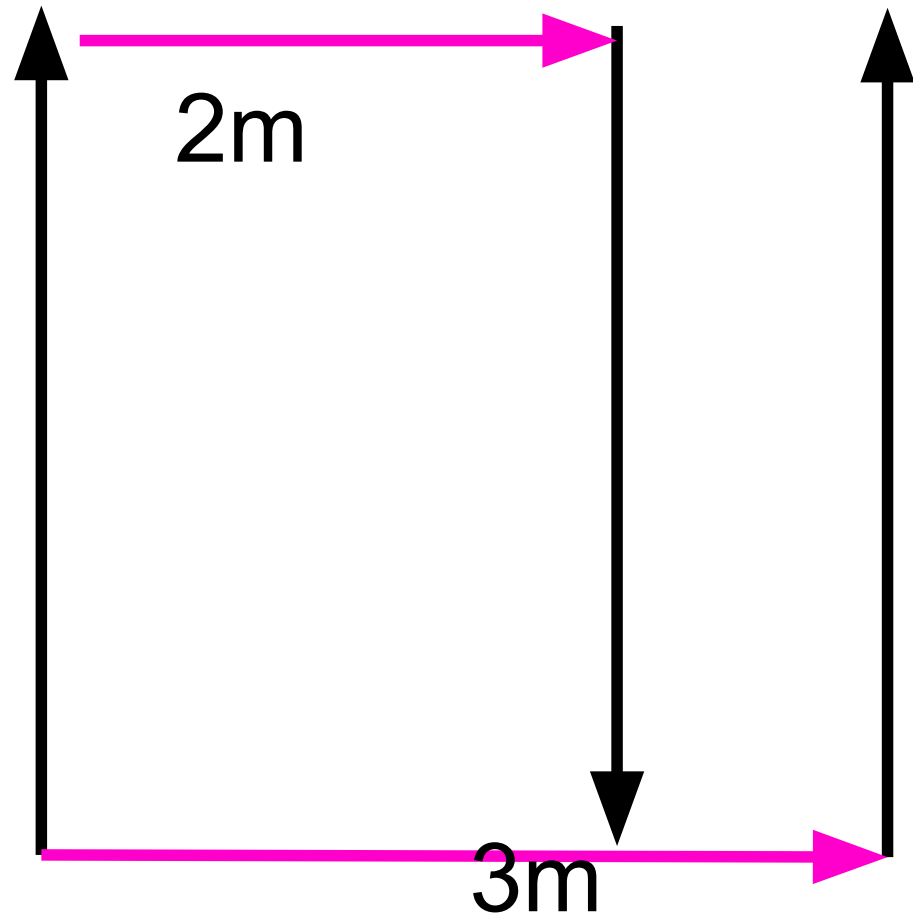
**$B_{\text{net}}$  at the location of the left wire ( $I_{\text{Left}}=1\text{A}$ ;  
 $I_{\text{Center}}=3\text{A}$ ;  $I_{\text{Right}}=4\text{A}$ ) is:**



- A)  $0.17\text{k}' (+z)$
- B)  $0.17\text{k}' (-z)$
- C)  $2.83\text{k}' (+z)$
- D)  $2.83\text{k}' (-z)$
- E) NOTA

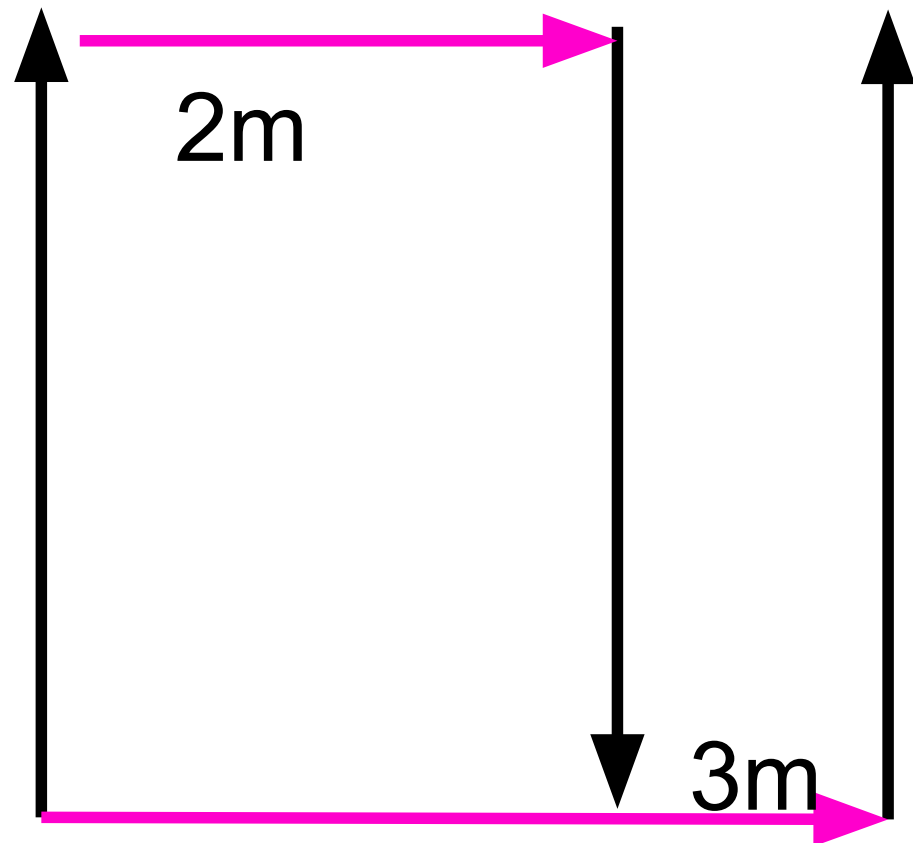
The directions of  $\mathbf{B}_{\text{net}}$  at the locations of the 3 wires ( $I_{\text{Left}}=1\text{A}$ ;  $I_{\text{Center}}=3\text{A}$ ;  $I_{\text{Right}}=4\text{A}$ ) are:

- A)  $-z, +z, +z$
- B)  $+z, -z, +z$
- C)  $+z, +z, -z$
- D)  $-z, -z, +z$
- E) NOTA



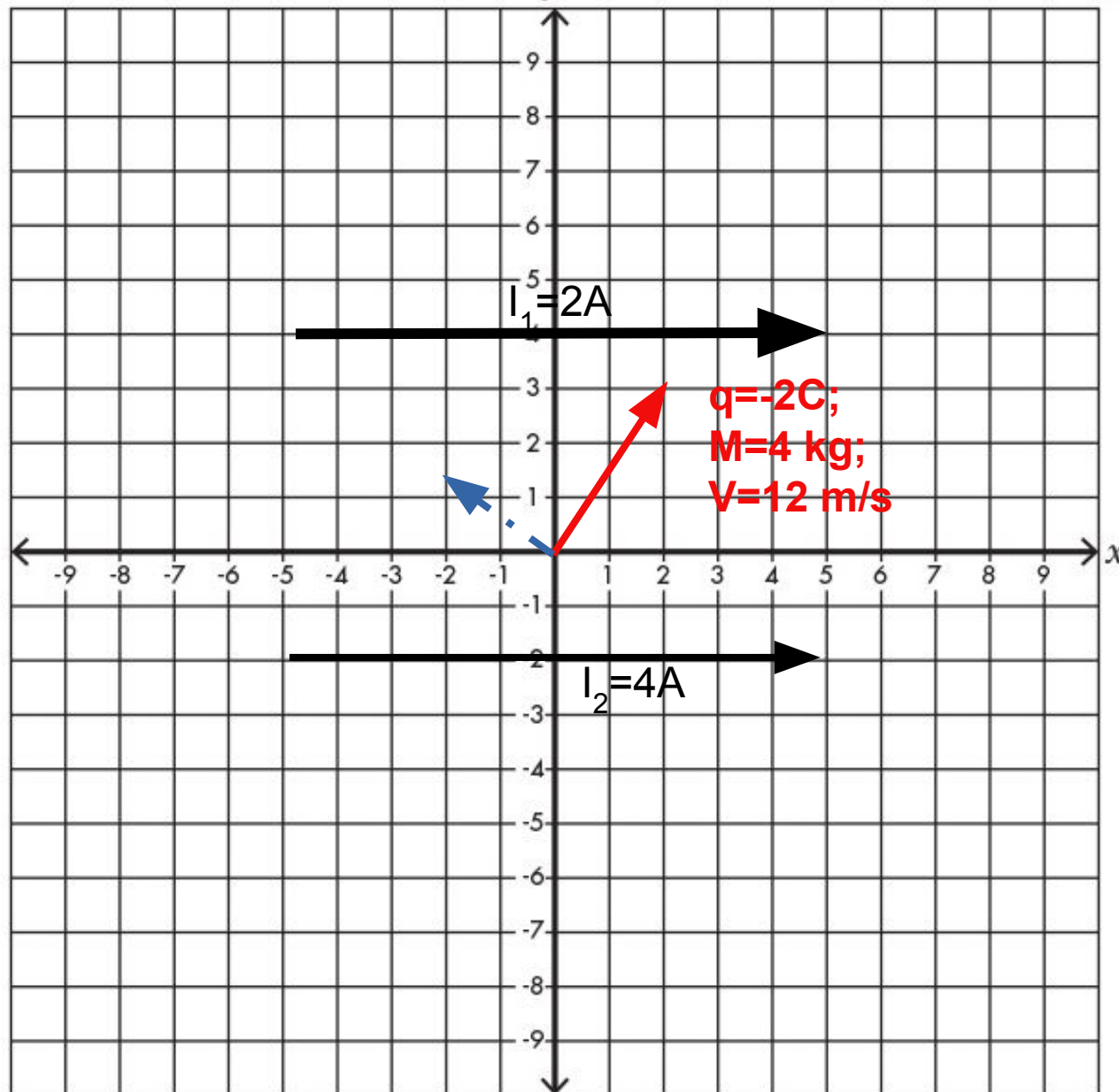
**The deflections of the 3 wires  
( $I_{\text{Left}}=1\text{A}$ ;  $I_{\text{Center}}=3\text{A}$ ;  $I_{\text{Right}}=4\text{A}$ ) are:**

- A) 0, L, R
- B) L, L, R
- C) L, 0, R
- D) R, L, 0
- E) NOTA



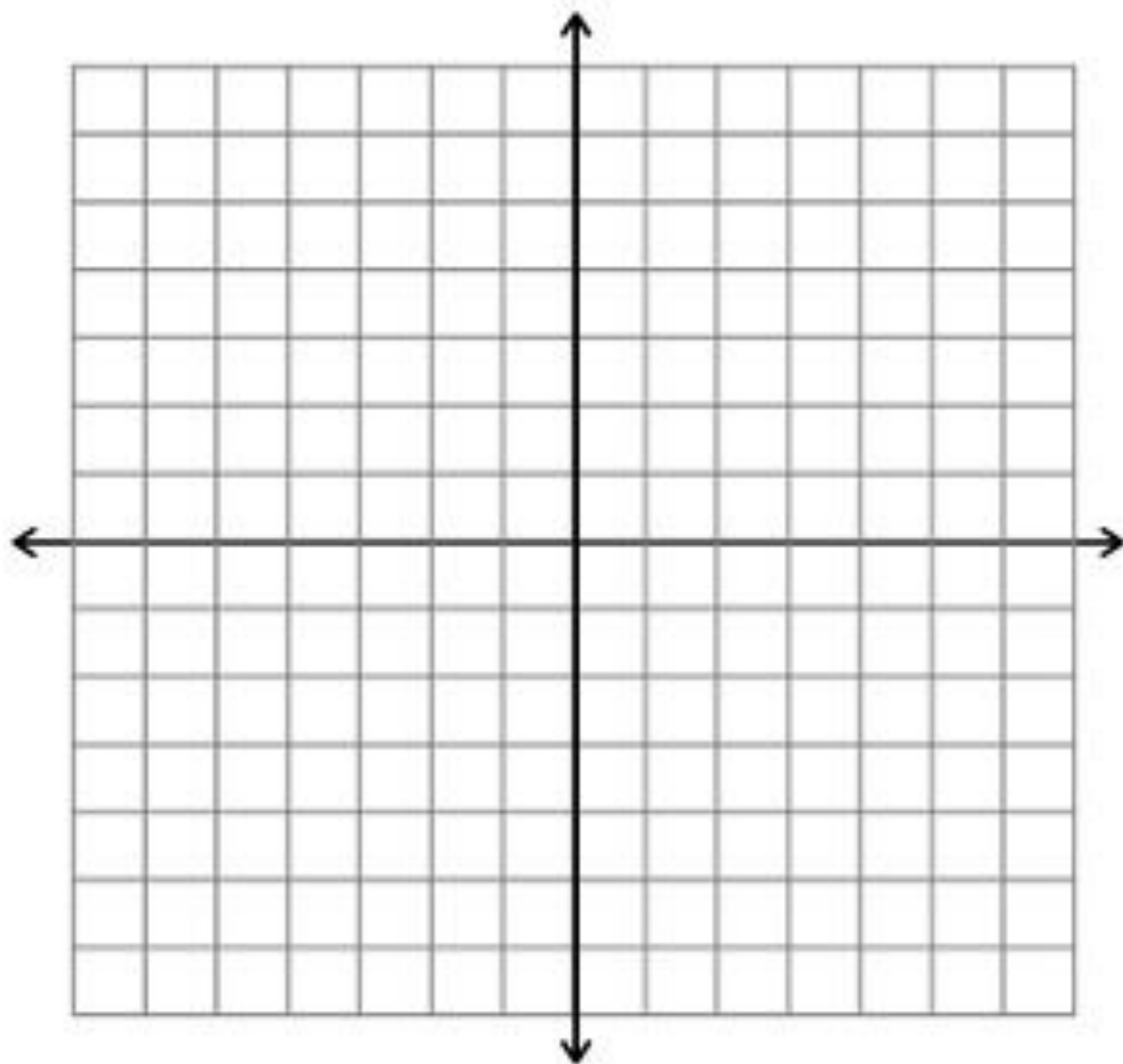


Want to find a) net magnetic field (via RHR-2) due to source currents, and then b) magnetic force (via RHR-1) on **q** at origin

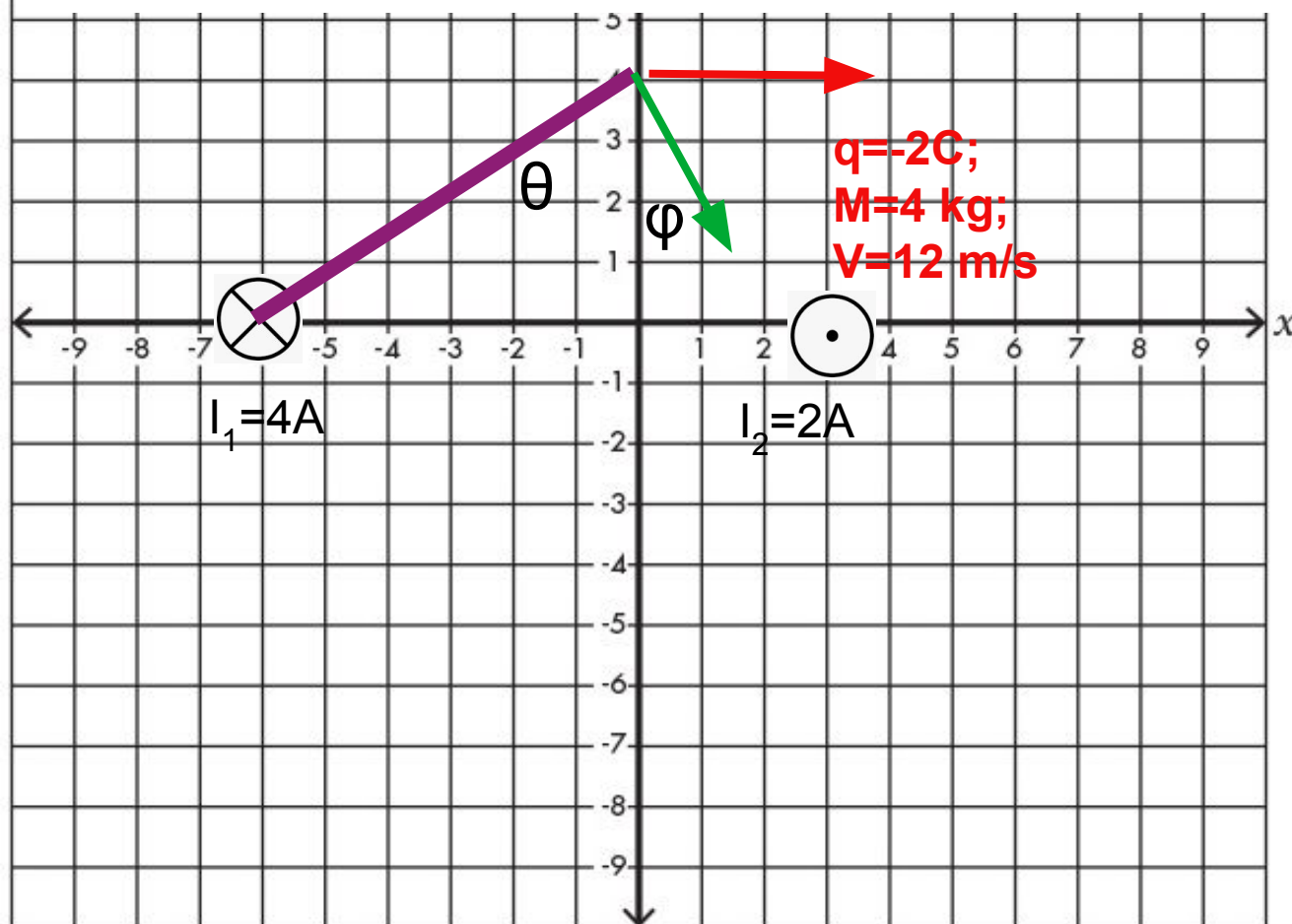


a) Add magnetic fields from two wires: from top wire,  $B_1 = k'(2A)/4m$  in -z direction (into page); bottom wire:  $B_2 = k'(4A)/2m$  in +z-direction (out-of-page), so net field =  $1.5k'$  Tesla in +z

b) magnetic force on **-2C** charge given by  $F_B = qvB\sin\theta$ , with  $\theta$  the angle between B and the velocity. In this case,  $90^\circ$  since  $B = B_z$  and velocity in xy-plane. So, find magnitude and direction in case where q is positive, then invert (since q actually negative).  
 $|F_B| = 2 \cdot 12 \cdot 1.5k' \sin(90) = 7.2e-6 = ma$ ,  
 so acceleration =  $1.8e-6 \text{ m/s}^2$   
 Direction as shown by blue arrow, since charge is negative

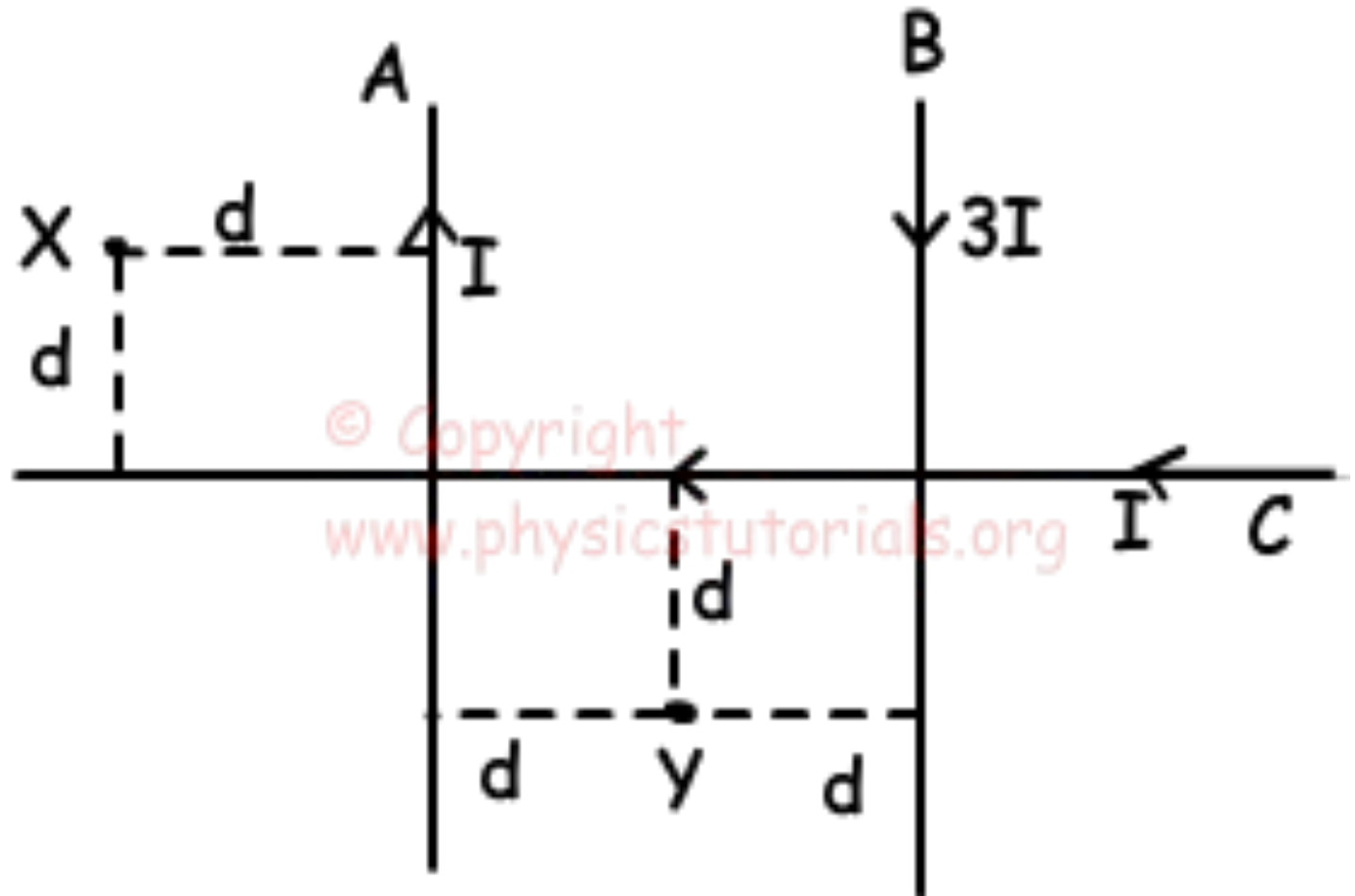


Want to find a) net magnetic field (via RHR-2) due to source currents, and then b) magnetic force (via RHR-1) on **q** at (0,4) a) Add magnetic fields from two wires: from top wire,  $B_1 = k'(4A)/r_1$ ;  $r_1 = \sqrt{36+16} = \sqrt{52}$ . To get direction, draw **line** from source to measurement point - line is radius of circular magnetic field line. Tangent would be 90 degrees from that line. In diagram,  $\theta$  is angle wrt y-axis, so  $\tan\theta = 6/4$ , or  $\theta = 56.3^\circ$ , B-field is at right angle wrt **line**, so  $\phi = 90 - 56.3 = 36.7^\circ$  - since, in this case velocity is directly along x-axis, the angle b/w B and v is also  $56.3^\circ$ ,  
b) magnetic force on **-2C** charge, due to 1<sup>st</sup> source only, is given by  $F_B = qvB\sin\theta$ , with  $\theta$  here =  $56.3^\circ$ . So  $F = 2C \cdot 12\text{m/s} \cdot k' \cdot 4A / \sqrt{52} \cdot \sin(56.3)$ ;  $k' = 2 \times 10^{-7}$ , so  $F = 2.21 \times 10^{-6}$  N; direction is into page!  
Similarly, add force from 2A source...



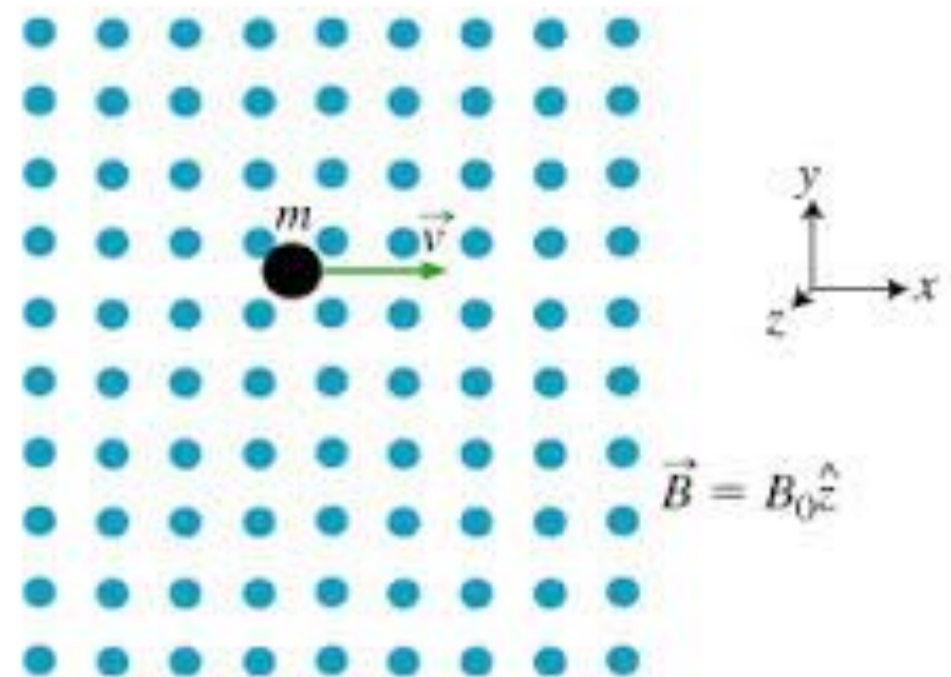
The ratio  $B(X)/B(Y)$  is:

- A) 1
- B)  $1/3$
- C) 3
- D) 9
- E) NOTA



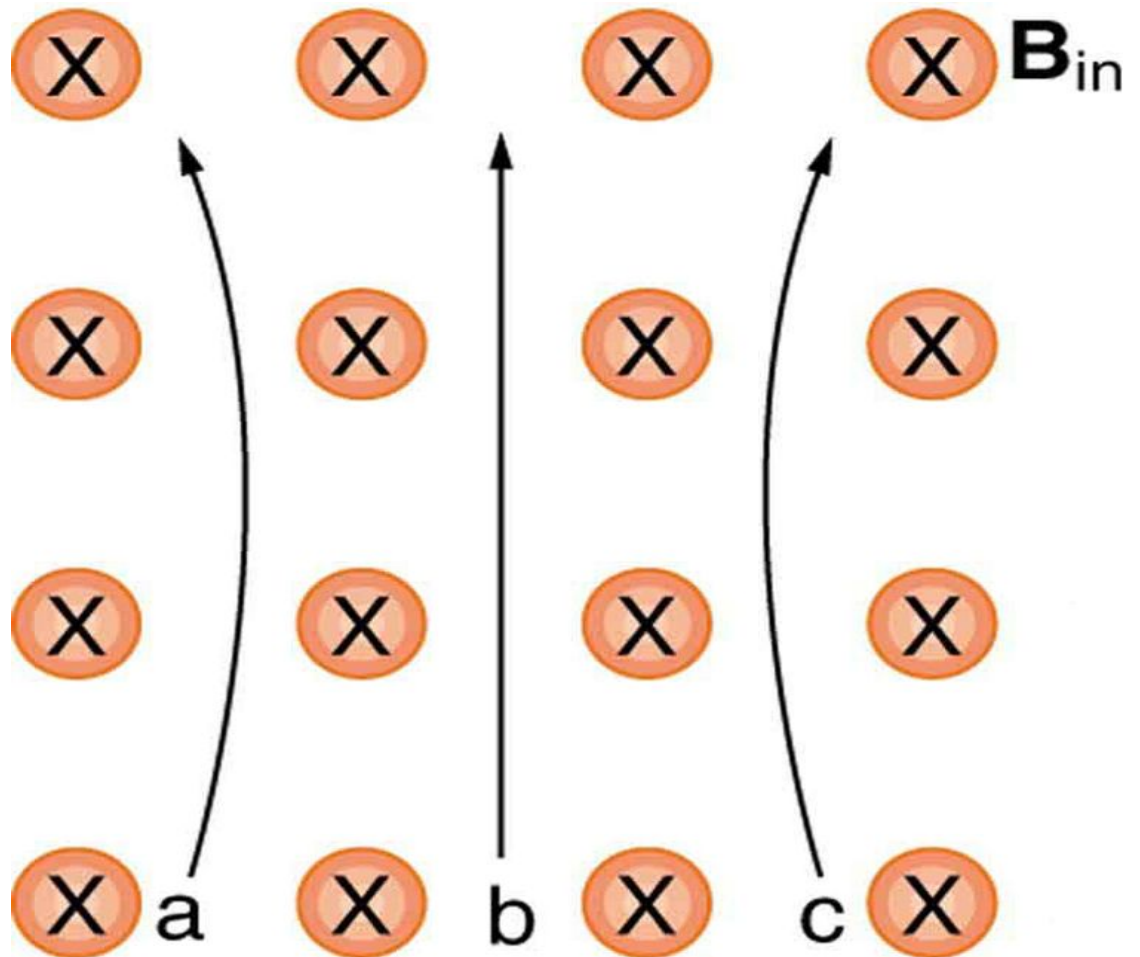
# An electron moves with initial velocity $v$ . As time proceeds...

- A) The  $e^-$  moves up, then continues straight
- B) The  $e^-$  moves down, then continues straight.
- C) The  $e^-$  moves up, then moves in a counter-clockwise circle
- D) The  $e^-$  moves down, then follows a circular motion clockwise
- E) NOTA

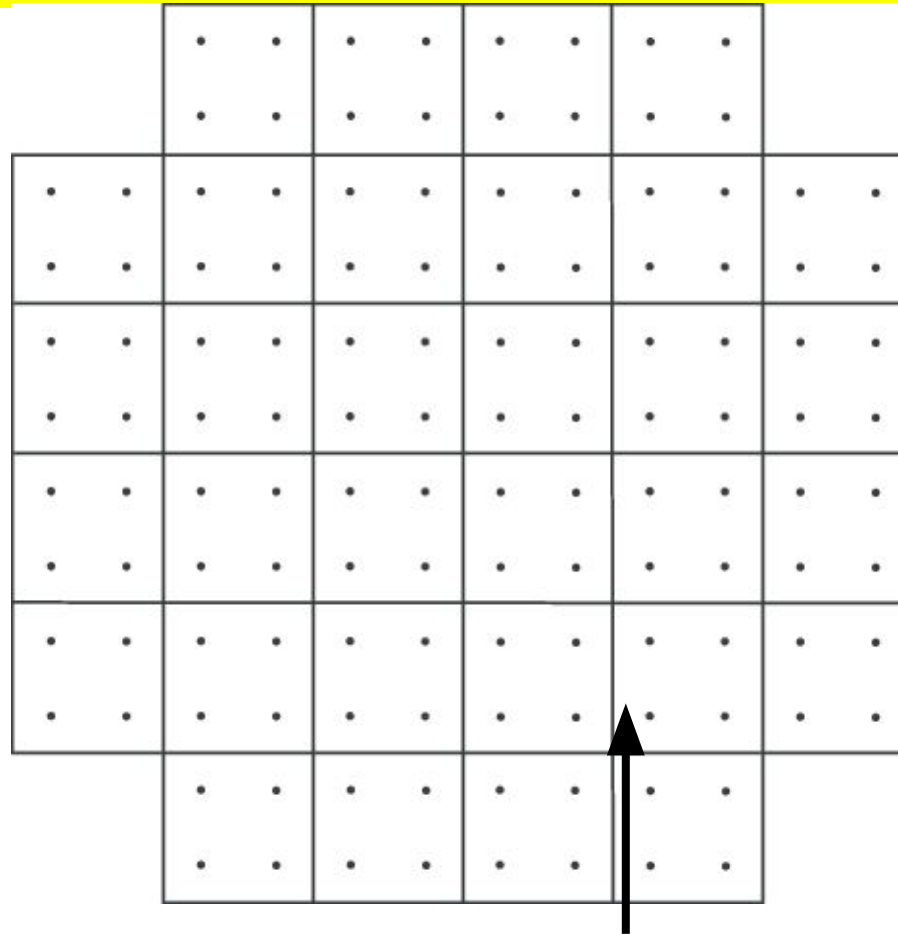


The charges of the particles shown are:

- A) +, 0, -
- B) -, 0, +
- C) -, 0, -
- D) +, 0, +
- E) NOTA



A particle of mass 2 kg and charge -2 Coulomb enters a magnetic field having magnitude 4 Tesla and direction +z. If it has an initial velocity 5 m/s in the +y-direction, the direction of its initial deflection is: A) +x B) -x C) +y D) -y E) NOTA

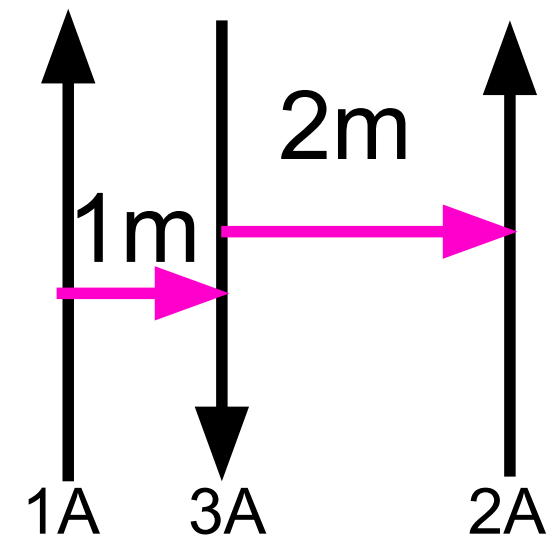


Assuming it is confined in the B-field, the radius of the particle's circular motion is:  
 A) 4 m B) 1.2 m C) 80 cm D) 40 cm E) NOTA

$$B_{2,3}(1A) = k'(3A/1m[-z] + 2A/3m[+z]) = 7k'/3[-z]; \text{ -x deflection (R)}$$

$$B_{1,2}(3A) = k'(1A/1m[-z] + 2A/2m[+z]) = 0 \text{ (no deflection)}$$

$$B_{1,3}(2A) = k'(1A/3m[-z] + 3A/2m[+z]) = 7k'/6[+z]; \text{ +x deflection (R)}$$



Orange arrow gives net deflection direction

Find  $B_{\text{tot}}$  at location of (2A) current:

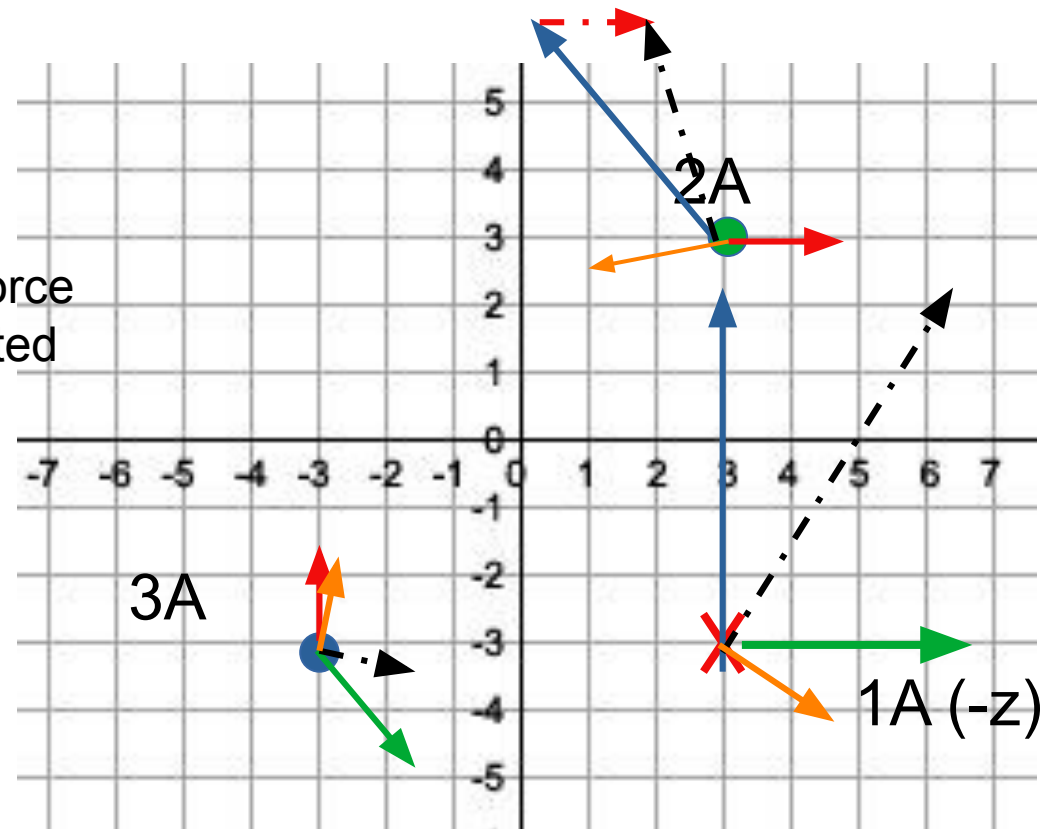
$$B_{1A}(2A) = k'/6 (+x);$$

$$B_{3A}(2A) = 3k'/8.48 \text{ (45}^\circ \text{ wrt +y);}$$

$B_{3A,x}(2A) = -0.25k'$ ;  $B_{3A,y}(2A) = +0.25k'$ ; resultant indicated by black dashed arrow. Deflection force indicated by orange arrow (RHR-1). 2A attracted to 3A, and repelled from 1A.





$$B_{\text{tot}}(3A) = 2k'/8.48 \text{ (-45}^\circ \text{ wrt x)} + k'/6(+y)$$

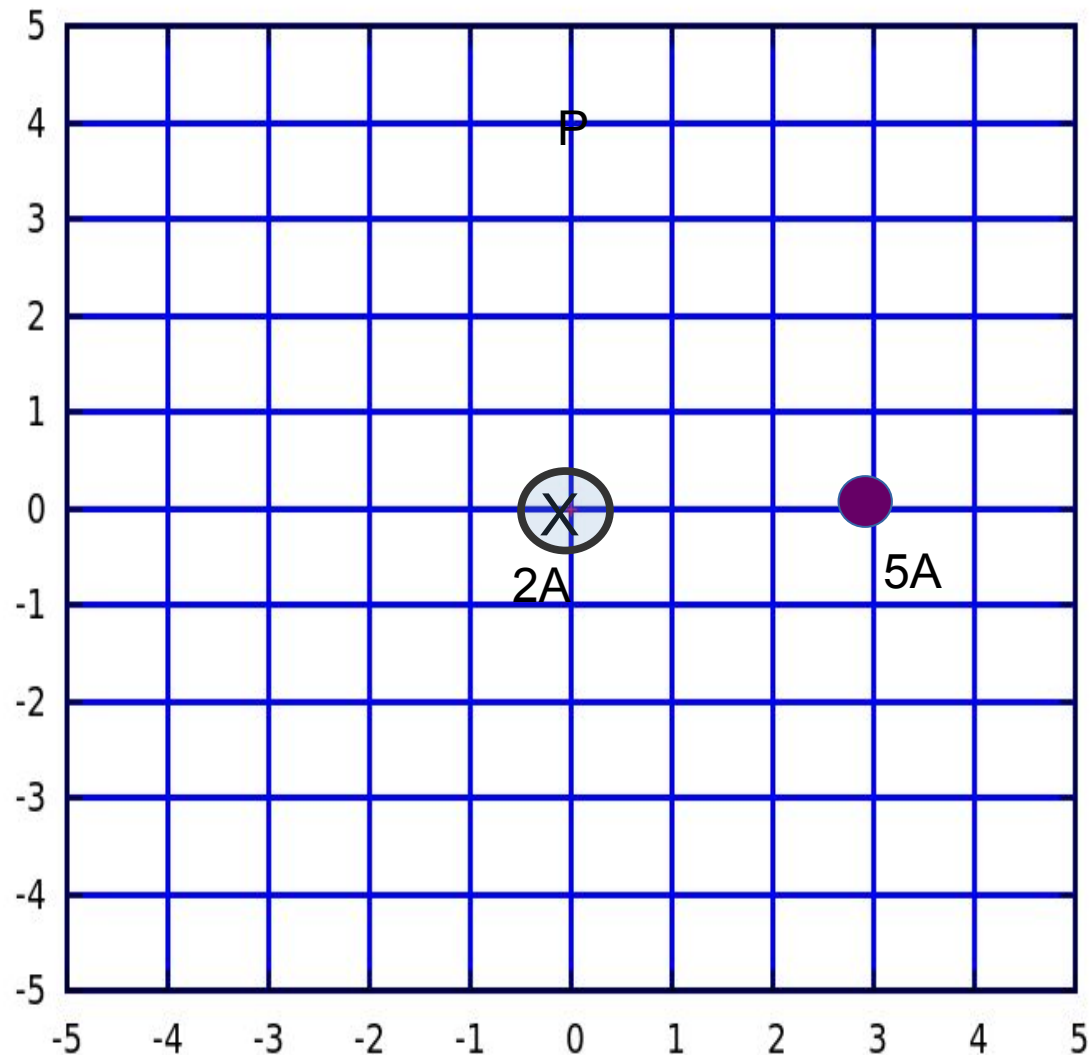
$$B_{\text{tot}}(1A) = 2k'/6(+x) + 3k'/6(+y)$$





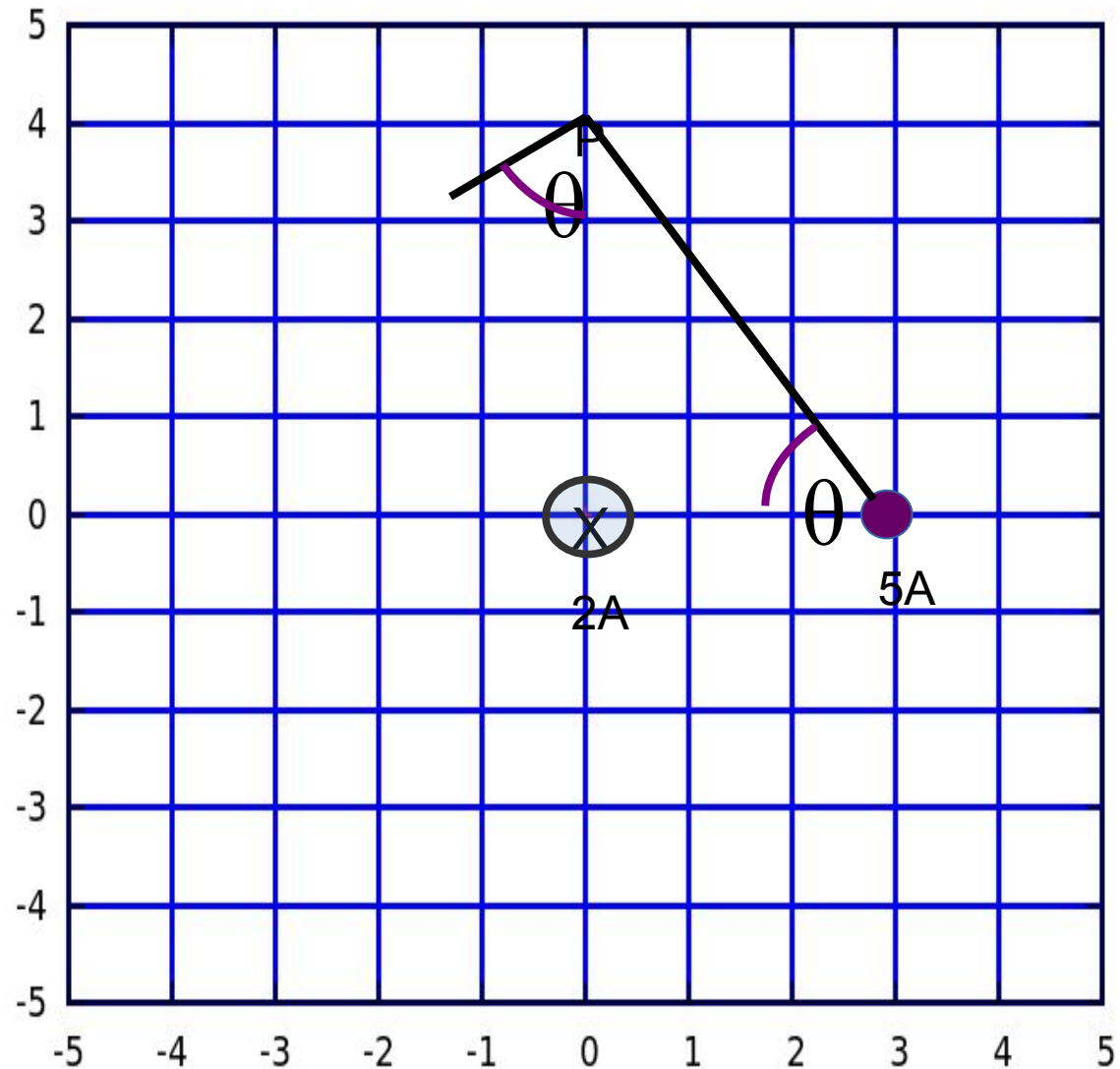
Given the current-carrying wires (2A in -z-direction at (0,0) and 5A in +z-direction at (3,0)) shown.  $B_{\text{net}}$  at (x,y)=(0,4) at P is in which direction?

- A) 
- B) 
- C) 
- D) 



**A wire carrying a current of 2A in the -z direction is now placed at (0,4)? In which direction does it deflect?**

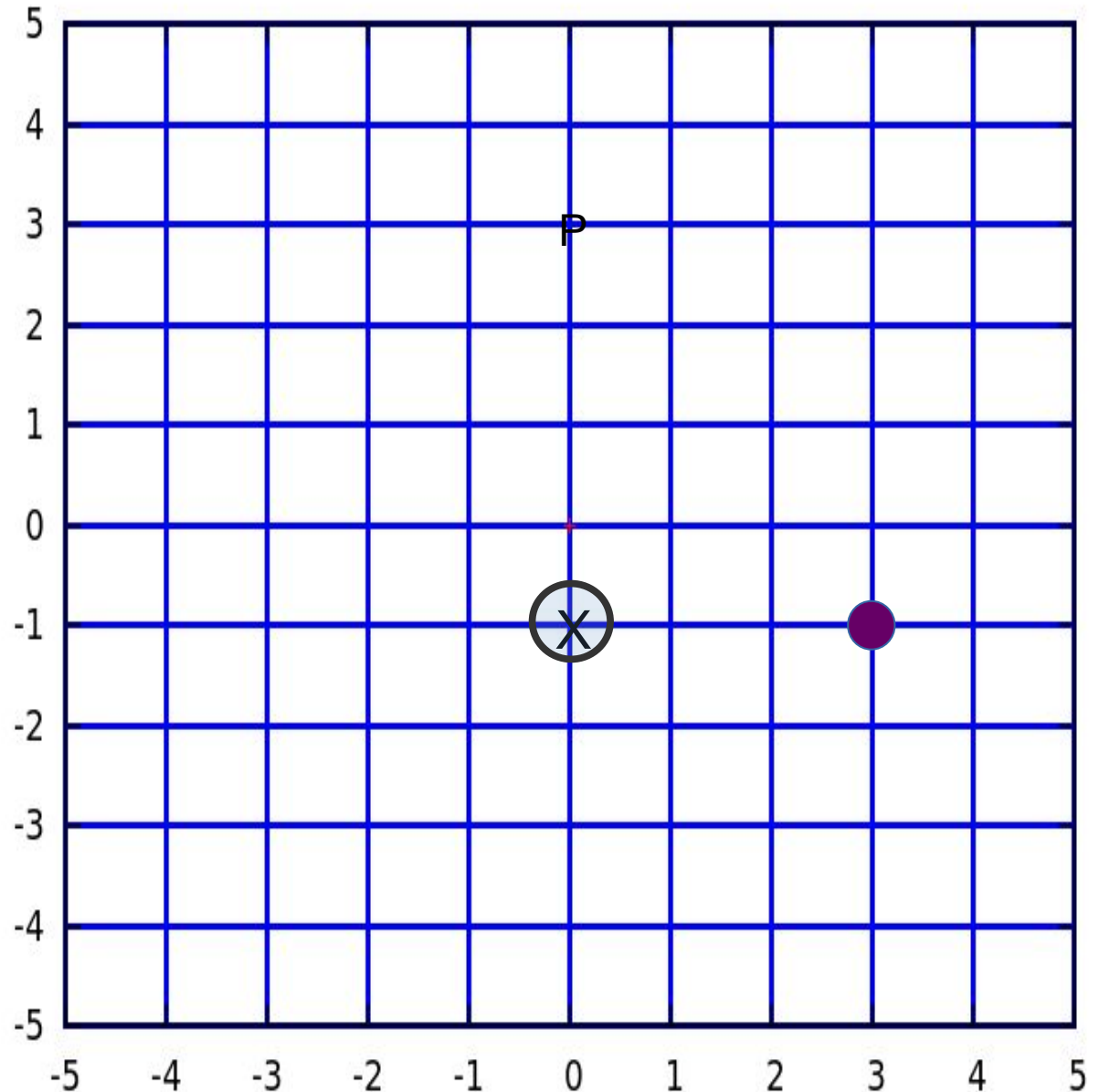
- A) ↗
- B) ↘
- C) ↙
- D) ↖



What is the value of  $B_x$  at  $(x,y)=(0,4)$ ?

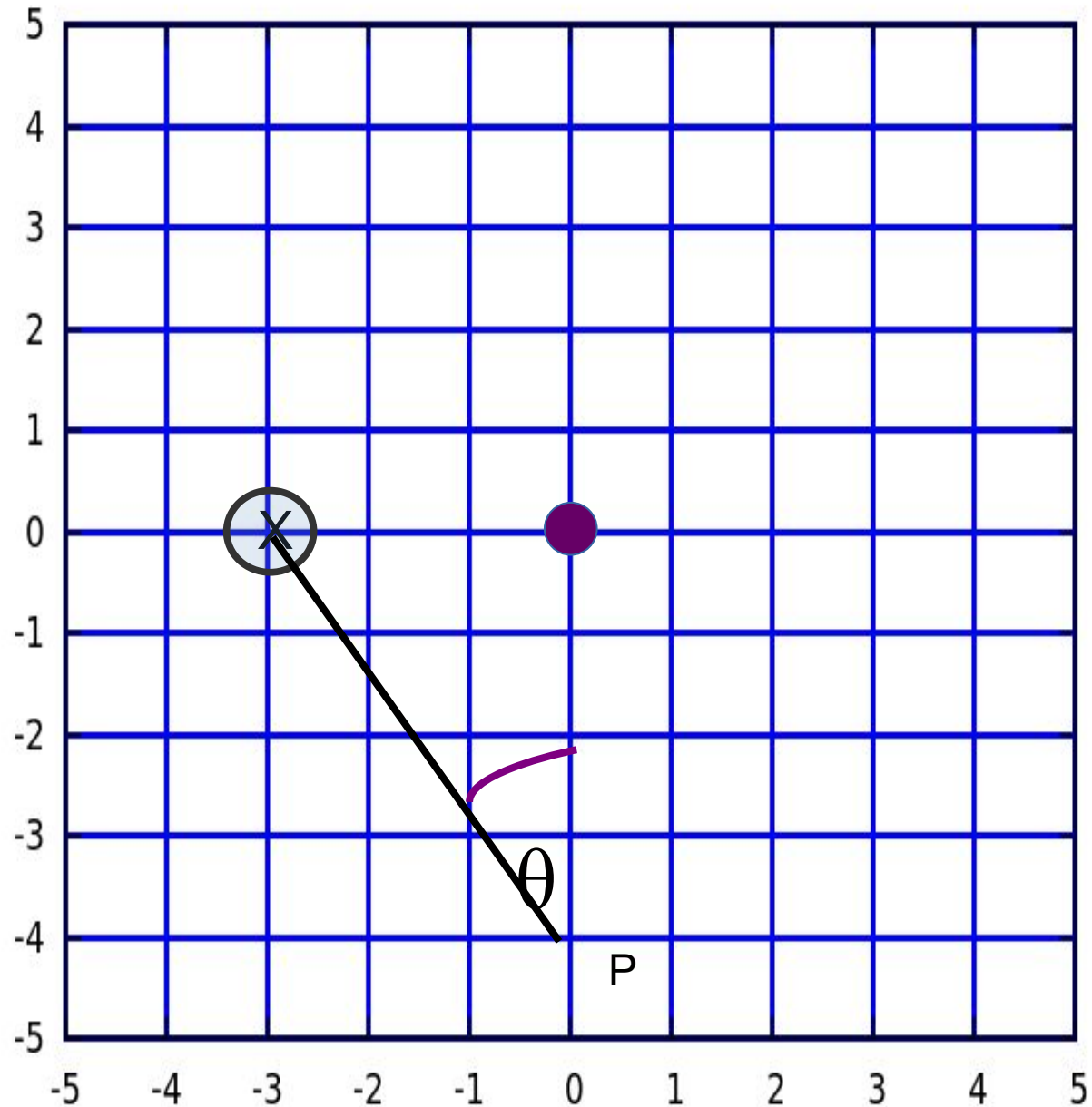
- A)  $0.8k' T$
- B)  $-0.1k' T$
- C)  $-0.3k' T$
- D)  $0.6k' T$
- E) NOTA

What is the value of  $B_y$  at  $(x,y)=(0,4)$ ?



What is  $B_{y,\text{net}}$  at the point P, if the left wire has 3A (-z) and the right wire has 4A (+z)

- A)  $-0.461k'$
- B)  $0.461k'$
- C)  $0.637k'$
- D)  $-0.637k'$
- E) NOTA



**What is  $B_{x,\text{net}}$  at the point P, if the left wire has 3A (-z) and the right wire has 4A (+z)**

- A)  $-0.461\text{k}'$
- B)  $0.461\text{k}'$
- C)  $0.637\text{k}'$
- D)  $-0.637\text{k}'$
- E) NOTA

## Faraday's Law:

Induced Voltage from changing external magnetic field  $B_{\text{ext}}$  within a loop  
(“EMF”)=Change in magnetic ‘flux’  $\Phi$  (#field lines penetrating loop) with time

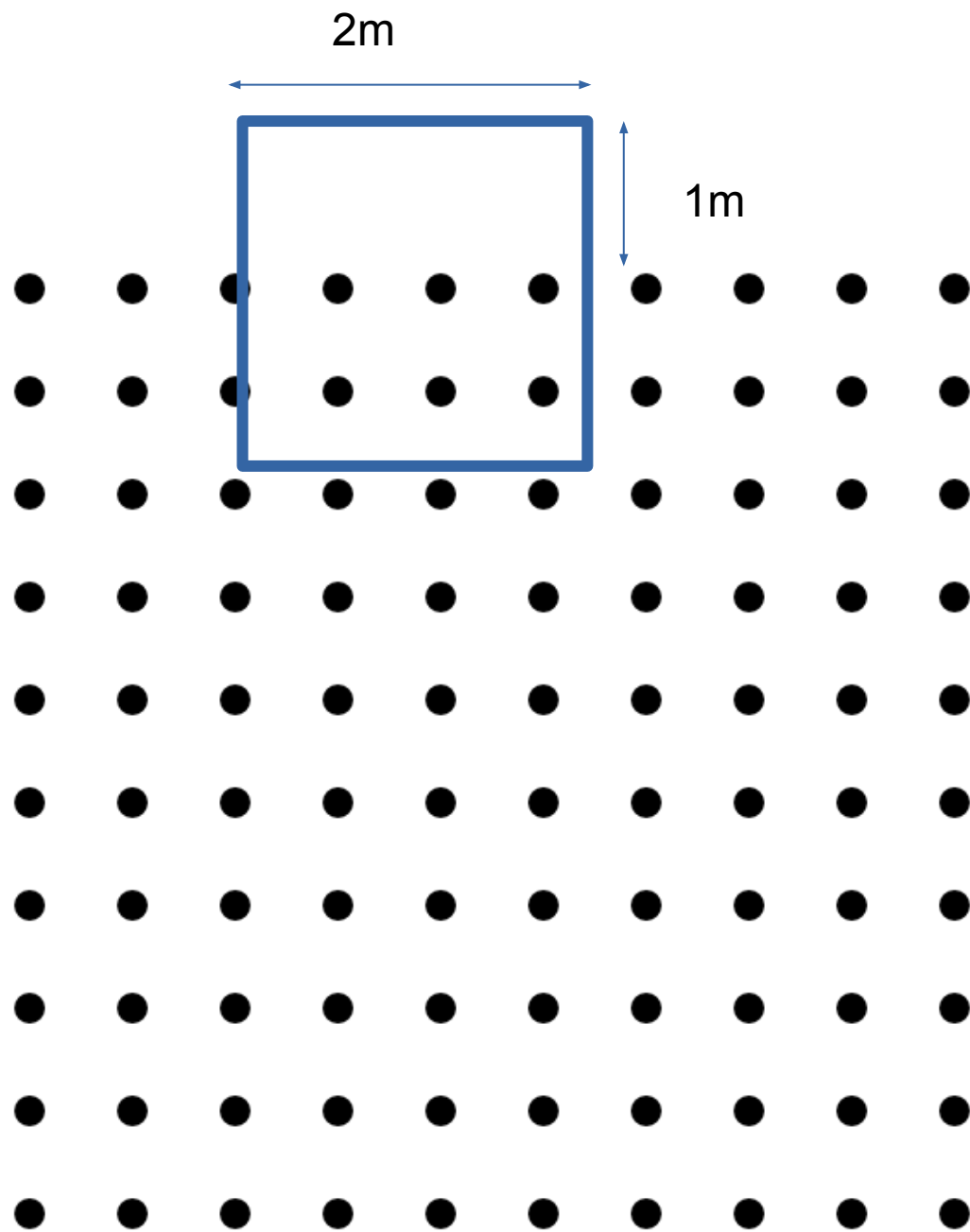
Magnetic flux  $\Phi = |B_{\text{ext}}| |A_{\text{loop}}| \cos \theta_{B,A}$ ;

$V = -\Delta\Phi/\Delta t$ ; (-: Lenz' Law: Induced current in loop creates  $B_{\text{induced}}$  which tries to negate change in  $B_{\text{ext}}$  )

If induced current= $I$  and length of side of loop= $d$ ,

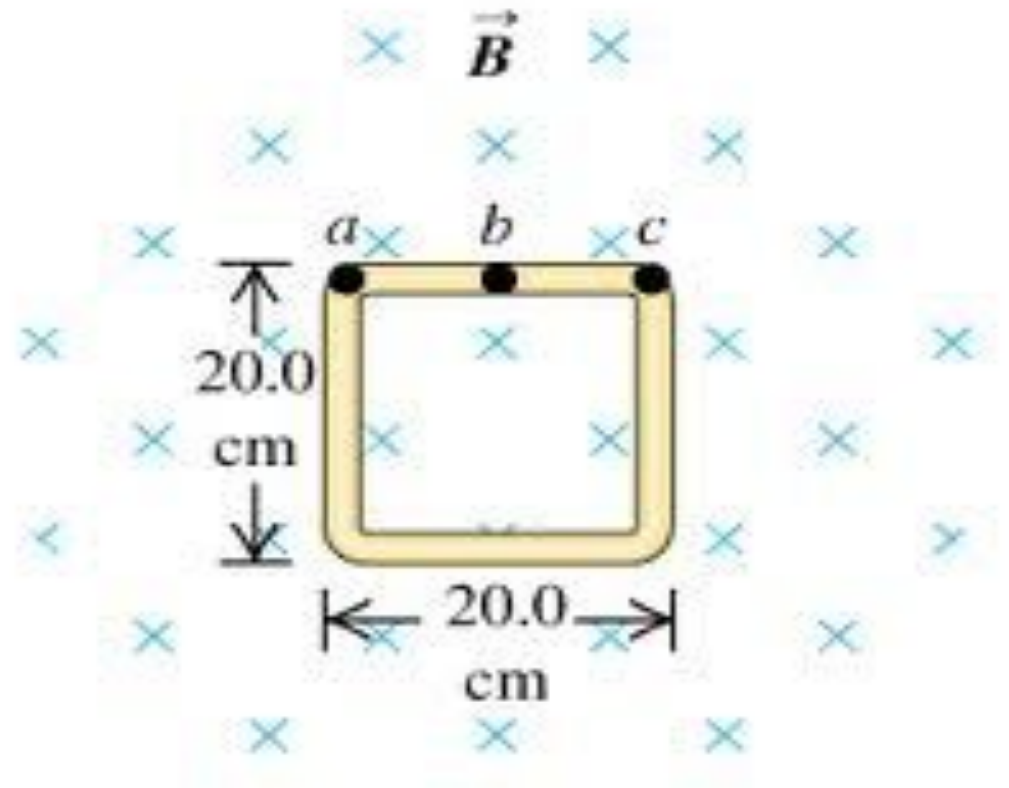
Magnetic deflection force from  $B_{\text{ext}}$  on  $I$  given by  $F = IdB$

$V = -L\Delta I/\Delta t$  is voltage across inductor



If  $B$  is decreasing at a rate of  $2\text{T/s}$ , and loop resistivity is  $\rho=4\Omega/\text{cm}$ , what is the current induced in the loop?

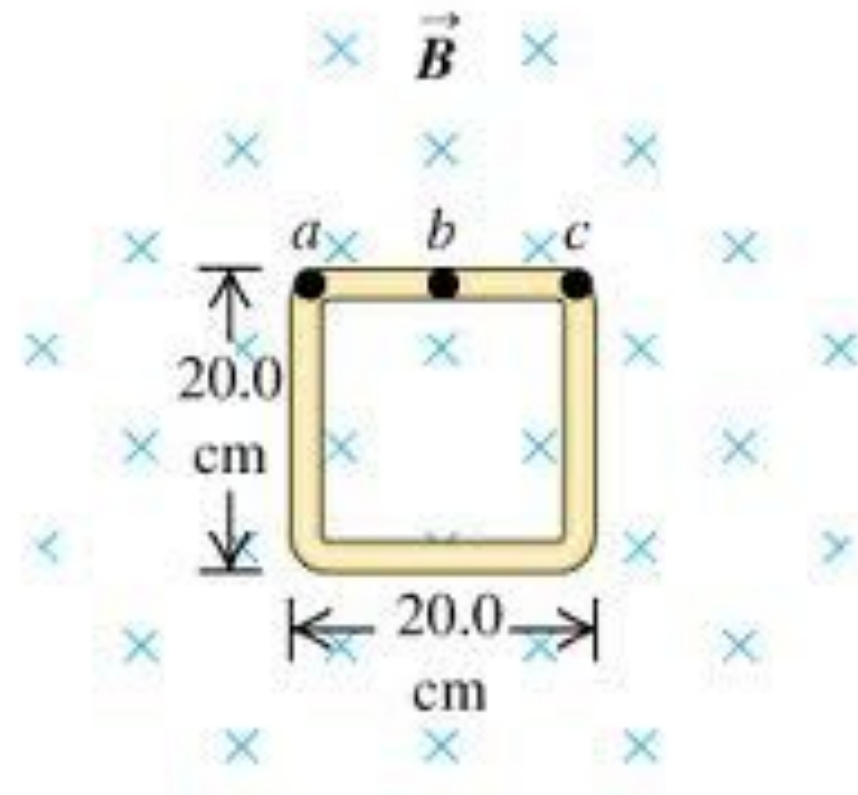
- A)  $2.5\text{ A}$ , counter-clockwise
- B)  $0.25\text{ mA}$ , clockwise
- C)  $1\text{ A}$ , c.
- D)  $4\text{ mA}$ , c.c.
- E) NOTA





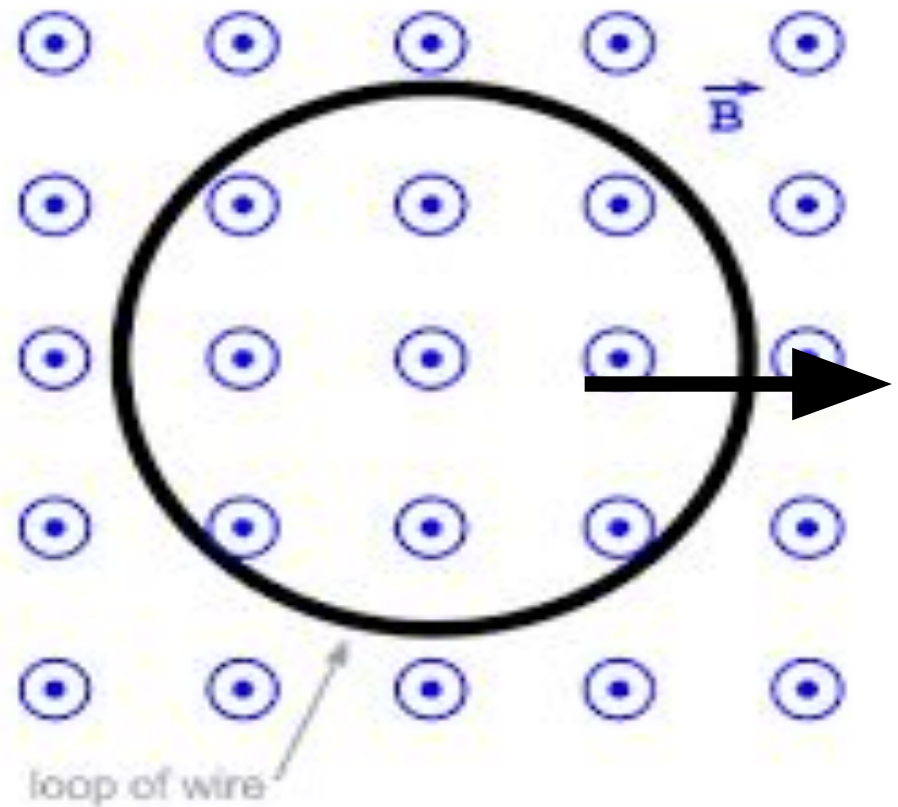
**In the previous problem, as the field decreases, the net  $F=ILB$  makes the loop**

- A) Spin in the plane of the page about its center
- B) Remain stationary
- C) Rotate about the y-axis
- D) Rotate about the x-axis
- E) NOTA



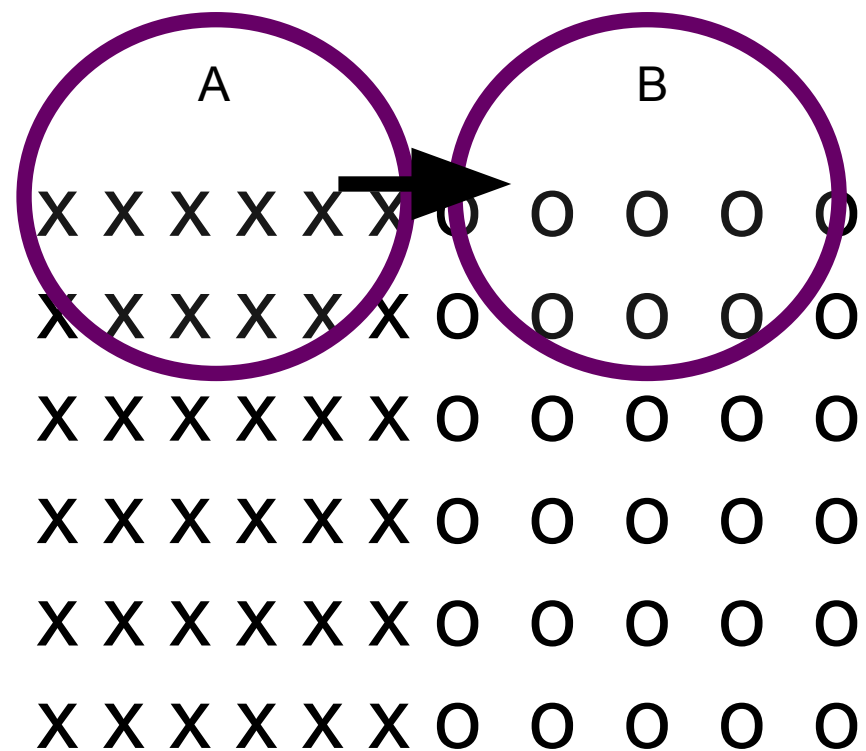
**As the loop moves out of the B-field region,  
what current direction is induced?**

- A) Clockwise
- B) Counter-clockwise
- C) Zero



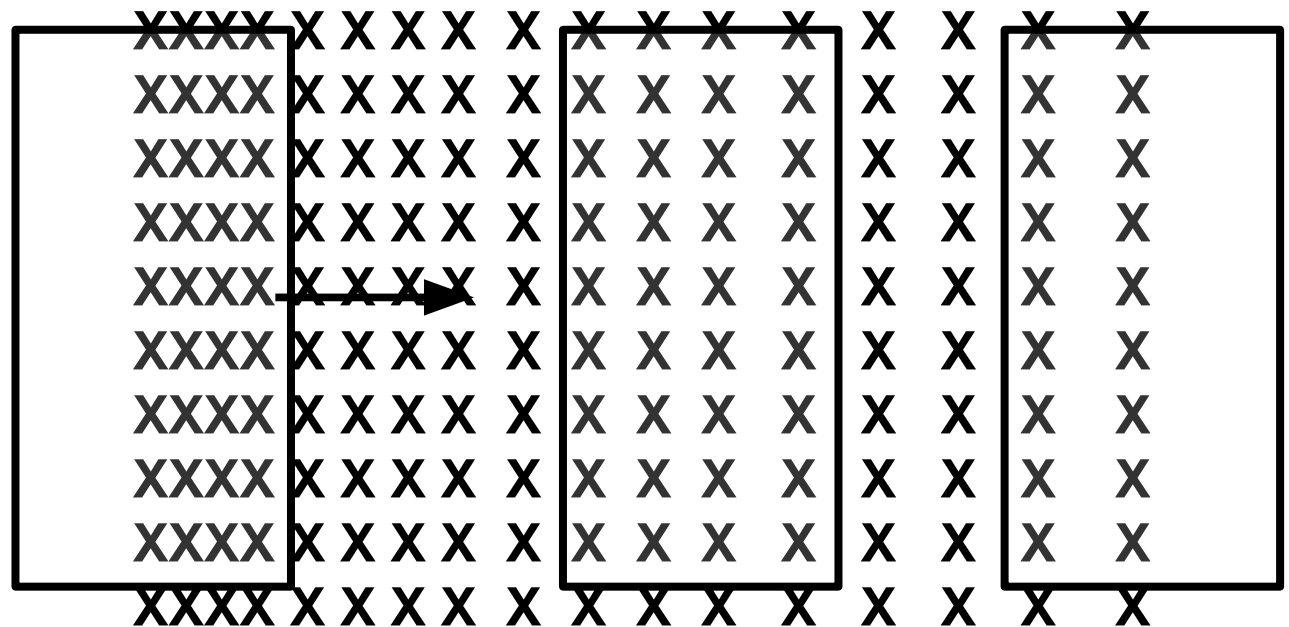
What is  $V_{\text{induced}}$  in the 1 m-radius circular loop below, which moves from  $A \rightarrow B$  in 4 s, given that  $B_A = 3\text{T}$  (-z) and  $B_B = 2\text{T}$  (+z)?

- A)  $5\pi/4$  V
- B)  $5\pi/8$  V
- C)  $3\pi/4$  V
- D)  $2\pi/4$  V
- E) NOTA



What is direction of  $I_{\text{induced}}$  as the loop is centered at the three points in the field region shown? Assume it is moving at constant  $v$ , and that the magnitude of  $B$  is indicated by the density of the field lines.

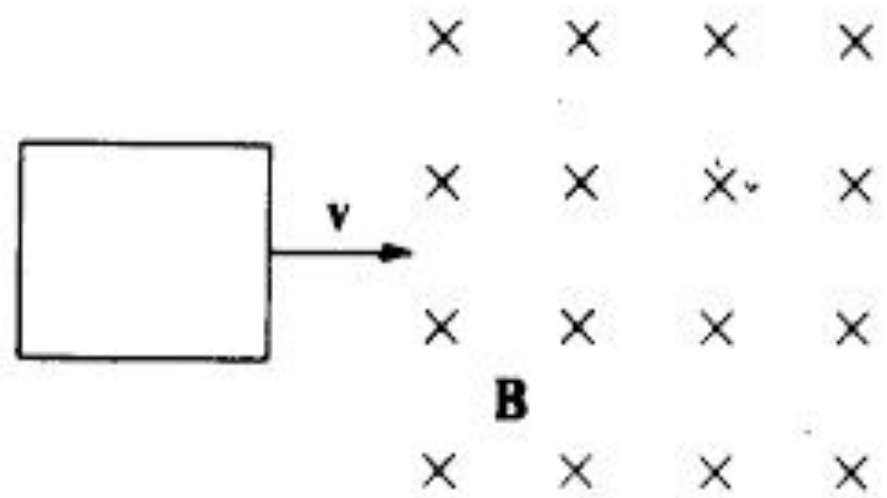
- A) C, CC, C
- B) CC, C, C
- C) C, C, C
- D) CC, CC, CC
- E) NOTA



In the previous problem, when the loop is fully immersed in the field region, the net force is to the A) left, B) right, C) zero

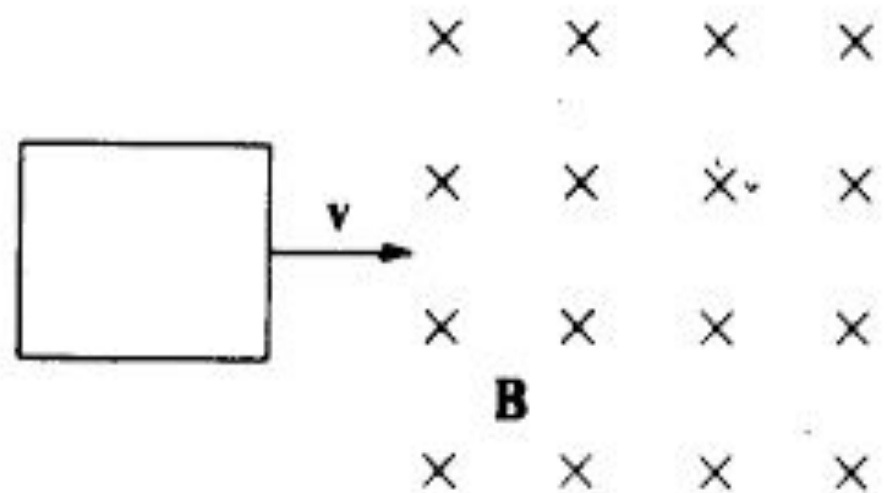
A 1m x 1m loop is dragged across the  $B=2\text{T}$  ( $-z$ ) region shown, with  $v=2\text{m/s}$ . Assuming the loop enters the region at  $t=0$ , what current directions are induced in the loop at times b/w  $0 \rightarrow 0.5\text{ s}$ ,  $1.5 \rightarrow 2.5\text{ s}$  and  $4 \rightarrow 4.5\text{ s}$ ?

- A) Clockwise, cc,  
clockwise
- B) Cc, 0, cc
- C) Cc, 0, clockwise
- D) Clockwise, 0, cc
- E) NOTA



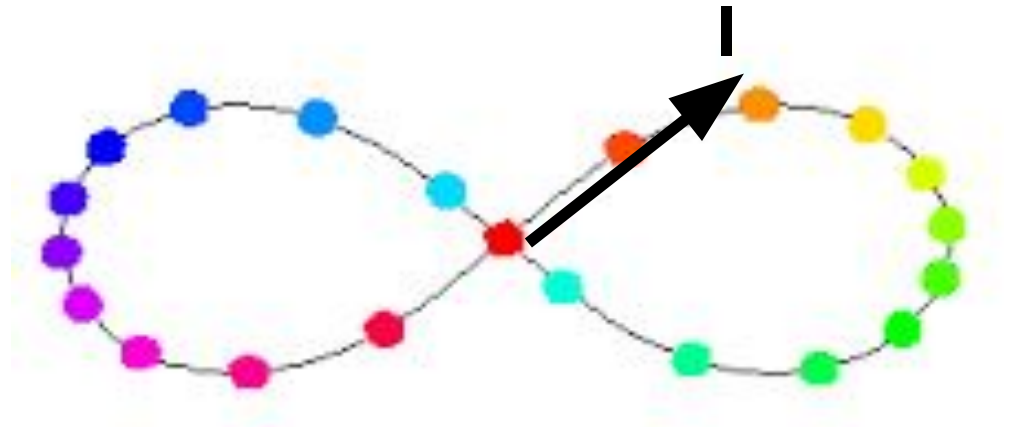
If the loop has a resistance of  $2\Omega$  and is 1 m on a side, the current induced from  $t=0 \rightarrow t=0.5$  s is:

- A) 0.25 A
- B) 0.5 A
- C) 1 A
- D) 2 A
- E) NOTA



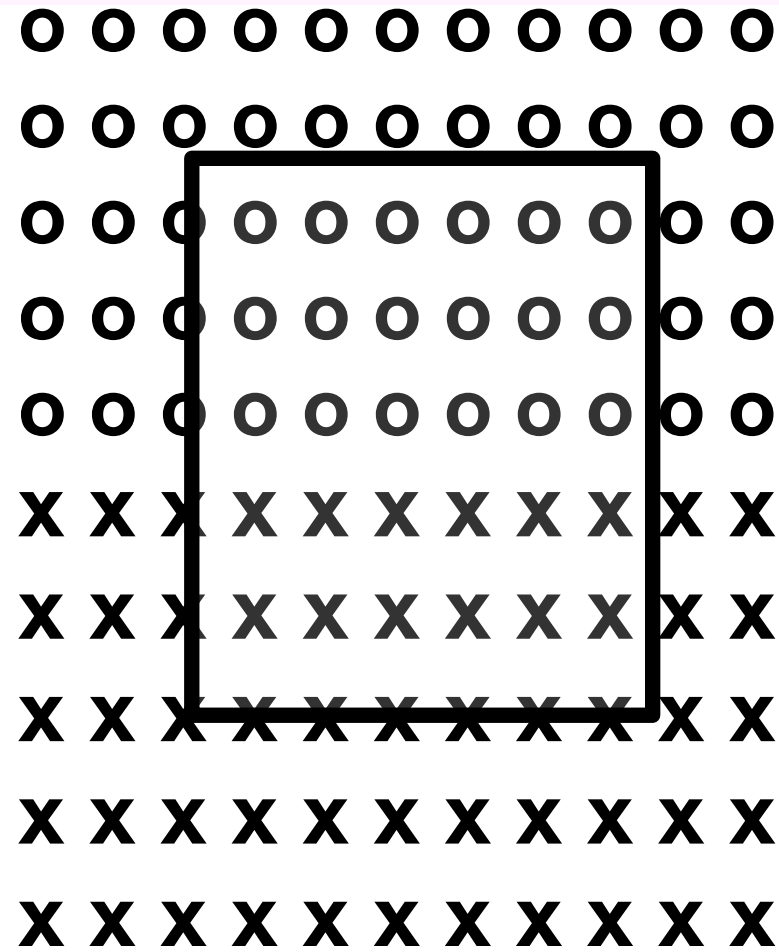
**Current follows a figure 8; the B fields at the centers of the left and right 'loops' point along, respectively:**

- A)  $+z, +z$
- B)  $-z, -z$
- C)  $+z, -z$
- D)  $-z, +z$
- E) NOTA



**A loop is centered on a B-field with  $B_z = +1$  T on top and  $B_z = -1$  T below. Which of the following result in current induced in the loop?**

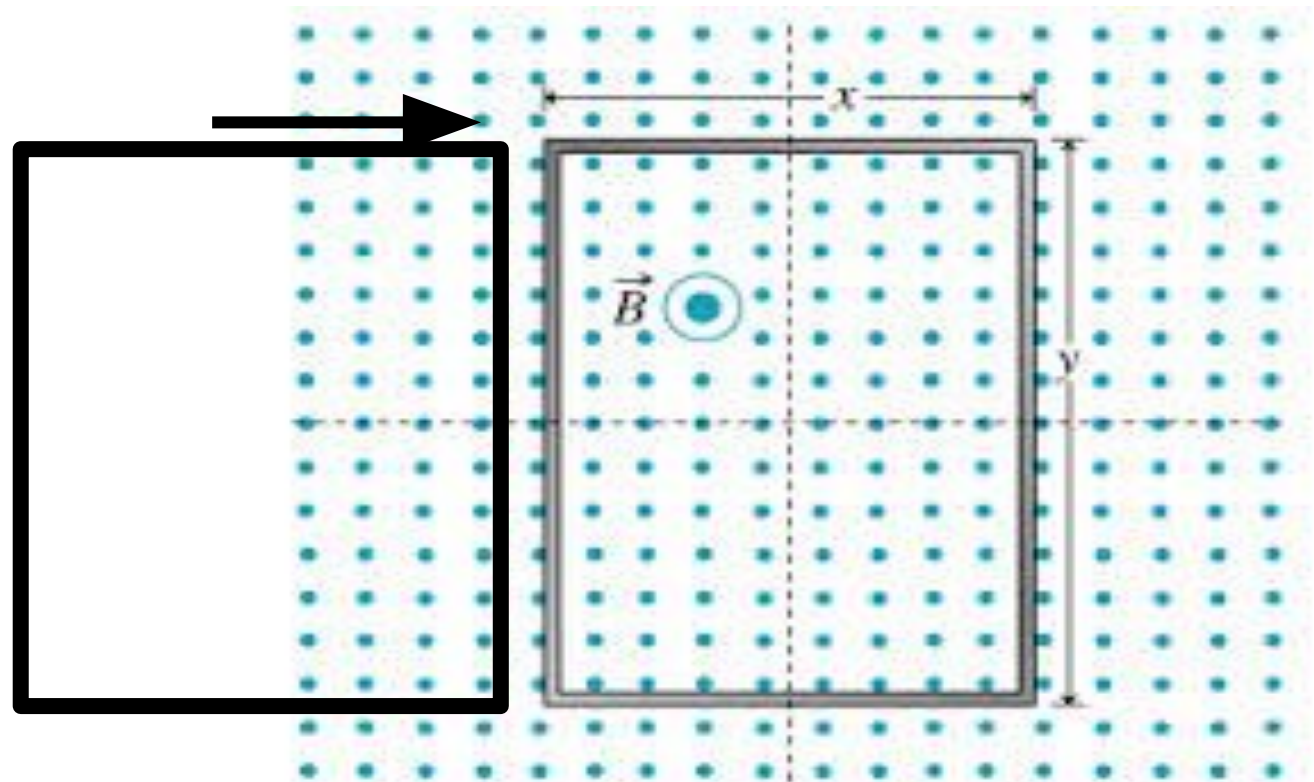
- A) Rotating about y-
- B) Moving along x-
- C) Moving along y-
- D) Increasing the magnitude of the B-field in both the upper and lower regions
- E) None of the above





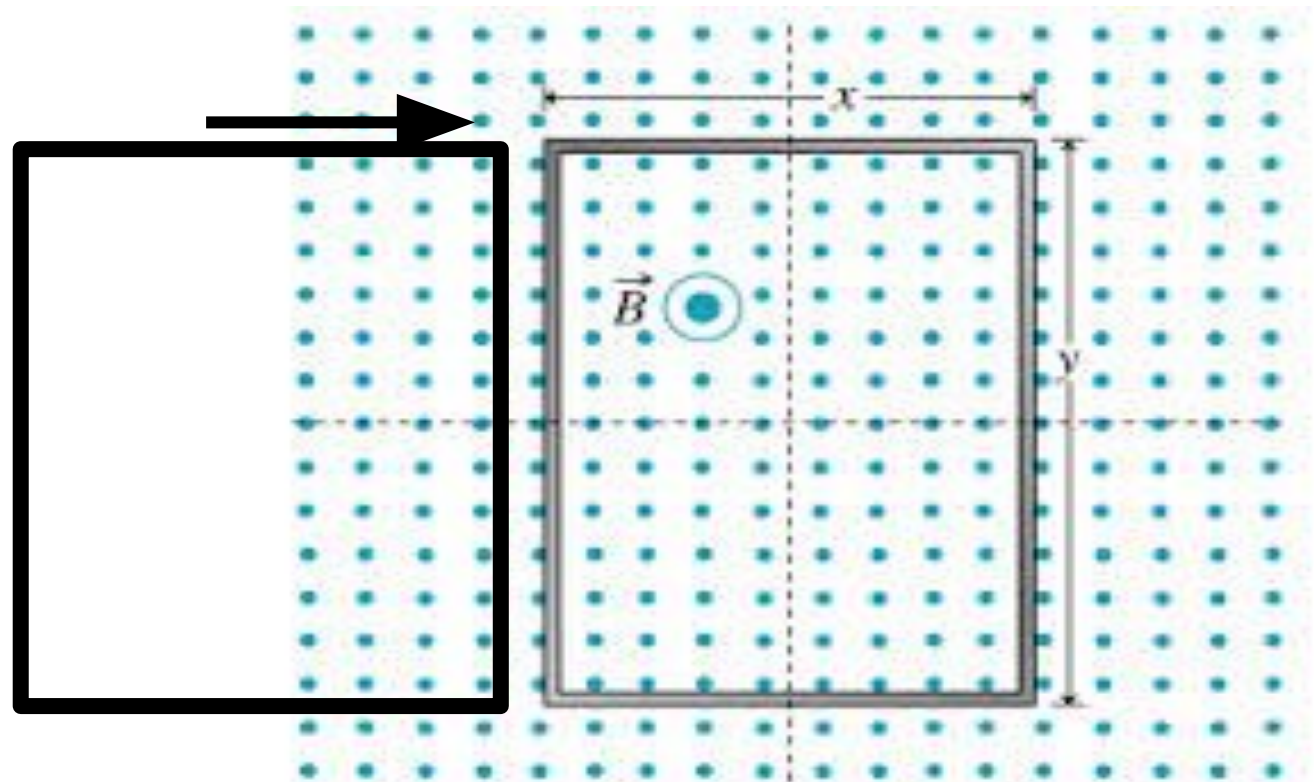
Imagine the loop is accelerating through the magnetic field region. At the times when the loop is entering, centered on, and exiting the external magnetic field, the direction of the induced current is:

- A) C, 0, CC
- B) CC, 0, CC
- C) C, C, CC
- D) CC, 0, C
- E) NOTA



A square/rectangular loop having size 3 meters x 4 meters and resistivity 4 Ohm/meter, moves in the +x direction at  $v=+2$  meters/second, through a region with a magnetic field having magnitude 4 T and pointing in the (+z) direction and an extent of 10 meters. The amount of time that it takes for the loop to fully enter the region is:

- A) 1.5 seconds
- B) 2 seconds
- C) 3 seconds
- D) 4 seconds
- E) NOTA



A square/rectangular loop having size 3 meters x 4 meters and resistivity 4 Ohm/meter, moves in the +x direction at  $v=+2$  meters/second, through a region with a magnetic field having magnitude 4 T and pointing in the (+z) direction and an extent of 10 meters. The current induced in the loop, during the time it enters the magnetic field region, is:

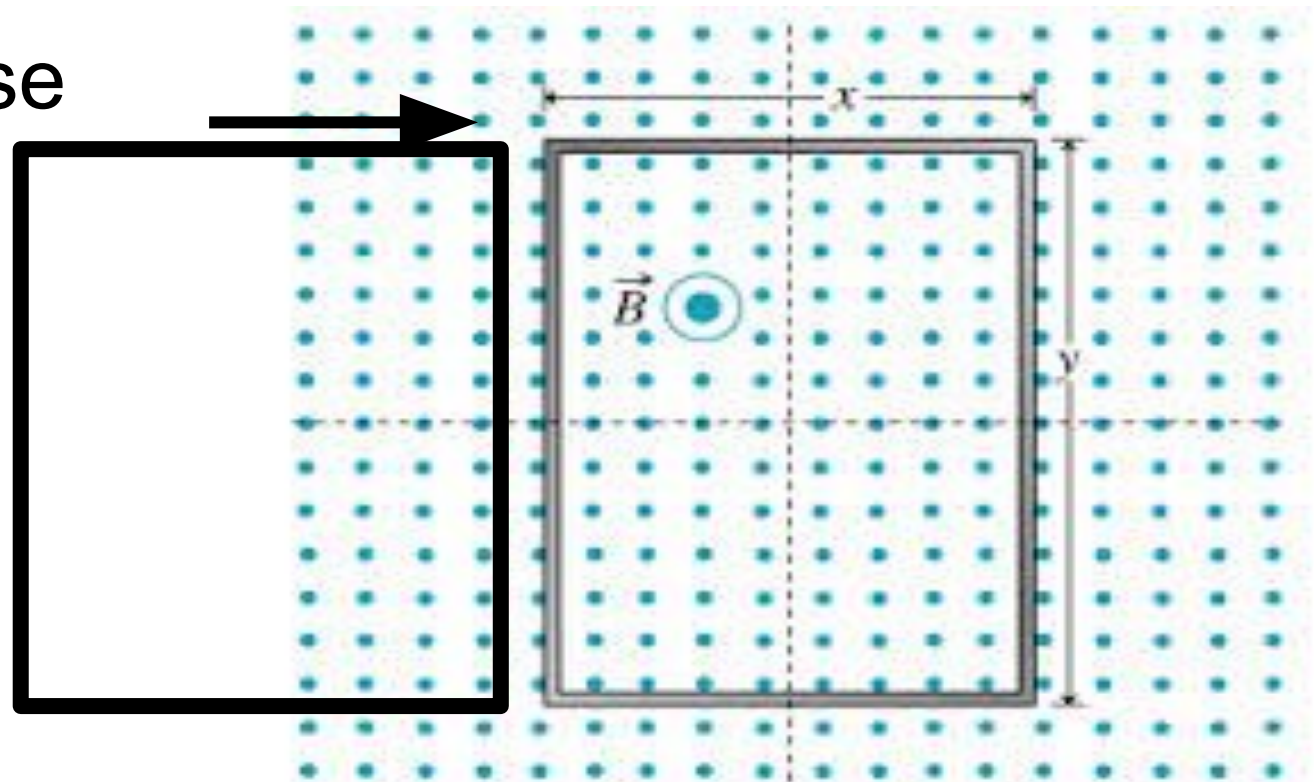
A) 1 A, clockwise

B) 1.25 A, cc

C) 0.57 A,  
clockwise

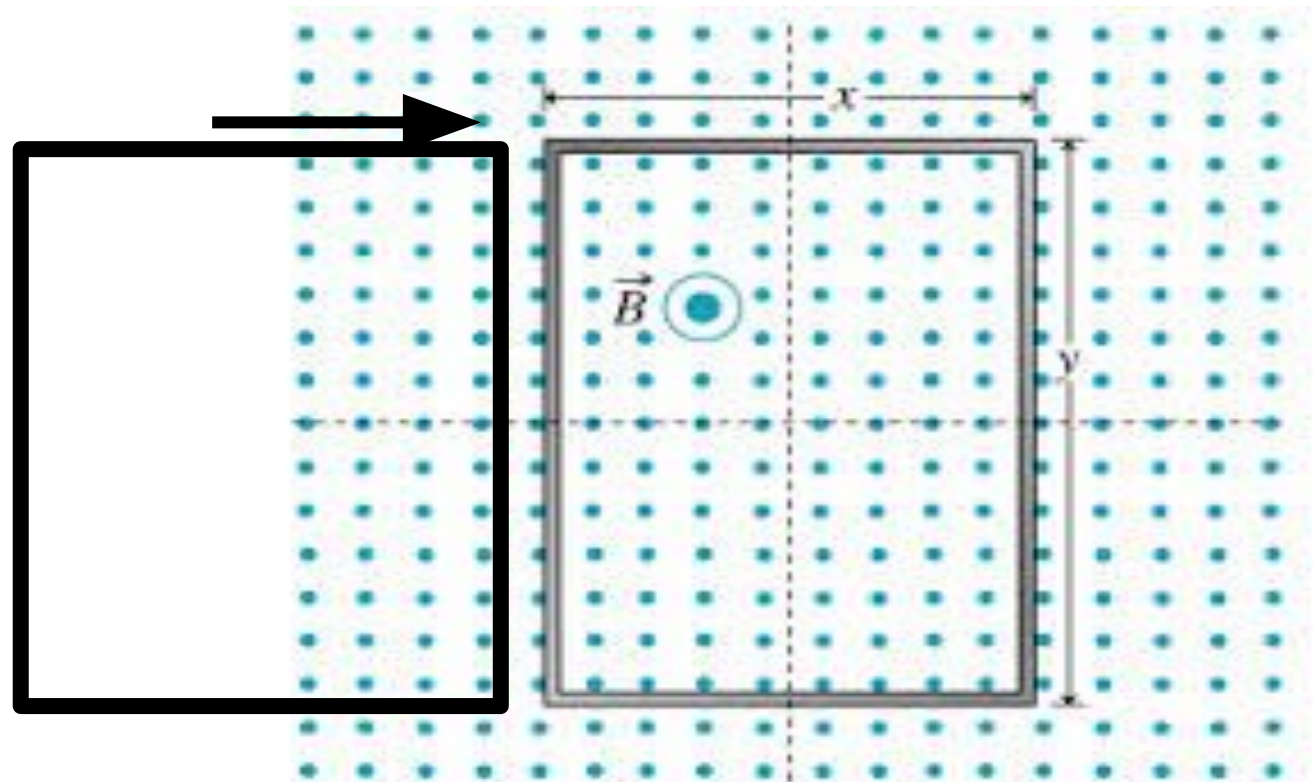
D) 2 A, cc

E) NOTA



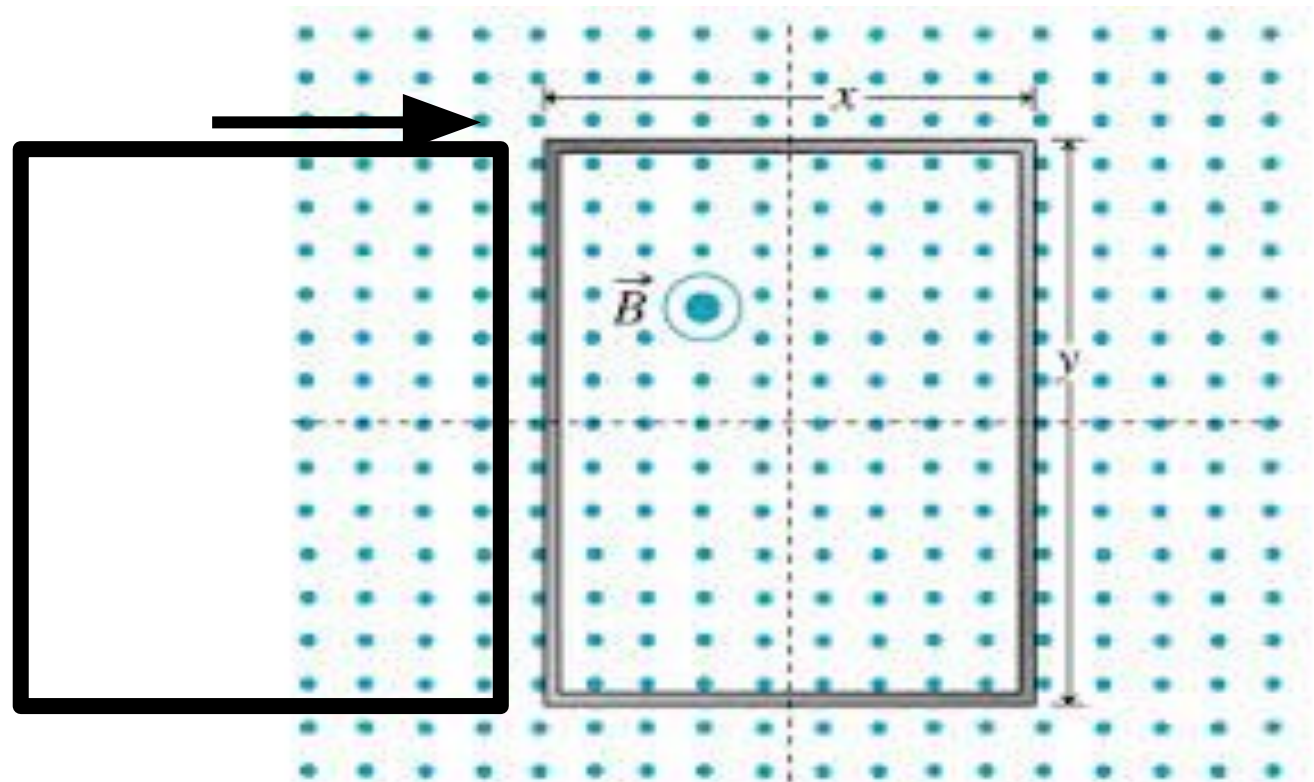
A square/rectangular loop having size 3 meters x 4 meters and resistivity 4 Ohm/meter, moves in the +x direction at  $v=+2$  meters/second, through a region with a magnetic field having magnitude 4 T and pointing in the (+z) direction and an extent of 10 meters. The magnetic force due to the external field on the right, bottom, left, and top sides of the loop as it enters, has direction:

- A) -x, +y, +x, -y
- B) -x, +y, 0, -y
- C) -x, -y, 0, +y
- D) +x, -y, -x, +y
- E) NOTA





A square/rectangular loop having size 3 meters x 4 meters and resistivity 4 Ohm/meter, moves in the  $+x$  direction at  $v=+1$  meters/second, through a region with a magnetic field having magnitude 4 T and pointing in the  $(+z)$  direction and an extent of 10 meters. Determine the time it takes for the loop to pass through the B-field region, and also the magnitude of the magnetic force due to the external field on the right, bottom, left, and top sides of the loop after 1.5 seconds



To move entirely into field region:  $d=3\text{ m}$ ;  $v=1\text{ m/s}$ ;  $t=3/v=3\text{ s}$

$V_{\text{emf}} = -\Delta\phi/\Delta t$ ; where Flux  $\phi=BA$ . So,  $|V_{\text{emf}}|=(4\text{ T})(12\text{ m}^2)/3\text{ s}=16\text{ V}$ ;  
 current  $=16\text{ V}/R_{\text{tot}}=16\text{ V}/[(4\Omega/\text{m})(14\text{ m})]=0.285\text{ Amp}$

Direction must be  $-z$ , so clockwise current

After 1.5s, loop half-in (as shown), force on each side from  $qvB\sin\theta_{v,B}$  –  $v$ =velocity of charges forming current in wire (not  $v_{\text{loop}}$ ),

For current  $I$  through wire of length  $l$

in field  $B$ :  $F=IlB\sin\theta_{v,B}$

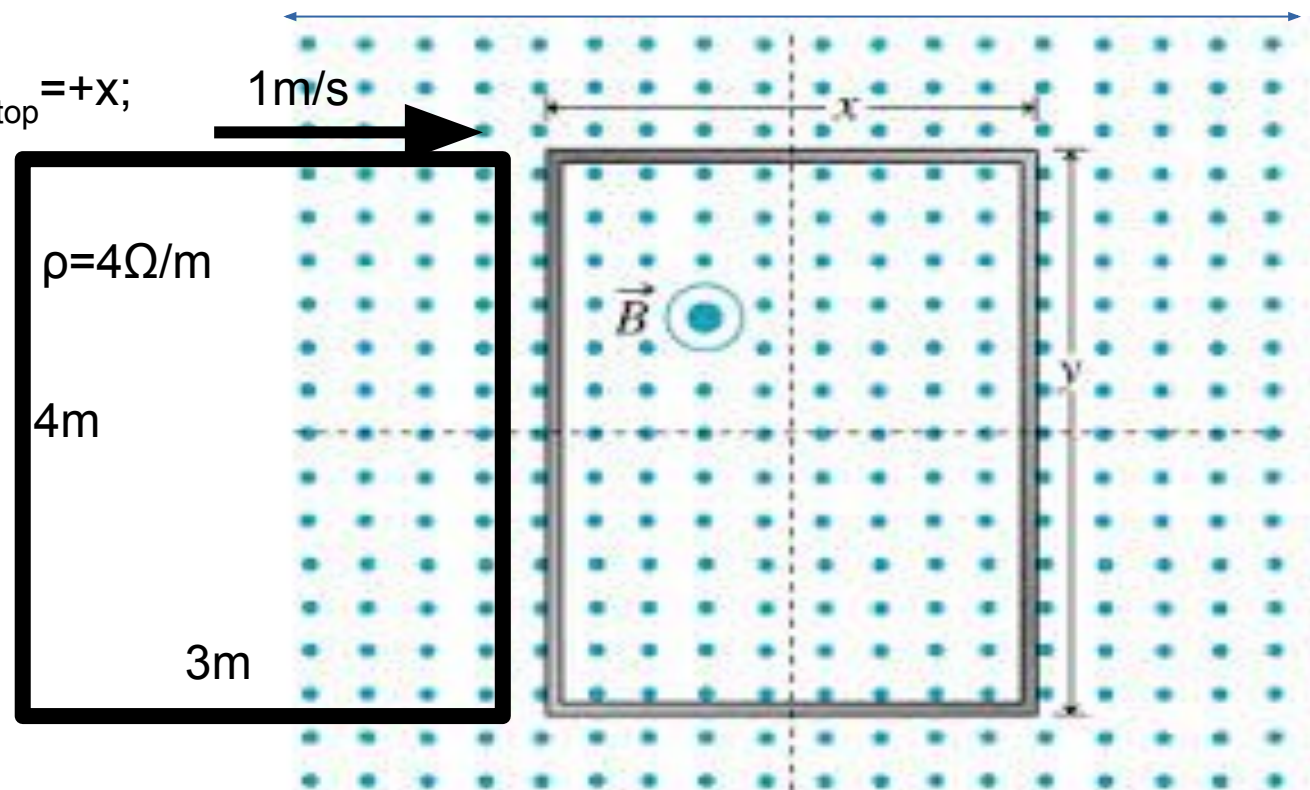
So  $I_{\text{left side}}=+y$ ;  $I_{\text{bottom}}=-x$ ;  $I_{\text{right}}=-y$ ;  $I_{\text{top}}=+x$ ;

$F_{\text{left}}=0$  (no  $B$ );

$F_{\text{bottom}}=(0.285\text{ A})(1.5\text{ m})(4\text{ T}) +y$

$F_{\text{right}}=(0.285\text{ A})(4\text{ m})(4\text{ T}) -x$

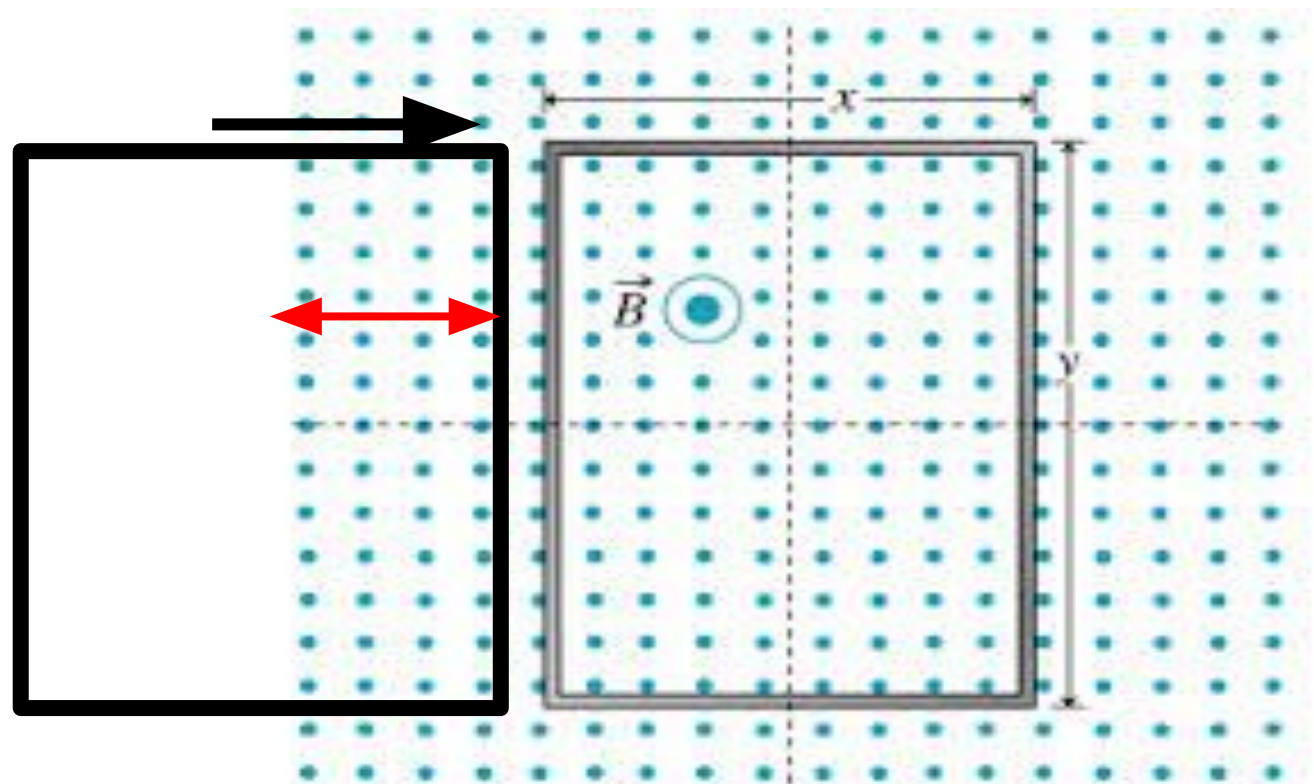
$F_{\text{top}}=(0.285\text{ A})(1.5\text{ m})(4\text{ T}) -y$



A square/rectangular loop having size 3 meters x 4 meters and resistivity 4 Ohm/meter, moves in the +x direction at  $v=+2$  meters/second, through a region with a magnetic field having magnitude 4 T and pointing in the (+z) direction and an extent of 10 meters. At the moment shown, the magnetic force due to the external field on the right and bottom sides, has magnitude:

A) 9.12 N, 4.66N

B) NOTA

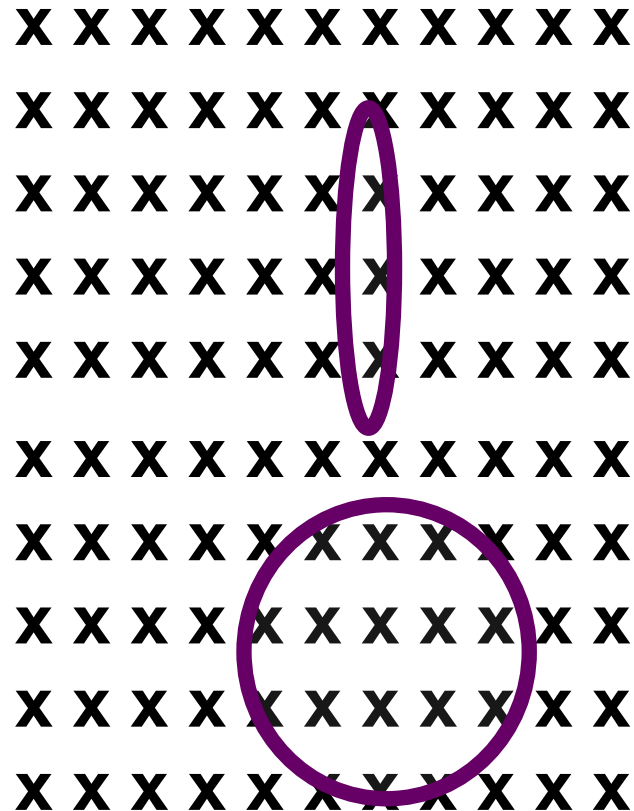


If the loop begins to enter the  $B_{\text{ext}}$  region at  $t_0$ , is fully immersed at  $t_1$ , then travels at constant velocity through the  $B_{\text{ext}}$  region over the time period  $t_1 \rightarrow t_2$ , then exits the  $B_{\text{ext}}$  region from  $t_2 \rightarrow t_3$ , which describes the shape of the  $\Phi(t)$  and also  $I_{\text{induced}}(t)$  curves?



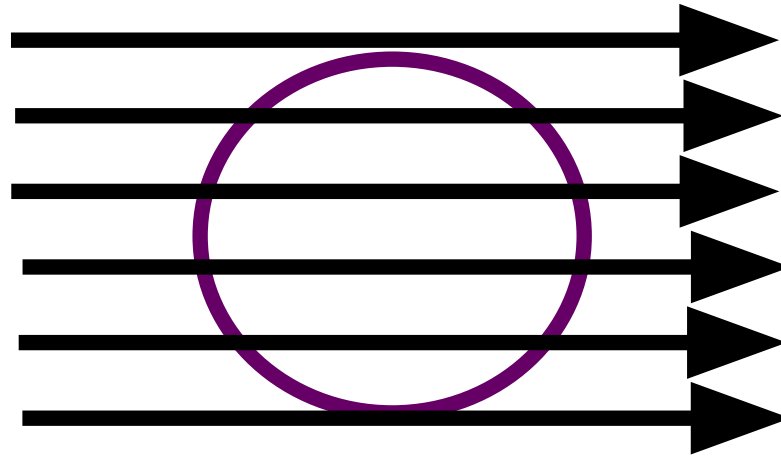
**A 50 cm radius wire loop rotates in a 2 T magnetic field every 8 seconds. The voltage generated through the loop after 2 seconds is:**

- A)  $0.25\pi$  V
- B) 0.5 V
- C)  $0.5\pi$  V
- D) NOTA



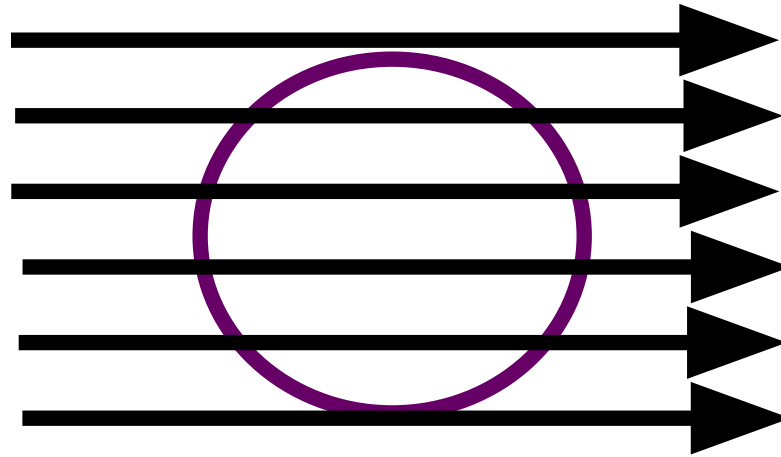
**The voltage generated from  $0 \rightarrow 1$  s,  
compared to  $1 \rightarrow 2$  s is:**

- A) smaller
- B) greater
- C) The same

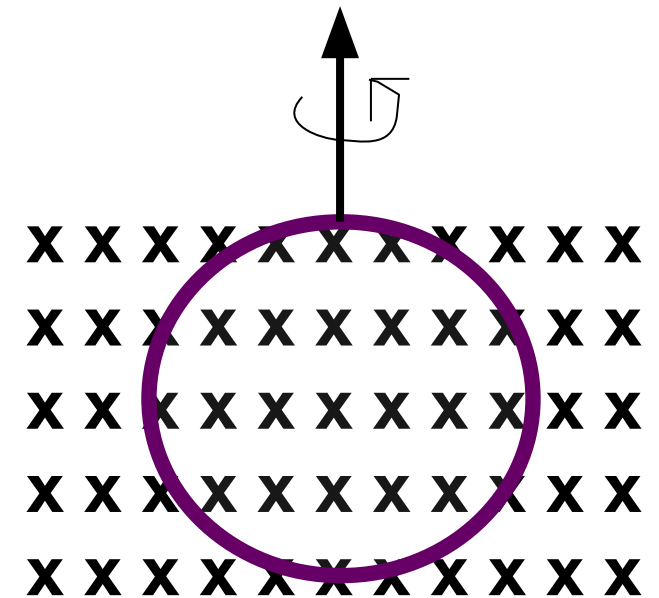
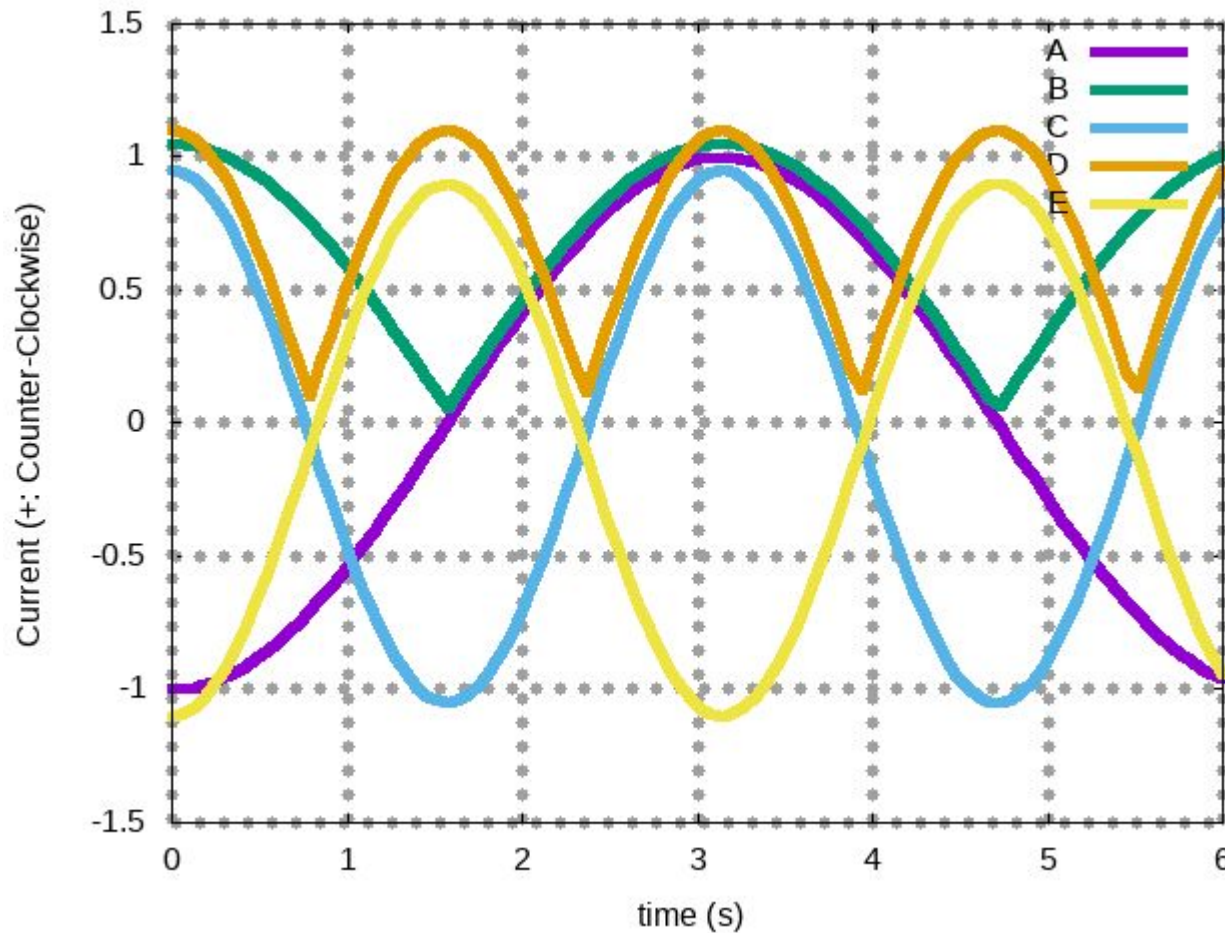


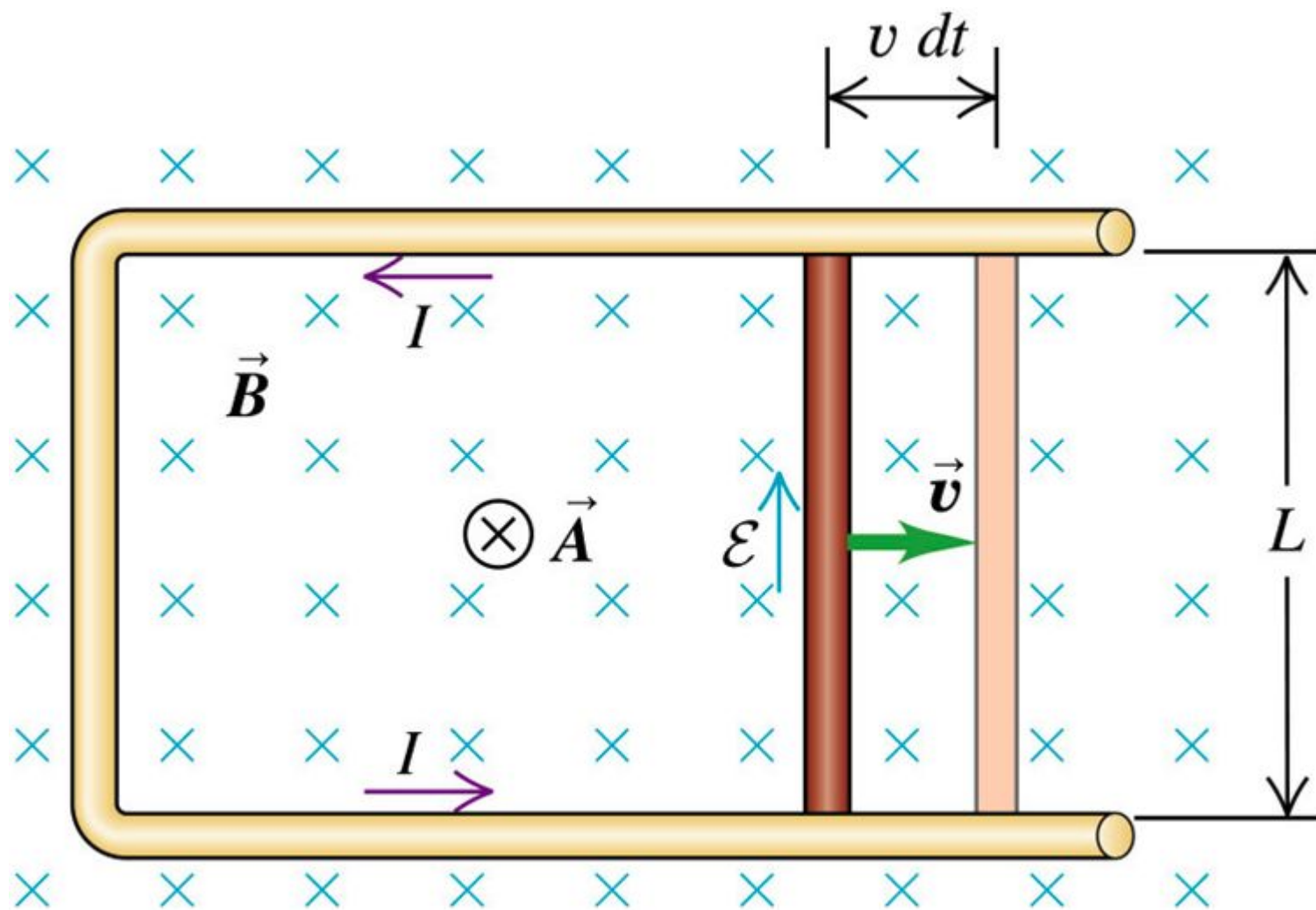
**The induced current direction from  
 $0 \rightarrow 2$  s, vs.  $2 \rightarrow 4$  s, is**

- A) Opposite
- B) The same

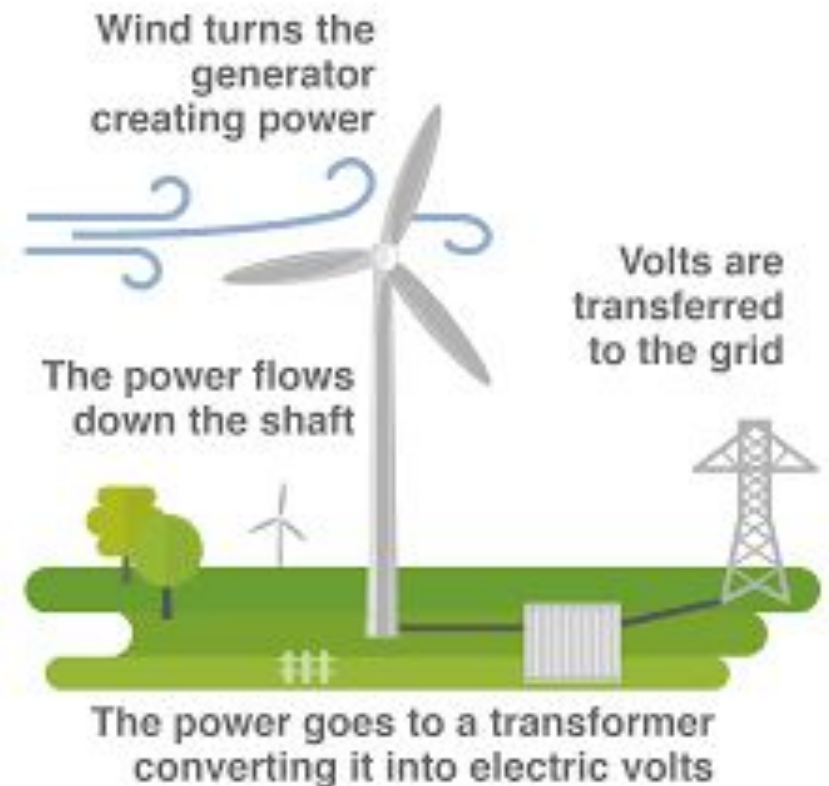
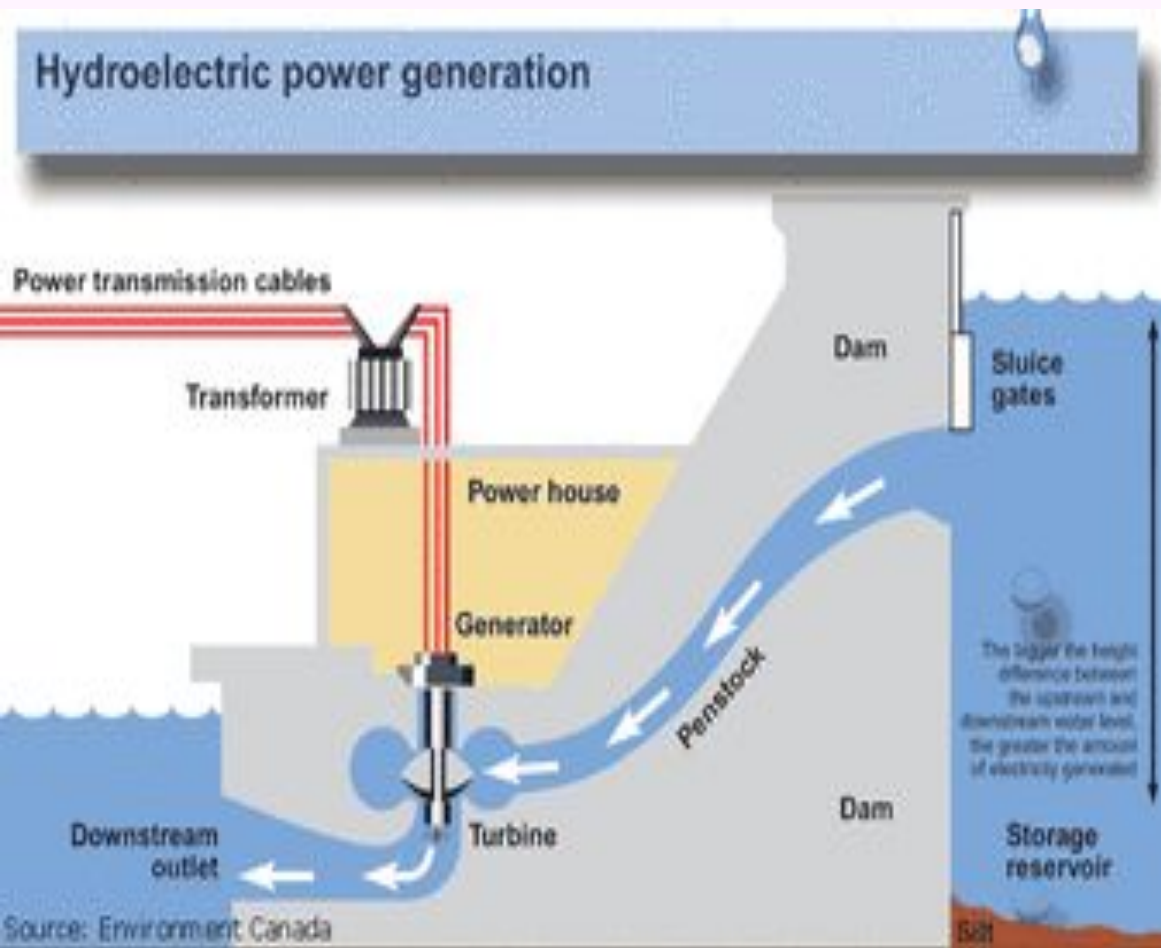


**Which of the curves shown correspond to a loop being rotated as shown, once every 6 seconds?**





# hydroelectric/wind-power energy



Qualitatively: the KE of flowing water/air turns a loop of wire in an external magnetic field, generating a voltage/current ('electromagnet')

$\rho$  (density)=500 kg/m<sup>3</sup>; h=5.1 m Volume fluid=4 m<sup>3</sup>;  $\Rightarrow KE=10^5$  J/s ;

loop: R=4 m; M=2.5 kg  $\Rightarrow l=20$   $\rho$  (resistivity)=2 $\Omega$ /m, so total resistance=

**At the Bowersock dam on the Kaw River,  $10 \text{ m}^3$  of water per second falls from a height of 2m, spinning a 20 kg square wire loop  $4\text{m} \times 4\text{m}$  (resistivity  $\rho=1 \text{ } \Omega/\text{m}$ ) immersed in an external 2T field. If the rotational inertia of the loop  $=\frac{1}{2} MR^2$ , converting KE to PE (recall that the density of water is  $10^3 \text{ kg/m}^3$ )**

The rotational velocity  $\omega$  of the loop is about:

A) 20 rads/s B) 16 rad/s C) 10 rad/s D) 99 rad/s E) NOTA



At the Bowersock dam on the Kaw River,  $10 \text{ m}^3$  of water per second falls from a height of 2m, spinning a 20 kg square wire loop 4m x 4m (resistivity  $\rho=1 \text{ } \Omega/\text{m}$ ) immersed in an external 2T field. If the rotational inertia of the loop  $=\frac{1}{2} MR^2$ , how much time does it take for the loop to make one full turn?

A) 99 seconds B) 15.75 seconds C) 622 seconds D) 0.0634 s seconds E) NOTA

At the Bowersock dam on the Kaw River,  $10 \text{ m}^3$  of water per second falls from a height of 2m, spinning a 20 kg square wire loop 4m x 4m (resistivity  $\rho=1 \text{ } \Omega/\text{m}$ ) immersed in an external 2T field. If the rotational inertia of the loop  $=\frac{1}{2} MR^2$ , given that there are 4 turns per rotation, what is the induced current over one full turn?

A) 31.5 A B) 63 A C) 126 A D) NOTA

**At the Bowersock dam on the Kaw River,  $10 \text{ m}^3$  of water per second falls from a height of 2m, spinning a 20 kg square wire loop 4m x 4m (resistivity  $\rho=1 \text{ } \Omega/\text{m}$ ) immersed in an external 2T field. If the rotational inertia of the loop  $=\frac{1}{2} MR^2$ , the voltage & current generated in the loop is:**

A) 100V, 4A B) 12V, 100A C) 1kV, 63 A D) 10000V, 100A E) NOTA

The rotational velocity  $\omega$  of the loop is about:

A) 20 rads/s B) 16 rad/s C) 10 rad/s D) 99 rad/s E) NOTA

Starting from zero, in one full turn of the loop, how many times does the induced current change direction? A) 1 B) 2 C) 3 D) 4 E) NOTA

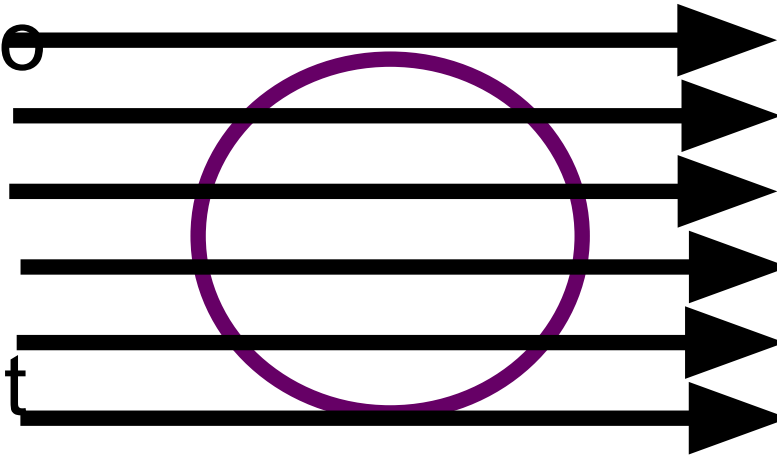
Calculating the change in area  $\Delta A$  in each turn and using  $\omega=2\pi/T$ , the induced voltage/current in the loop is: A) 1008V, 63A B) 4V, 4A C) 4V, 64A D) 64V, 4A E) NOTA

**If the loop is at 45 degrees wrt to the field with a clockwise current as viewed from behind the loop, which is true of the force on the loop?**

A) Torque tends to rotate the loop into the field

B) Torque tends to rotate the loop out of the field

C) There is no torque



**Find the magnetic field generated by an electron ( $q=1.6 \times 10^{-19}$  C) orbiting in a circle of radius  $5.29 \times 10^{-11}$  m with velocity  $2.19 \times 10^6$  m/s around a proton in Hydrogen**

- A)  $10^{-12}$  T
- B)  $10^{-9}$  T
- C)  $10^{-6}$  T
- D)  $10^{-3}$  T
- E) 1 T

**An average current of 0.1 A flows, over a period of 0.02 seconds, across an 800 turn, 10 cm diameter solenoid of the Davemobile\*. What is the induced voltage?**

- A) 0.0004 V
- B) 0.004 V
- C) 0.04 V
- D) 0.4 V
- E) 4 V



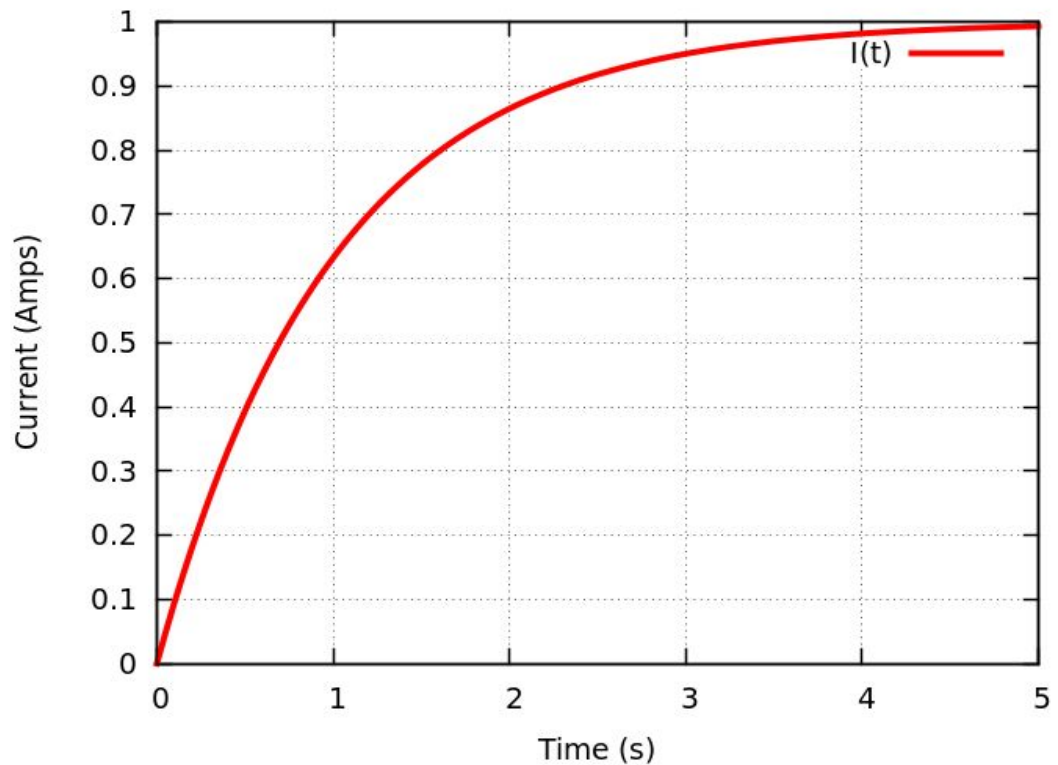
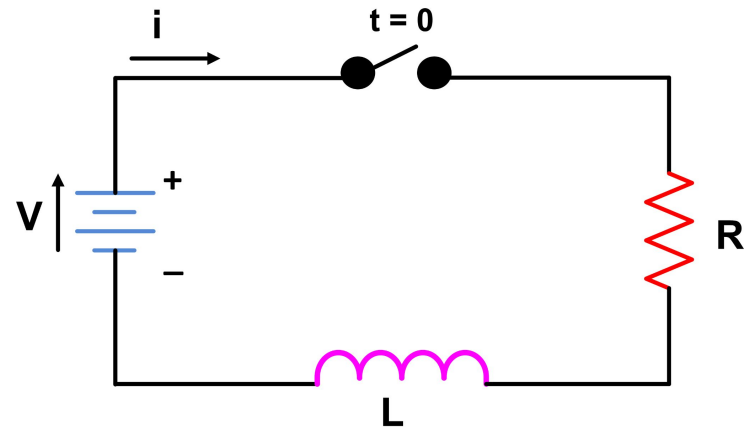
See my e-how-to-pop-a-solenoid online video

**Inductors (symbol='L'; units=Henries)!**

**The magnetic equivalent of capacitors – capacitors work by storing electrical energy ( $PE=Q^2/2C$ ) in the charge that accumulates on the capacitor plates. Inductors, which are just loops of wire, work by storing magnetic energy. The voltage magnitude through an inductor at some time is given by:  $V_L=L\Delta I/\Delta t$  Note that the effect of an inductor in a circuit is greatest when the circuit is initially connected, since at that time the flux through the loop increases from zero to some non-zero value. Time dependence of an inductor is thus opposite to a capacitor – at  $t=0$ , inductor blocks current into branch containing it, and has all the voltage drop in that branch. As  $t\rightarrow\infty$ , inductor is equivalent to a 'straight wire'.  $\tau(RC)=RC$ ;  $\tau(LR)=L/R$**

Which of the following would have the effect of steepening the  $I(t)$  curve shown below?

- A) increasing the size of the circuit
- B) increasing the resistance in the circuit
- C) both of the above
- D) NOTA



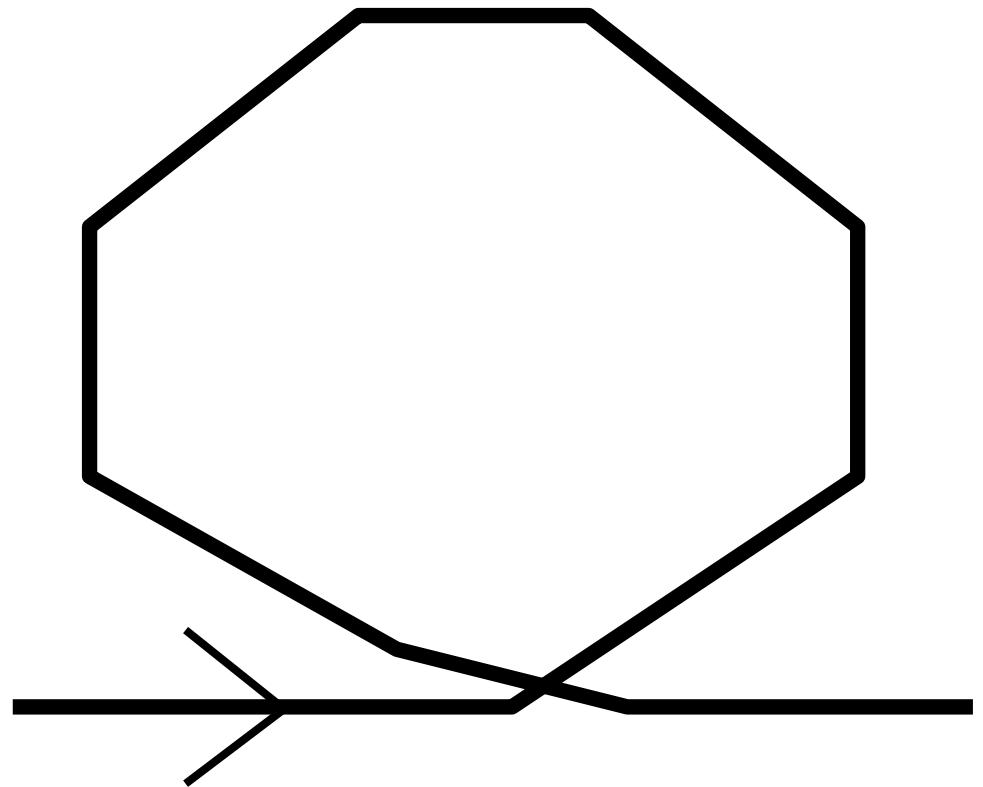


**Which of the following would best describe the inductance of a square loop of wire length  $l$  on a side, with resistance  $R$ ?**

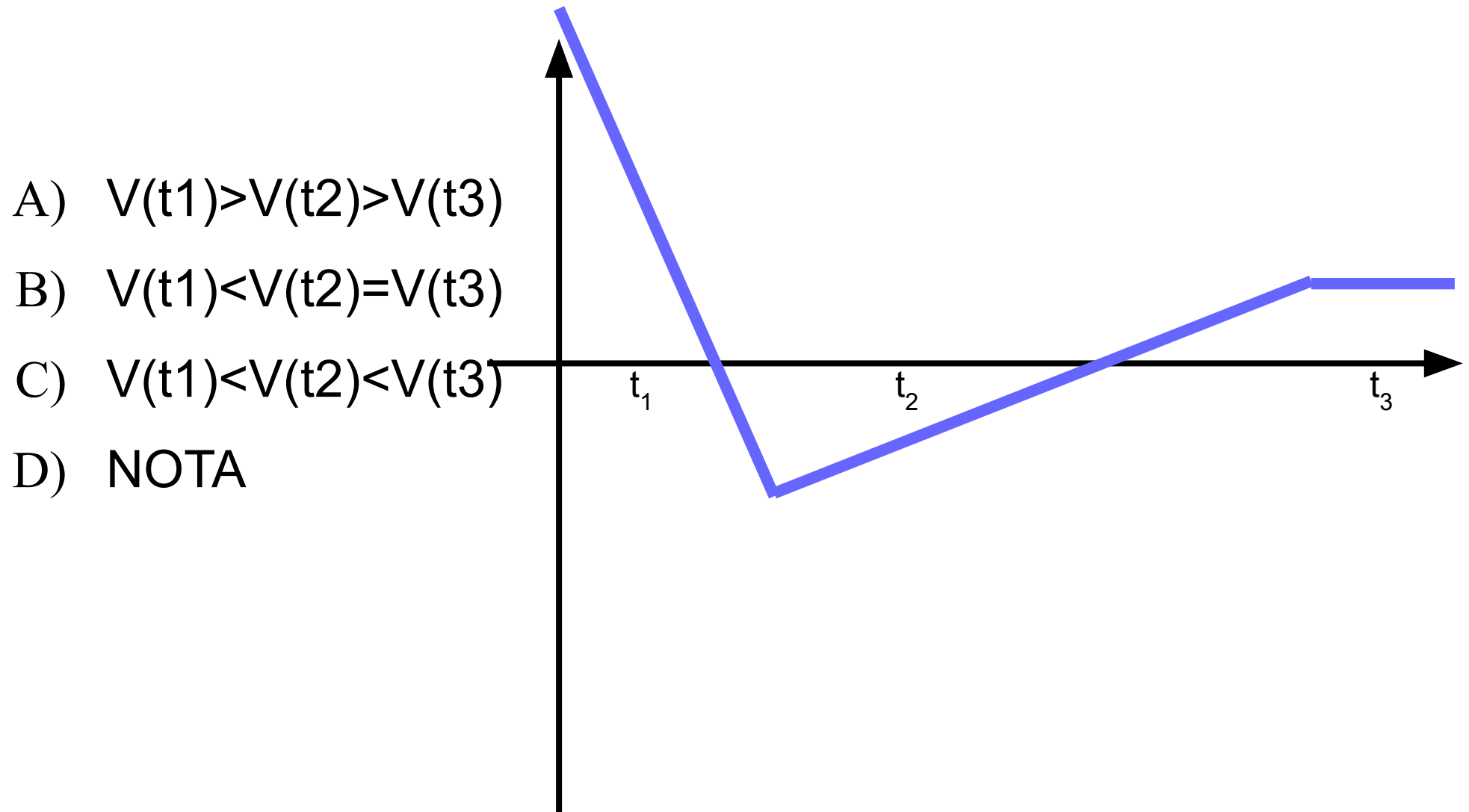
- A)  $L \sim l/R$
- B)  $L \sim 1/(Rl)$
- C)  $L \sim Rl$
- D)  $L \sim R/l$

**A current begins to flow through the loop shown. The loop responds with a voltage in which direction?**

- A) Clockwise
- B) CC

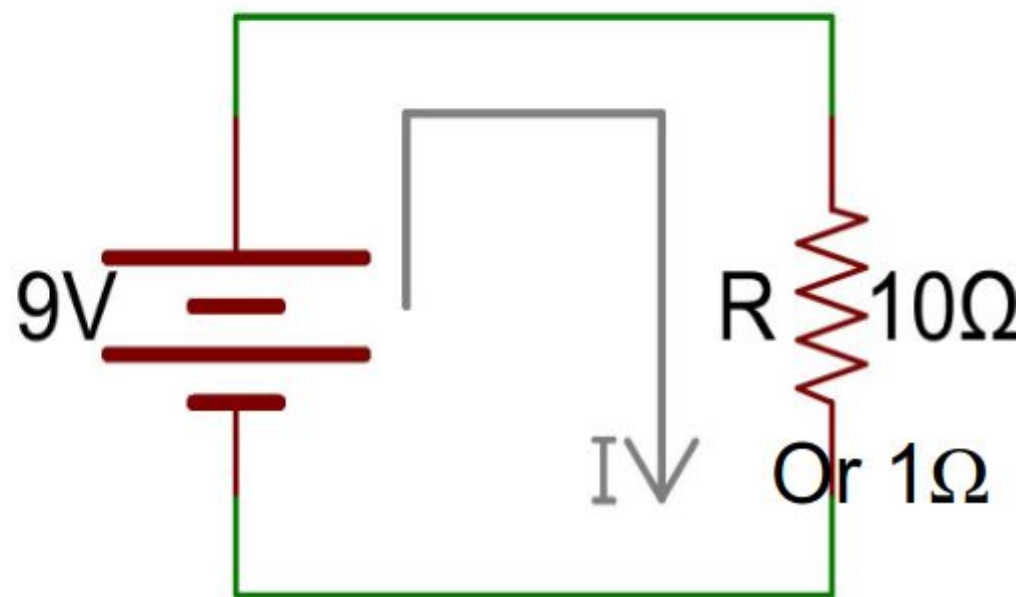


**Given the graph shown of  $B(t)$  through a loop, the ranking of the induced voltage magnitude is:**



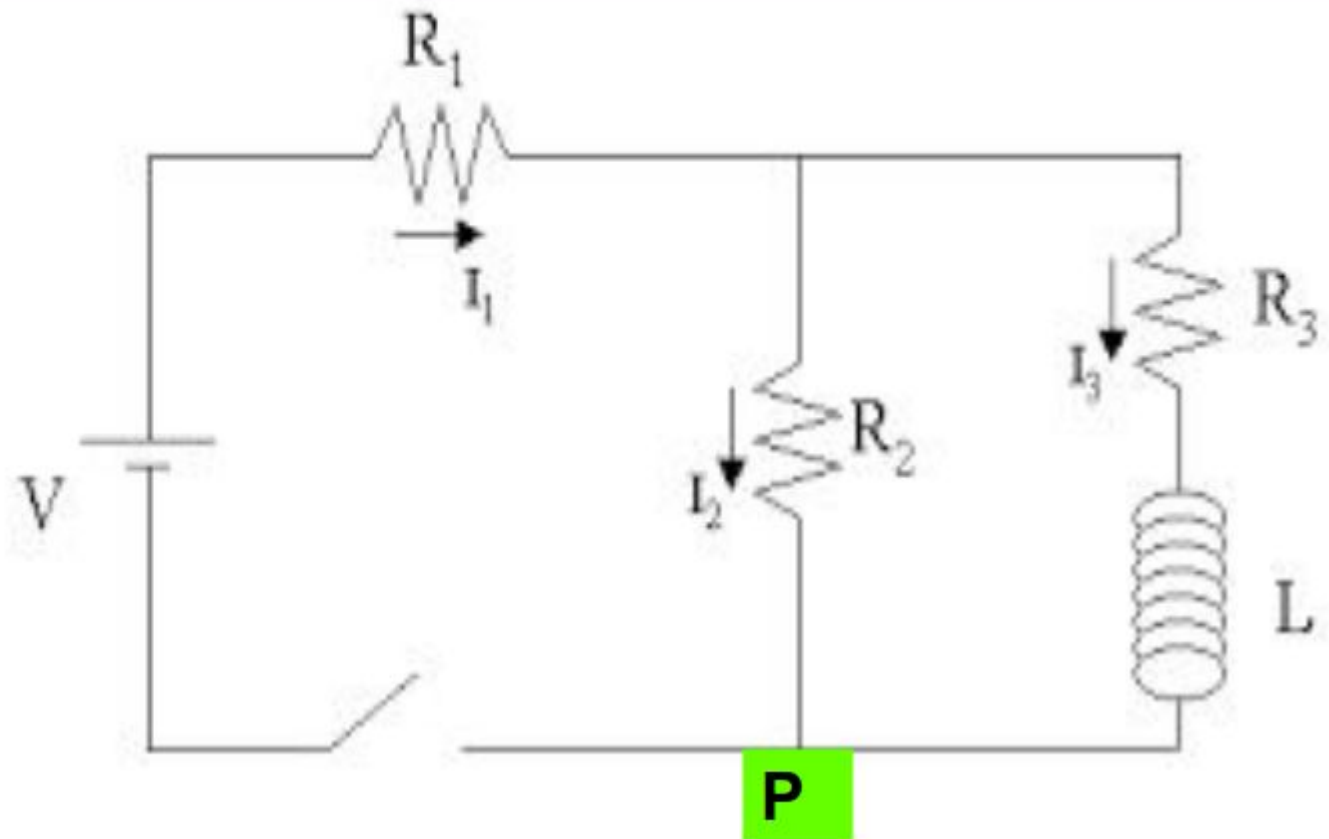
# In which of the circuits below does current rise fastest\*?

- A)  $V=9V$  connected to  $R=1\Omega$  using 0.1m long wires.
- B)  $V=9V$  connected to  $R=10\Omega$  using 0.1m long wires.
- C)  $V=9V$  connected to  $R=1\Omega$  using 1m long wires.
- D)  $V=9V$  connected to  $R=10\Omega$  using 1m long wires.
- E) All are the same



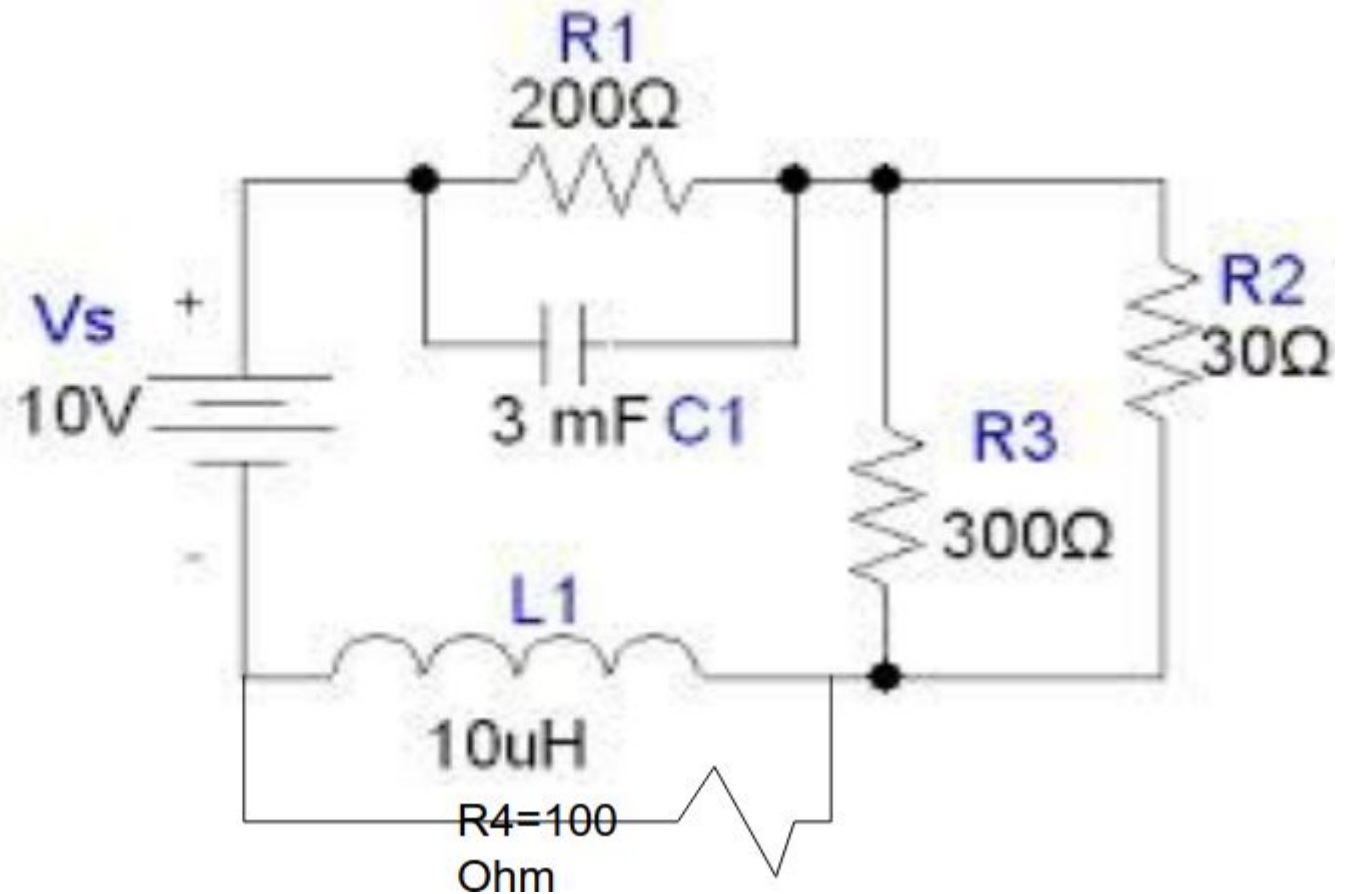
If  $V=12\text{ V}$ ,  $R_1=6\Omega$ ,  $R_2=12\Omega$  and  $R_3=24\Omega$ ,  
then  $V_2(t=0)/V_2(t\rightarrow\infty)=$

- A)  $2/3$
- B)  $1.5$
- C)  $1.167$
- D)  $0.857$
- E) NOTA



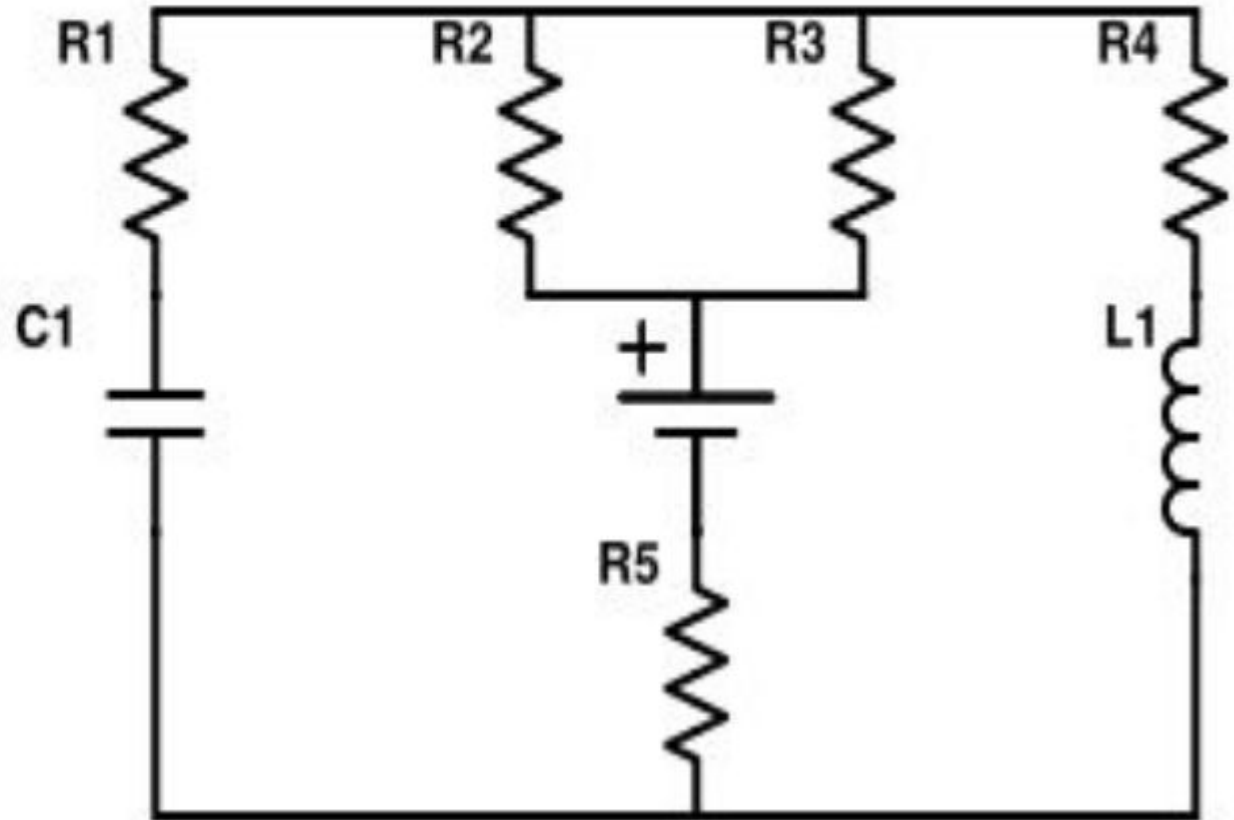
Comparing  $t=0$  to  $t \rightarrow \infty$ , which statement is true?

- A)  $V_{2,0} < V_{2,\infty}$
- B)  $V_{3,0} < V_{3,\infty}$
- C)  $V_{L1,0} < V_{L1,\infty}$
- D) Two of the above
- E) NOTA



**Given:  $R_1=6\ \Omega$ ,  $R_2=6\ \Omega$ ,  $R_3=12\ \Omega$ ,  $R_4=12\ \Omega$ ,  
 $R_5=2\ \Omega$ ,  $C_4=2\text{F}$ ,  $V=36\text{ V}$ ;  $V_{L1}(t=0)=?$**

- A) 3 V
- B) 6 V
- C) 9 V
- D) 18 V
- E) NOTA

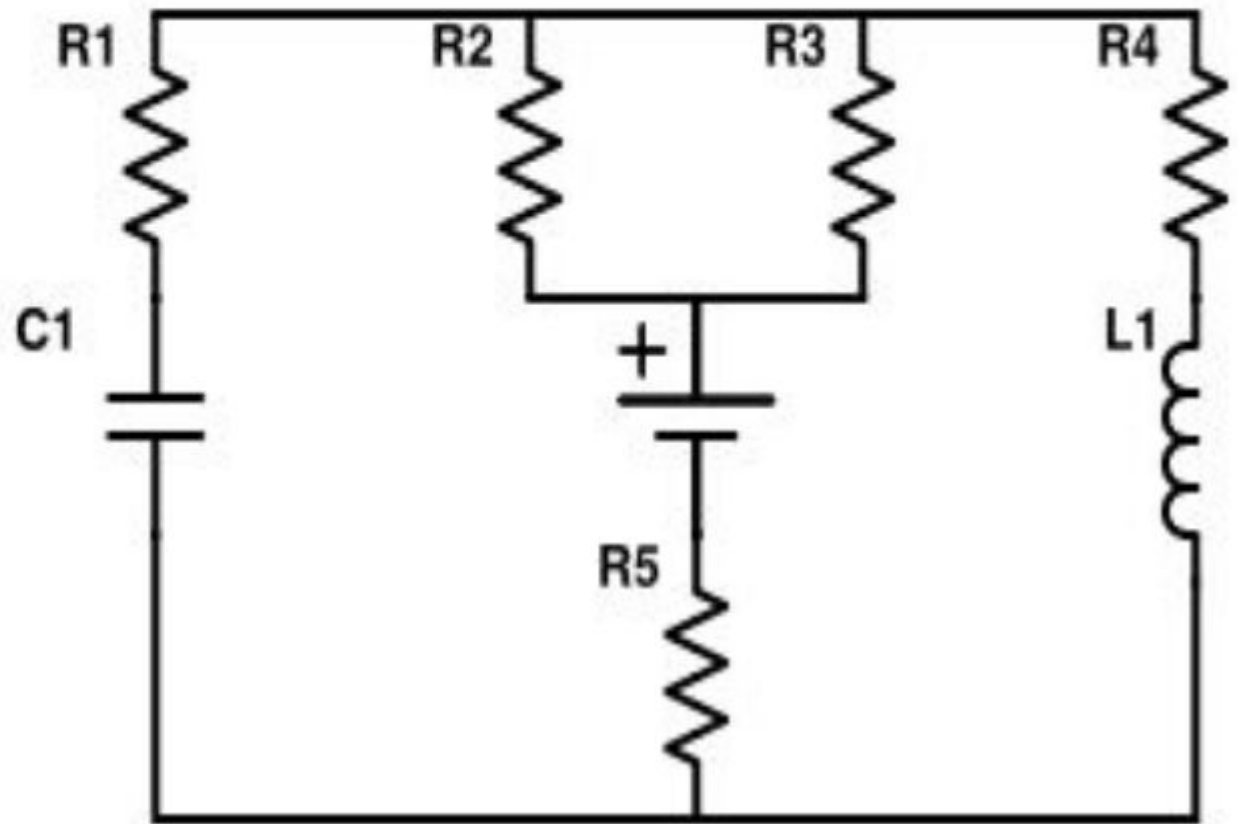


If  $C1$  and  $L1$  were replaced by straight wires, how many combinations of resistors (as drawn) are there? A) 2 parallel, 0 series B) 1 ||, 2 series, C) 1 ||, 1 series D) 0 ||, 2 series, E) NOTA



**Given:  $R_1=6\ \Omega$ ,  $R_2=6\ \Omega$ ,  $R_3=12\ \Omega$ ,  $R_4=12\ \Omega$ ,  
 $R_5=2\ \Omega$ ,  $C_4=2\text{F}$ ,  $V=36\text{ V}$ ;  $Q_{C_1}(t \rightarrow \infty)=?$**

- A) 8 C
- B) 16 C
- C) 24 C
- D) 48 C
- E) NOTA

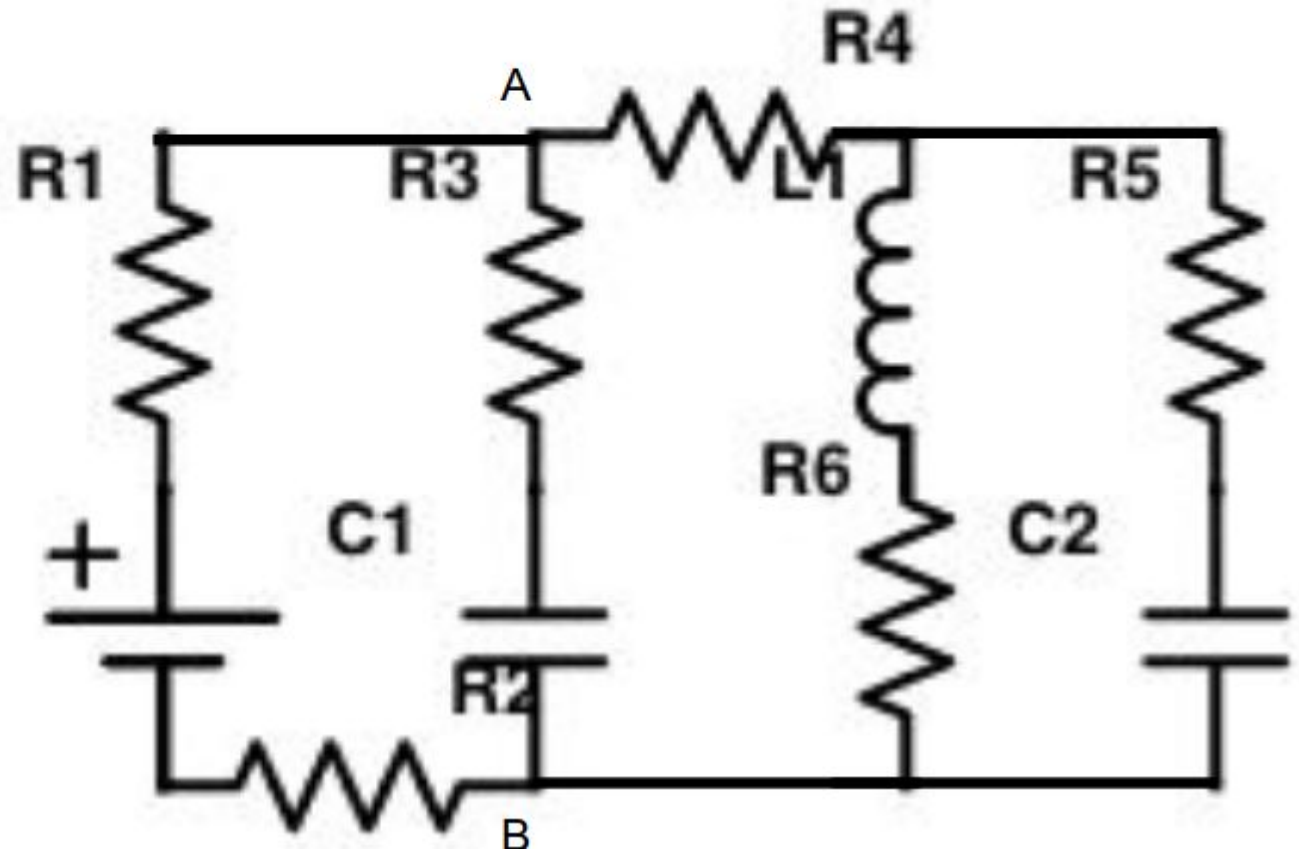




If all resistors have  $R=24\Omega$ ,  $C_1=C_2=4F$  and  $L=2H$ , then  $V_1(t=0)/V_1(t\rightarrow\infty)=$

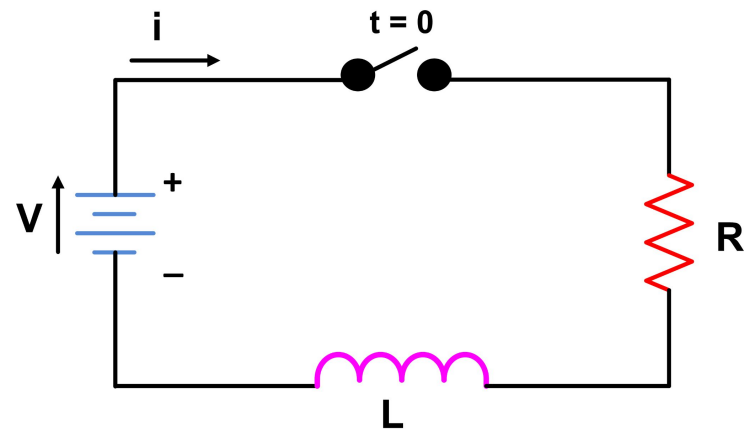
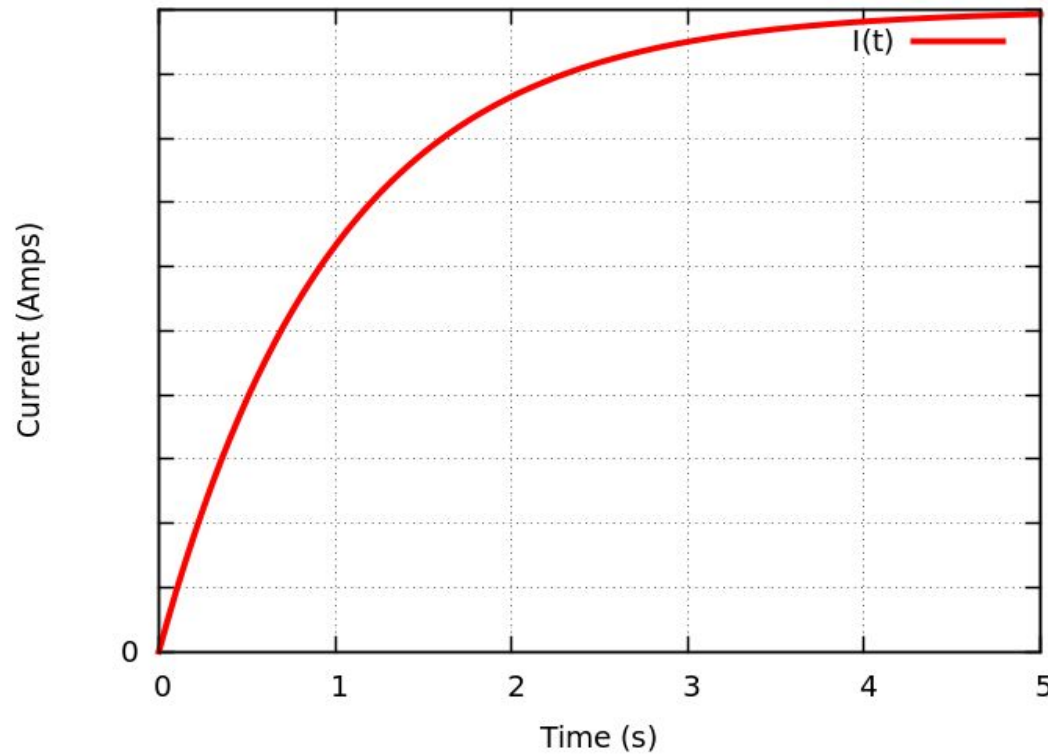
- A) 1.0
- B) 2/3
- C) 3/2
- D) 2.0
- E) NOTA

$V=128V$

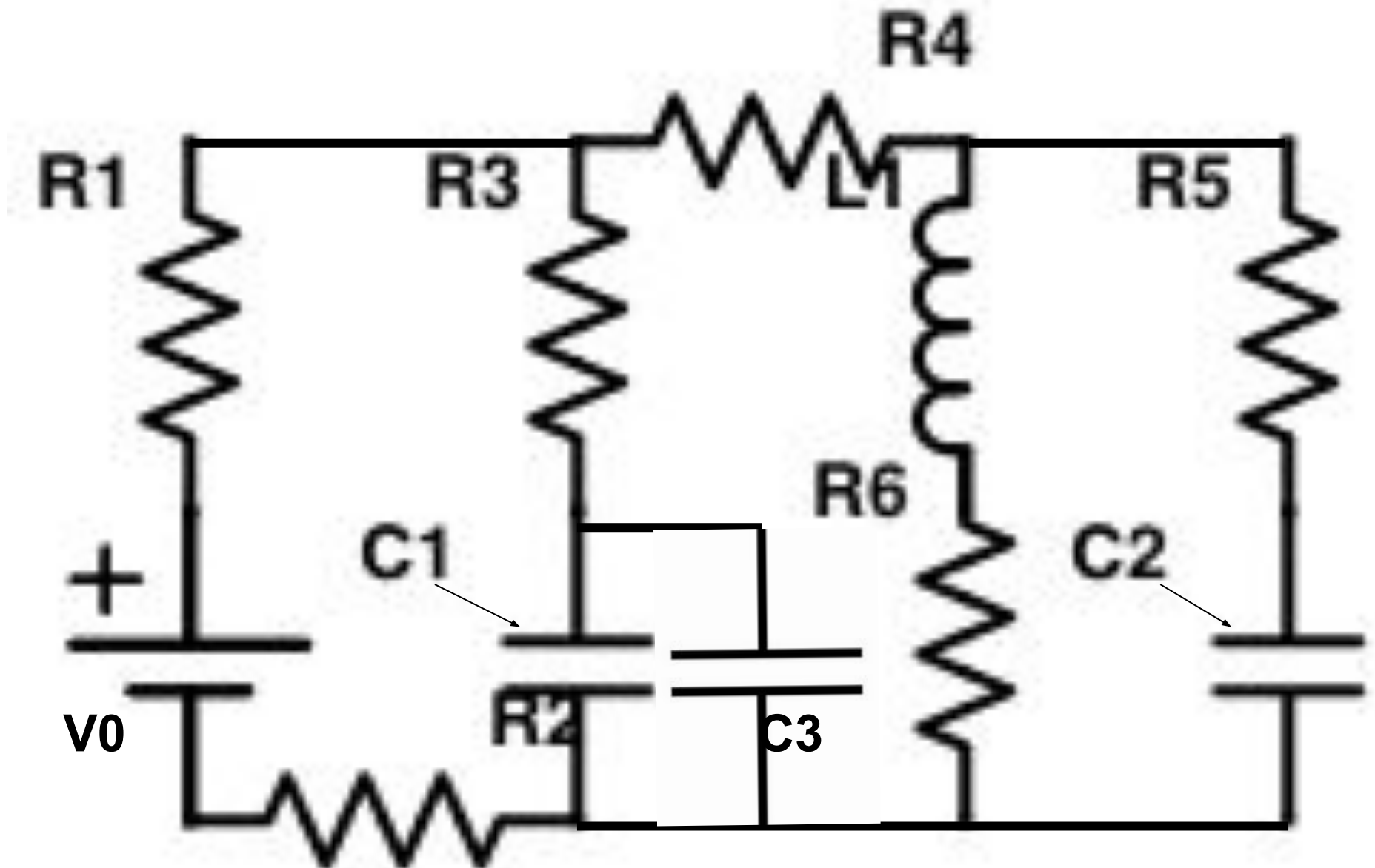


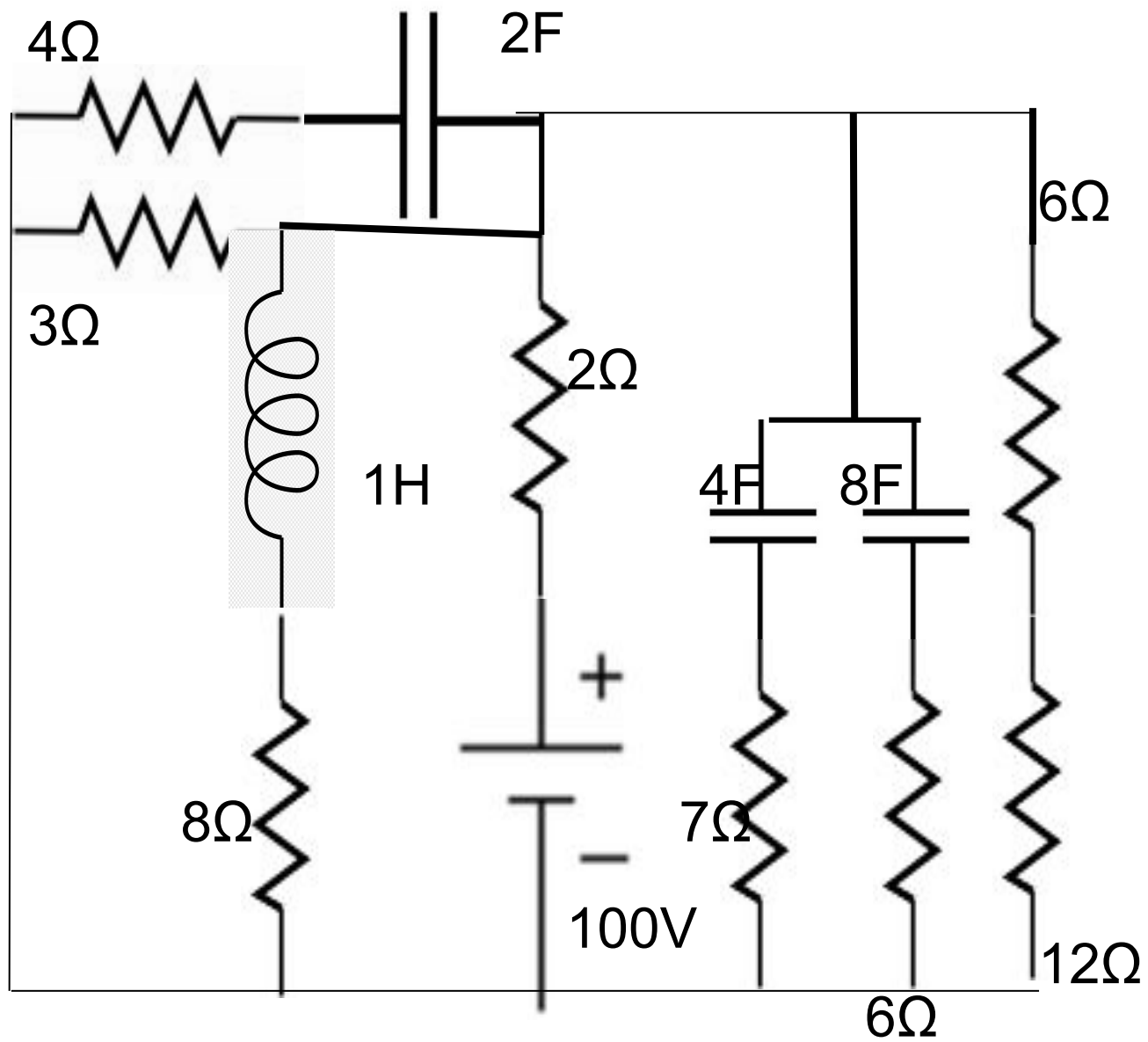
Given  $I(t)$  for an RL circuit with  $V=4V$ , what is the approximate value of  $V_L(2\text{ s})$ ?

- A) 0.8 V
- B) 0.3 V
- C) 4 V
- D) 1.8 V
- E) 10 V



$R_1=2$ ;  $R_2=4$ ;  $R_3=8$ ;  $R_4=8$ ;  $R_6=R_5=12$ .  $V_0=100$  V,  $C_1=2$ F= $C_2$ ;  $C_3=4$ F.





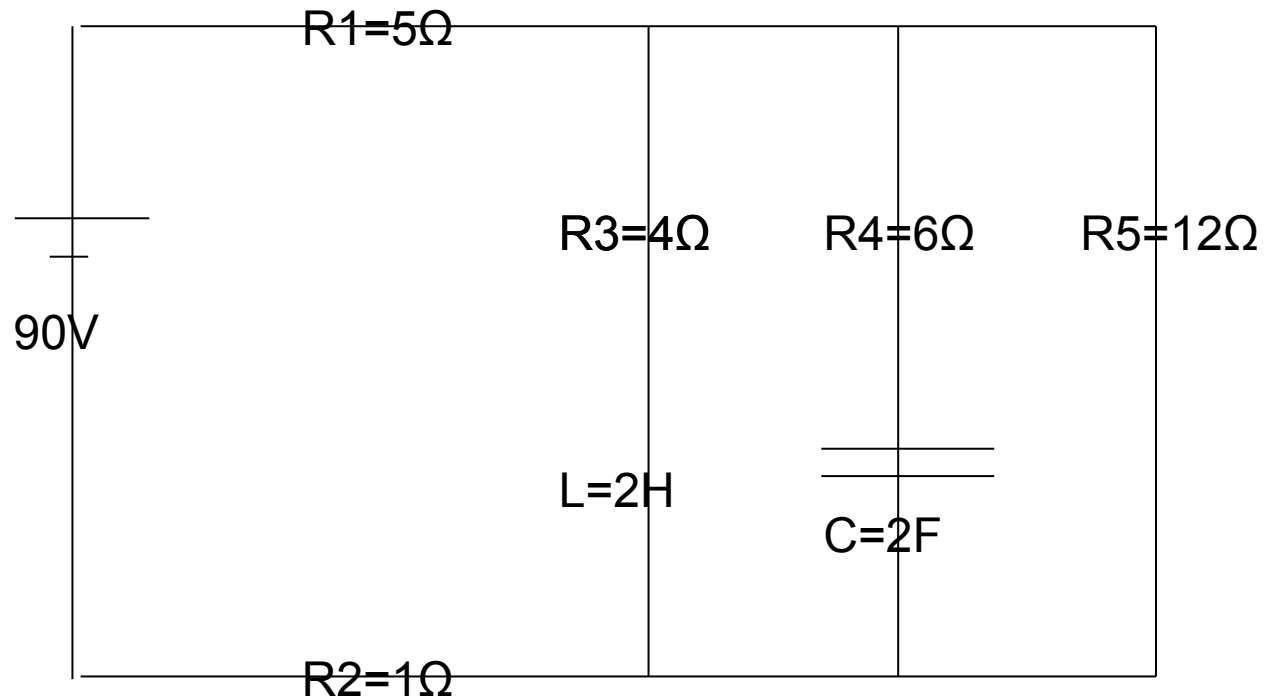
$T=0$ : C has no effect; inductor blocks current through its branch:  $R_{\text{tot}}=10\Omega \Rightarrow I_{\text{tot}}=9\text{A}$ .

$$\Delta V_1=45\text{V}; \Delta V_2=9\text{V}; \Delta V_3=0; \Delta V_L=36\text{V}=\Delta V_4=\Delta V_5$$

$t \rightarrow \text{infinity}$ : L has no effect; Capacitor blocks current through its branch:

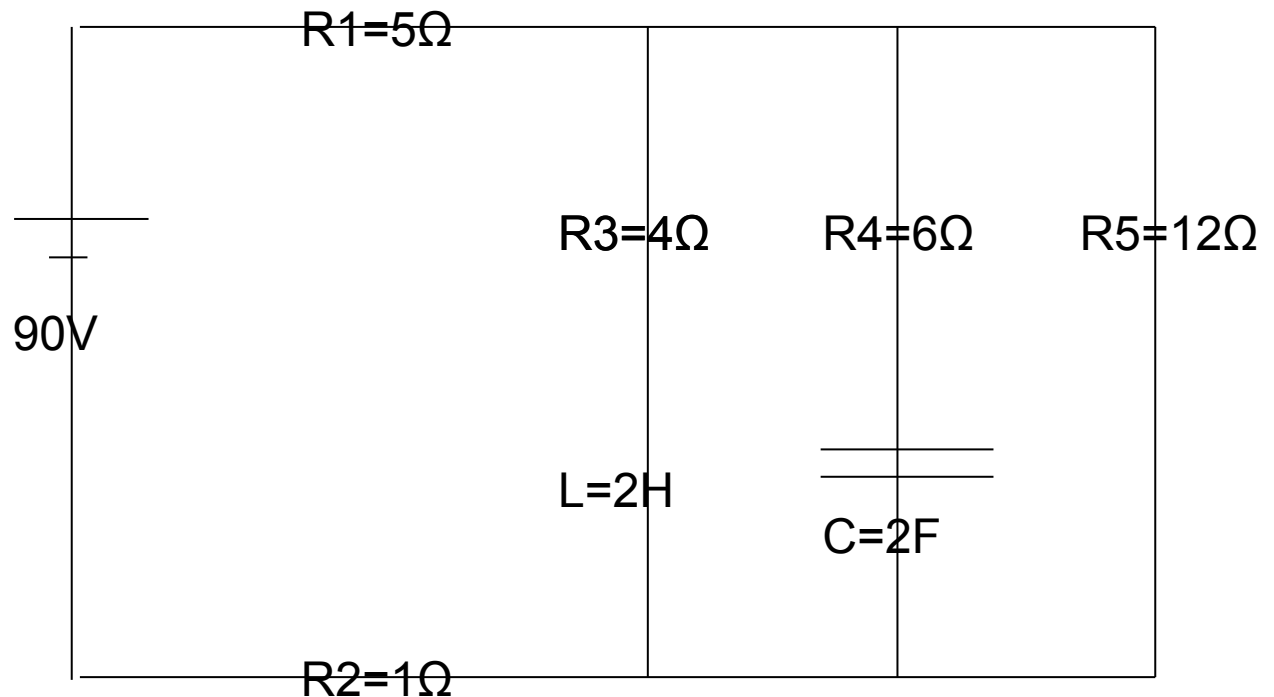
$$R_{\text{tot}}=9\Omega \Rightarrow I_{\text{tot}}=10\text{A}$$

$$\Delta V_1=50\text{V}; \Delta V_2=10\text{V}; \Delta V_L=V_4=0 \quad \Delta V_3=30\text{V}=\Delta V_C=\Delta V_5, \text{ so Charge on Capacitor}=60\text{C}$$

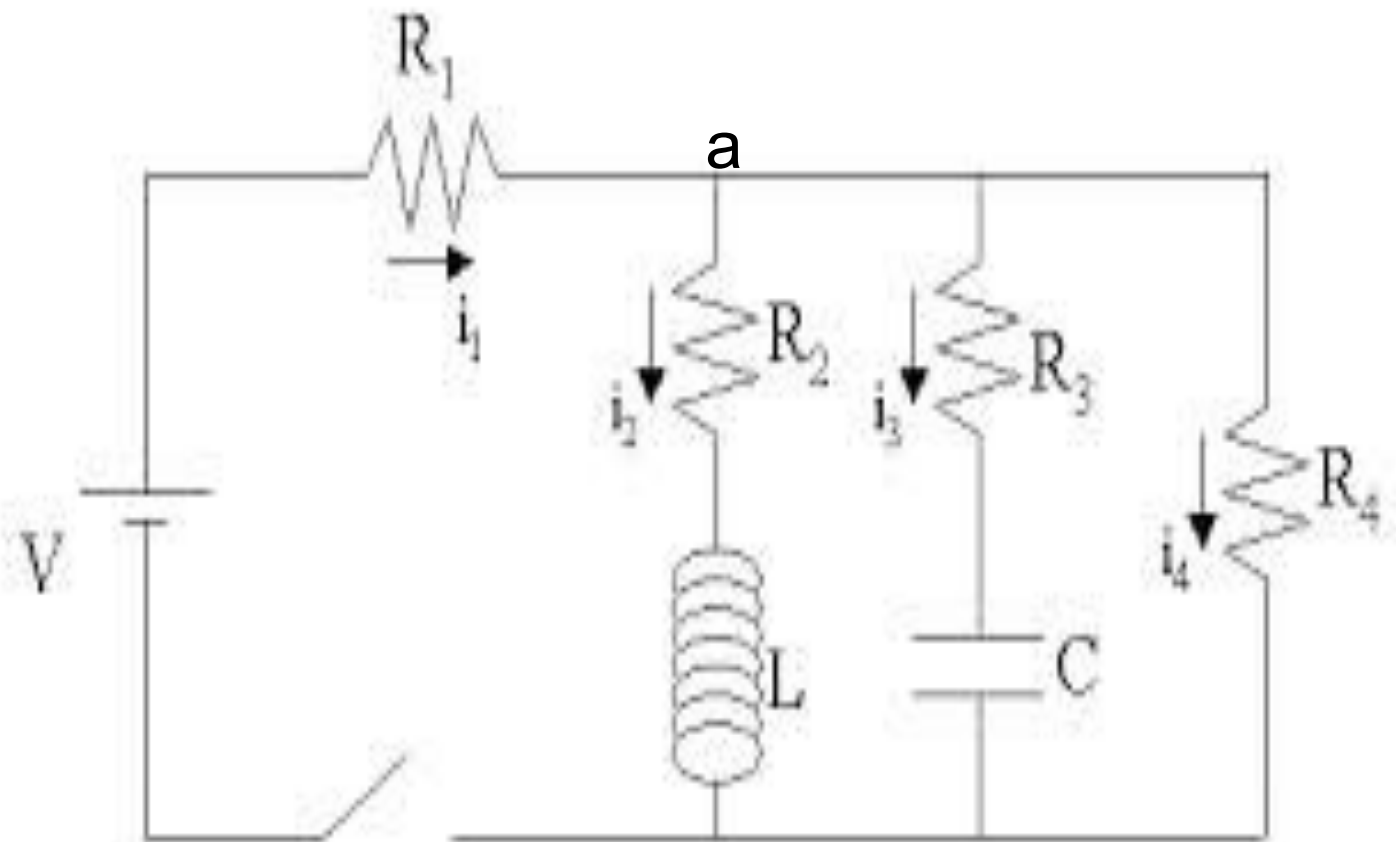


In the circuit below, what is the current  $I_{\text{tot}}(t=0)$  and also  $t \rightarrow \infty$ ?

- A) 9A, 10A
- B) 10A, 9A
- C) 8A, 8A
- D) 9A, 9A
- E) NOTA



**Write Kirchhoff's laws for junction a and the outer and middle loops.**



**Which of the following would you expect to be the correct formula for the resonant frequency in an RLC circuit?**

- A) Frequency  $\sim 1/(LC)$
- B) Frequency  $\sim L/C$
- C) Frequency  $\sim C/L$
- D) Frequency  $\sim LC$



Given an RLC circuit with  $L=0.2$  Henries and  $C=0.4$  Farads, being driven at a frequency of 2 Hz. To bring the circuit closer to resonance, you would:

- A) Add a capacitor in series with the existing capacitor
- B) Add a capacitor in parallel with the existing capacitor

**In the analogy with a 'driven' mechanical oscillator, which eqn. makes 'sense' for  $\omega$ ?**

- A)  $\omega \sim km$
- B)  $\omega \sim k/m$
- C)  $\omega \sim m/k$
- D)  $\omega \sim 1/(km)$

## Electromagnetic Radiation:

$$E_{\text{rad}} = kqa(\sin\theta)/c^2r; \theta = \text{angle}(a,r)$$

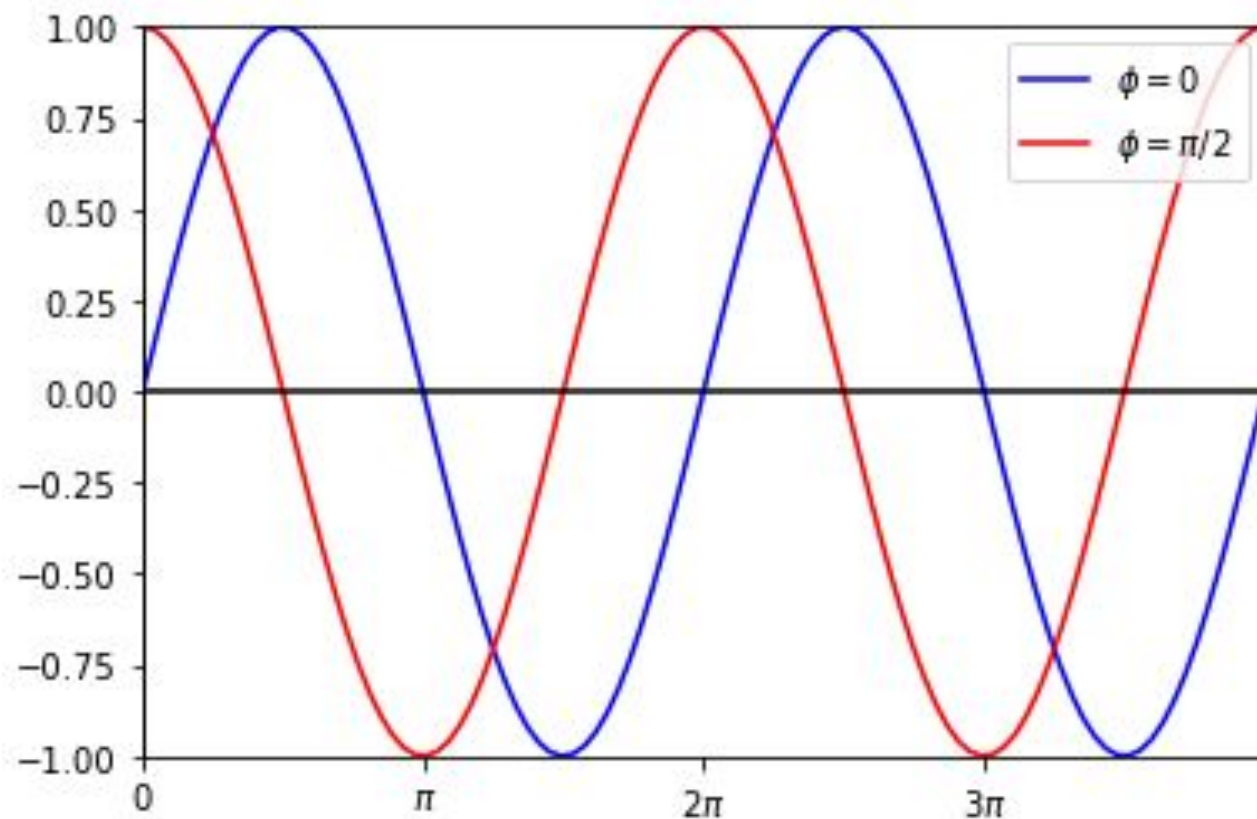
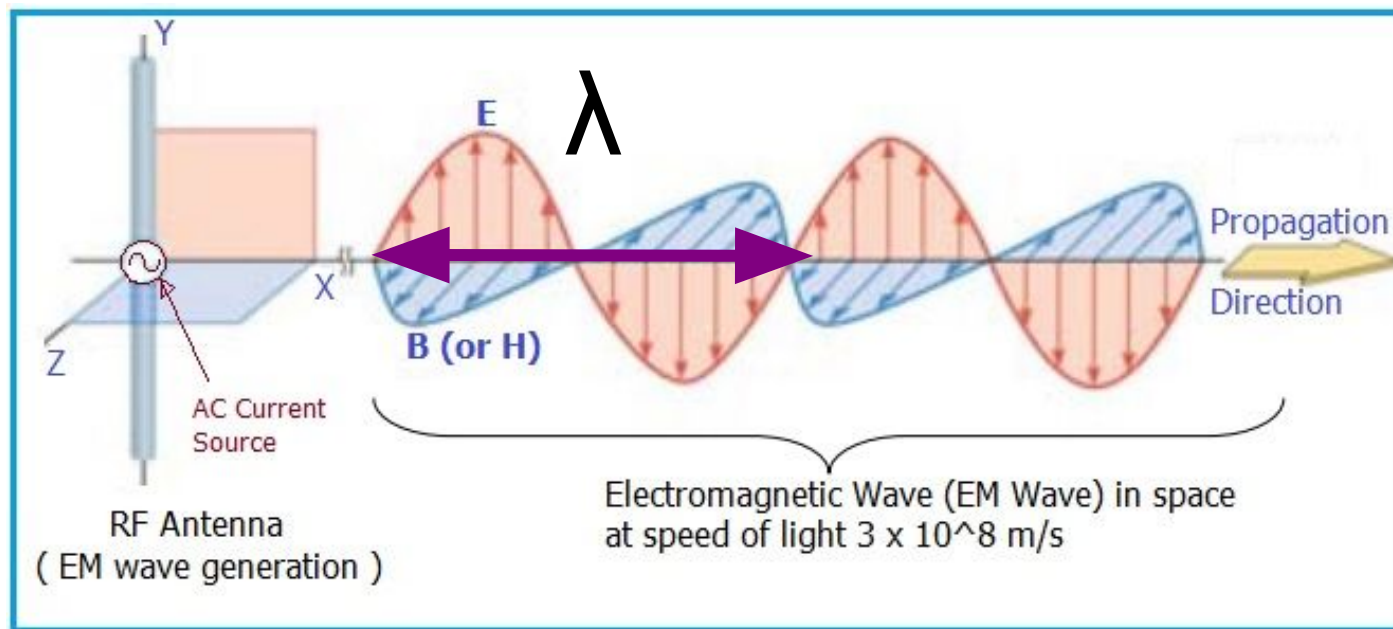
$E_{\text{rad}}$  always perpendicular to  $k \Rightarrow$  perp to  $E_{\text{coulomb}}$   
and in direction of current constituted by source motion

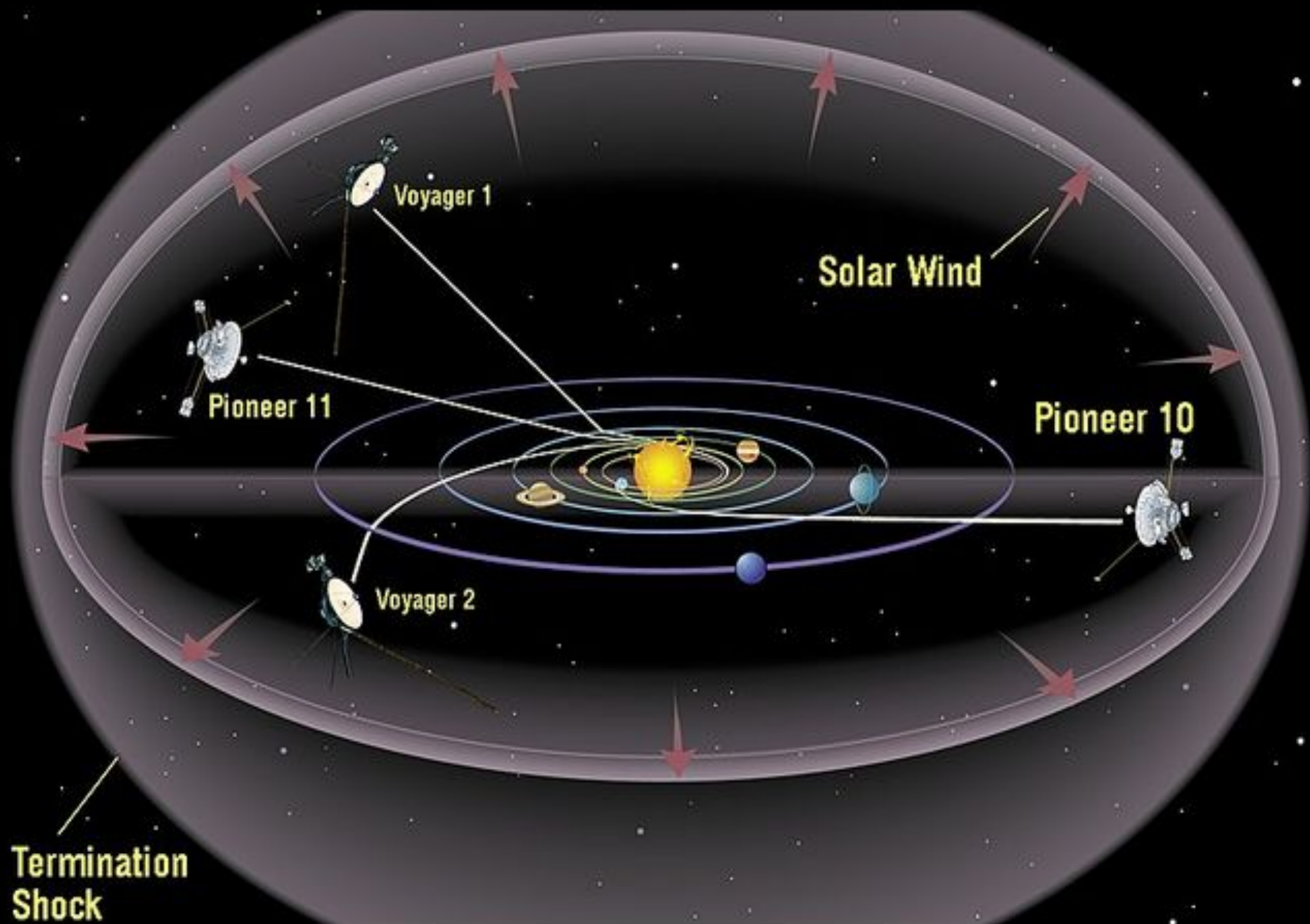
$E_{\text{rad}}$  is vector, so add as vectors  
(*case of multiple sources, e.g.*)

$c$  = velocity of light in vacuum & is limit to  
information transfer (N.B: **EPR paradox!**)

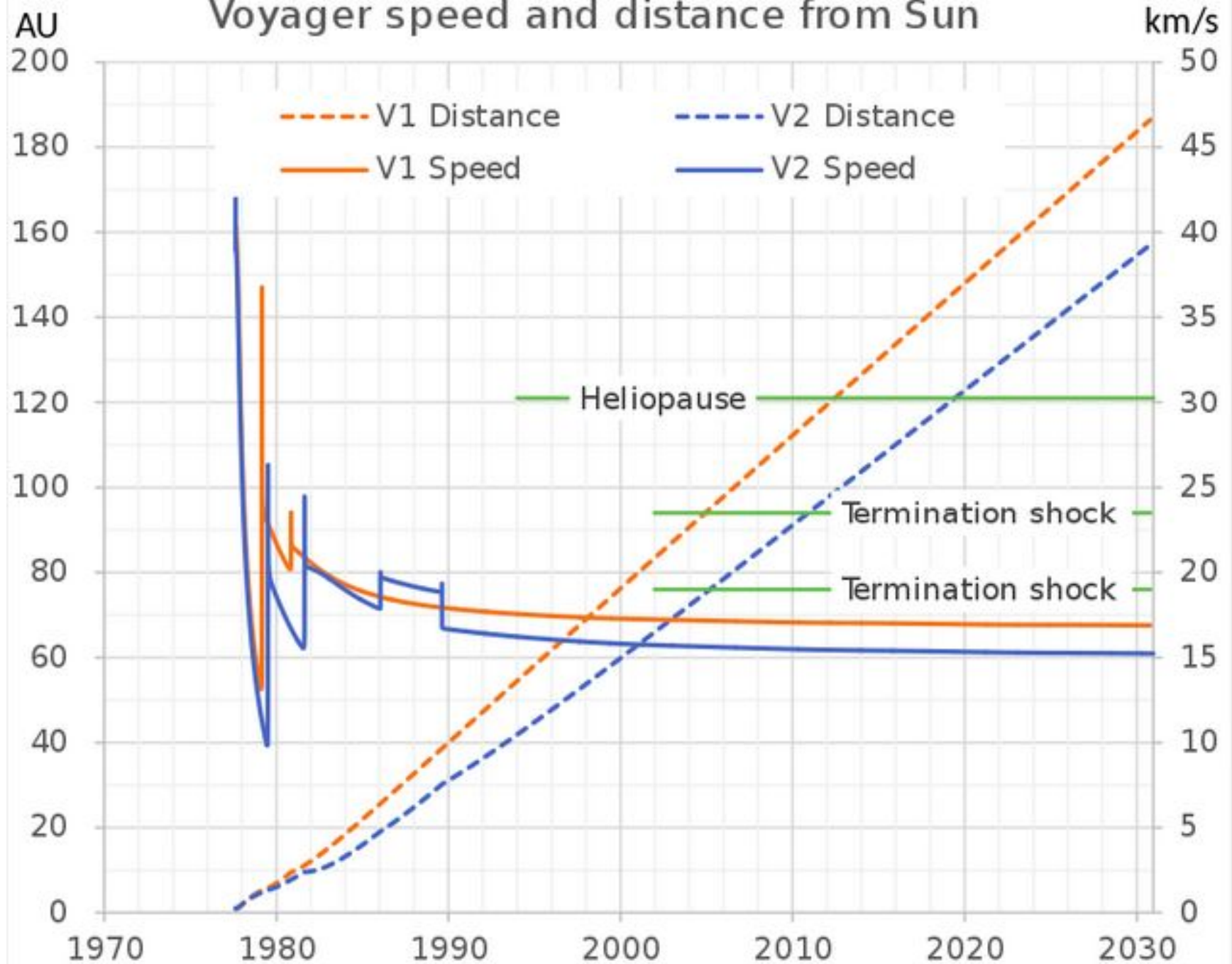
$|B_{\text{rad}}| = |E_{\text{rad}}|/c$ ;  $B$  direction given by  $k \times E$  -  
check directions:  $E \times B = k$ ?

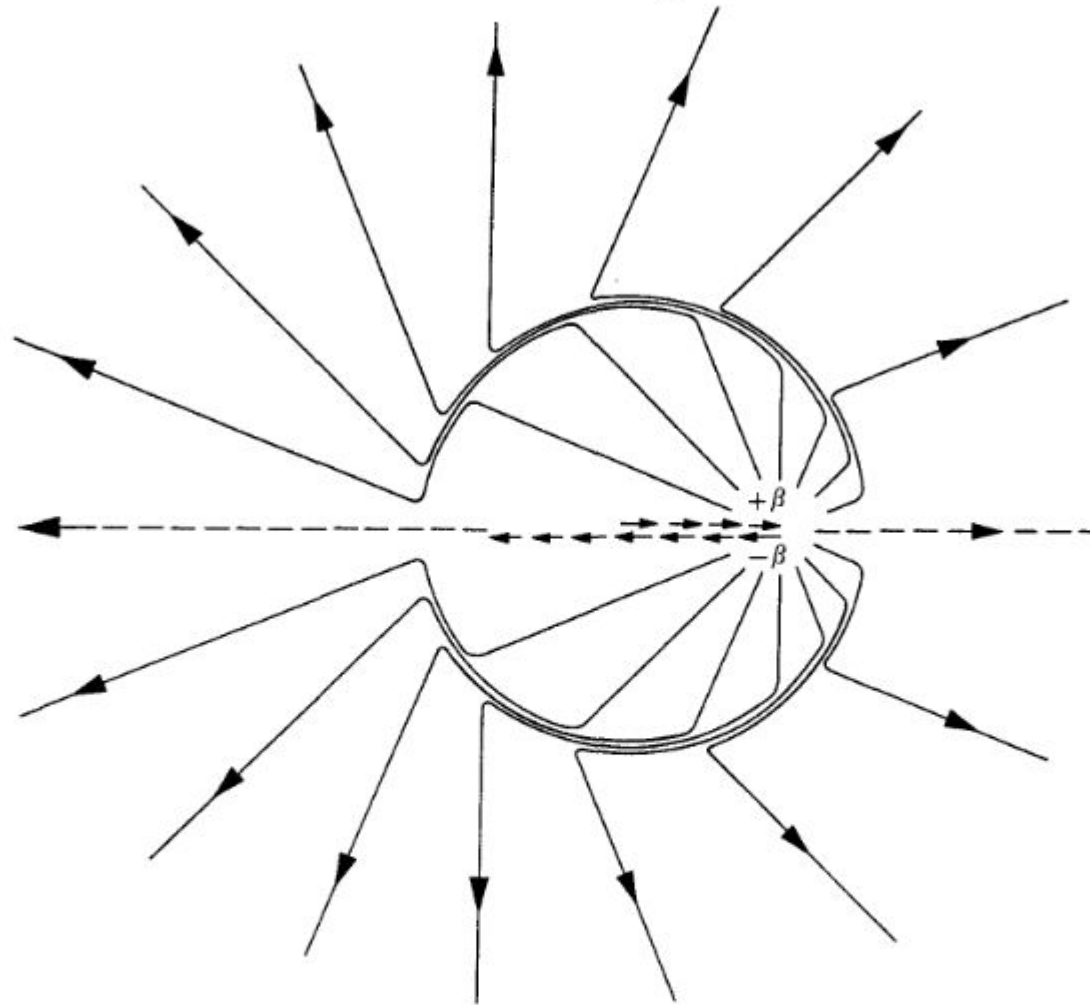
$$S [\text{Energy}/\text{m}^2/\text{sec} = \text{W}/\text{m}^2] = E_{\text{rad}}^2/2Z_0 \sim E_{\text{rad}}^2/240\pi$$





# Voyager speed and distance from Sun



**Figure 4.6.**

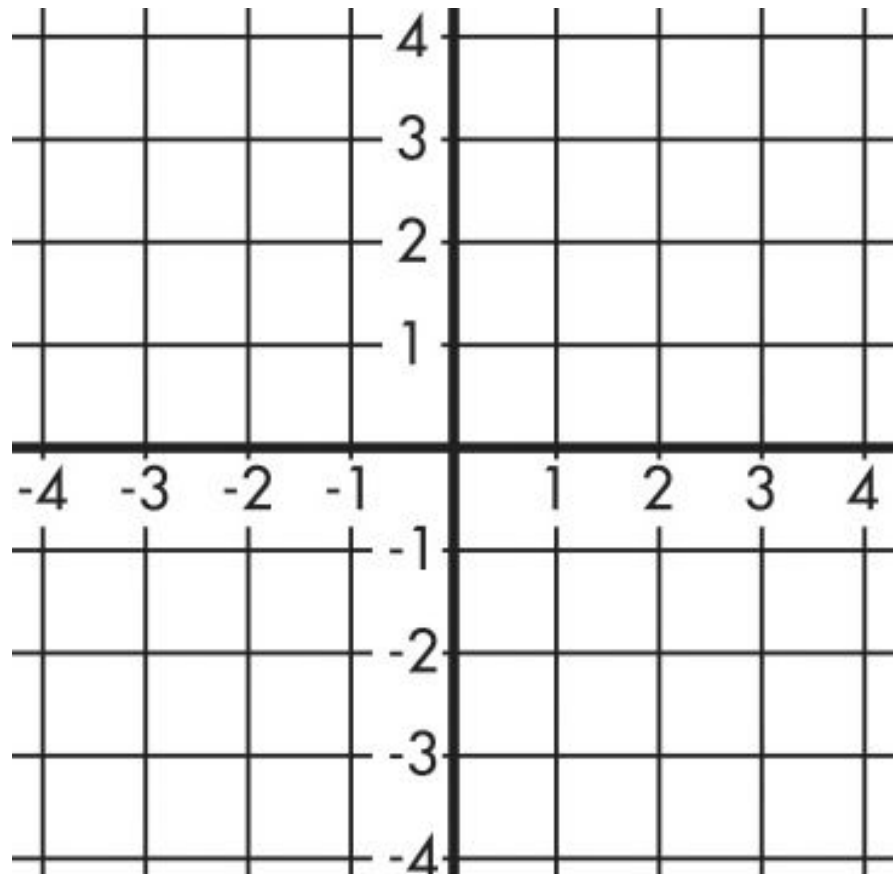
*Mechanism of radiation.* J. J. Thomson's way to understand why the strength of an electromagnetic wave falls only as the inverse first power of distance  $r$  and why the amplitude of the wave varies (for low velocities) as  $\sin \theta$  (maximum in the plane perpendicular to the line of acceleration). The charge was moving to the left at uniform velocity. Far away from it, the lines of force continue to move as if this uniform velocity were going to continue forever (Coulomb field of point-charge in slow motion). However, closer up the field is that of a point-charge moving to the right with uniform velocity ( $1/r^2$  dependence of strength upon distance). The change from the one field pattern to another is confined to a shell of thickness  $\Delta r$  located at a distance  $r$  from the point of acceleration (amplification of field by "stretching factor"  $r \sin \theta \Delta \beta / \Delta r$ ; see text). We thank C. Teitelboim for the construction of this diagram.



A -0.4 C, 2 gram charge at (0,0) is in an  $E_{+y}$  field 250 N/C, as a result of which it accelerates and begins radiating.

The produced  $E_{\text{rad}}(800 \text{ m}, 0) =$

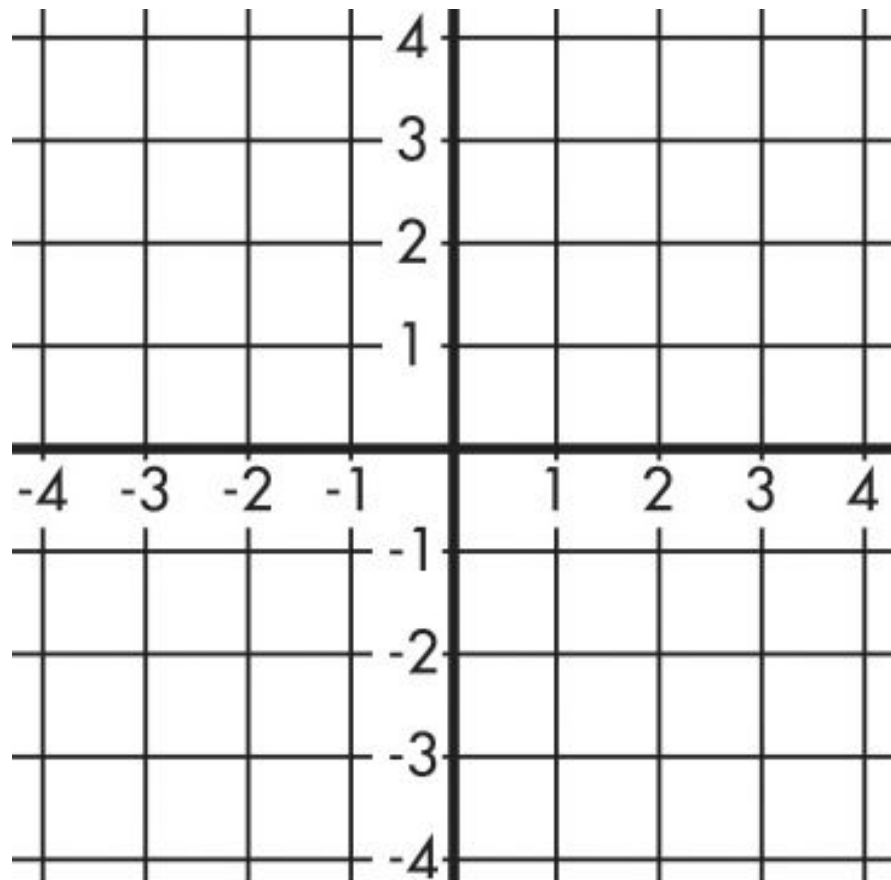
- A)  $2.5 \times 10^{-2} \text{ N/C (+y)}$
- B)  $2.5 \times 10^{-3} \text{ N/C (+y)}$
- C)  $2.5 \times 10^{-5} \text{ N/C (-y)}$
- D)  $2.5 \times 10^{-6} \text{ N/C (-y)}$
- E) NOTA





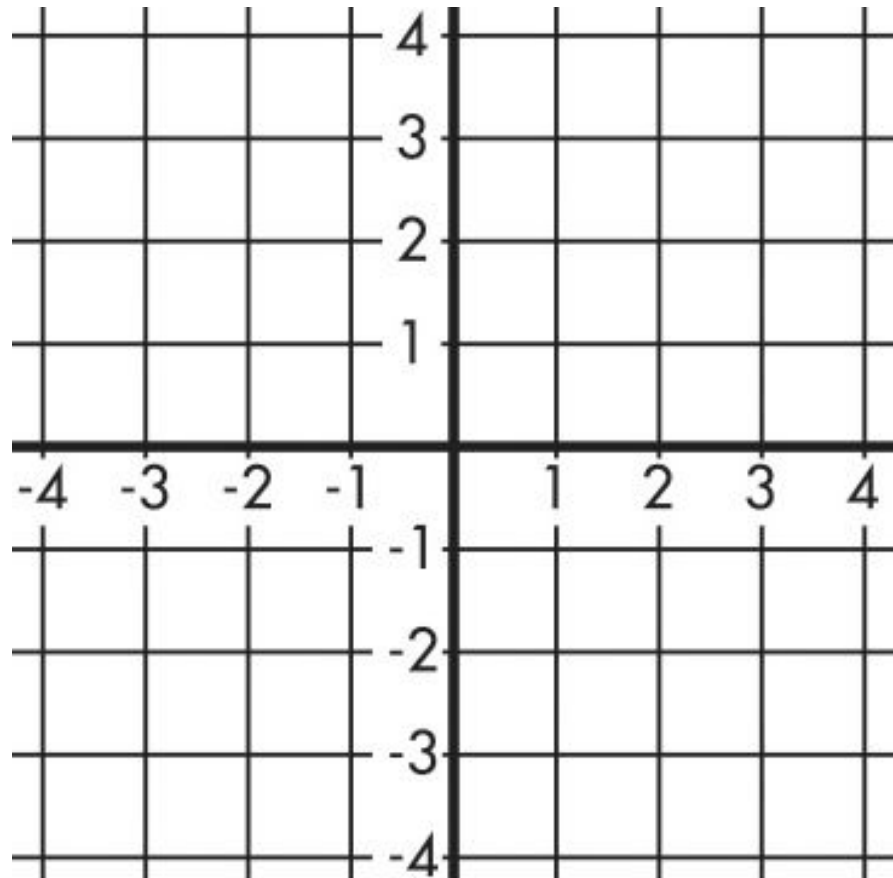
**The directions of  $B_{\text{rad}}$  at  $(800,0)$ ,  $(-800,0)$  and  $(0,800)$ , respectively, are:**

- A)  $+z, -z, 0$
- B)  $-z, +z, 0$
- C)  $+x, -x, 0$
- D)  $-x, +x, 0$
- E) NOTA



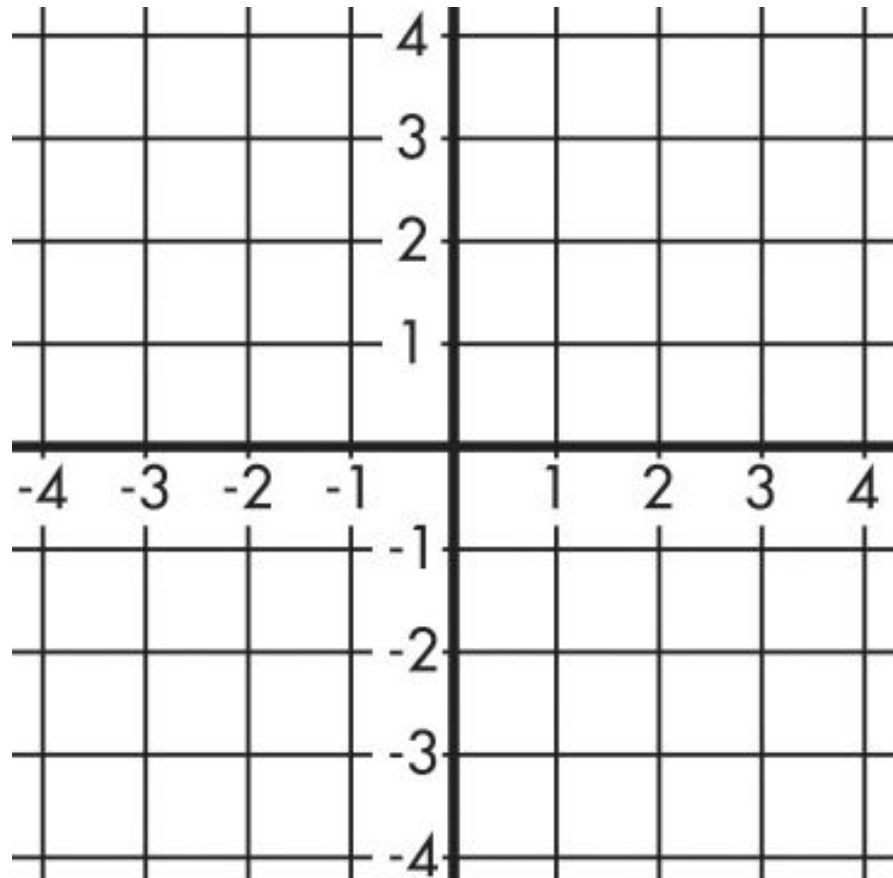
**A 0.8 C, 4 g charge at (0,0) feels a force  $F_y=200\text{ N}$ .  $E_{\text{rad}}(1200\text{ m}, 1600\text{ m})=$**

- A)  $1.5 \times 10^{-2}\text{ N/C}$
- B)  $1.5 \times 10^{-3}\text{ N/C}$
- C)  $1.2 \times 10^{-6}\text{ N/C}$
- D)  $6 \times 10^{-7}\text{ N/C}$
- E) NOTA



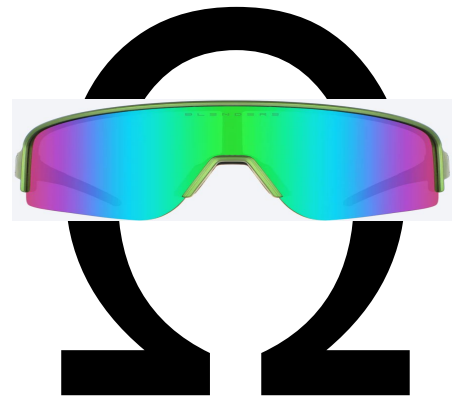
**The average current  $I = \Delta q / \Delta t$ , after the charge has moved 1 mm, is:**

- A) 2 mA
- B) 0.2 mA
- C) 4 mA
- D) 4000 A
- E) NOTA



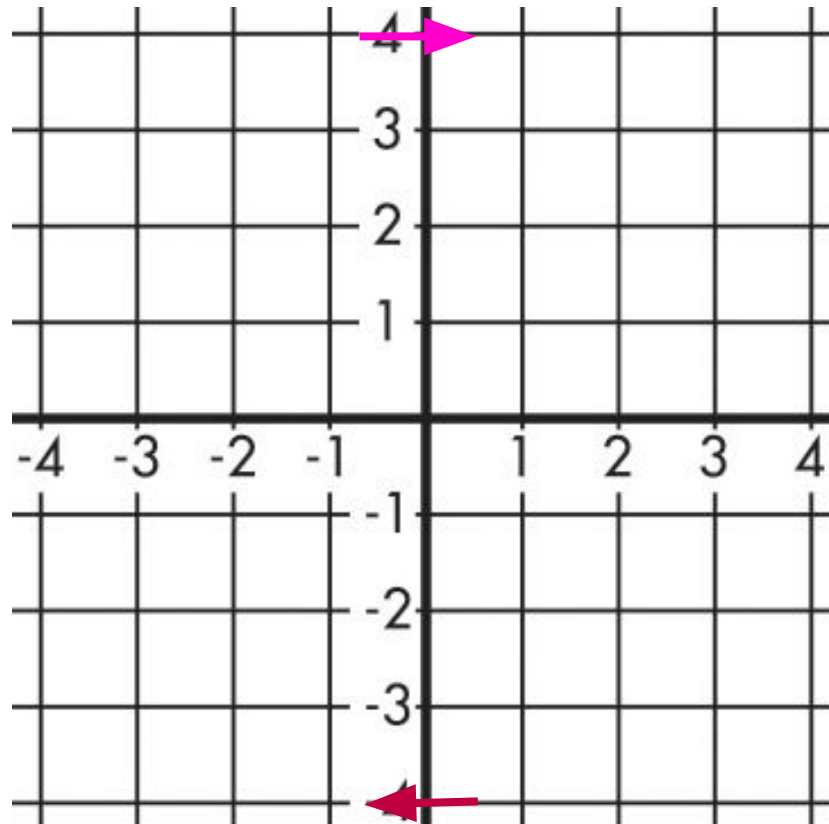


Soy la  
**pera**



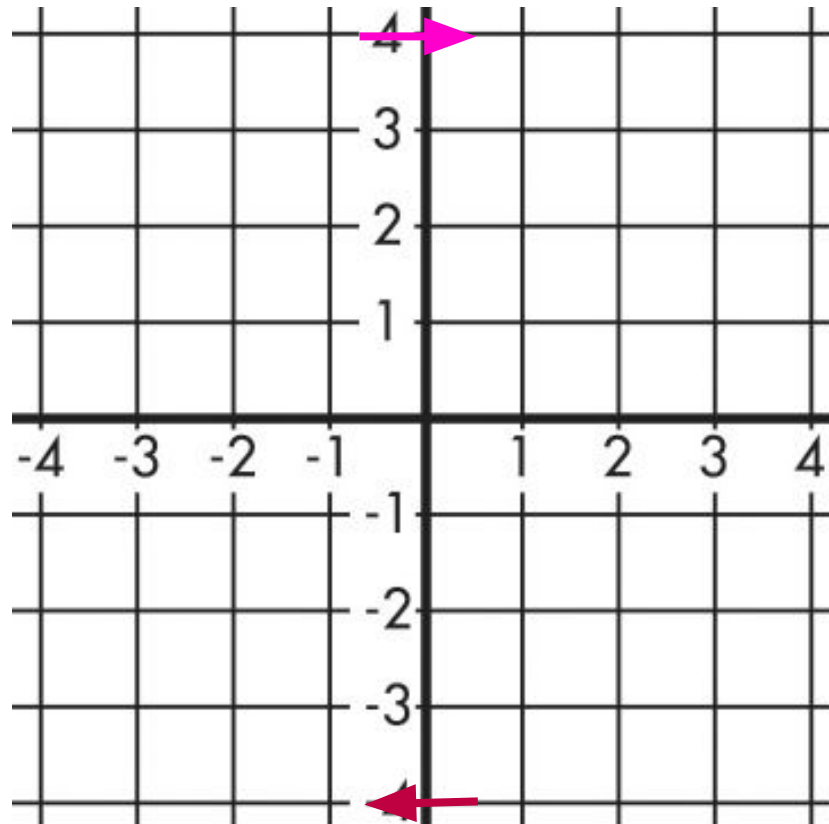
A  $-1\text{C}$  charge accelerates in  $+x$  at  $(0,4\text{m})$ ; a  $+1\text{C}$  charge has the same magnitude acceleration, in  $-x$  at  $(x,y)=(0,-4\text{m})$ . A negative test charge at the origin will accelerate in which direction, under the influence of the net radiation electric field at the origin?

- A)  $+x$
- B)  $+y$
- C)  $-x$
- D)  $-y$
- E) NOTA or  $E_{\text{net,rad}} \sim 0$



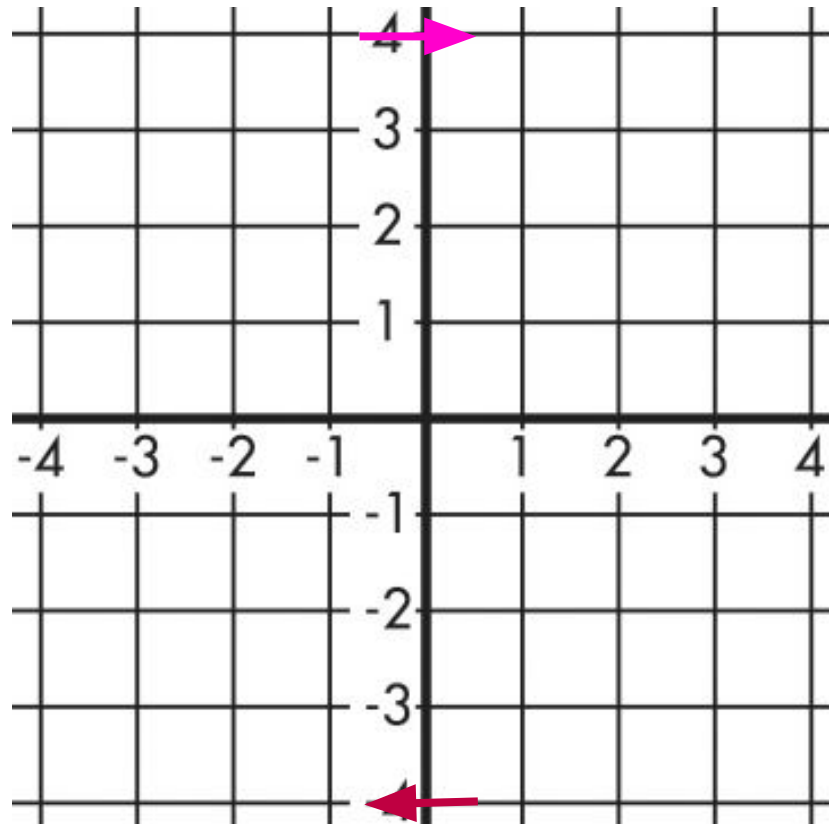
A  $-1\text{C}$  charge accelerates in  $+x$  at  $(0,4\text{m})$ ; a  $+1\text{C}$  charge has the same magnitude acceleration, in  $-x$  at  $(x,y)=(0,-4\text{m})$ . The net radiation magnetic field at the origin points in

- A)  $+x$
- B)  $+y$
- C)  $-x$
- D)  $-y$
- E) NOTA or  $B_{\text{net,rad}} \sim 0$



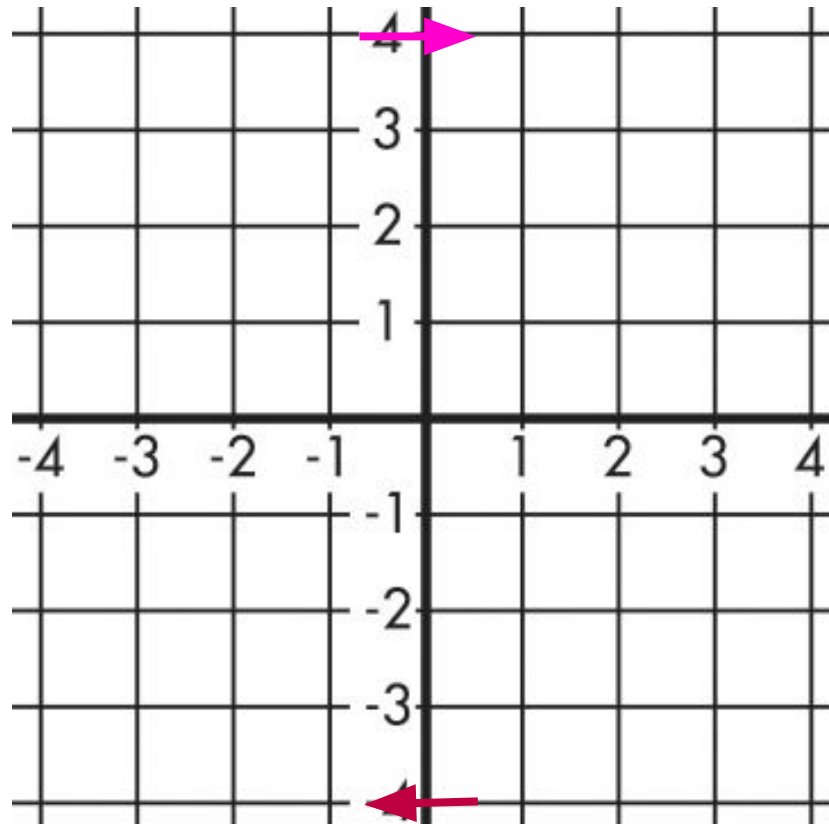
A  $-1\text{C}$  charge accelerates in  $+x$  at  $(0,4\text{m})$ ; a  $+1\text{C}$  charge has the same magnitude acceleration, in  $-x$  at  $(x,y)=(0,-4\text{m})$ . The net static electric field at the origin points in

- A)  $+x$
- B)  $+y$
- C)  $-x$
- D)  $-y$
- E) NOTA or  $E_{\text{net,static}} \sim 0$



A  $-1\text{C}$  charge accelerates in  $+x$  at  $(0,4\text{m})$ ; a  $+1\text{C}$  charge has the same magnitude acceleration, in  $-x$  at  $(x,y)=(0,-4\text{m})$ . The net current magnetic field at the origin points in

- A)  $+x$
- B)  $+y$
- C)  $-x$
- D)  $-y$
- E) NOTA or  $B_{\text{net,current}} \sim 0$



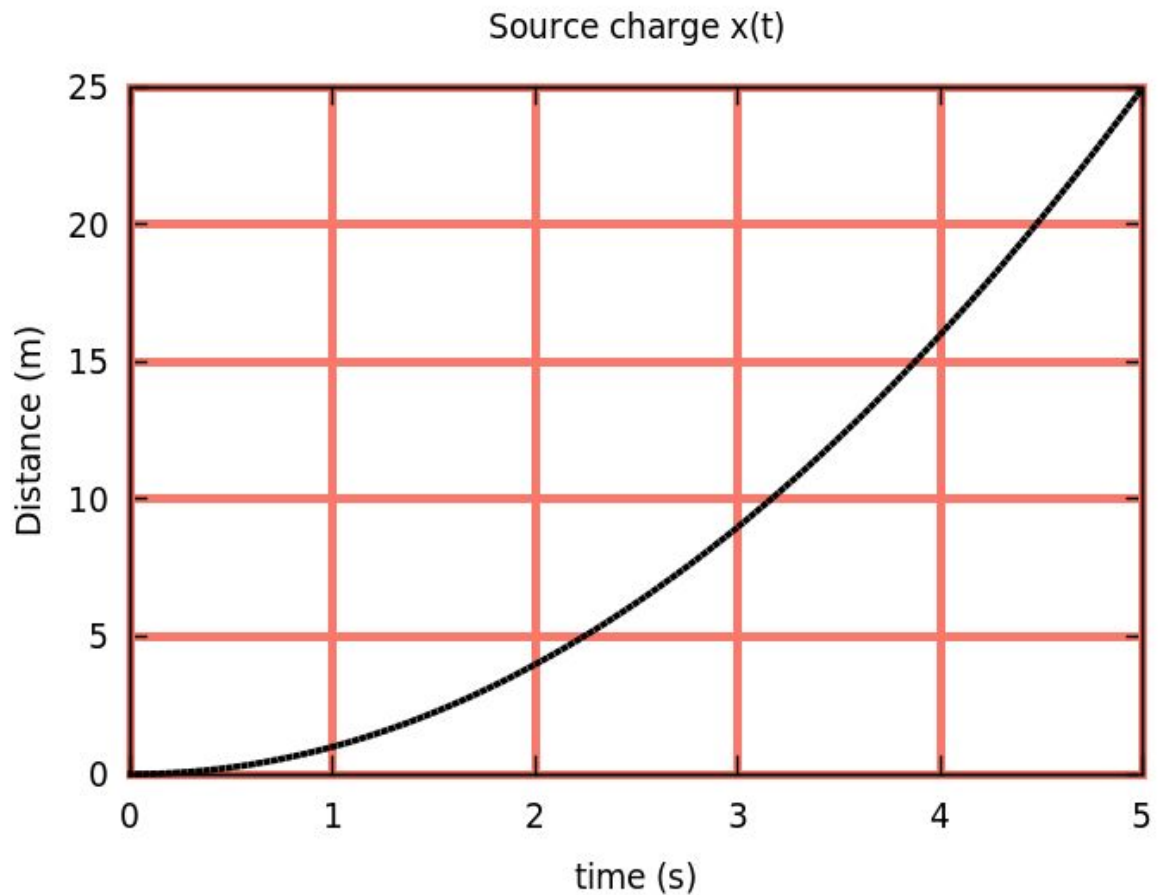


Given that  $E_{\text{rad}} = kqas \sin \theta / c^2 r$ , the acceleration needed such that  $E_{\text{rad}}$  from a source at the origin, measured at  $r=100$  m equals  $E_{\text{Coulomb}}$

- A)  $10^3$  g
- B)  $10^4$  g
- C)  $10^5$  g
- D)  $10^9$  g
- E)  $10^{15}$  g

Given the  $x(t)$  graph for a 0.1 C source charge, rank the  $E_{\text{Coulomb}}$ ,  $B_{\text{current}}$ ,  $E_{\text{rad}}$ , and  $B_{\text{rad}}$  fields at a distance of 2 km

- A)  $E_C > B_C > E_r > B_r$
- B)  $E_r > B_r > E_C > B_C$
- C)  $E_C < B_C < E_r < B_r$
- D) NOTA



A 2 C charge accelerates in +x for  $10^{-6}$  seconds at  $10^6$  g;  $F_{\text{net}}$  on a stationary 0.1 C test charge at  $(x,y)=(0,10^7 \text{ km})=$  ( $k=9 \times 10^9$  N-m<sup>2</sup>/C<sup>2</sup>)

- A)  $1.8 \times 10^{-11} \text{ N}$
- B)  $1.96 \times 10^{-11} \text{ N}$
- C)  $2.66 \times 10^{-11} \text{ N}$
- D) NOTA

The direction of  $F_{\text{net}}$  is:

**$B_{\text{current}}$  after the charge has moved 1 mm,  
is (assume it starts from rest):**

- A)  $0.16k' \text{ T}$
- B)  $1.6k' \text{ T}$
- C)  $2k' \text{ T}$
- D)  $2 \times 10^{-5}k' \text{ T}$
- E) NOTA

If the 0.1 C test charge moves at 0.2c, then  
 $F_{\text{net, magnetic}}$  on the charge is:

- A) 10.5 N
- B) 0.00024 N
- C) 0.1 N
- D)  $3 \times 10^{-13}$  N
- E) NOTA

A 4 kg, -2 C charged particle is located at the point (2m,0); it is in an E-field: 6N/C in the +y direction; as a result of the electrical force, the -2C charge moves a distance of 4 m. Determine, in sequence: i) the acceleration of the charge, ii) the time required for the charge to move the 4 m, iii) the current constituted by the charge's motion (charge divided by the time you just calculated). iv) From your answer to i), determine  $E_{\text{rad}}$  (magnitude and direction) at the measurement point (-4m,0), as well as  $B_{\text{rad}}$  (magnitude and direction) at the same point. As a reminder, the magnitude of  $B_{\text{rad}} = E_{\text{rad}}/c$  (see sect. 24.2 from the online text). v) from your answer to iii), calculate  $B_{\text{current}}$  (magnitude and direction) at the same measurement point (0,4m).

$a=F/m=12\text{N}/4\text{kg} = 3\text{m/s}^2$  (-y); t to travel 4 m from  $y_f=y_i+v_{i,y}t+\frac{1}{2}at^2$  (ignore gravity), so  $t=1.64$  s;  
 current  $I=q*x/t=(2\text{C}/1.64\text{s})$  (+y) / What is  $E_{\text{rad}}(-4\text{m},0)$  due to initial acceleration?

$E_{\text{rad}}=kqa(\sin\theta/c^2r)$ :  $\theta$ =angle b/w  $\mathbf{a}$  and  $\mathbf{r}$  ( $90^\circ$  in this case),  $r$ =vector b/w source of  $E_{\text{rad}}$  and measurement pt X (6m in this case), so  $E_{\text{rad}}=1\text{e-}07$  N/C;  $E_{\text{rad}}\parallel\mathbf{a}$  for positive source charge, so here in +y-direction.

$B_{\text{rad}}=E_{\text{rad}}/c$ ; direction given by RHR(1) – first vector= $\mathbf{r}$ ; second vector= $\mathbf{E}_{\text{rad}}$ , so  $B_{\text{rad}}$  in -z

$B_{\text{current}}=k'I/r=2\text{e-}7*(2\text{C}/1.64\text{s})/6\text{m}=4.54\text{e-}8$  T (+z)

Energy considerations

(CAP22): Two takes: 1.

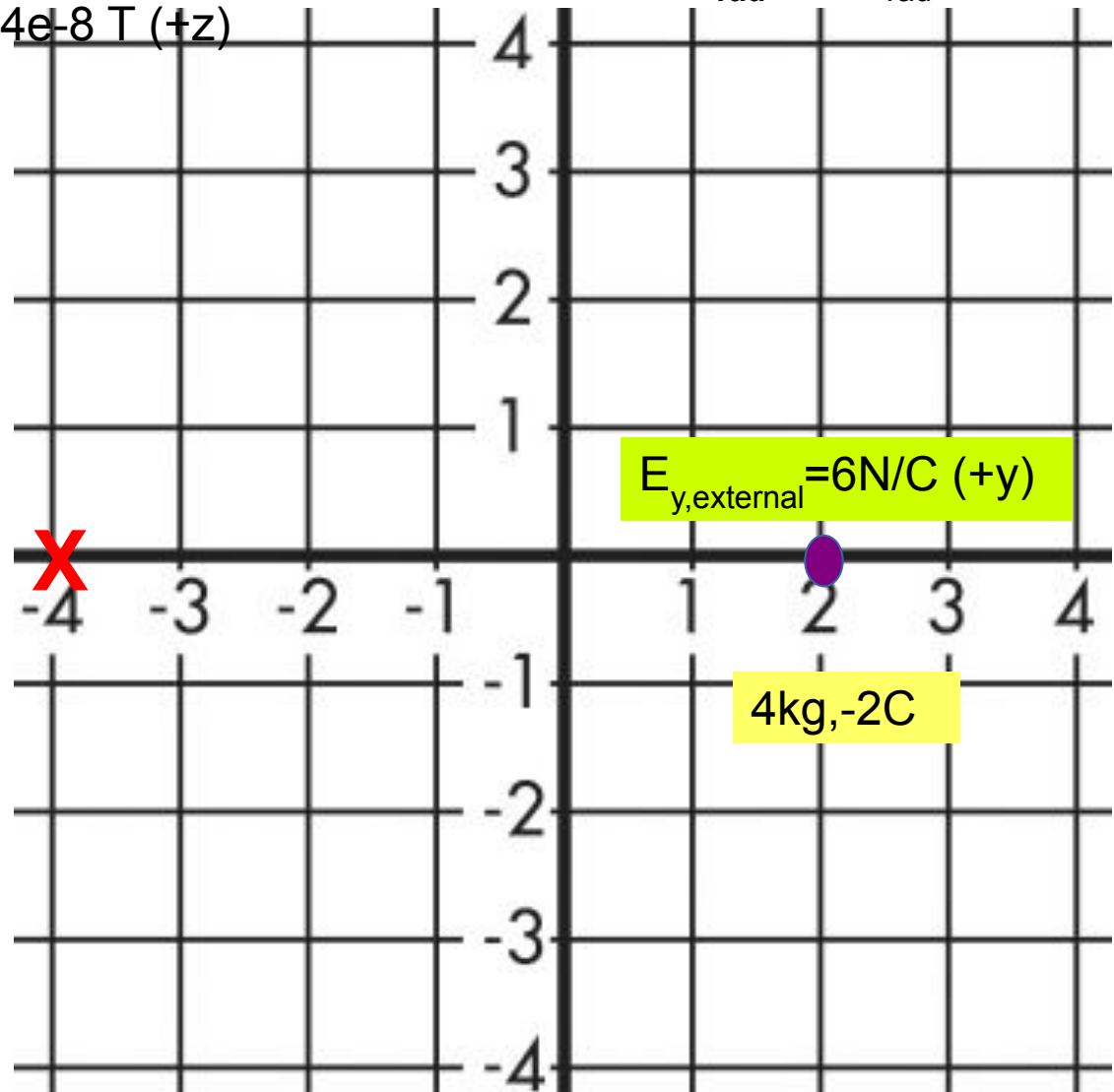
Mechanical power required to accelerate charge over 4 meters: Power=Work/time;

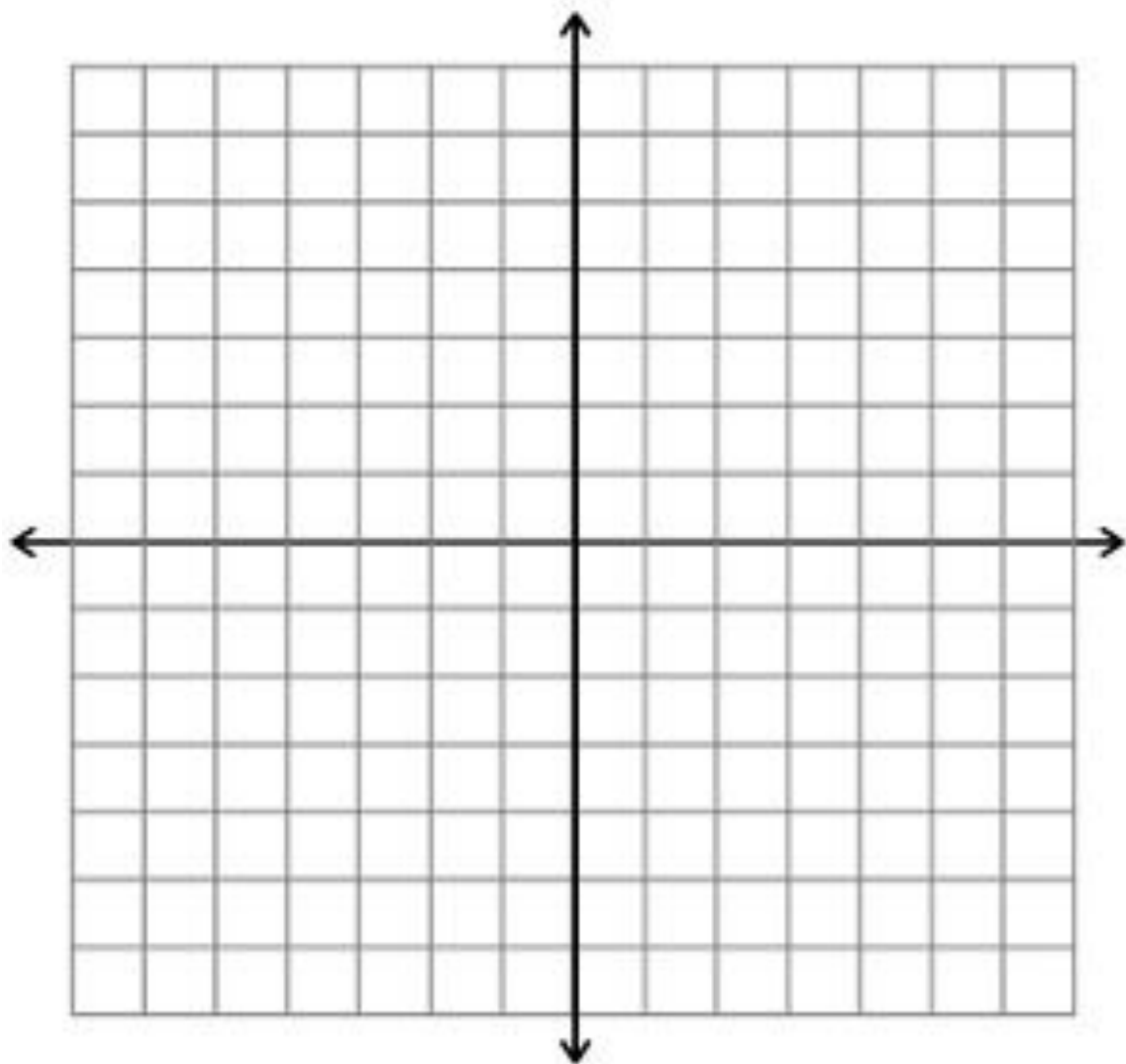
Work= $0.5*4\text{kg}*v_f^2$ ;  $v(f)=at=4.92$  m/s, so  $W=48.4$  J, so  $P=48.4$  J/1.64s=29.52 W

2. Calculate  $E_{\text{rad}}$  at measurement point and work backwards:  $E_{\text{rad}}=1\text{e-}7$ ;

$S=1\text{e-}18/240\pi$ , so

$S=1.327\text{e-}17$  W/m<sup>2</sup>; output power= $S*4\pi*36=6\text{e-}15$







A radiation electric field, due to a charge  $Q$  (having mass  $m$ ) at the origin, acted on by an external electric field  $E$ , is measured to be  $0.0025 \text{ N/C}$  at  $x=800 \text{ m}$ . The radiation *Intensity* at  $x=800 \text{ m}$  is:

- A)  $1.48 \times 10^{-3} \text{ W/m}^2$
- B)  $3.34 \times 10^{-6} \text{ W/m}^2$
- C)  $8.3 \times 10^{-9} \text{ W/m}^2$
- D)  $2.44 \text{ W/m}^2$
- E) NOTA

**Assuming full-spherical spreading, the total amount of energy radiated by the accelerating charge, in 2 seconds, is about:**

- A) 0.13 J
- B) 1.3 J
- C) 0.013 J
- D) 13 J
- E) NOTA

**Assuming  $E_{\text{external}}$  doubled, then the amount of radiated power would:**

- A) Be the same
- B) Double
- C) Quadruple
- D) Halve
- E) NOTA

**Assuming the charge  $Q$  and its mass both doubled,  
then the amount of radiated power would:**

- A) Be the same
- B) Double
- C) Quadruple
- D) Halve
- E) NOTA

**Solar energy radiates the Earth at  $1368 \text{ W/m}^2$ ; estimate the efficiency of the human eye (assume circular pupil of diameter 1 cm) if it generates a voltage of about 1 mV and a current of about 1 pA for one second.**

- A)  $10^{-14}$
- B)  $10^{-8}$
- C)  $10^{-3}$
- D) 0.4

**Estimate the energy delivered (full moon) to a 50 cm x 50 cm solar panel via moonshine in 5 seconds, knowing that the moon is about  $1^\circ$  in the sky, with an albedo of 0.11.  $P_s=1368 \text{ W/m}^2$ )**

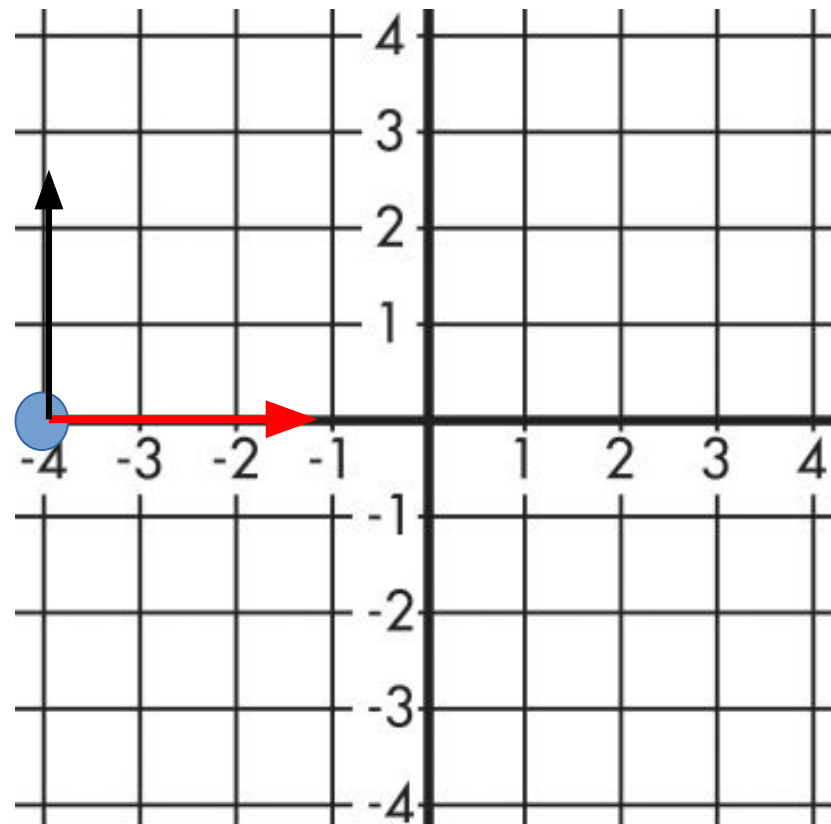
- A) 15 J
- B) 0.15 J
- C) 0.015 J
- D) 0.0015 J
- E) 0.00015 J

Standing at  $x = -4$  m, you measure  $E_{\text{rad}}$ , in  $+x$ ; you also measure  $E_{\text{coulomb}}$  in  $+y$ -direction. From that and knowing  $E_{\text{rad}} = kQa(\sin\theta)/c^2r$ , you conclude that the source  $Q$  is:

- A) Negative, at  $y > 0$ , and  $a$  in  $-x$
- B) Negative, at  $y < 0$ , and  $a$  in  $+x$
- C) Positive, at  $y > 0$ , and  $a$  in  $+x$
- D) Positive, at  $y < 0$  and  $a$  in  $-x$
- E) Two of the above

In the same problem,  $B_{\text{current}}$  and  $B_{\text{rad}}$  point along:

- A)  $+z$ ,  $-z$
- B)  $+z$ ,  $+z$
- C)  $-z$ ,  $+z$
- D)  $-z$ ,  $-z$
- E) NOTA



Given  $S_0 = 1368 \text{ W/m}^2$ , and  $1 \text{ W} = 1 \text{ J/s}$ , the total solar electromagnetic radiation energy intercepted by a solar panel  $2\text{m} \times 2\text{m}$  directly facing the sun, over a time  $0.25$  seconds, =

- A)  $1368 \text{ J}$
- B)  $2736 \text{ J}$
- C)  $684 \text{ J}$
- D) NOTA





What is  $E_{\text{rad}}$  for solar EM radiation at the Earth given that  $S = E^2/240\pi$  ( $S = I_{\text{solar}} = 1368 \text{ W/m}^2$ ;  $r_{\text{Earth,Sun}} = 146 \times 10^9 \text{ m}$ )

- A)  $E_{\text{rad}} = 34.3 \text{ V/m}$
- B)  $E_{\text{rad}} = 211.1 \text{ V/m}$
- C)  $E_{\text{rad}} = 1015 \text{ V/m}$ ;
- D) NOTA

What is the total solar power radiated at the source, assuming 3-d spherical spreading?

- A)  $P_0 = 3.66 \times 10^{26} \text{ Watts}$
- B)  $P_0 = 6.2 \text{ MegaWatts}$
- C)  $P_0 = 7.89 \text{ TeraWatts}$
- D) NOTA

Suppose the sun only beamed energy into the hemisphere containing the Earth, and you measured the same  $I_{\text{solar}}$ . In that case, your estimate for the total solar power at the source, relative to the previous problem, would be

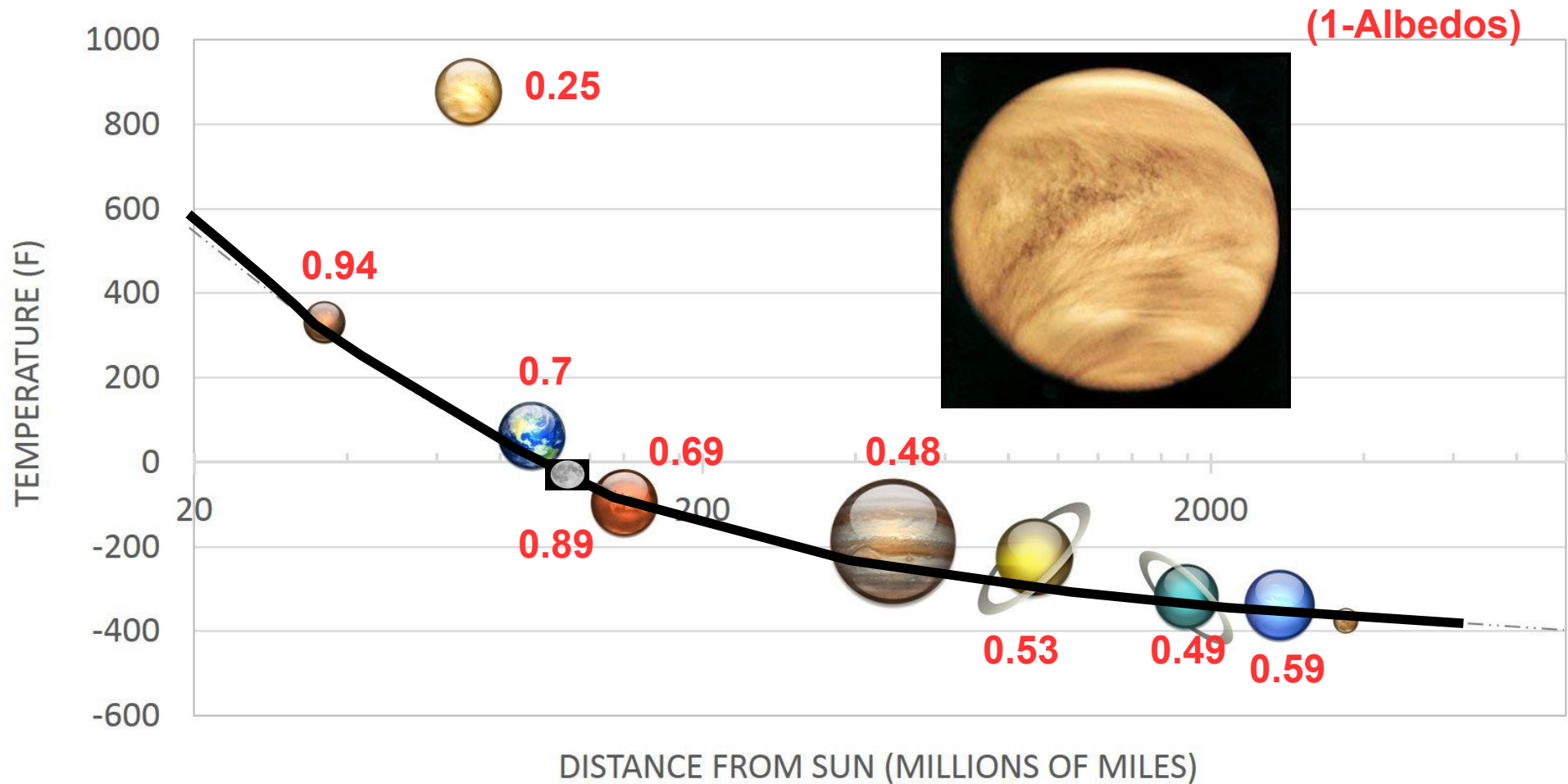
- A) Smaller
- B) Larger
- C) unchanged

(cf high-gain antenna)

Planet Temperatures: a) distance to sun, b) “albedo”, c) greenhouse effect

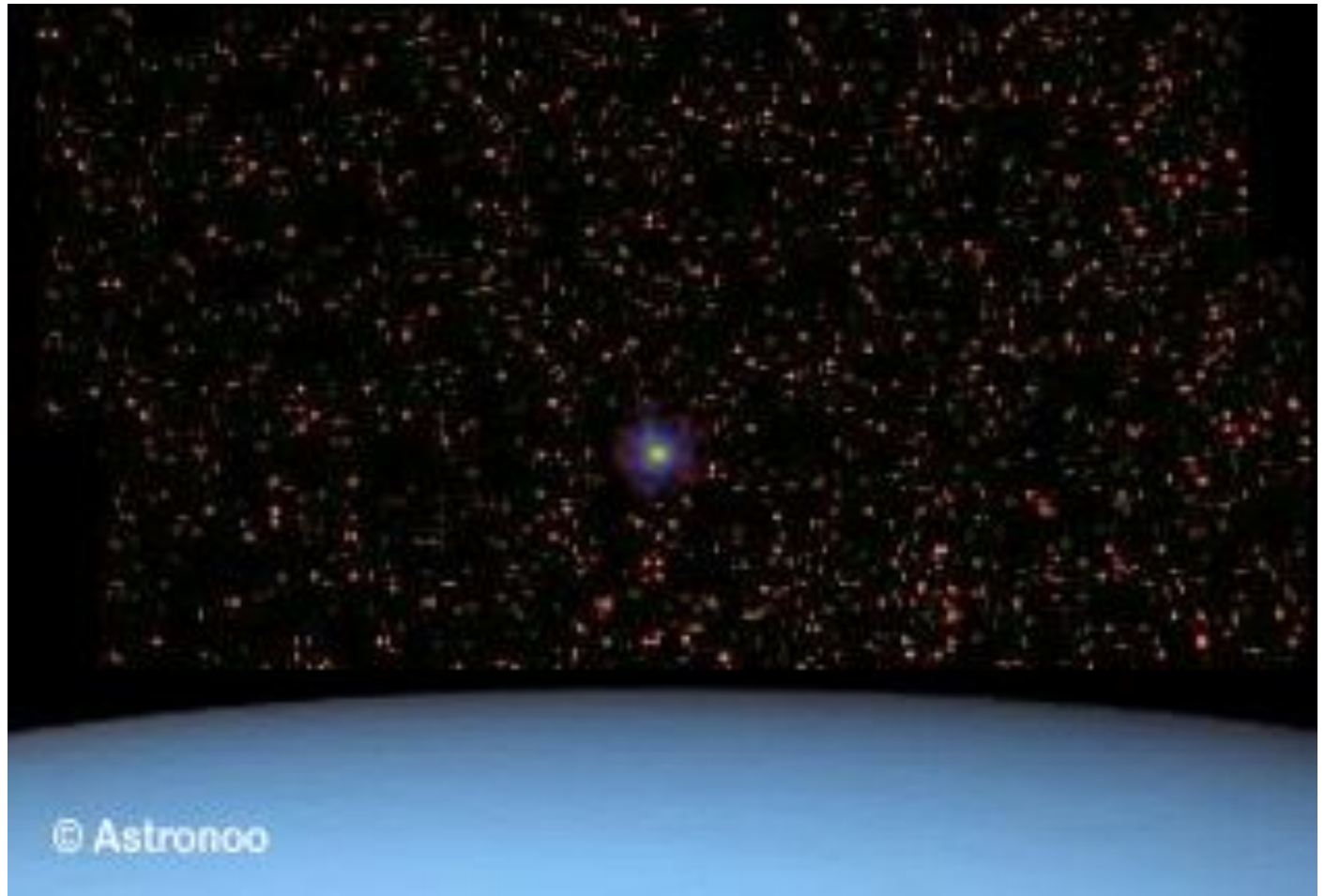
<http://www.climatefiles.com/climate-change-evidence/presidents-report-atmospher-carbon-dioxide/>

## Average Planetary Temperatures



- Without  $\text{CO}_2 + \text{CH}_4 + \text{N}_2\text{O}$  warming, Earth surface temperature would be  $\sim 0^\circ\text{F} / -18^\circ\text{C}$  (giss.nasa.gov)

# View of sun from Uranus



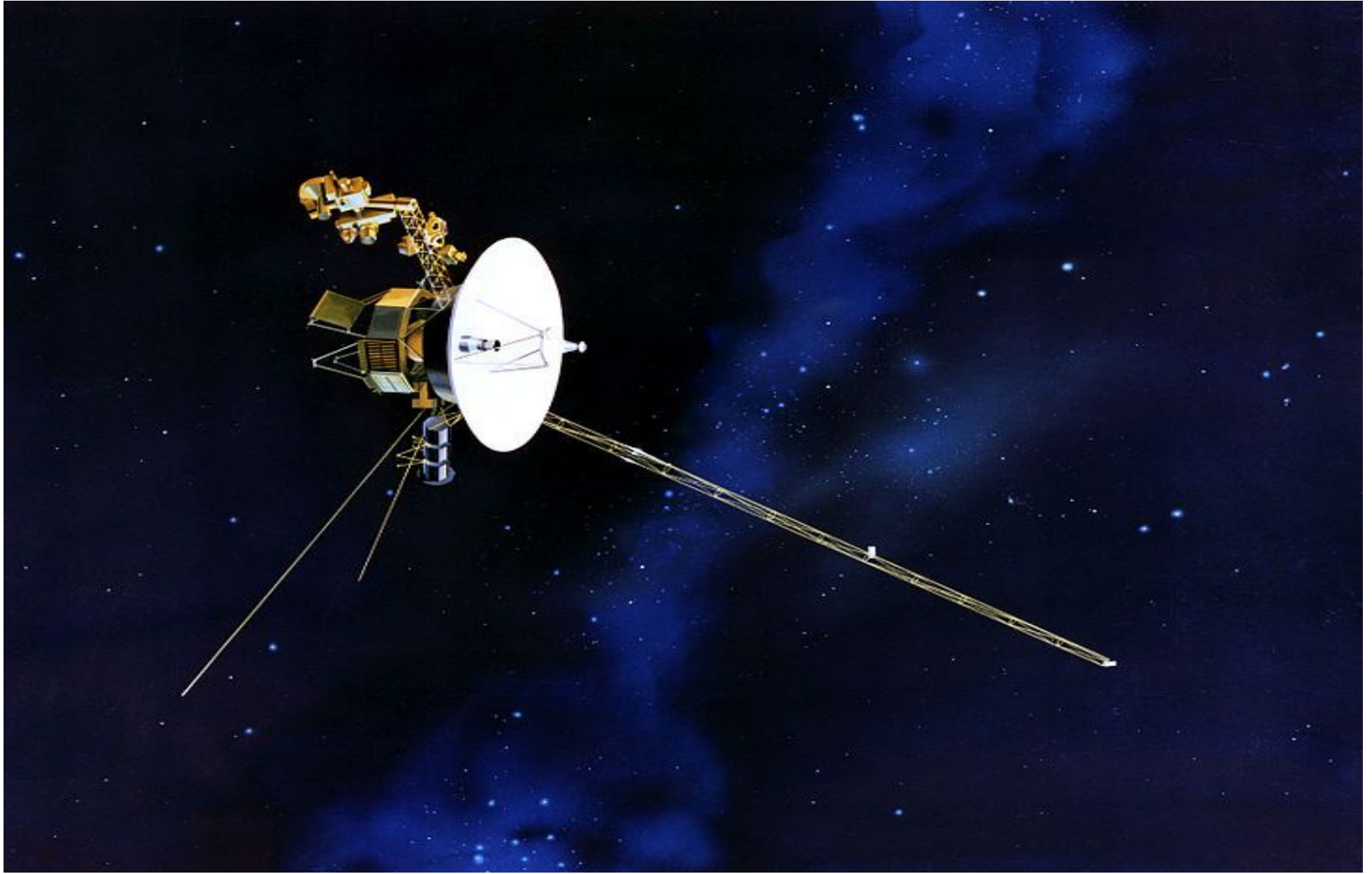
**In the previous problem, the total electromagnetic energy delivered to a 50 cm x 50 cm antenna as the echo is measured is closest to:**

- A) 2.44 J
- B)  $3.3 \times 10^{-19}$  J
- C)  $3.3 \times 10^{-11}$  J
- D) 98.33 mJ

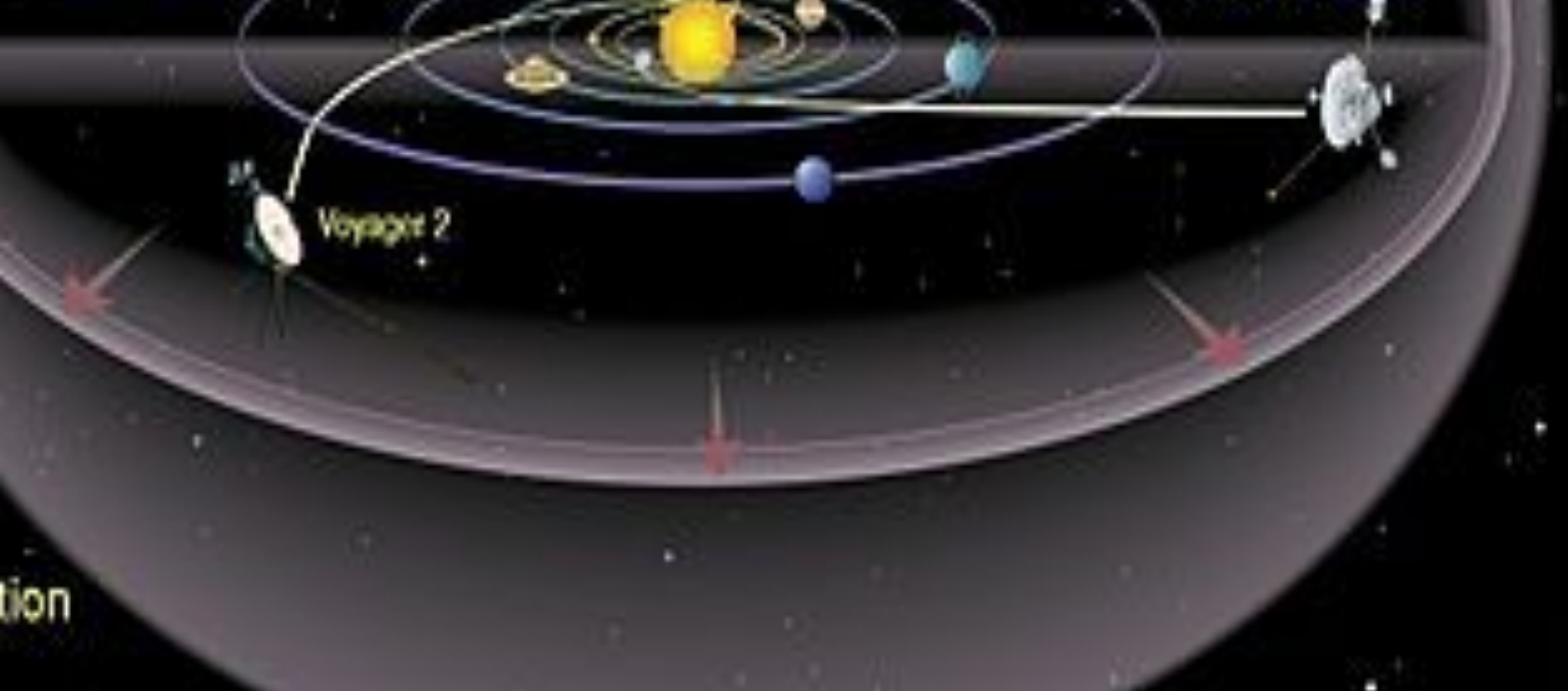
**By finding the fraction **f** of total emitted power intercepted, the maximum power output at the source is approximately (assume spherical spreading)**

- A) 2.44 W
- B)  $5.2 \times 10^{-2}$  W
- C)  $5.2 \times 10^{-10}$  W
- D) 98.33 mW
- E) NOTA

**Voyager 1, launched on 9/5/77, communicates with Earth from a distance of 133 A.U. ( $20 \times 10^9$  km) at a frequency of 2.1-2.3 GHz**







**The Voyager spacecraft is currently  
20 billion km from Earth  
How long does it take signals to  
reach the Earth from its current  
location?**

- A) About 2 seconds
- B) About 9 minutes
- C) Almost one day
- D) One year



**Voyager communicates via 9V solar panel “battery” that charges up a 4F capacitor, which discharges (via relay) in 0.01 ms across a 1 meter long “antenna”;  
What is the  $E_{\text{rad}}$  intensity  $20 \times 10^{10}$  m away?**

- A)  $17 \times 10^{-9} \text{ W}$
- B) 17 mW
- C) 0.17 mW
- D)  $10^{-15}$  Watts
- E) NOTA

**KANU-FM broadcasts 100 kW at 91.5 MHz. Assuming the power is broadcast spherically, the amount of energy intercepted by a 100-m distant antenna with 2 m<sup>2</sup> coverage, over 4 seconds is:**

- A) 1 J
- B) 6.36 J
- C) 12.72 J
- D) NOTA

**The ANITA experiment measures 10 pW of radio energy over  $10^{-7}$  s. If the balloon is 38 km above the ice surface, the CR interacts 3 km above the surface, and the reflection point is 50 km away, what is the distance to the CR interaction?**

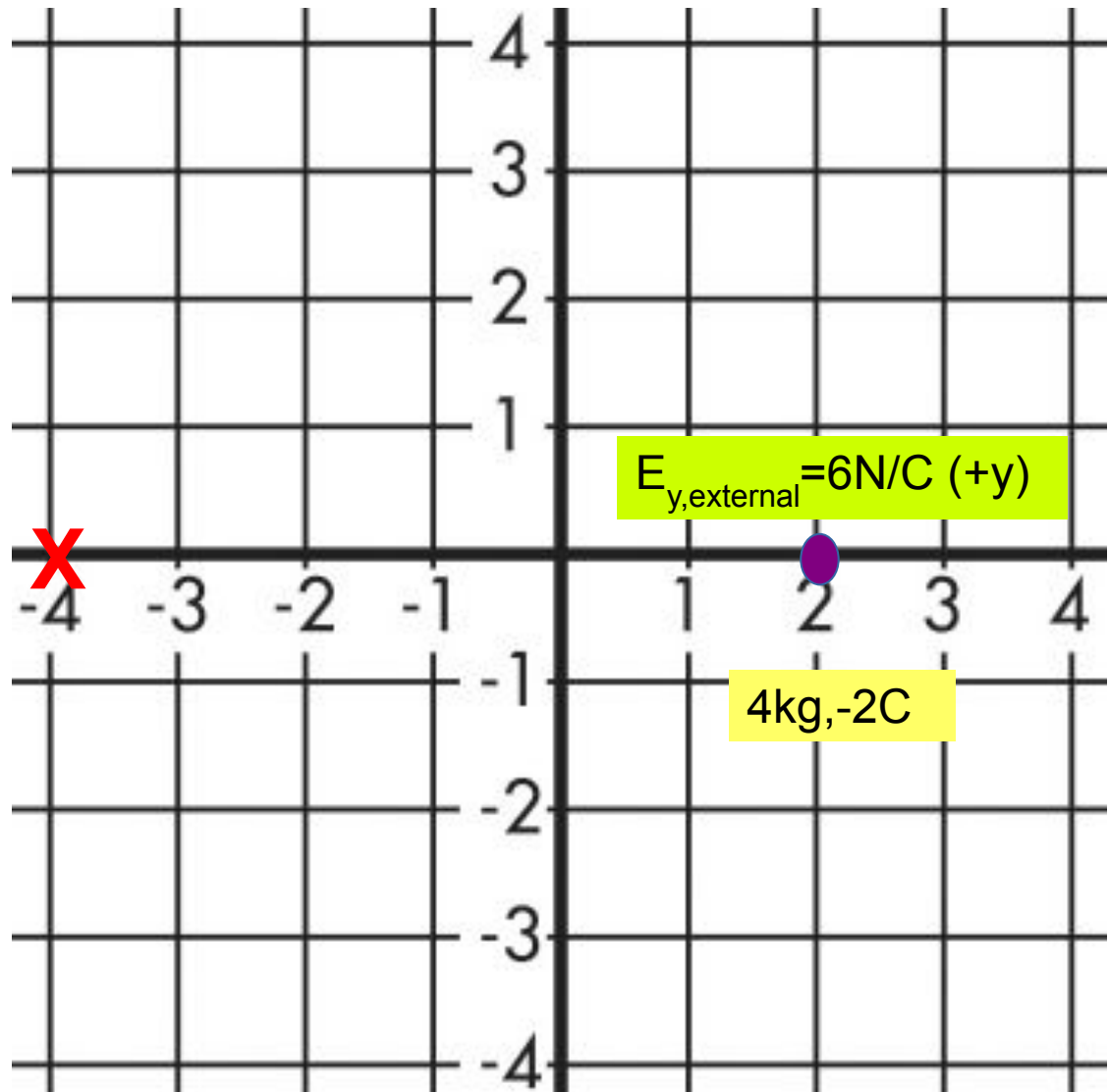
**The total energy of the CR (assuming spherical spreading) is:**

**If the power is not beamed into the ground, the previous value**

- A) Increases
- B) Decreases
- C) Is unchanged

# Propagation of electromagnetic radiation, examples (radio-frequency)

$a = F/m = 12\text{N}/4\text{kg} = 3\text{m/s}^2$  (-y); t to travel 4 m from  $y_f = y_i + v_{i,y}t + \frac{1}{2}at^2$  (ignore gravity), so  $t = 1.64$  s;







Wave Optics and interference:  $v=c/n$ ;  
 $A(x,t)=A\cos(kx-\omega t+\phi_0)$ ; argument describes  
**constant phase** condition for  $x=ct$ ;  $A(x,t)$  gives  
amplitude at some given value of  $x$  and  $t$ .

Interference, including thin film interference,  
assuming two waves traveling with same velocity

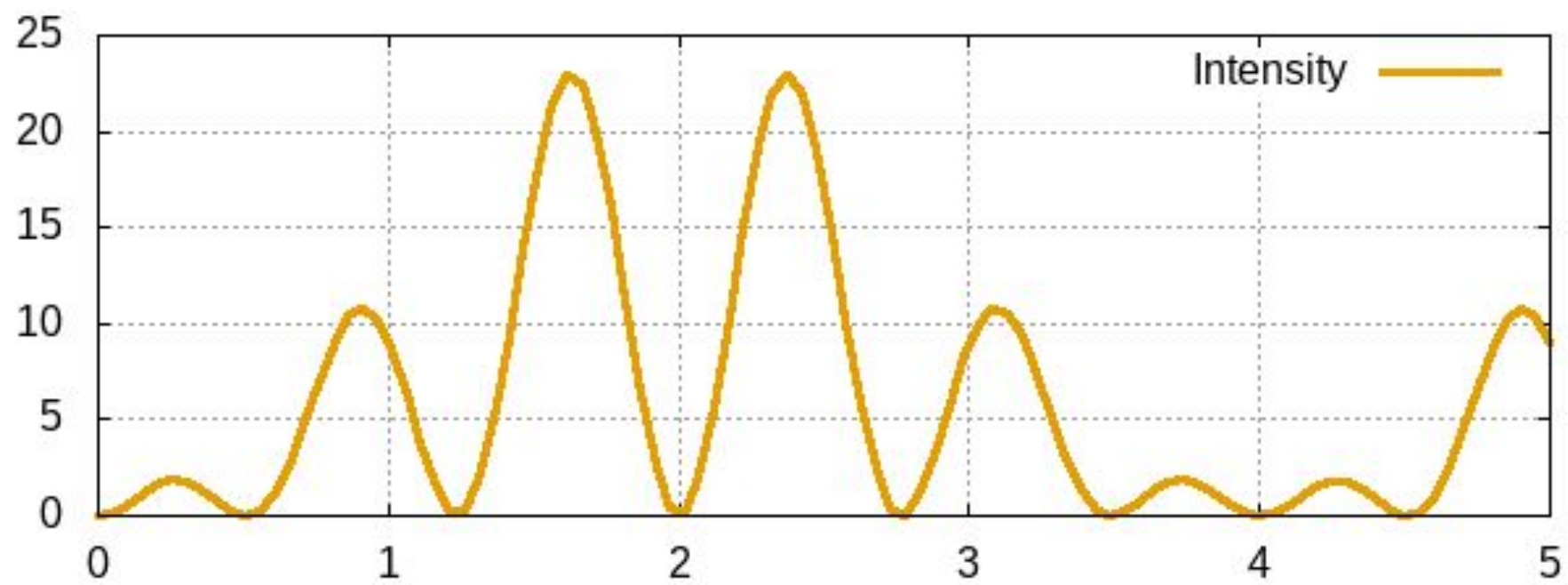
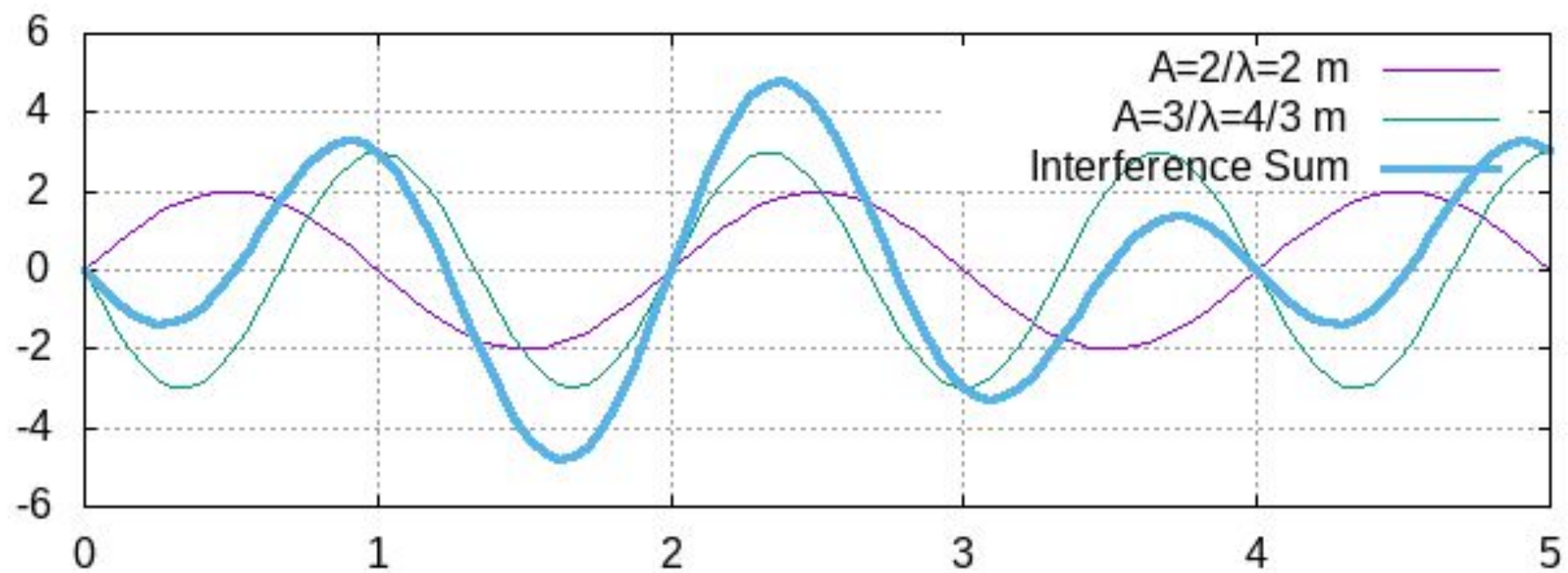
- 1) Count total number of inversions
- 2) If number=even=> minimum thickness from

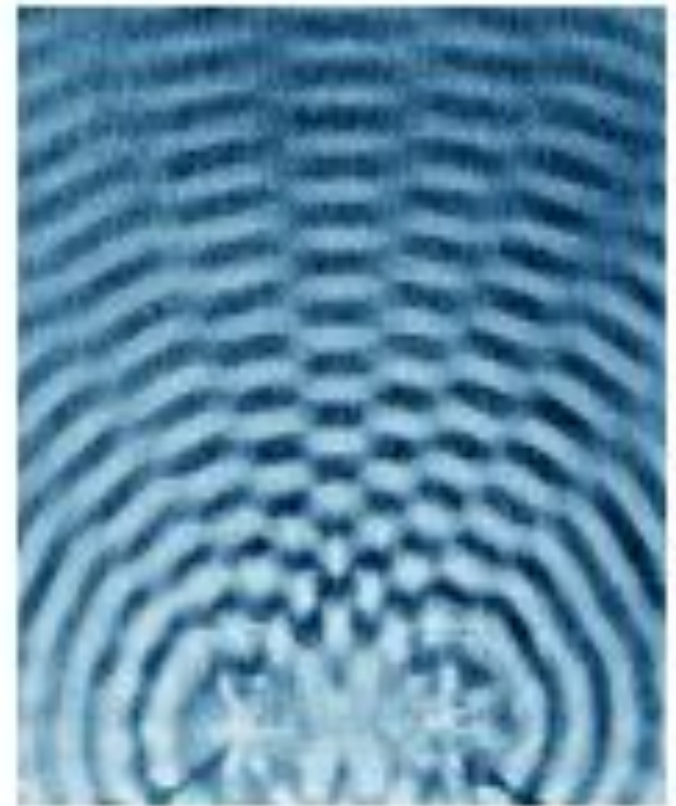
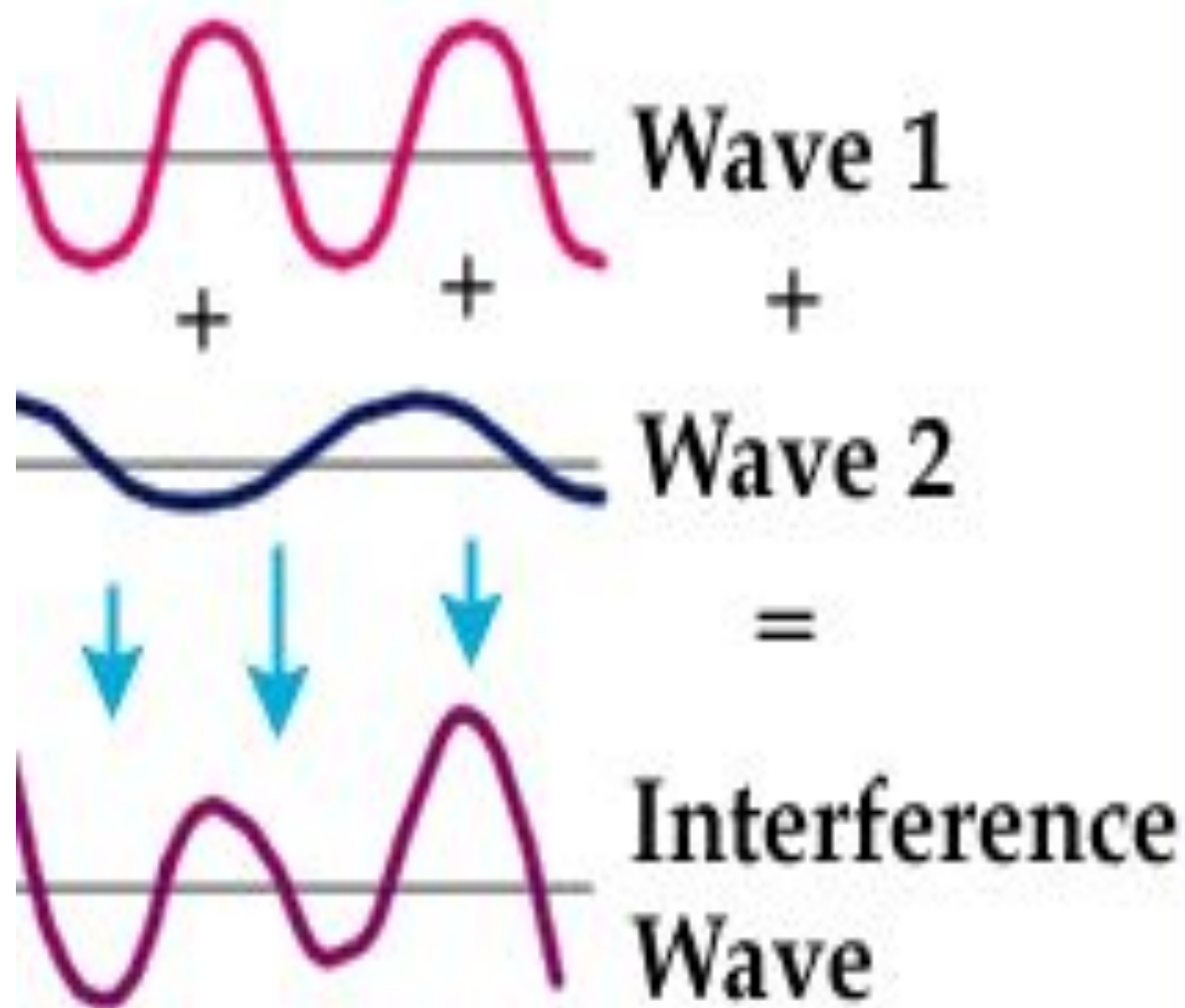
$$2t=\lambda_{\text{medium}}$$

If number odd=>minimum thickness given by

$$2t=\lambda_{\text{medium}}/2$$

Next higher thickness: add 1 unit

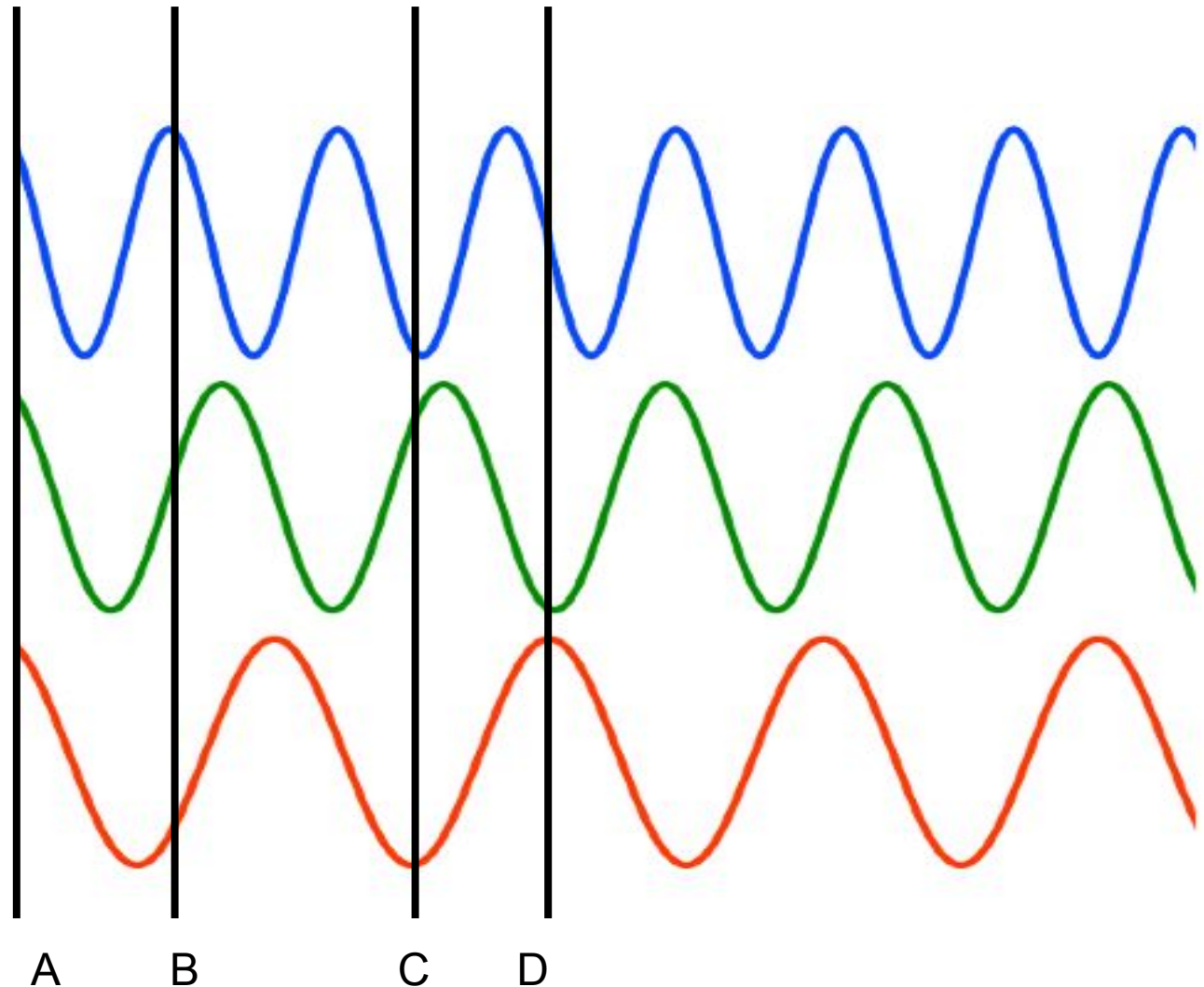




The interference of water waves coming from two sources.

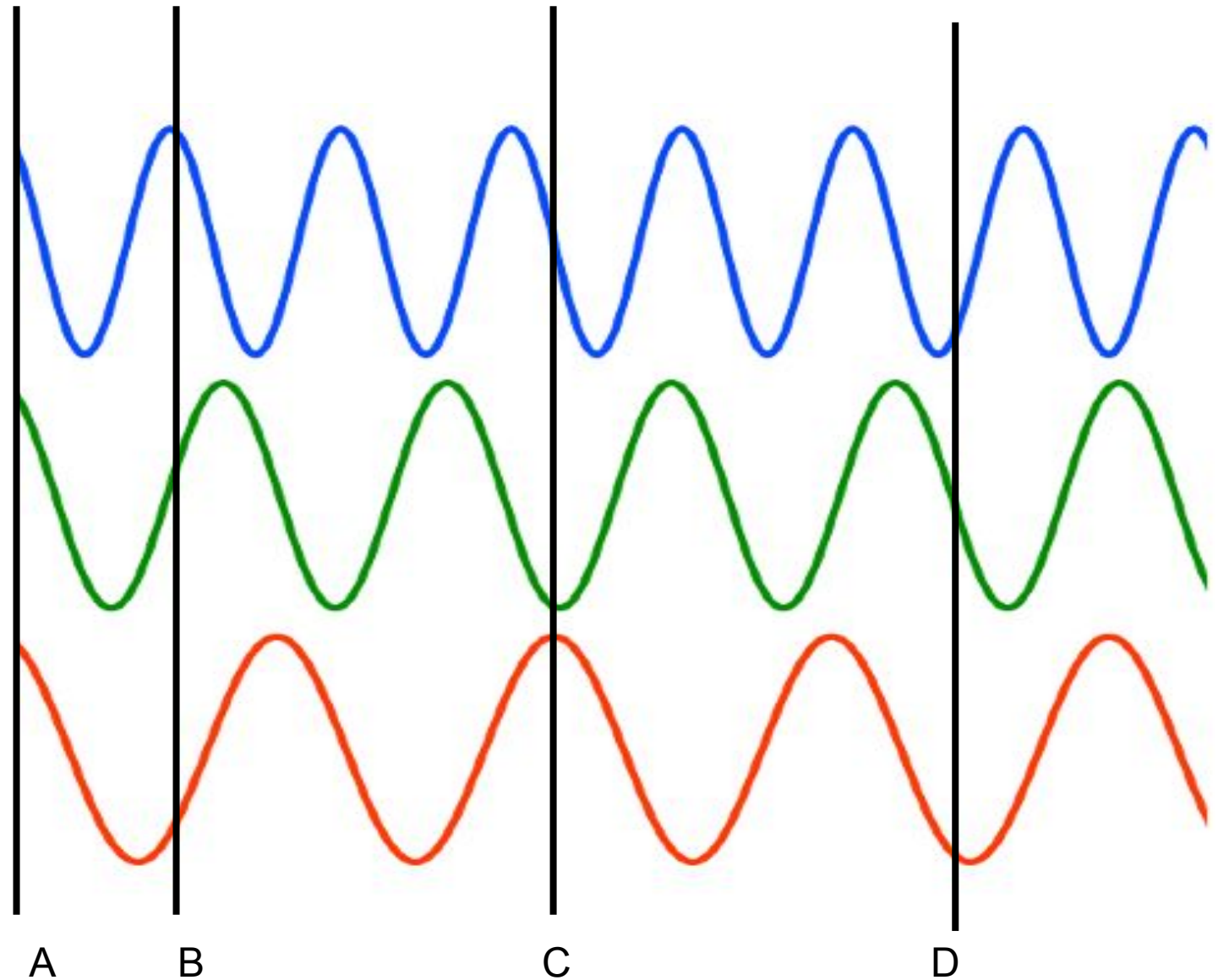
**What is the correct ordering (most negative to most positive) of the net electric field at the points shown?**

- A) A,B,C,D
- B) B,C,A,D
- C) C,D,B,A
- D) D,B,C,A
- E) NOTA



**What is the correct ordering (smallest to highest) of the net intensity?**

- A) A,B,C,D
- B) B,C,A,D
- C) C,D,B,A
- D) D,B,C,A
- E) NOTA





Electrons a) and b) are accelerating/oscillating in +/-y at a) 600 MHz b) 150 MHz, with the same amplitude, in two antennas, exactly OUT of phase and sending electromagnetic waves into space. At  $t_0$ , electron 1 is at a crest. Given  $n_{\text{waves}} = d/\lambda$ , the ratio of the EM intensity of the two waves, interfering at  $x=8$  m, compared to the EM intensity of just one wave, is closest to:

- A) 0
- B) 1
- C) 2
- D) 4

Electrons a) and b) are accelerating/oscillating in  $\pm y$  at a) 600 MHz b) 150 MHz, with the same amplitude, in two antennas, exactly OUT of phase and sending electromagnetic waves into air. At  $t_0$ , electron 1 is at a crest. The ratio of the EM intensity of the two waves, interfering at  $x=7$  m, compared to the EM intensity of just one wave, is:

- A) 0
- B) 1
- C) 2
- D) 4

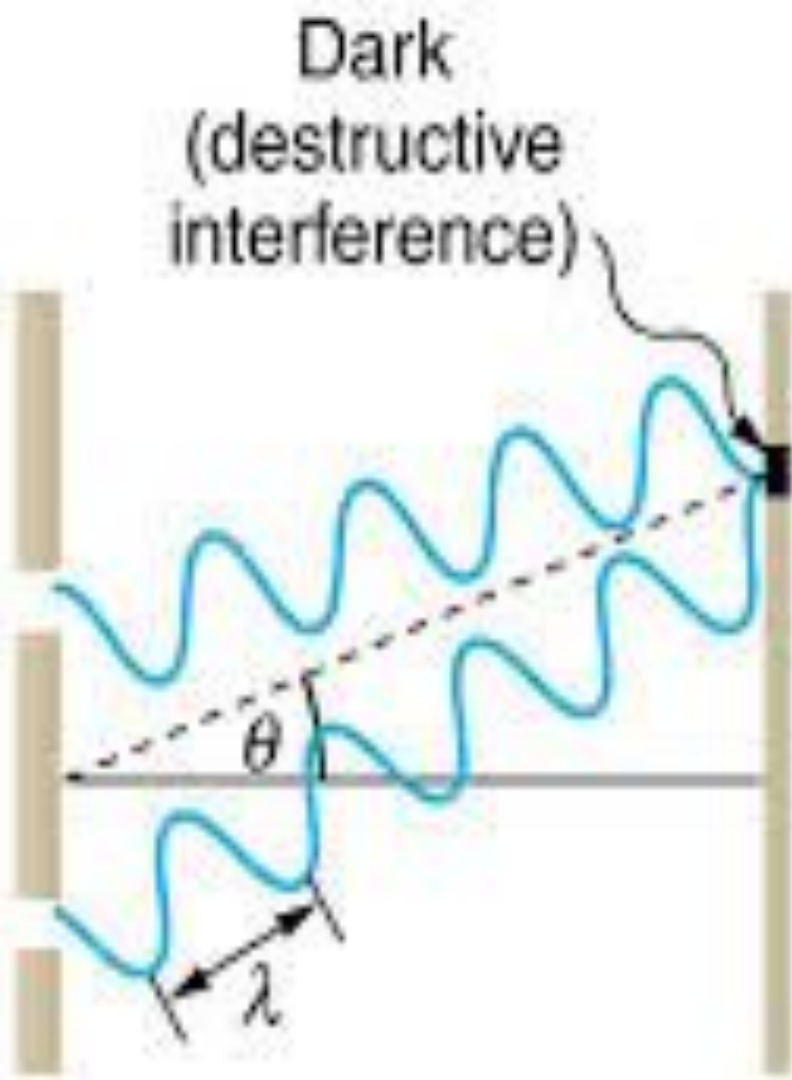
Two identical charges oscillate in phase, along y, at the origin, and the intensity measured at a point P along the x-axis. Now the second is rotated to oscillate along z, and the maximum intensity re-measured. What is the ratio:

$$I_{\text{both y}} : I_{\text{one y, one z}} : I_{\text{one only}} ?$$

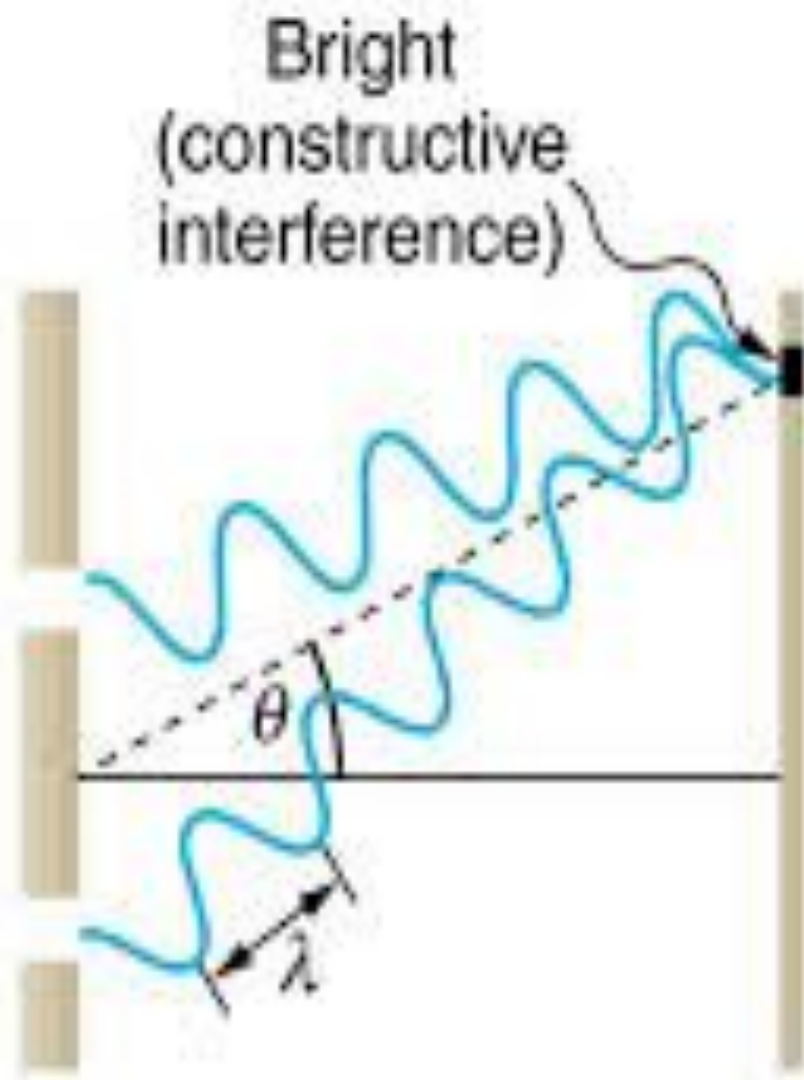
- A) 4:2:1
- B) 1:1:1
- C) 1:2:1
- D) 4:1:1



# Double Slit interference



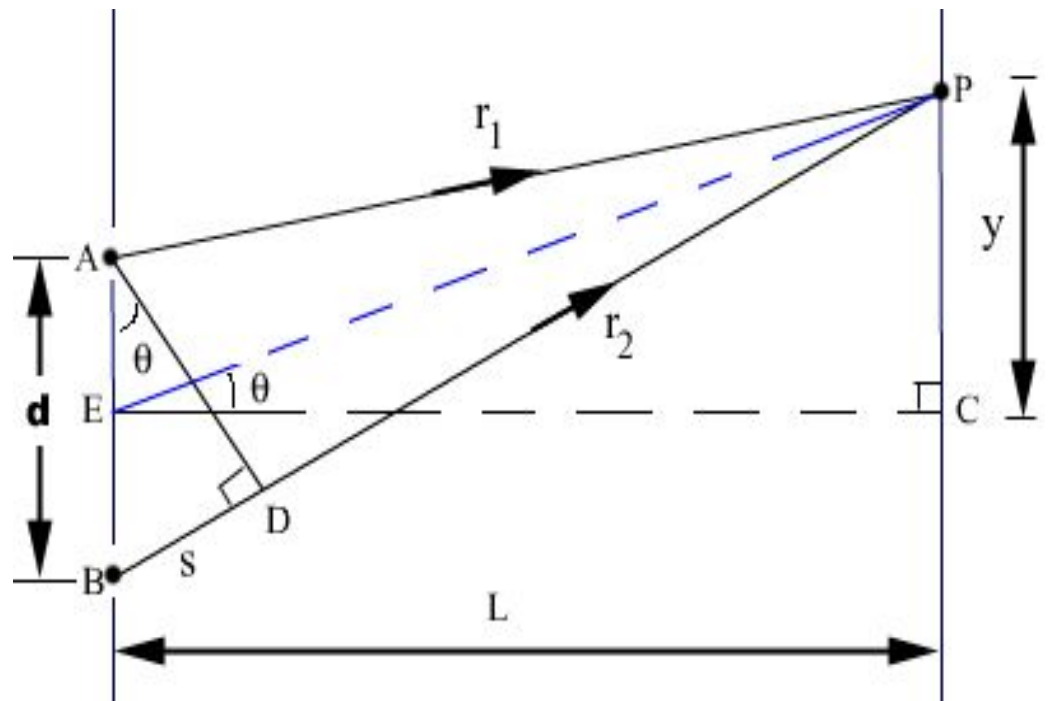
(a)



(b)

**In the diagram below, which formula corresponds to the angles  $\theta$  at which bright spots are observed?**

- A) A)  $m\lambda = d\sin(\theta/2)$
- B) B)  $m\lambda = d\sin(\theta)$
- C) C)  $m\lambda = d\cos(\theta/2)$
- D) D)  $m\lambda = d\cos(\theta)$

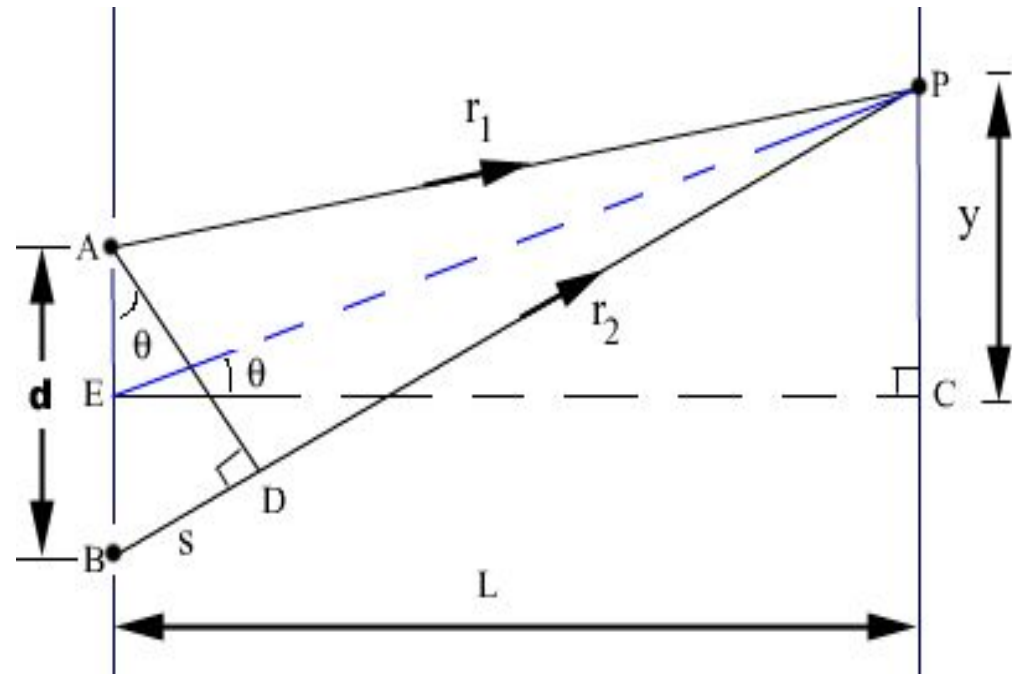


**50 MHz radio waves oscillate out of phase, at  $y=-1$  m and  $y=1$  m. The point P at the midpoint of a screen is**

- A) always bright
- B) always dark
- C) alternately bright and dark

**If they are oscillating in-phase,  
what distances  $r_1$  and  $r_2 (=r_1+s)$   
will give a dark spot at P?**

- A)  $r_1=18$  m;  $r_2=24$  m
- B)  $r_1=21$  m;  $r_2=27$  m
- C)  $r_1=21$  m;  $r_2=24$  m
- D) NOTA

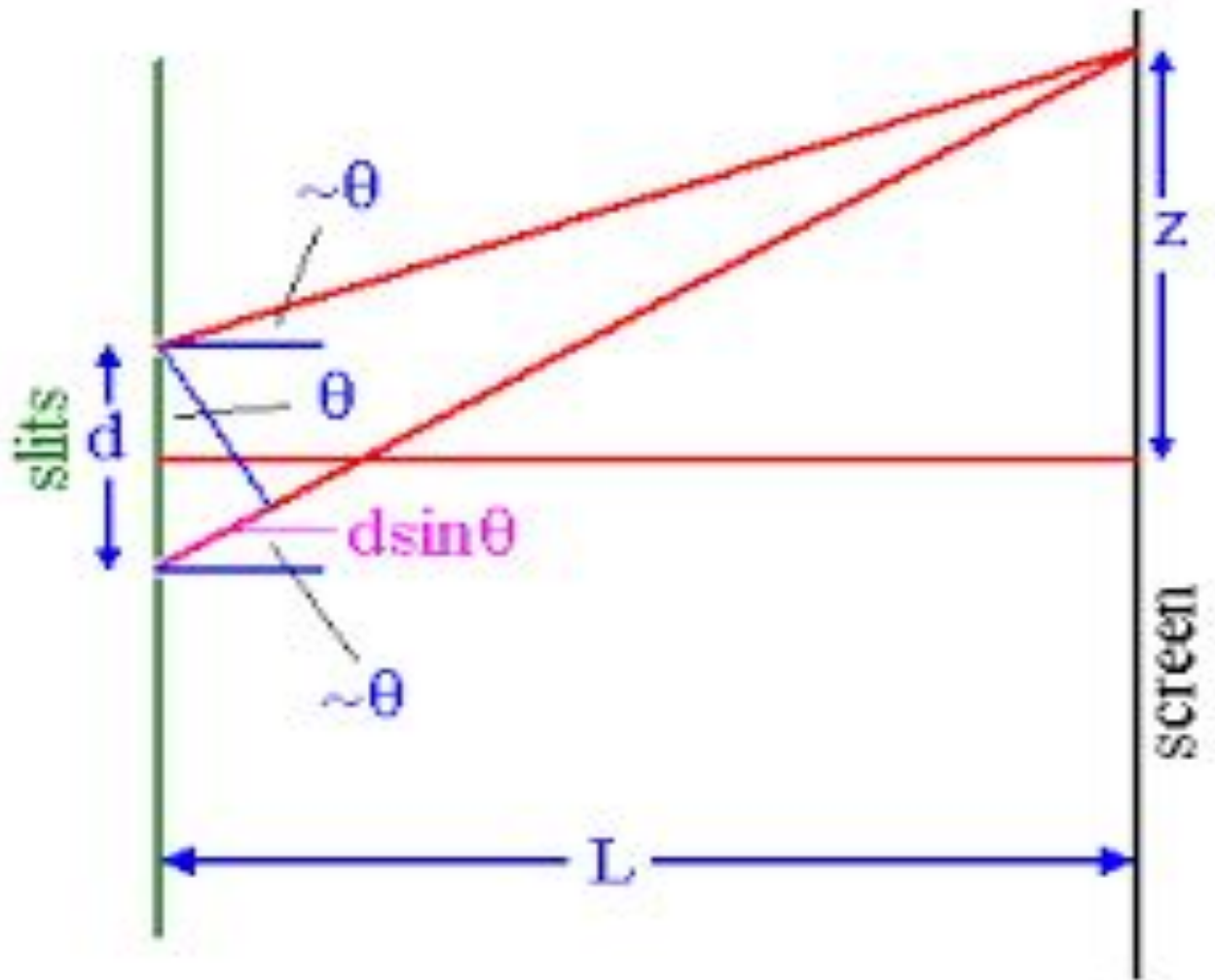


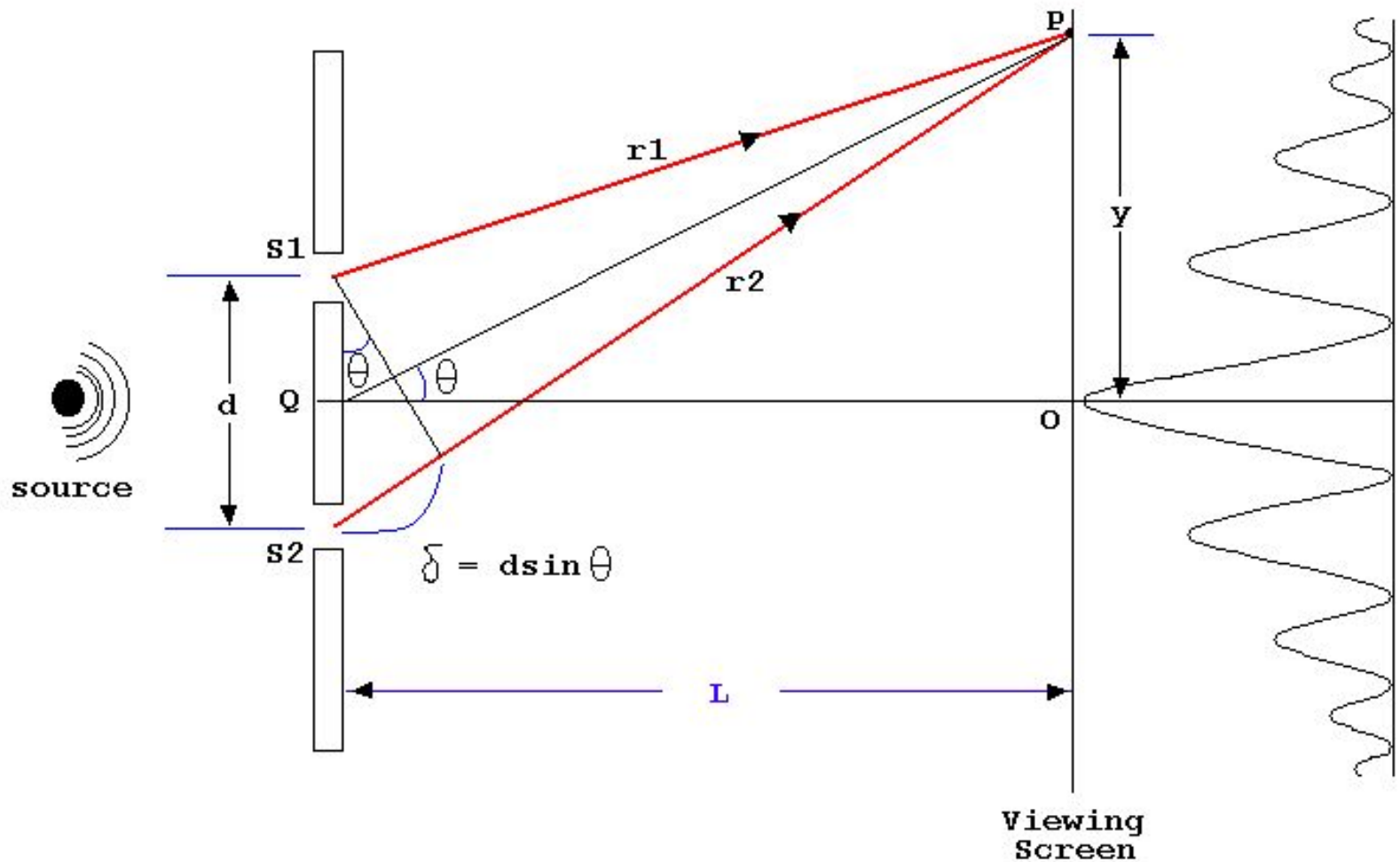
**Two spherical radio sources, emitting in-phase at 600 MHz, are placed 2 m apart. Find the angle (above x-) at which the radio waves first interfere constructively on a distant screen.**

- A)  $14.4^{\circ}$
- B)  $22.6^{\circ}$
- C)  $33.9^{\circ}$
- D)  $44.4^{\circ}$
- E) NOTA

**If  $d=1$  cm,  $\lambda=4$  mm,  $L=10$  cm, what is  $z$  such that there is destructive interference at the distance  $z$ ?**

- A) 1.32 cm
- B) 4.36 cm
- C) 2.18 cm
- D) 3.33 cm
- E) NOTA





Suppose  $L=4$  m,  $f=1500$  MHz ( $\lambda=c/f=20$  cm),  $d=120$  cm.

Angle of 2<sup>nd</sup> bright spot on screen:  $m\lambda=d\sin\theta$ , with  $m=2$ , so  $\theta=\arcsin(2*20\text{cm}/120\text{cm})=0.34$  rad, so  $y=1.41$  m

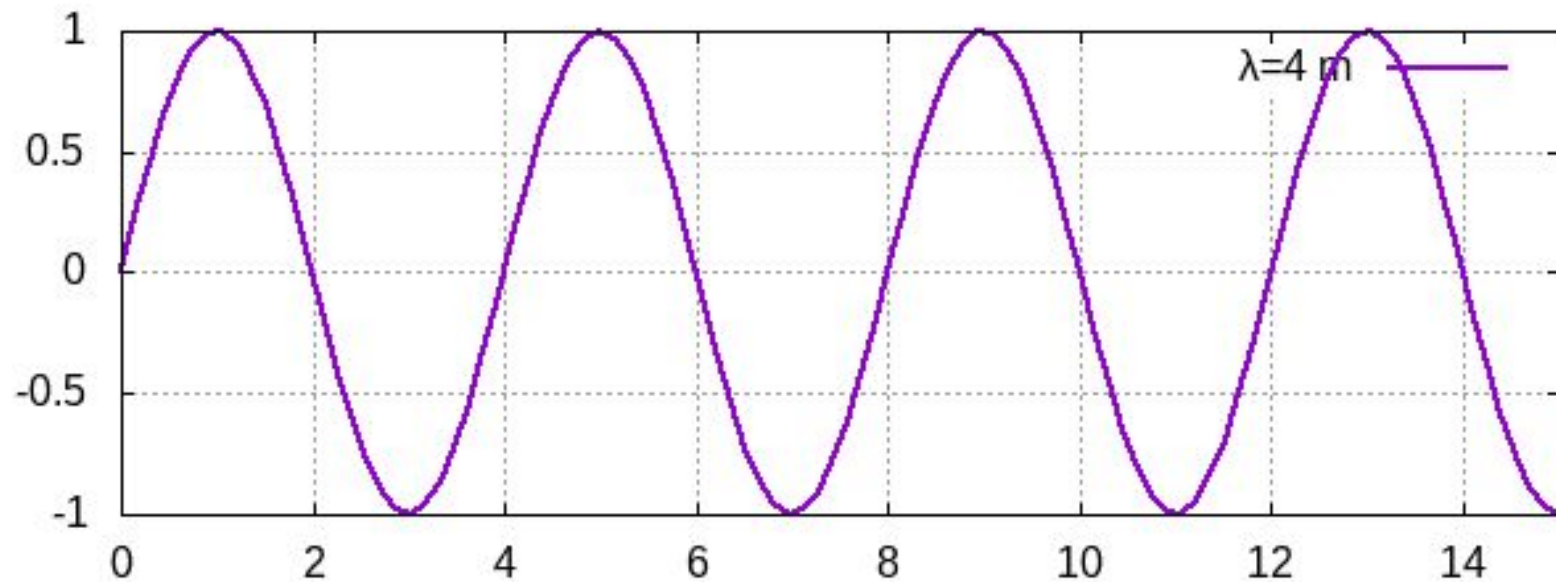
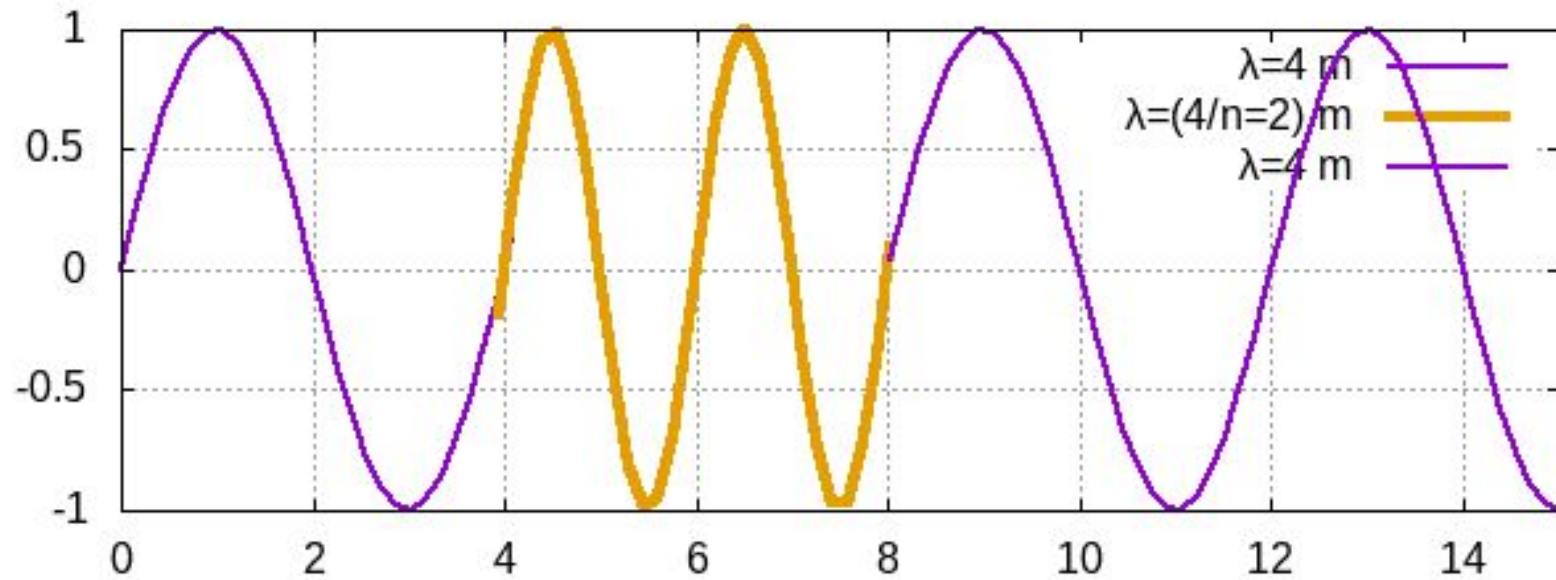
Angle 3<sup>rd</sup> dark spot on screen:  $(m+\frac{1}{2})\lambda=d\sin\theta$ , with  $m=2$ , as well;  $\theta=0.43$  rad, so  $\tan\theta=y/L$ , or  $y=1.83$  m

In this case, waves are in phase at source. We want phase relationship 15 m from source:

Top path:  $n_{\text{waves}} = (4\text{m in } x)/(4\text{m } \lambda) + (4\text{ m})/(2\text{m } \lambda) + (7\text{m in } x)/(4\text{m } \lambda) = 5.75$

Bottom path:  $(15\text{m}/4\text{m wavelength}) = 4.75$

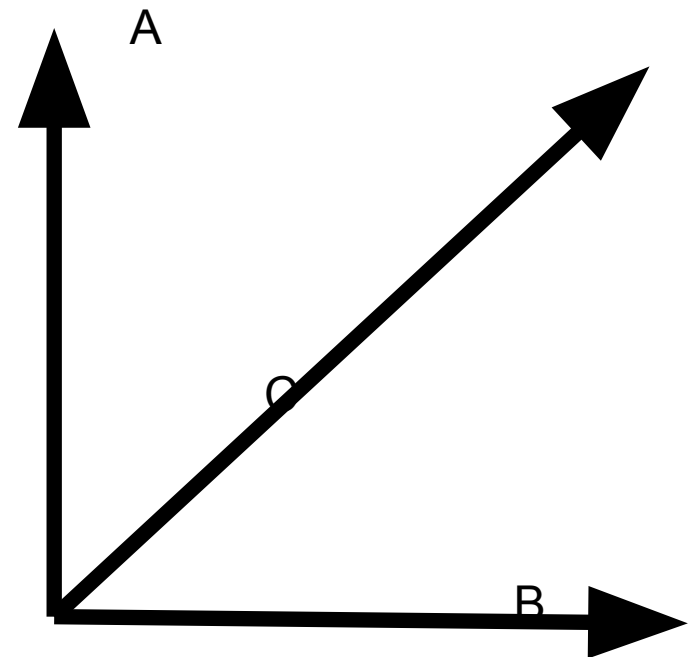
Difference=one wavelength, so phase relationship identical to phase at source!





**Given the three electric field vectors shown, after passing through polarizers. The ratio of the EM power  $(A+B+C)/(A+B)$  is about:**

- A) 1
- B) 2
- C) 4
- D) 8
- E) 16



**Vertically polarized light is passed through: a) two successive polarizers at  $45^\circ$ , b) one polarizer at  $90^\circ$ , and c) two successive polarizers at  $30^\circ$ . The relative intensities a:b:c are:**

- A)  $\frac{1}{4}:0:9/16$
- B)  $\frac{1}{2}:0/0.866$
- C)  $\frac{1}{2}: \frac{1}{4} : \frac{1}{9}$
- D) 1:2:4
- E) NOTA

Waves are produced at the origin, which then travel through glass ( $n=1.667$ ). At  $x=4$  m, Billy Zoom measures  $6 \times 10^7$  waves passing per second, with  $|E_{\text{rad,max}}|=4$  V/m. What is  $E_{\text{rad}}$  for this wave at  $x=8$  m,  $t=6$  ns (assuming spherical spreading)?

- A) 4V/m
- B) 2V/m
- C) -0.69 V/m
- D) -1.38 V/m
- E) NOTA

Charge 1 oscillates with a frequency of 1200 MHz at  $x=-1$  m; charge 2 oscillates with a frequency of 150 MHz at  $x=-3$  m. At  $x=1$  m, you measure an  $E_{\text{rad},1}$  magnitude (amplitude) of 1 V/m, and an  $E_{\text{rad},2}$  magnitude of 2 V/m. At  $t=3.33$  nanoseconds, what is the ratio of the intensity of wave 2 only, relative to the intensity of wave 1 only?

- A) 1
- B) 2
- C) 4
- D) 8
- E) NOTA

Charge 1 oscillates with a frequency of 1200 MHz at  $x=-1$  m; charge 2 oscillates with a frequency of 150 MHz at  $x=-3$  m. At  $x=1$  m, you measure an  $E_{\text{rad},1}$  magnitude (amplitude) of 1 V/m, and an  $E_{\text{rad},2}$  magnitude of 2 V/m. At  $t=3.33$  nanoseconds, what is the ratio of the intensity of waves 1+2 together, relative to the intensity of wave 1 only?

- A) 1
- B) 2
- C) 4
- D) 8
- E) NOTA

**In the previous problem, if source 2  
were oscillating along z-, the net  
intensity  $I_{1+2}$  would:**

- A) Increase
- B) Decrease
- C) Be unchanged

**A charge  $Q$  at  $(0,0)$  oscillates at 50 MHz into a 'waveguide'; at  $x=1$  m,  $|E_{\max}|=4$  V/m. At  $x=8$  m, O measures  $E=-4$  V. At what time did O make the measurement?**

- A) 44.9 ms
- B) 36.6 ns
- C) 1.02 seconds
- E) NOTA

# Which statement is true?

- A) At a slight time later, the measured value of electric field is still negative, but smaller in magnitude.
- B) At a slight time later, the measured value of electric field is still negative, but larger in magnitude
- C) At a slight time later, the measured value of electric field is now positive.
- D) At a slight time later, the measured value of electric field is still negative, but may be either larger or smaller in magnitude.



**At  $t=60$  ns, someone measures  $E=-2V$ . What is the position at which the measurement is made?**

# Which statement is true?

- A) At a slight time later, the measured value of electric field is still negative, but smaller in magnitude.
- B) At a slight time later, the measured value of electric field is still negative, but larger in magnitude
- C) At a slight time later, the measured value of electric field is now positive.
- D) At a slight time later, the measured value of electric field is still negative, but may be either larger or smaller in magnitude.

If, instead of being channeled into a waveguide, energy is emitted spherically, if  $|E_{\max}|$  at  $x=1$  is  $4\text{V/m}$ , what is  $|E_{\max}|$  at  $x=4\text{m}$ ?

- A)  $4\text{V/m}$
- B)  $2\text{V/m}$
- C)  $1\text{V/m}$
- D)  $0.5\text{ V/m}$
- E) NOTA

**The peak electric field strength in case 1, relative to case 2 is**

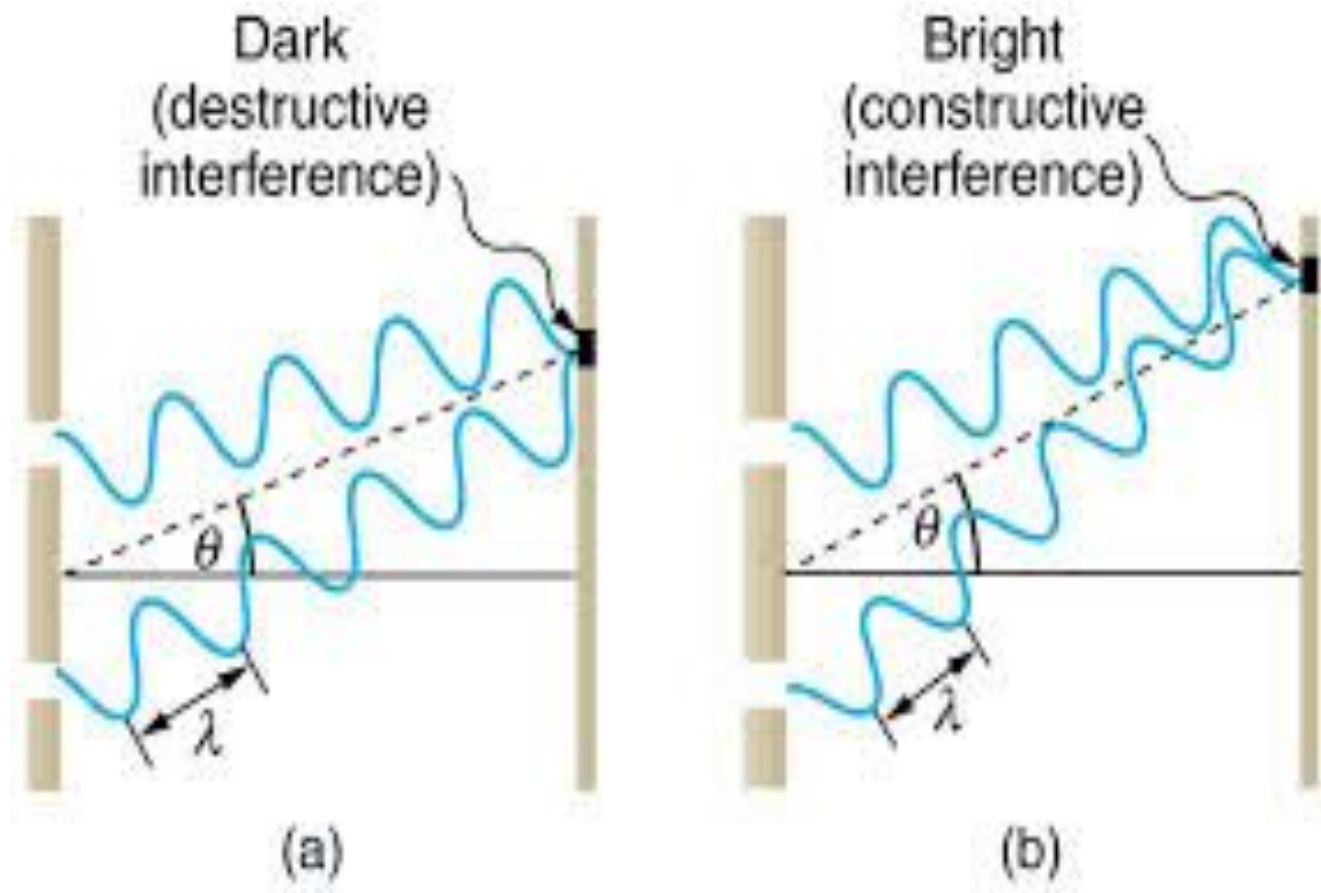
- A) Smaller by a factor of 2
- B) Smaller by a factor of 4
- C) Smaller by a factor of  $\sqrt{2}$
- D) unchanged

$A_1=2$  m,  $f=400$  MHz;  $A_2=1$  m,  $f=150$  MHz;  
out-of-phase at source,  $x=9$  m, so  $\lambda_1=0.75$  m;  $\lambda_2=2$  m;  
 $n_1=9/0.75=12$  waves;  $n_2=4.5$  waves so  $A_1$ =crest,  $A_2$   
also at crest, so  $A_{\text{tot}}=3$ , so  $I_{\text{tot}}=9$ ; verify:  
 $A_1(x=9)=2*\cos(2\pi*9/0.75+0)=2$  (check!)  
 $A_2(x=9)=1*\cos(2\pi*9/2+\pi)=1.0$  (check!)

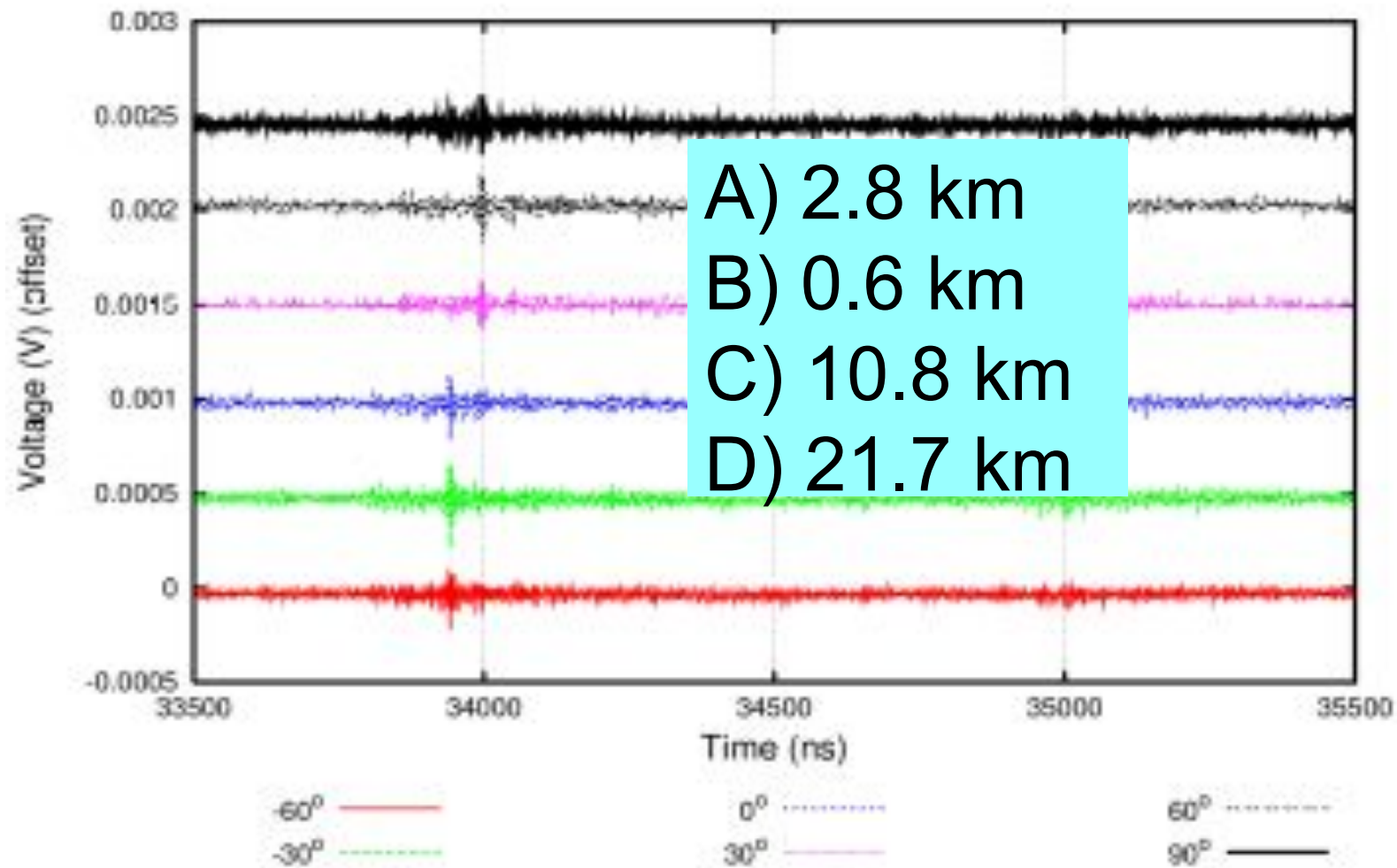
$d=1$  m;  $f=3000$  MHz; screen 2 m away, so solve for  $\theta$ :  
 $0.1=\sin\theta$ , so  $\theta=5.74$  degrees  
so height along screen given by  $\tan\theta=h/2m$ , so  $h=20$   
cm for first interference maximum

- 1) Accelerating charges  $\Rightarrow E_{\text{rad}}$  and  $B_{\text{rad}}$
- 2) Oscillating, accelerating  $Q \Rightarrow$  EM waves (radiation)
  - A) Characterized by Amplitude  $A$ , frequency  $f$ , wavelength  $\lambda$ , wavespeed (“ $c$ ” or “ $v$ ”):  $f\lambda=c$ 
    - A) Also ‘phase’ (at crest or trough at starting point, e.g.)
  - B) “Intensity”  $\sim A^2$
  - C) Waves overlapping ‘interfere’
    - A)  $A_{\text{tot}}=A_1+A_2$
- 3) “index-of-refraction”  $n$ :  $c'=c/n$ ;  $\lambda'=\lambda/n$  ( $f=f'$ );  $n_{\text{vacuum}}=1$ 
  - A) Waves travel more slowly if  $n>1$
  - B) Waves ‘refract’ when crossing from  $n_1 \rightarrow n_2$ ;  
 $n_1 \sin \theta_1 = n_2 \sin \theta_2$

**Which of the following would increase the spacing between maxima on the screen? A) Increasing the slit distance, B) Increasing the wavelength, C) Both A) and B) D) NOTA**



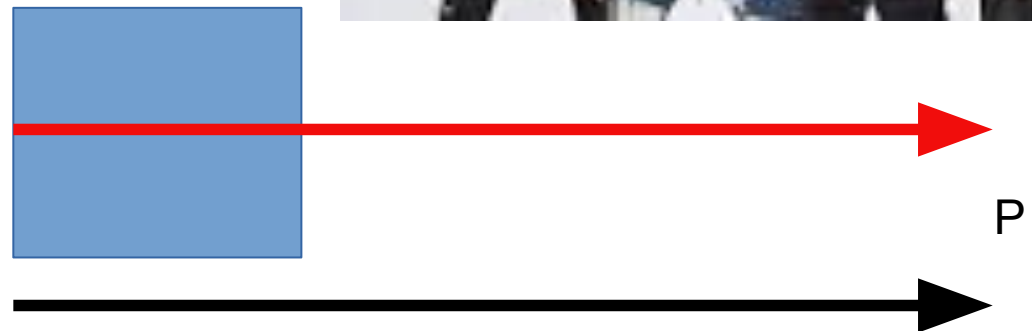
**What is the ice thickness at South Pole based on the radar data below ( $n_{\text{ice}}=1.75$ )?**





Assuming a single source, the minimum thickness of polycarbonate ( $n=2$ ) for which radio waves of frequency 150 MHz give zero intensity (darkness) at a pt 8 m away is:

- A) 2.33 m
- B) 3 m
- C) 1 m
- D) 2 m
- E) NOTA

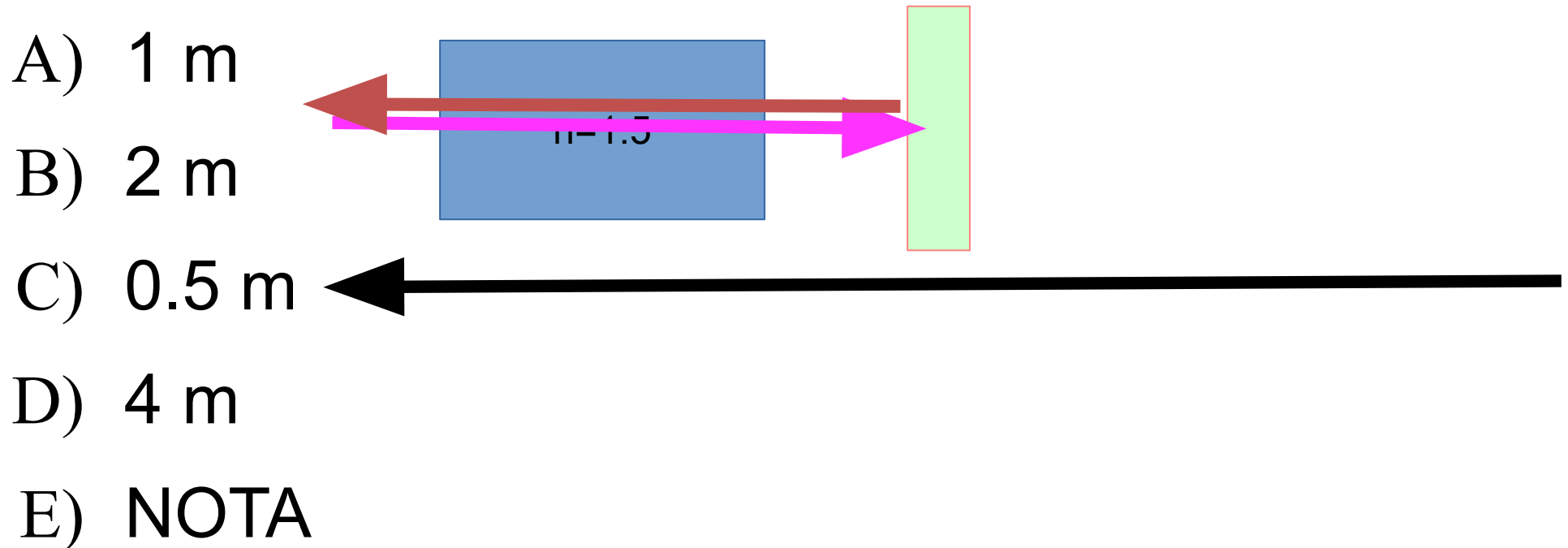


**Assuming a single source, the second smallest thickness of polycarbonate ( $n=2$ ) for which radio waves of frequency 300 MHz give zero intensity (darkness) at a pt 8 m away is:**

- A) 2.33 m
- B) 0.5 m
- C) 1 m
- D) 2 m
- E) NOTA



Now a mirror is placed, as shown, so the interference is between one reflected wave, and one direct wave.  $t_{\min}$  for constructive interference at a point 8 m to the left of the mirror is (assume the rays are emitted in phase, and both travel a total distance of 16 m)



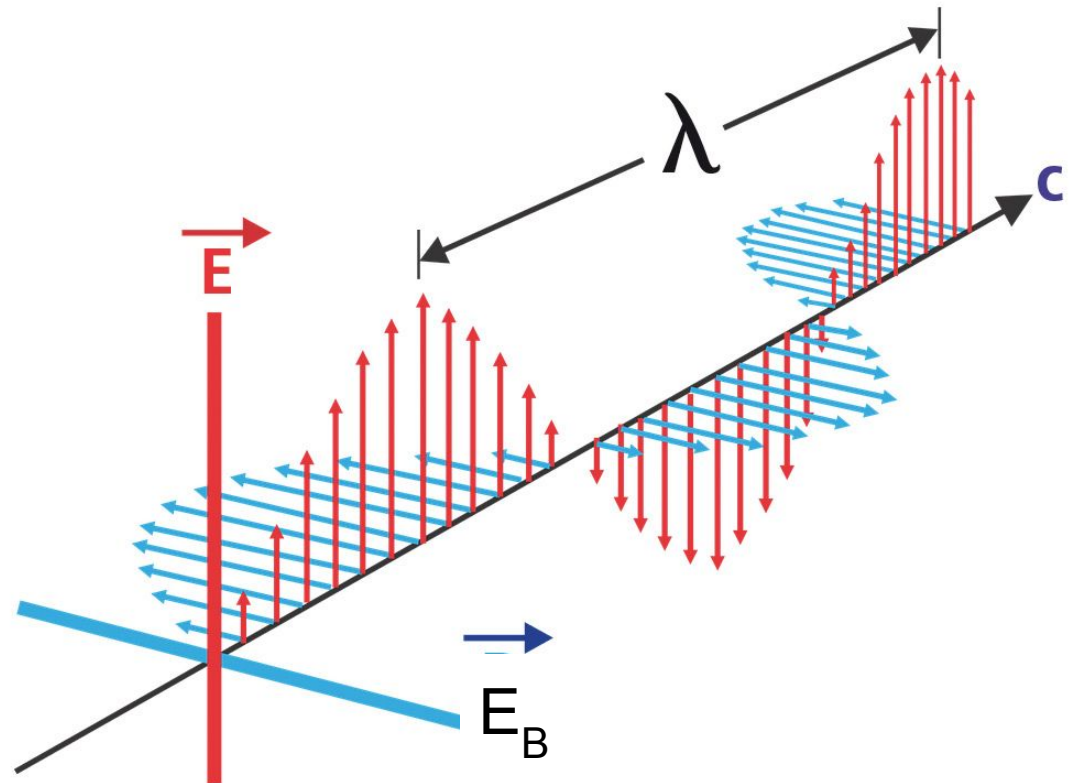
**If the two sources oscillate out-of-phase at 50 MHz, find the minimum non-zero thickness of glass ( $n=2$ ) such that P is 'dark', given that  $d=24$  m**

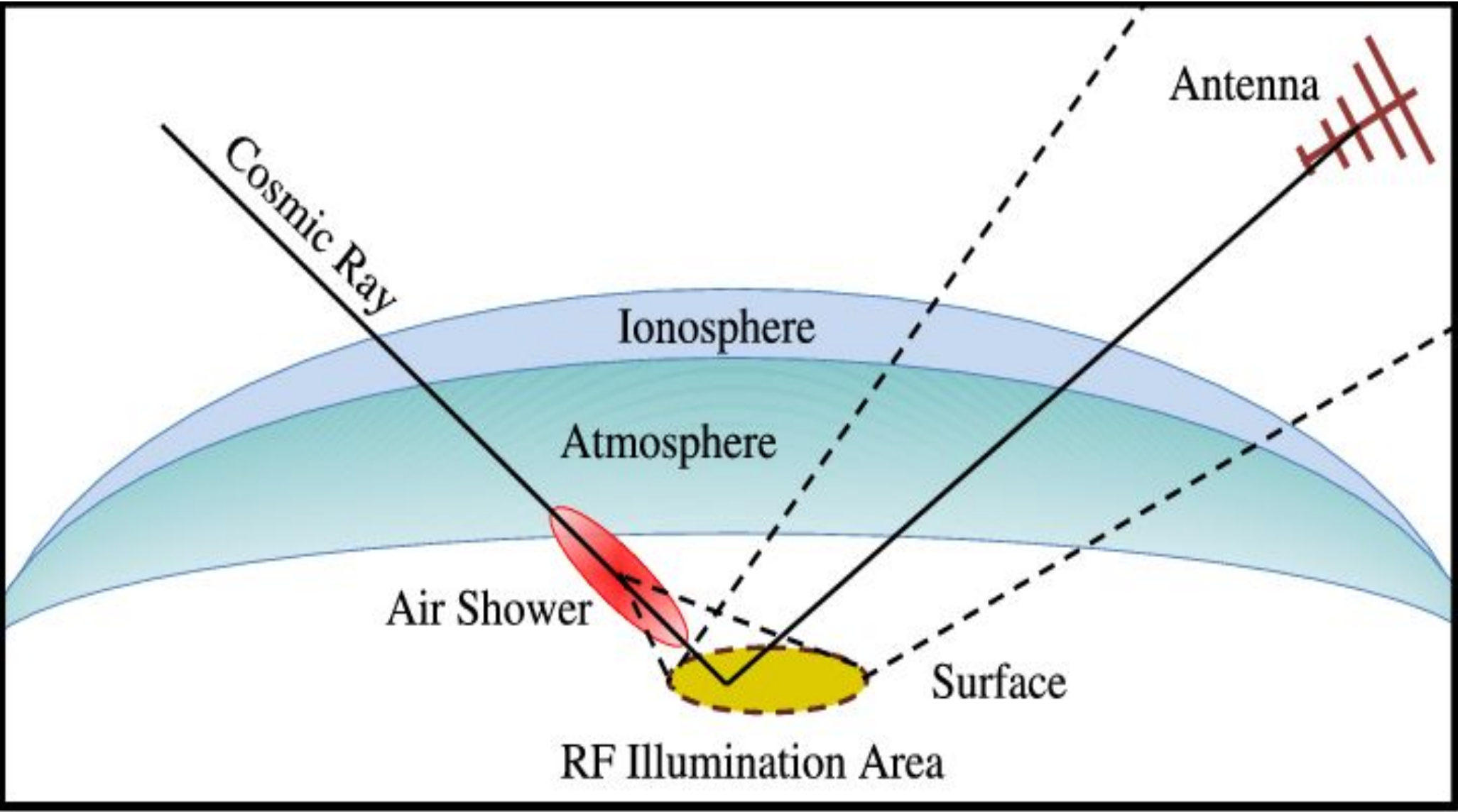
- A) 1 m
- B) 6 m
- C) 12 m
- D) 24 m
- E) NOTA



If wave A is polarized in the y direction and wave B is polarized in the x direction, the intensity of A+B, relative to A only is:

- A) 1
- B) 2
- C) 4
- D) 8
- E) NOTA





interferometric  
payload



B. Hill /Hawaii

balloon

Not to scale,  
angles don't  
reflect reality

direct UHECR

reflected UHECR

6°

geomagnetic + Askaryan  
Emission

geomagnetic + Askaryan  
emission

Askaryan  
emission

Ice

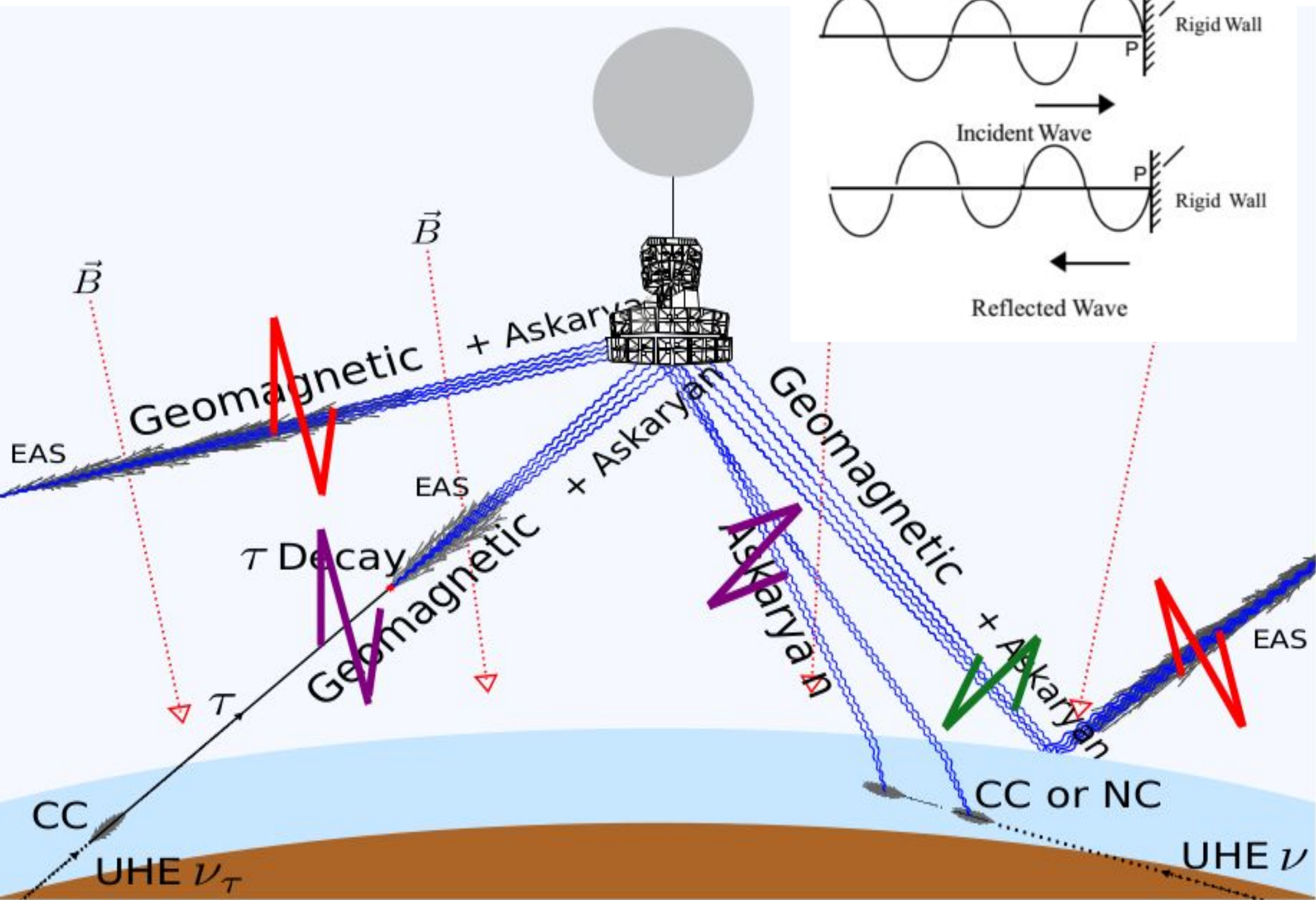
$\sim \text{EeV } \nu$

$\nu_\tau$

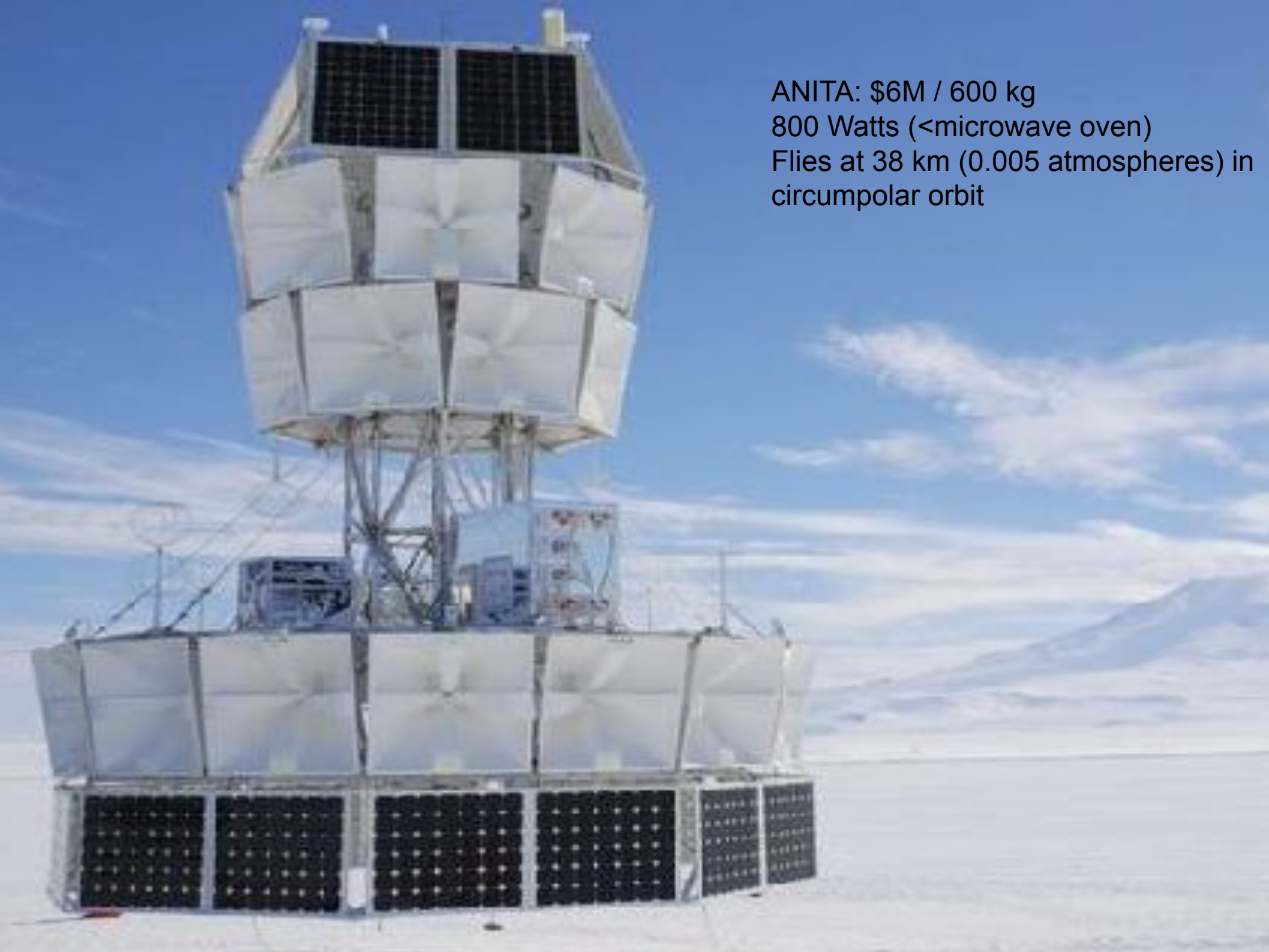
$\tau$



EM waves invert when reflecting off higher  $n$  surface!







ANITA: \$6M / 600 kg  
800 Watts (<microwave oven)  
Flies at 38 km (0.005 atmospheres) in  
circumpolar orbit

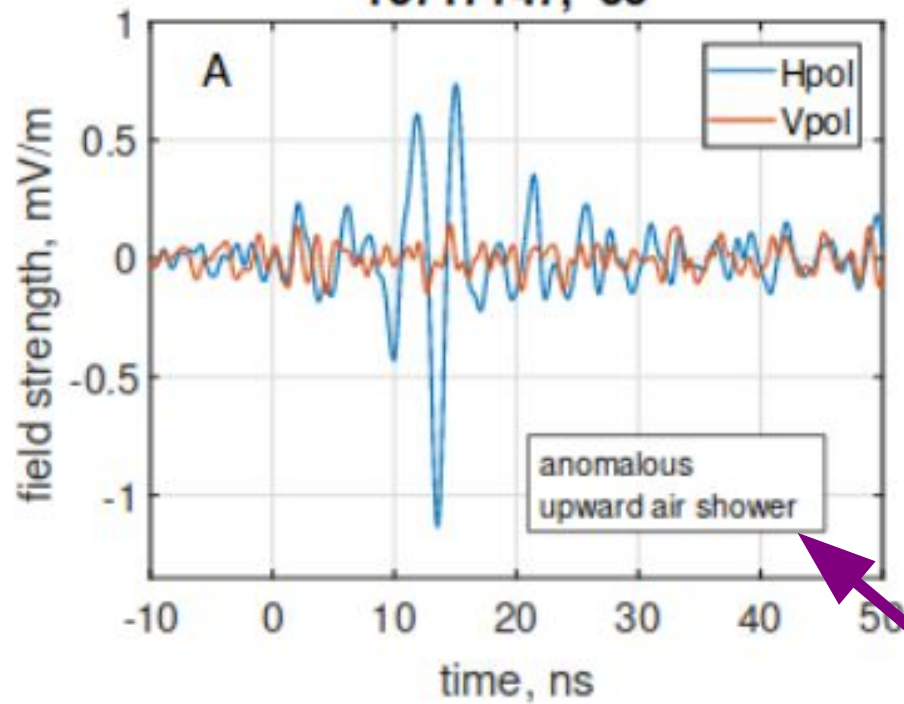




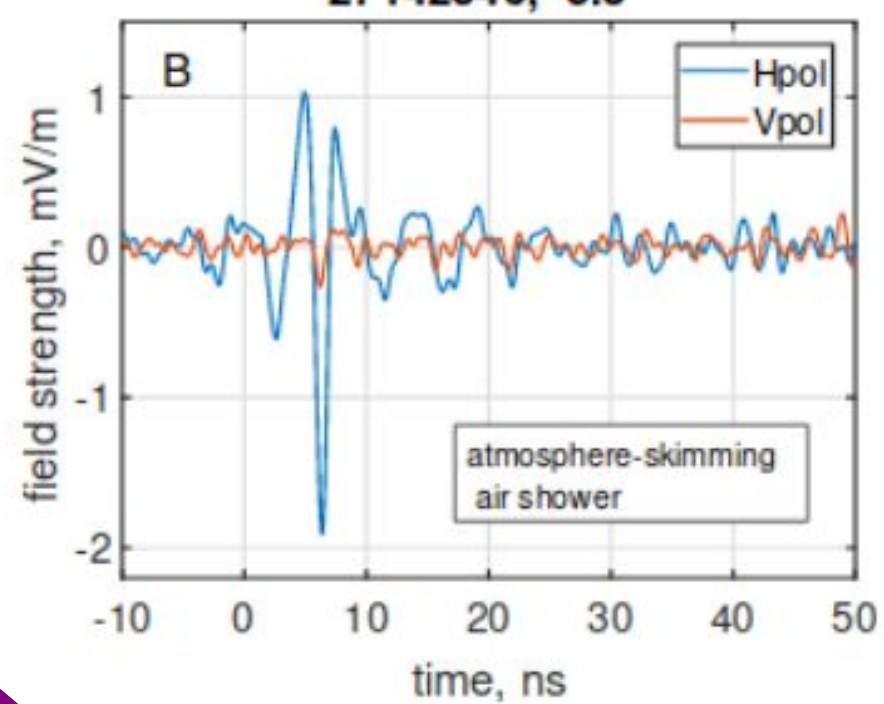


# ANITA-III UHECR Air Showers

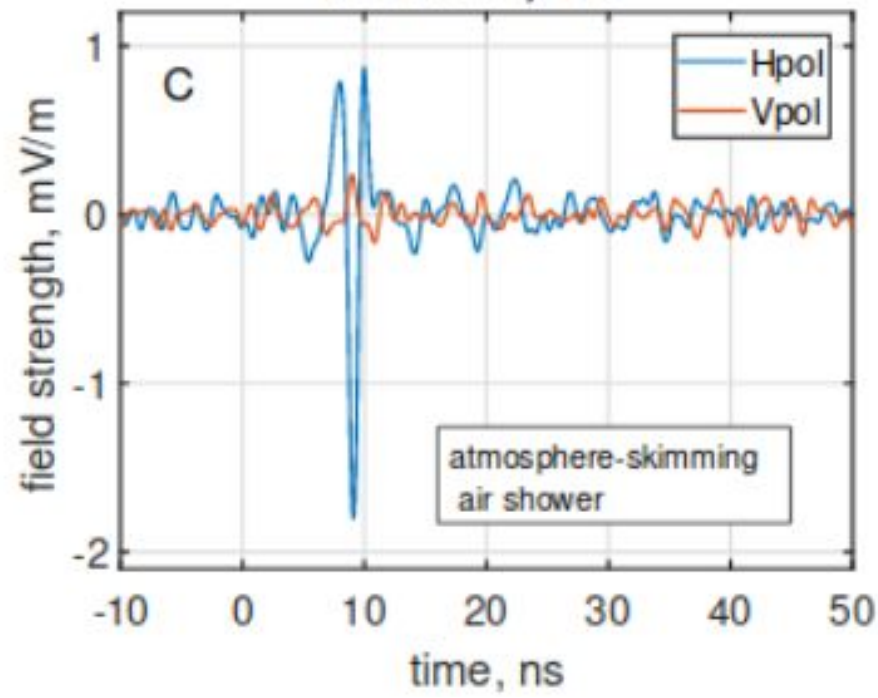
15717147,  $-35^\circ$



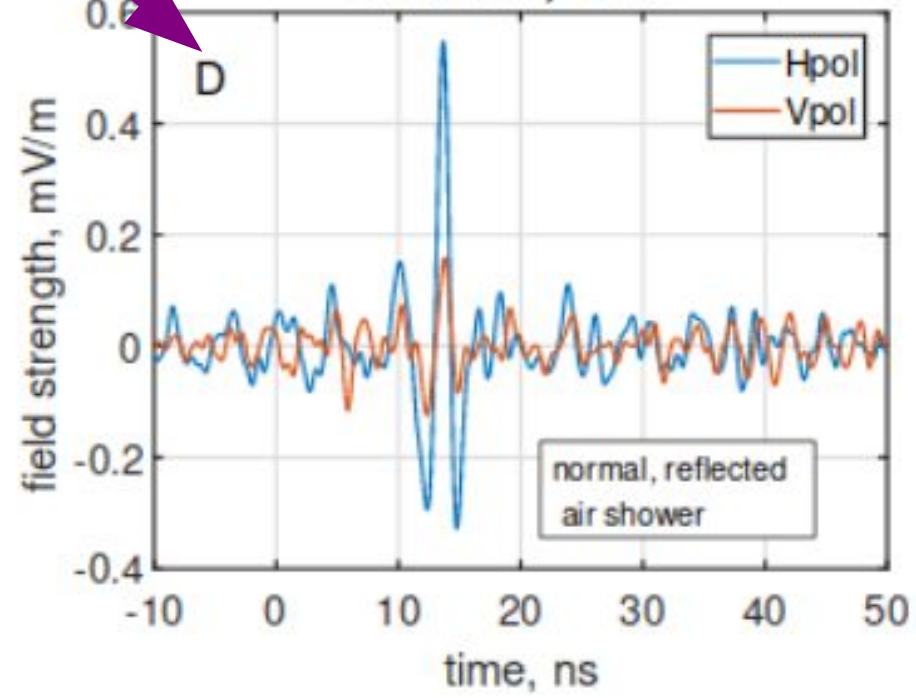
27142546,  $-5.5^\circ$



39599205,  $-3.6^\circ$



68298837,  $-36.7^\circ$



## We may have spotted a parallel universe going backwards in time

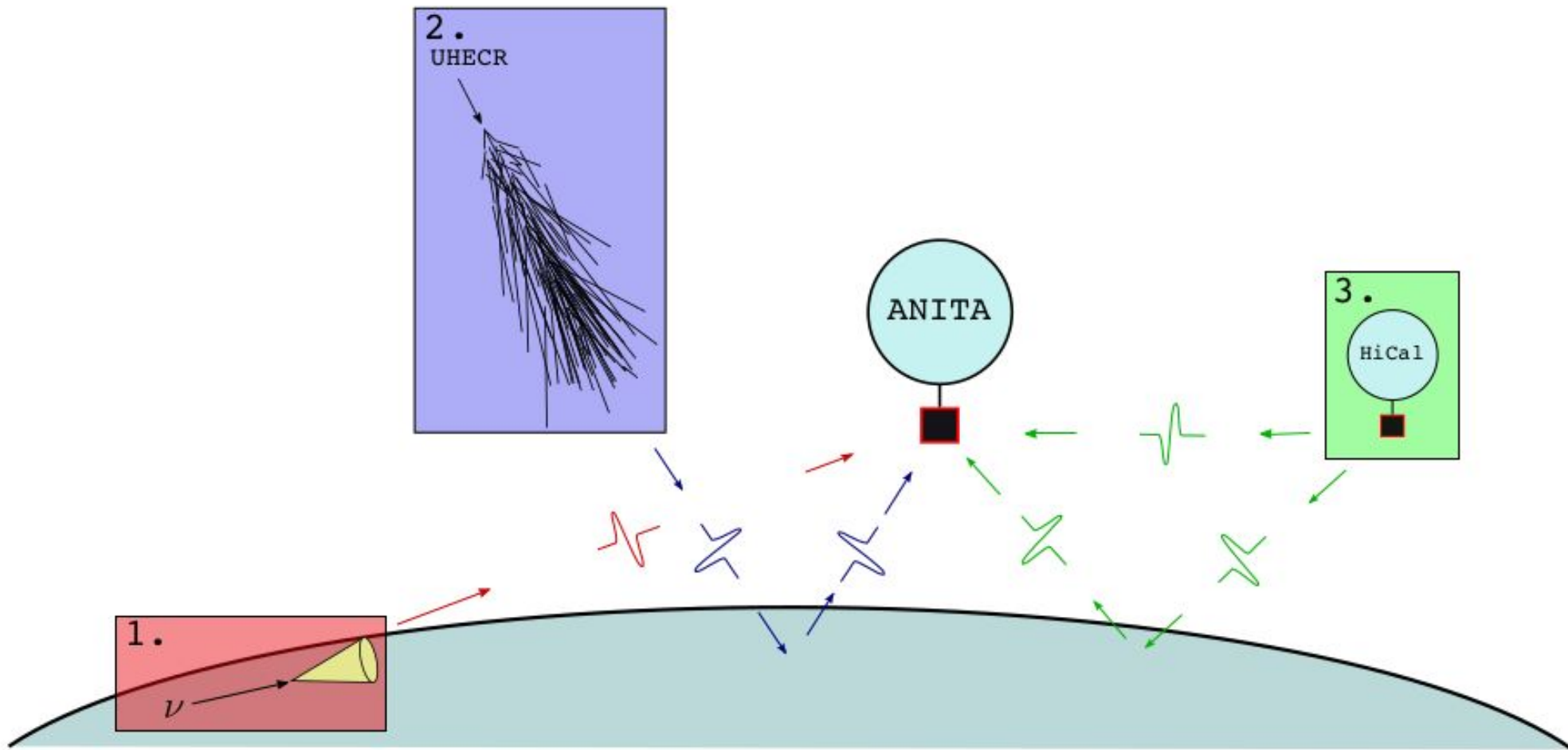
Strange particles observed by an experiment in Antarctica could be evidence of an alternative reality where everything is upside down



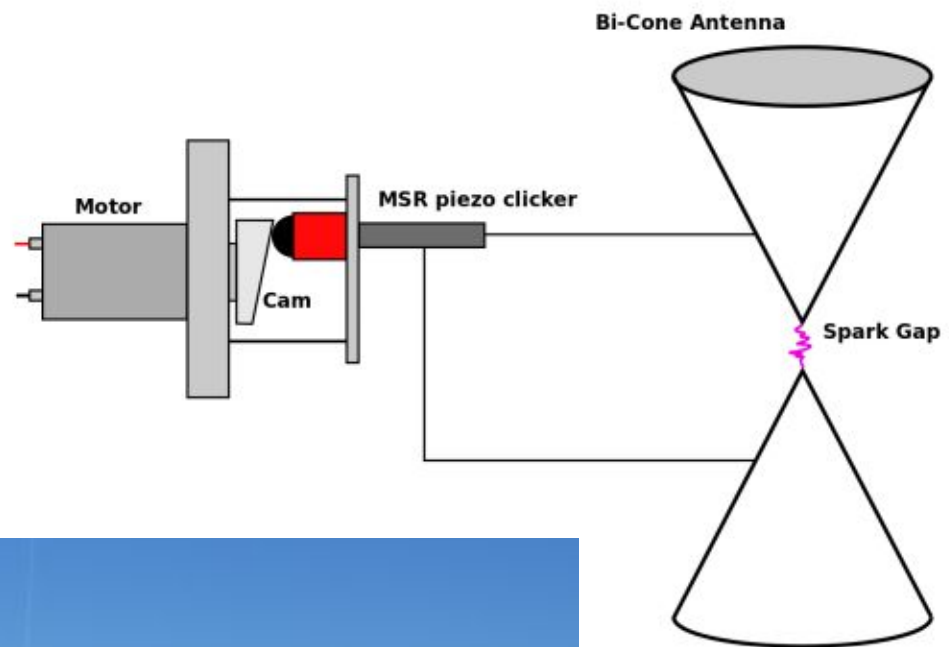
SPACE 8 April 2020

By [Jon Cartwright](#)

The HiCal concept: test whether signals really invert if  $n_2 > n_1$







# D Event 79998774

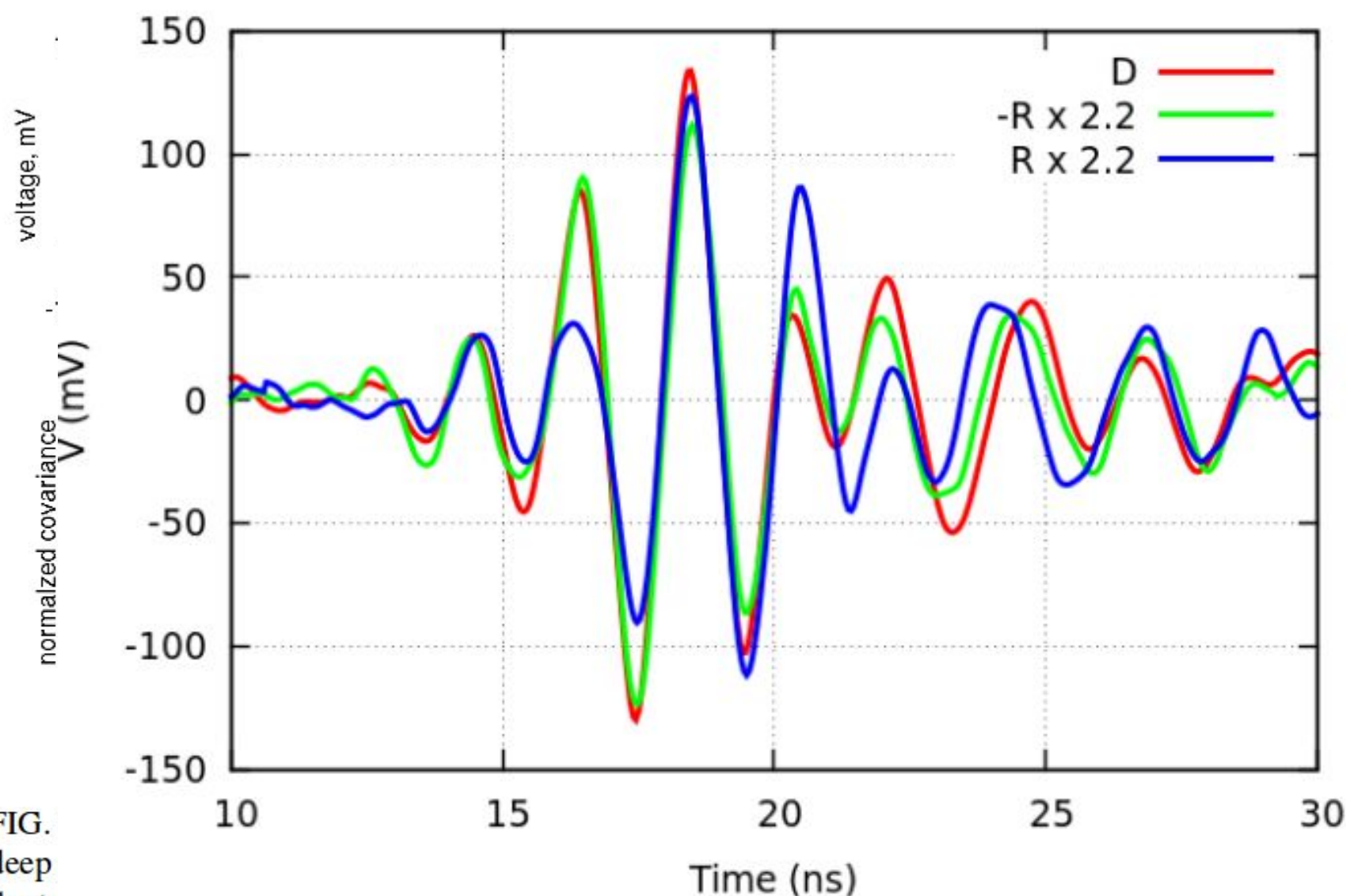


FIG.  
deep  
the tr  
wave  
pulse  
122 r  
reflec

FIG. 10: *Overlay of HiCal-1 Direct event 79998774 (red) overlaid with inverted event 79998775 (green) and non-*





**You look down at an oil slick on water and see a bright yellow ( $\lambda=600$  nm) circle. Assuming 'normal' incidence, what is the next larger thickness which results in a bright color?**

- A) 450 nm
- B) 300 nm
- C) 200 nm
- D) 100 nm
- E) NOTA

**You look down at an oil slick on water and see a bright yellow ( $\lambda=600$  nm) circle. Assuming 'normal' incidence, what is the next smaller wavelength which appears bright?**

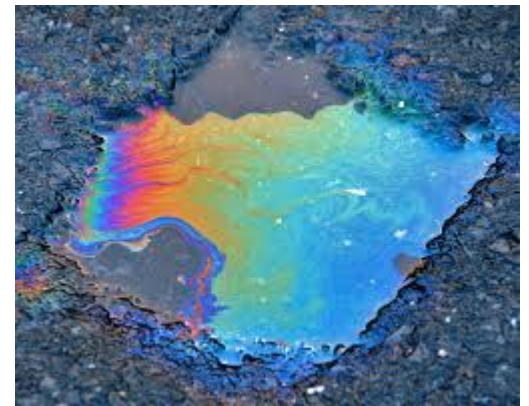
- A) 400 nm
- B) 300 nm
- C) 200 nm
- D) 100 nm

**In the previous problem, as you moved further from 'normal' incidence, the spacing between bright colors would get:**

- A) Smaller
- B) Larger
- C) Be unchanged

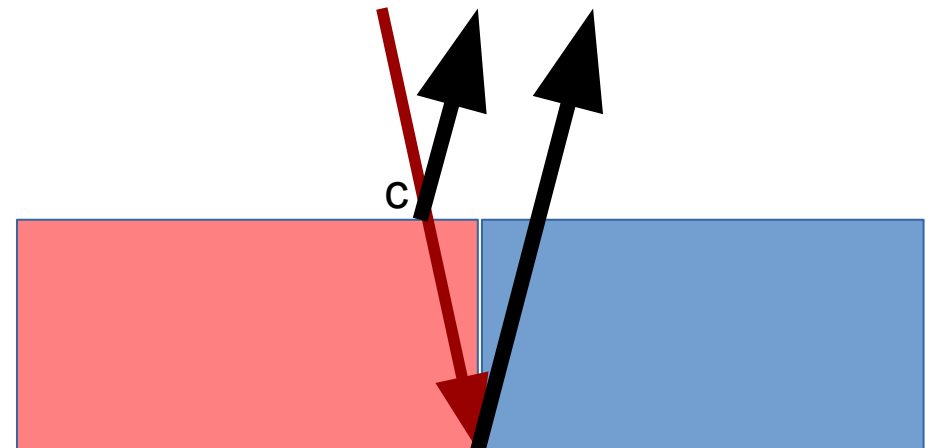
# What would have the effect of making the colors 'closer' in the oil slick shown?

- A) Making the oil slick thicker
- B) Increasing index-of-refraction of oil
- C) Decreasing index-of-refraction of underlying water



**Yellow light ( $f=5 \times 10^{14}$ ) shines (starting in air) on a 600 nm thick film with  $n_1=2$  on the left, and  $n_2=1.5$  on the right, on top of air, nearly vertical. By tracing out crests and troughs, the interference at the observer is:**

- A) Constructive
- B) Destructive
- C) Halfway between constructive and destructive



What is the total time required for the light that refracts to travel through the pink/blue blocks?

**Observing a bright blue  
( $f=7.5 \times 10^{14}/\text{sec}$ ) circle on an oil slick  
( $n_{\text{oil}}=2$ ) sitting on top of water ( $n=1.5$ ),  
you would conclude that the oil slick  
has a minimum thickness of:**

- A) 100 nm
- B) 200 nm
- C) 450 nm
- D) 50 nm
- E) NOTA

**Observing a bright blue ( $\lambda=400$  nm) circle on an oil slick sitting on top of glycol ( $n_{\text{glycol}}=2.5$ ), you conclude that the oil slick has a minimum thickness of:**

- A) 100 nm
- B) 200 nm
- C) 450 nm
- D) 50 nm
- E) NOTA



if  $n_1=2$ ,  $n_2=1.25$  and  $n_3=2.5$ , and if the frequency is  $3 \times 10^{14}$ , the second smallest thickness for constructive interference is:

- A) 600 nm
- B) 200 nm
- C) 450 nm
- D) 800 nm
- E) NOTA

**In the previous problem, what is the minimum thickness for 'darkness'?**

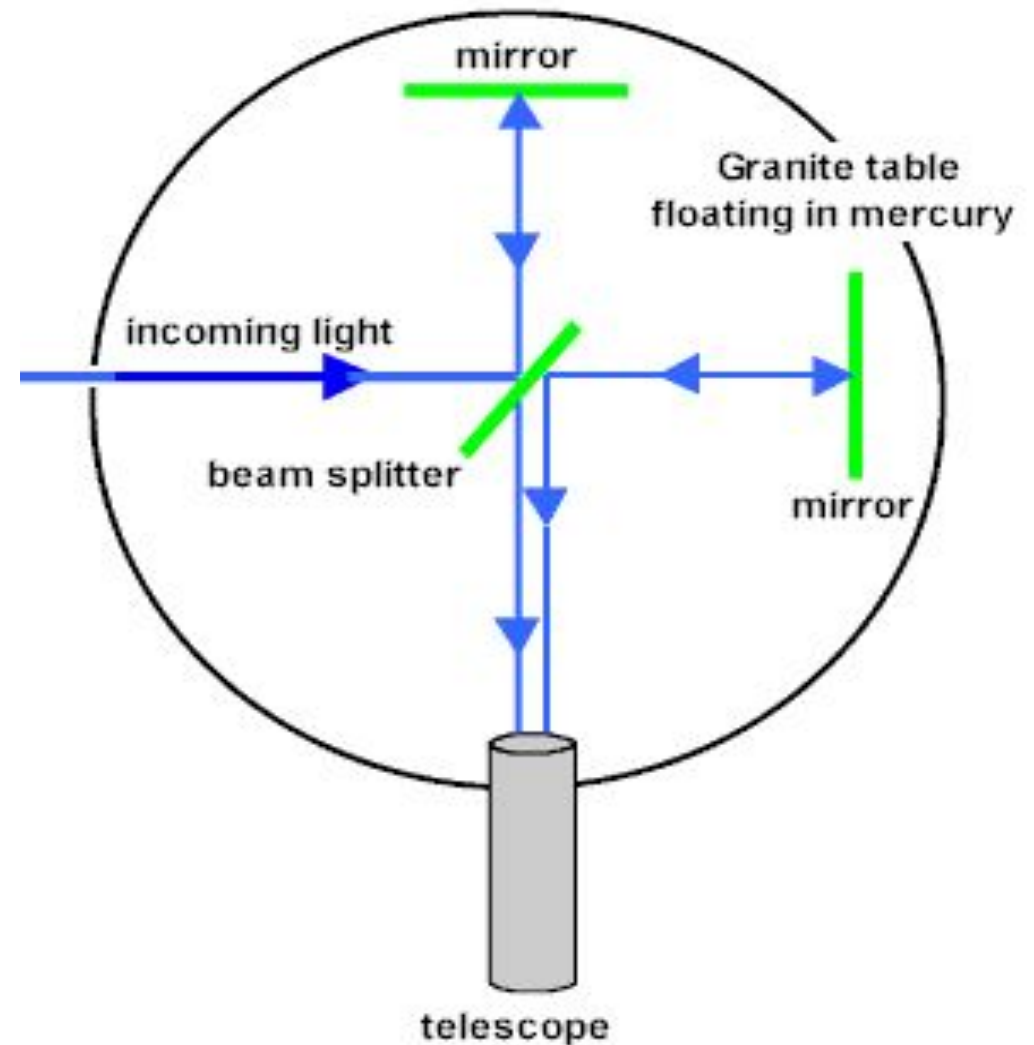
**MM expt: Given two trains, one moving at  $v_x = +20$  km/s and another moving at  $v_x = -40$  km/s. At the time they cross each other at  $x=0$ , each fires a bullet\* at observer O at  $x=+80$  km. How much earlier does the bullet from train 1 arrive?**

- A) 2 s
- B) 0.8 s
- C) 2.8 s
- D) 1.2 s
- E) NOTA

Now allowed in your coat pocket in our phsx class!

**In the Michelson-Morley expt., with  $v_x = +300$  km/s, and given  $d_1 = d_2 = 300$  m which ray arrives at the telescope first?**

- A) Vertical ray
- B) Horizontal ray
- C) They arrive at the same time



A **400 MHz**  $F_y = 6$  N force acts on a **+2 C**, 1 kg charge at (0,0), an identical **in-phase** force, but at **300 MHz**, acts on a **3 C**, 1.5 kg charge, at **x=-1**. At **x=2**, the ratio  $|E_{\text{rad},1}|/|E_{\text{rad},2}| =$

- A) 1.5
- B) 1.0
- C) 0.75
- D) 0.66
- E) NOTA

**At  $x=2$  m and  $t=10^{-9}$  s, the ratio of intensity from charge 1 only to the net intensity from both charges is:**

- A) 0.230
- B) 0.193
- C) 0.108
- D) 0.914
- E) NOTA

**In the previous problem, if the second source was oscillating out-of-phase, then the ratio of intensities would be:**

- A) 0.230
- B) 0.193
- C) 0.108
- D) 0.914
- E) NOTA

**In the previous problem, if the second source charge was negative, then the ratio of intensities would be:**

- A) 0.230
- B) 0.193
- C) 0.108
- D) 0.914
- E) NOTA

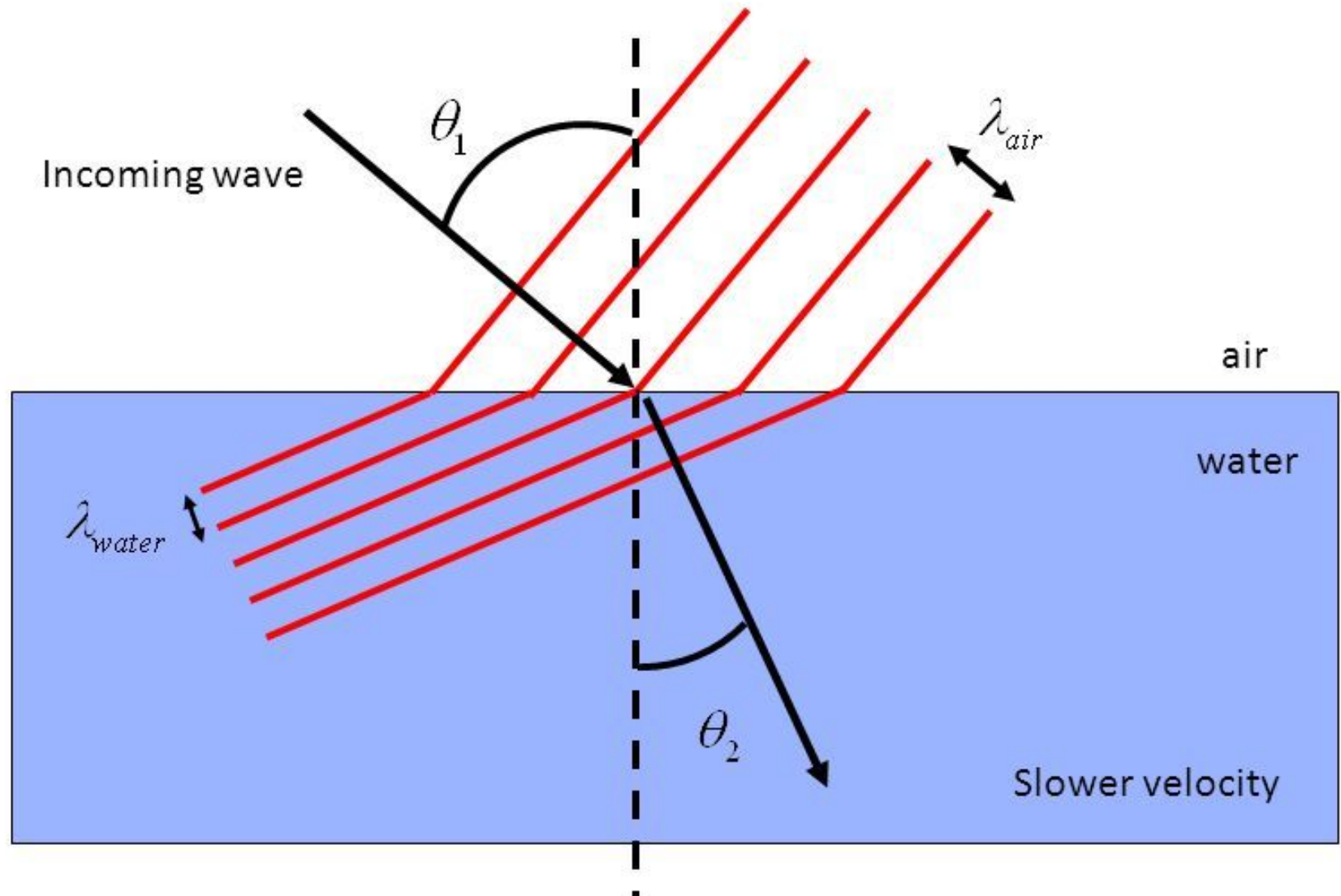


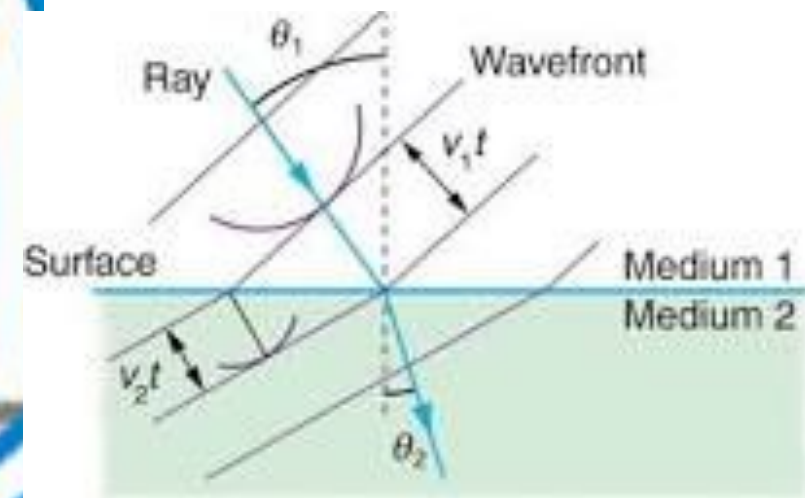
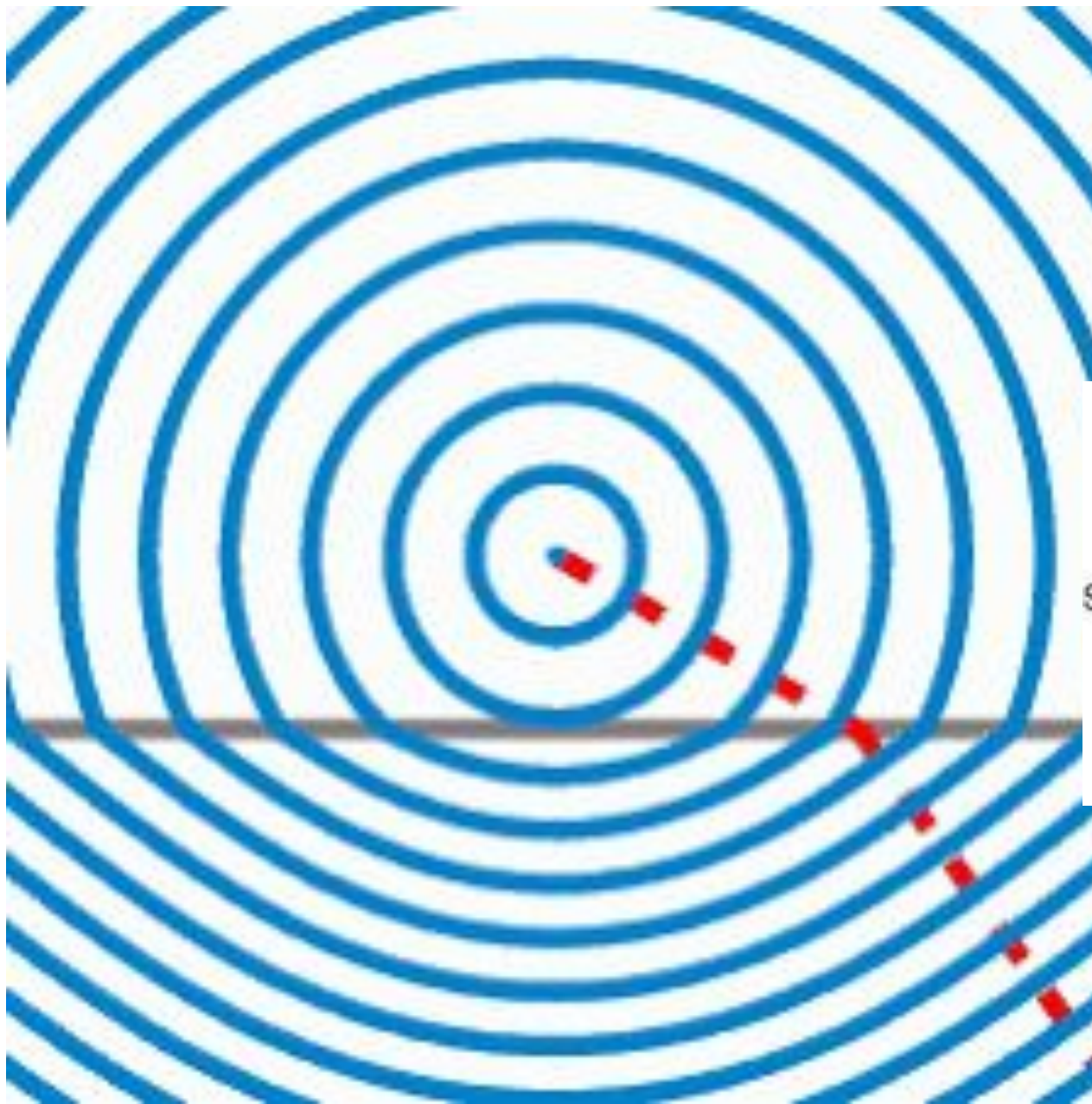
**In the previous problem, if the 3C charge was oscillating in the  $\pm z$  direction, what is the new intensity ratio at  $x=2$  m?**

- A) 0.230
- B) 0.193
- C) 0.108
- D) 0.914
- E) NOTA

# Refraction

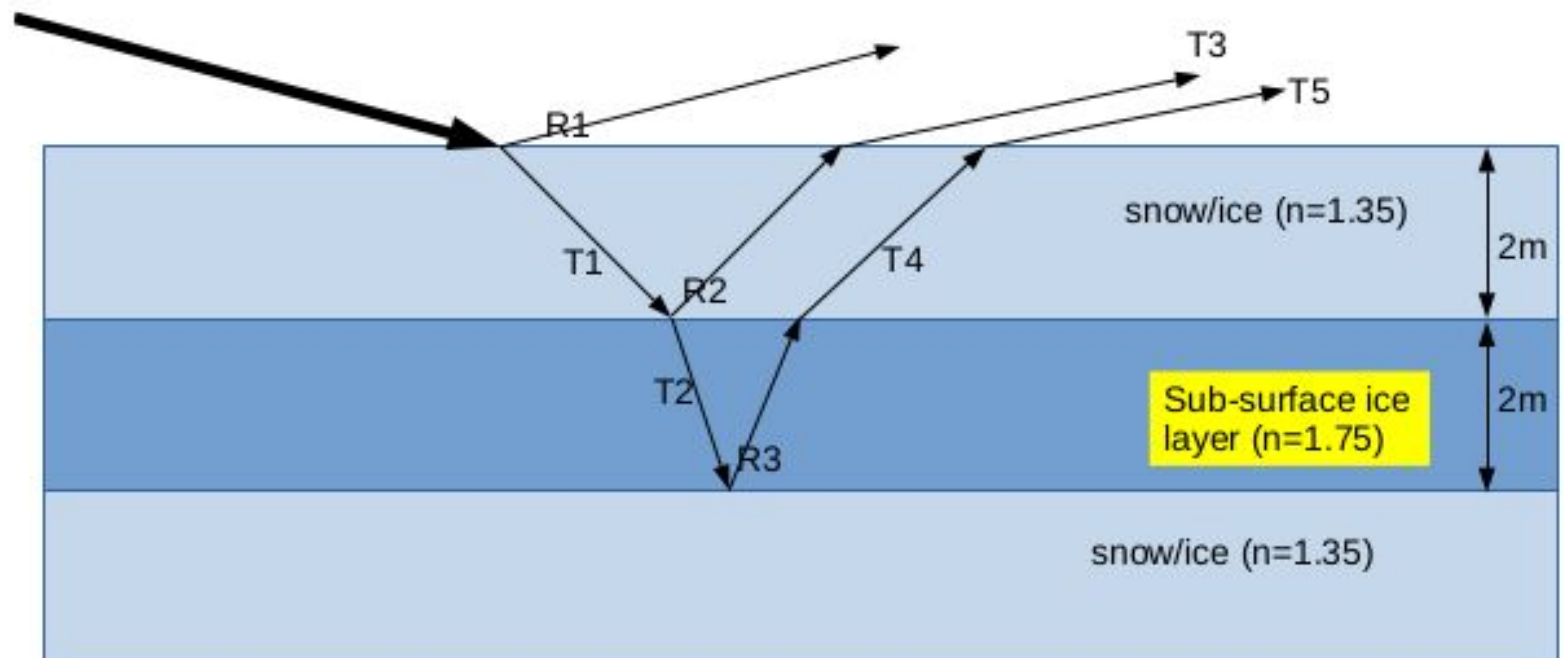
**Refraction:** bending of a wavefront at a boundary between two media, due to the change in velocity





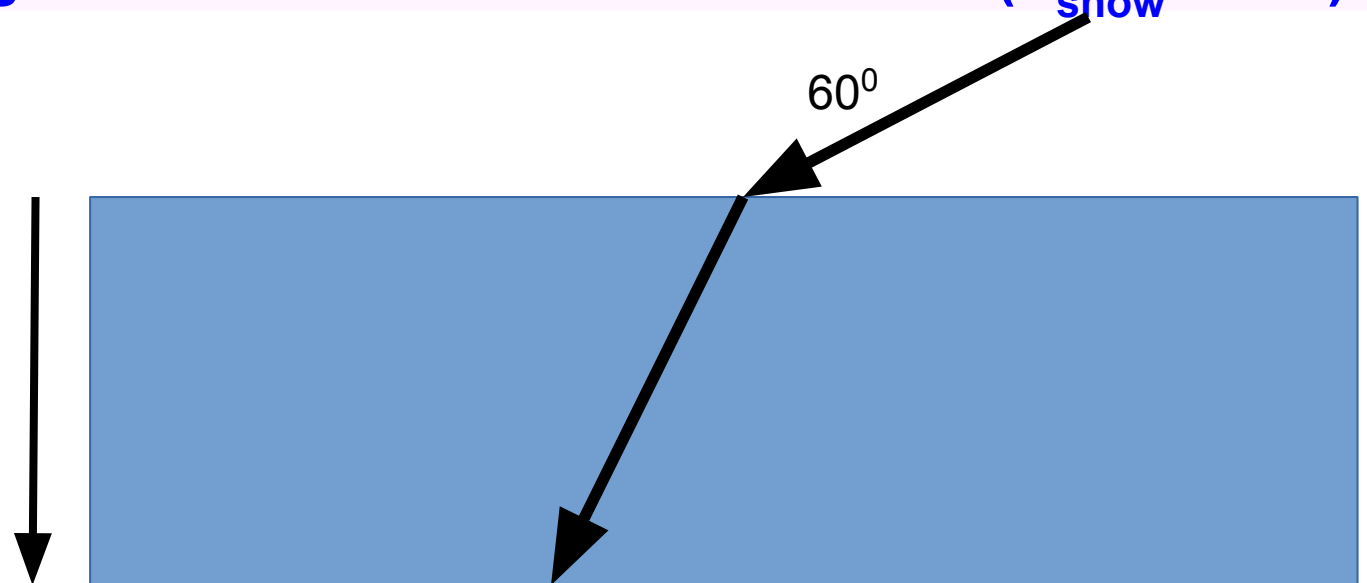
**Suppose the incident ray is at a crest as it intercepts the surface, the phase of T3 and T5 are**

- A) Crest, trough
- B) Crest, crest
- C) Trough, trough
- D) Trough, crest
- E) NOTA



Speculation to explain 'mystery' events:  
Maybe there is an embedded ice layer within the Antarctic ice sheet?

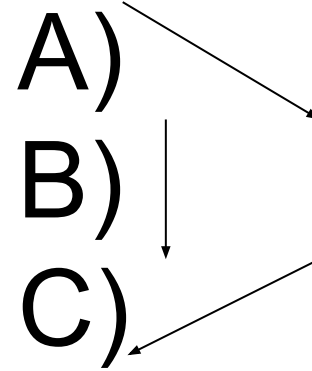
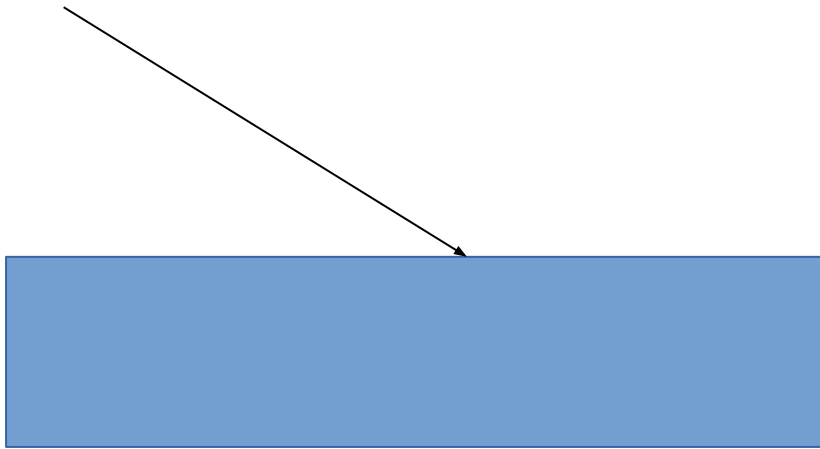
A 600 MHz radio wave is incident from air into a 6 meter thickness of snow, at a  $60^\circ$  angle relative to vertical (“normal”). The total number of wavelengths the wave travels in snow is: ( $n_{\text{snow}} = 1.35$ )



- A) 5.27
- B) 6.0
- C) 3.0
- D) 21.1
- E) NOTA

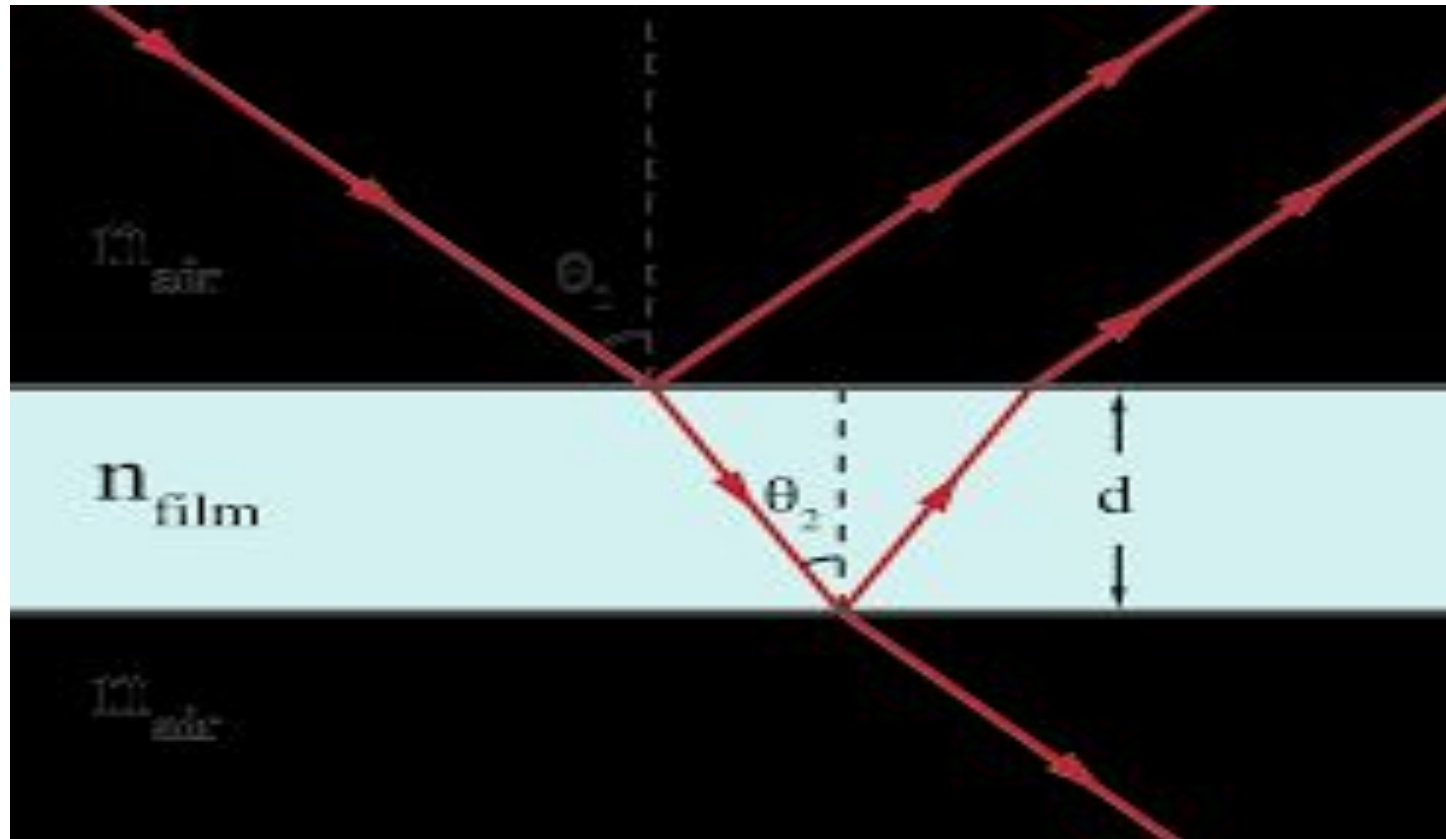
What is the total time required for the wave to travel through the snow?

Given  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ , which arrow best represents the direction that the ray shown emerges on the bottom side of the glass block ( $n=1.5$ )?

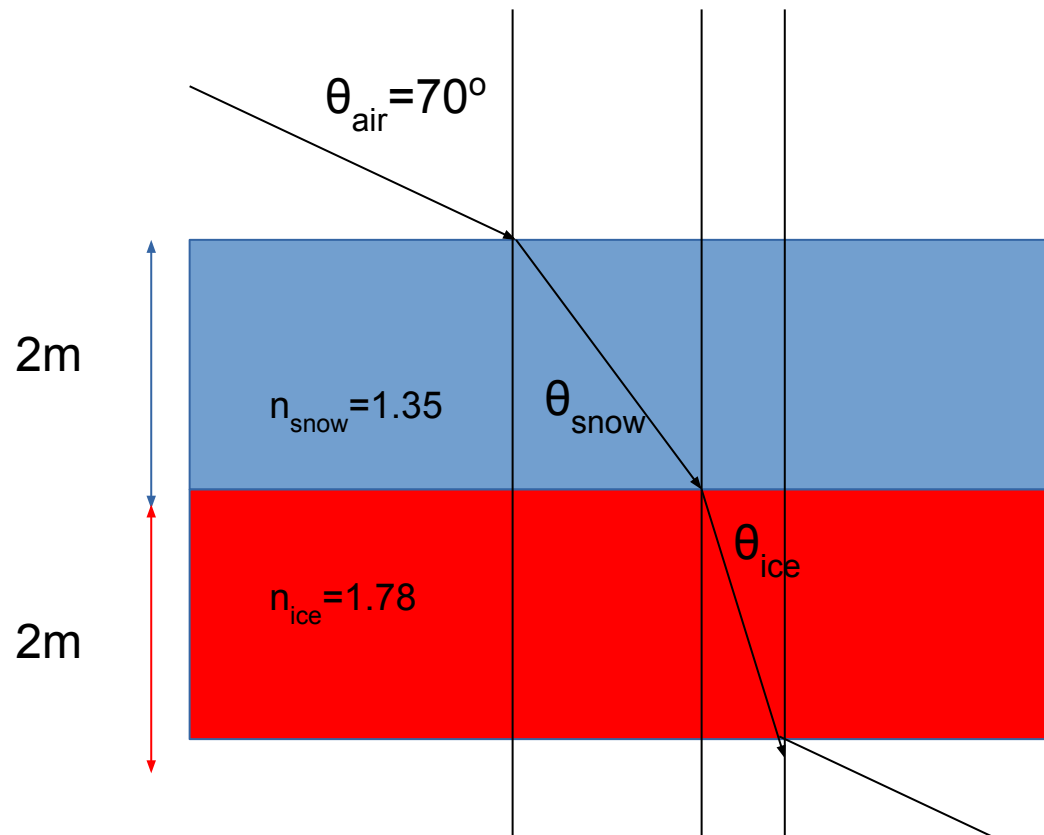


If  $\theta_1 = 53.1^\circ$  and  $n_{\text{film}} = 2$  and  $\lambda = 400 \text{ nm}$ , what is the minimum thickness of the film such that the two emerging rays in the upper right are in phase?

- A) 80 nm
- B) 91.5 nm
- C) 200 nm
- D) 400 nm
- E) NOTA







Use geometry + successive applications of Snell's Law to determine angles, distances traveled and propagation times.

At air/snow boundary:

$n_{\text{air}} \sin(\theta_{\text{air}}) = n_{\text{snow}} \sin(\theta_{\text{snow}})$ , solve for  $\theta_{\text{snow}} = 44.11^\circ$ ; now input that value to:  
 $n_{\text{snow}} \sin(\theta_{\text{snow}}) = n_{\text{ice}} \sin(\theta_{\text{ice}})$  to get  $\theta_{\text{ice}} = 31.86^\circ$

To determine total transit time:

Total distance through snow =  $2\text{m} / \cos\theta_{\text{snow}} = 2.78$  meters; similarly, total distance through ice =  $2\text{m} / \cos\theta_{\text{ice}} = 2.35$  m (more vertical).

Total travel time =  $d_{\text{snow}}/c + d_{\text{ice}}/c = 26.45$  ns

How many waves over that 4 meter distance? Depends on frequency, since  $f\lambda = c$ . Suppose  $f = 900$  MHz, then  $\lambda(\text{air}) = 33.33$  cm;  $\lambda(\text{snow}) = 33.33$  cm/1.35;

$N(\text{waves, snow}) = 2.78\text{ m} / (0.333\text{m}/1.35) = 11.34$  waves

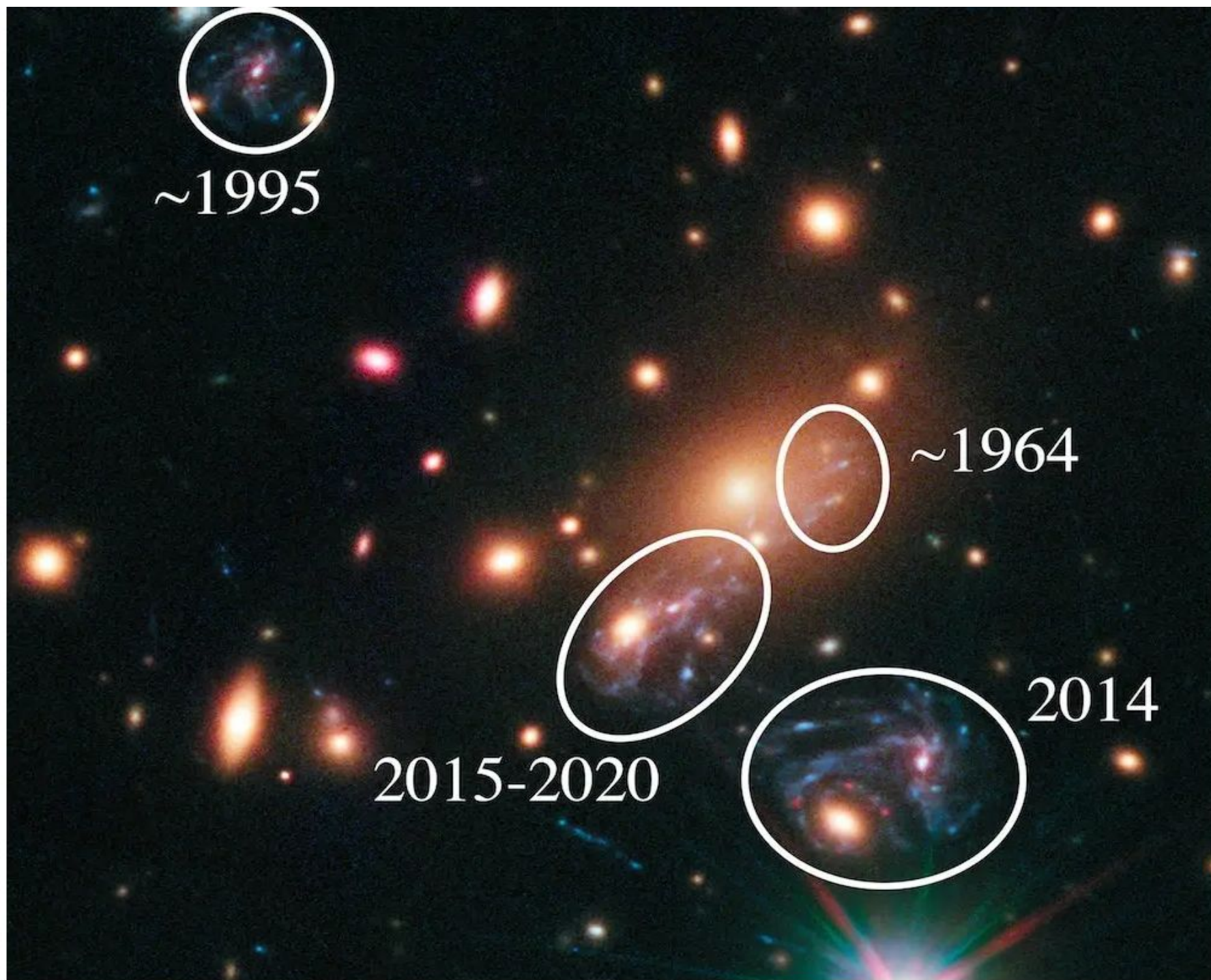
$N(\text{waves, ice}) = 2.35\text{ m} / (0.333\text{m}/1.78) = 12.68$  waves (more waves since lower wavelength)



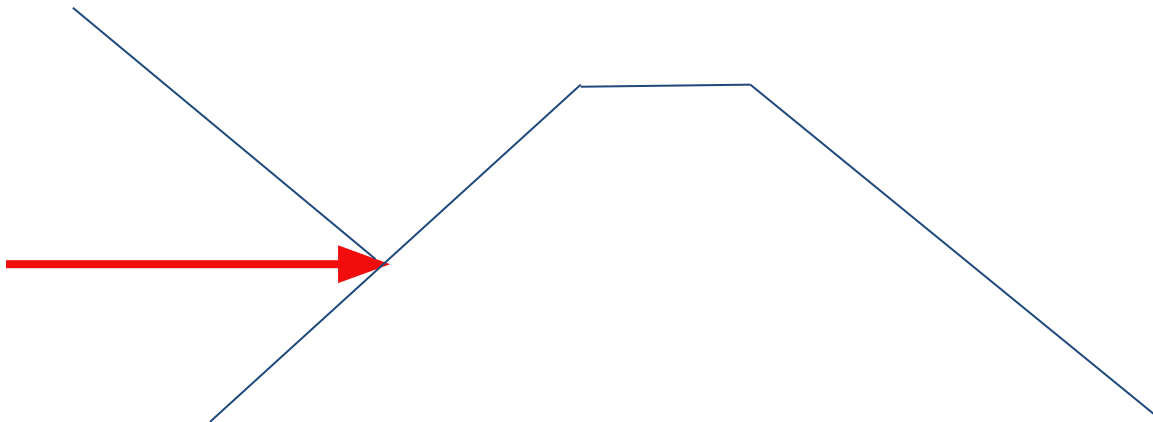
**Optics: Dark disk in front of light source.  
Center of shadow is brighter/darker than  
surrounding? (Huygens)**

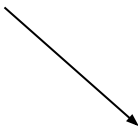

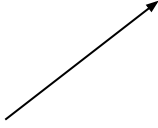
**Ray/Geometric optics: Fermat's Least Time  
principle:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ ;  
lenses and mirrors**

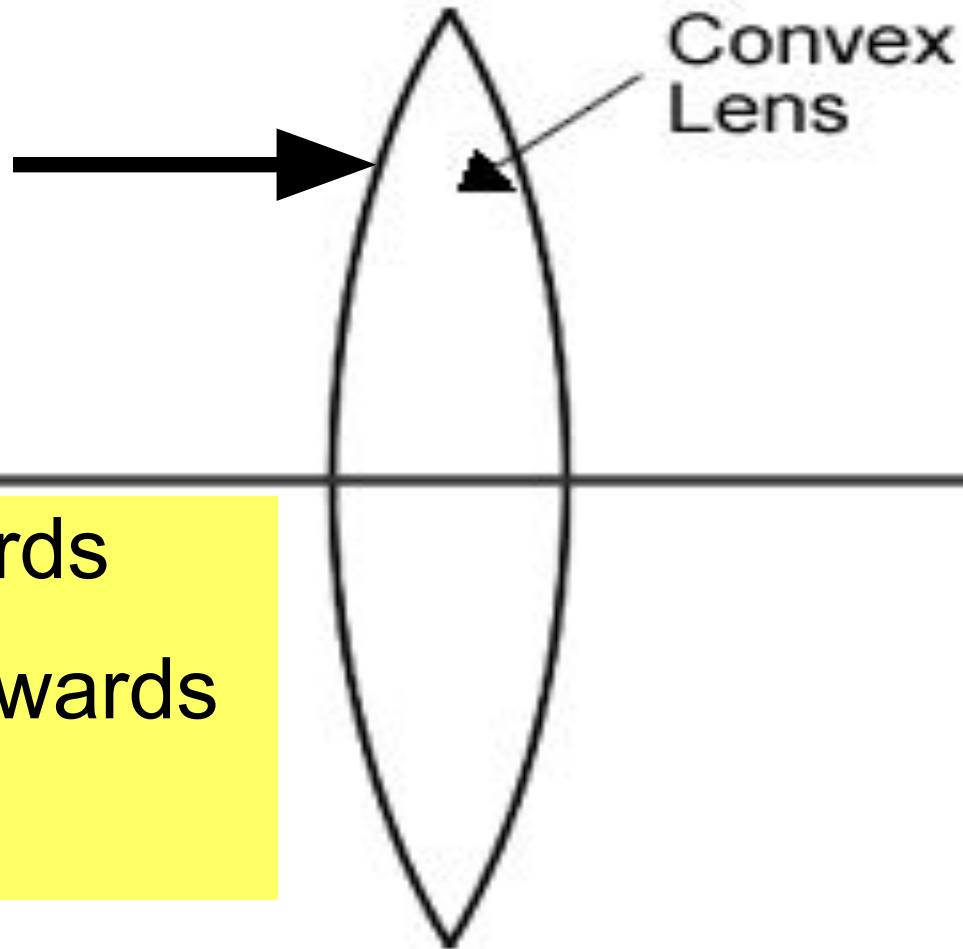
**focal lengths: positive and negative;  
 $1/i + 1/o = 1/f$ ;  $m = -i/o$ ; focal length determined by  
crossing-point of horizontal ray with x-axis;  
light rays **CROSS** the normal on refraction**



Given  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ , which arrow best represents the direction that the ray shown emerges on the right side of the glass block ( $n=1.5$ )?

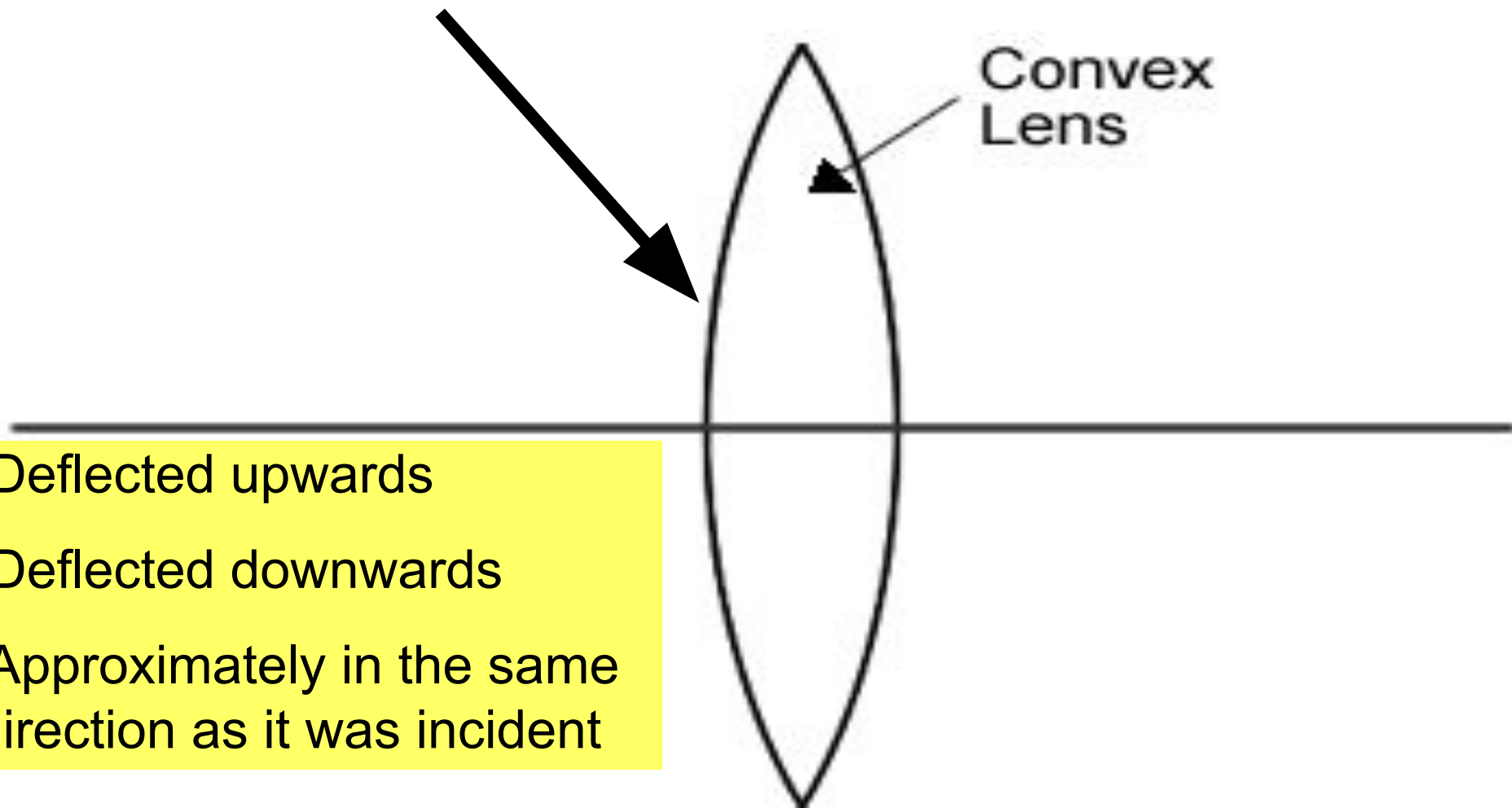


- A) 
- B) 
- C) 



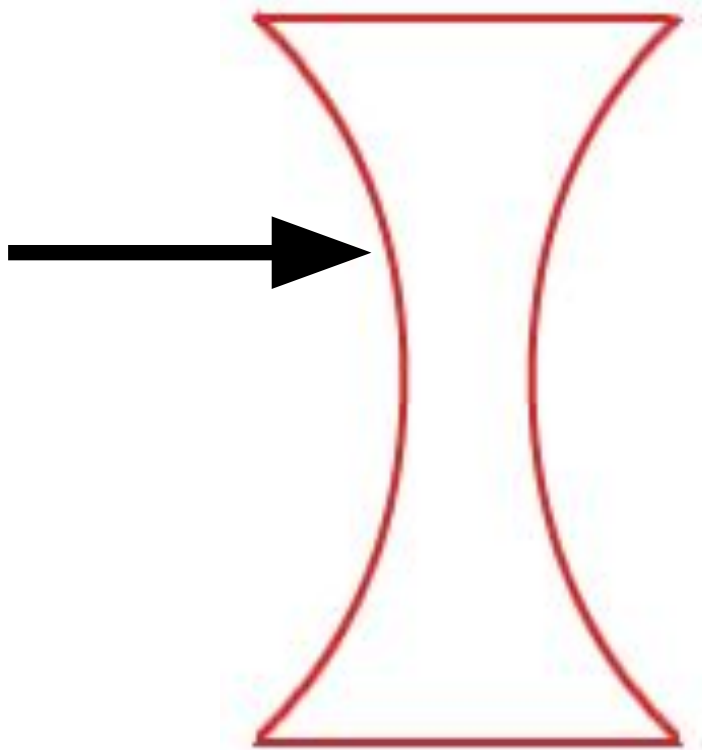
- A) Pointing upwards
- B) Pointing downwards
- C) horizontal

**A horizontal light goes from air onto glass ( $n=1.5$ ) and emerges out the back side. The ray shown emerges:**



- A) Deflected upwards
- B) Deflected downwards
- C) Approximately in the same direction as it was incident

**An angled light ray goes from air onto glass ( $n=1.5$ ) and emerges out the back side. The ray shown emerges:**



- A) Pointing upwards
- B) Pointing downwards
- C) horizontal

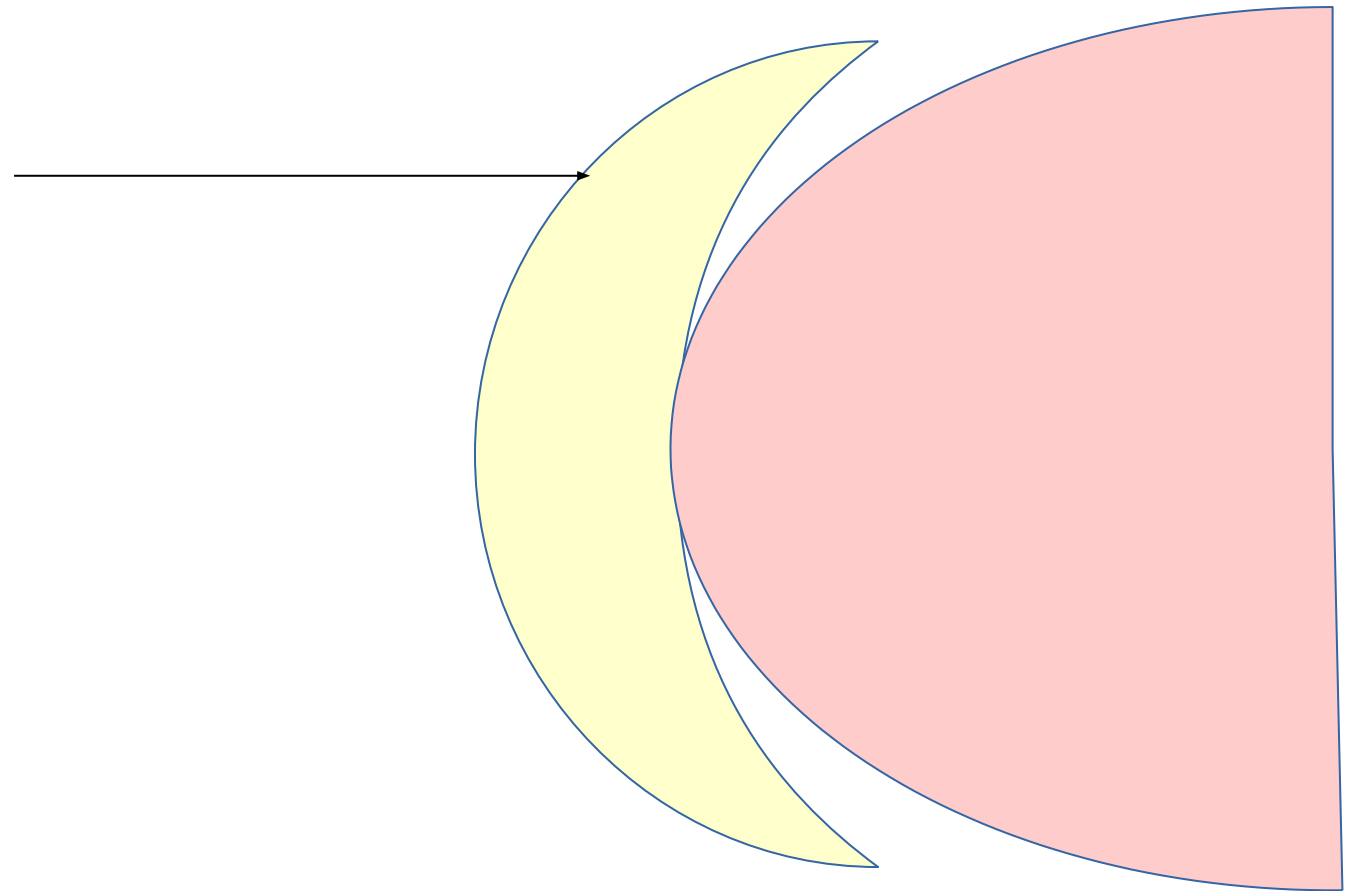
**Light goes from air onto glass ( $n=1.5$ ). The ray shown emerges:**

**Looking down on I-80, you see a 'puddle', from which you conclude:**

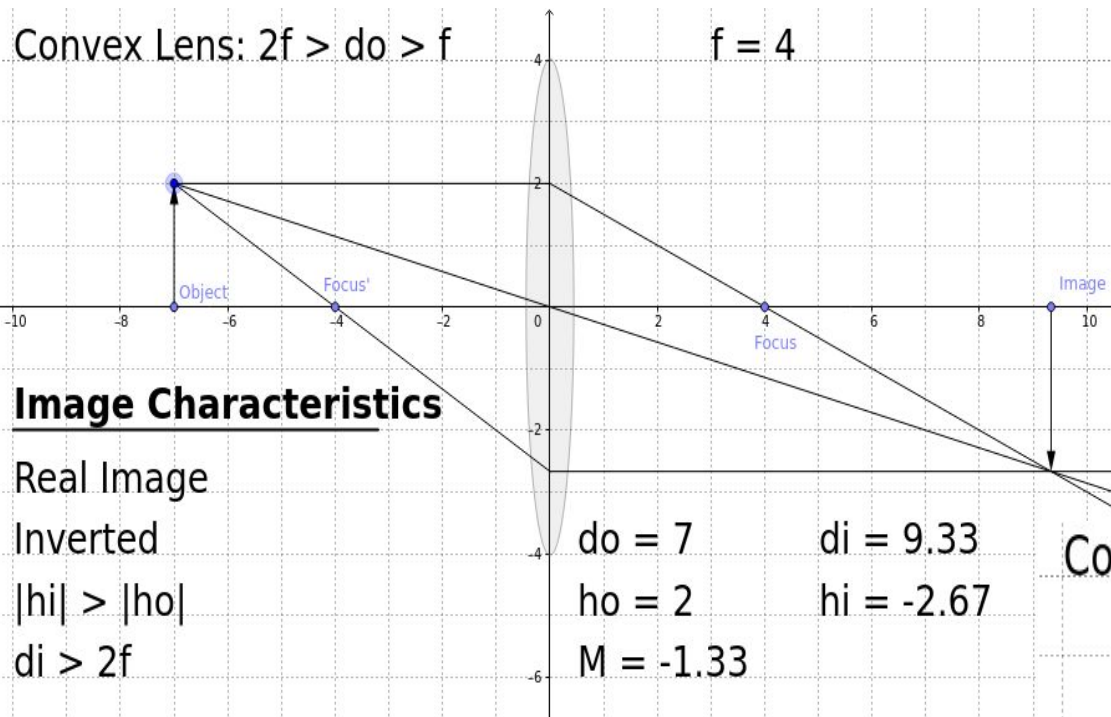
- A) The index of refraction of air increases with temperature
- B) The index of refraction of air decreases with temperature

**Light is incident from glycol ( $n=2$ ) onto a plate of glass ( $n=1.5$ ), in front of a layer of snow ( $n=1.35$ ). The ray shown emerges:**

- A) Pointing upwards
- B) Pointing downwards
- C) horizontal



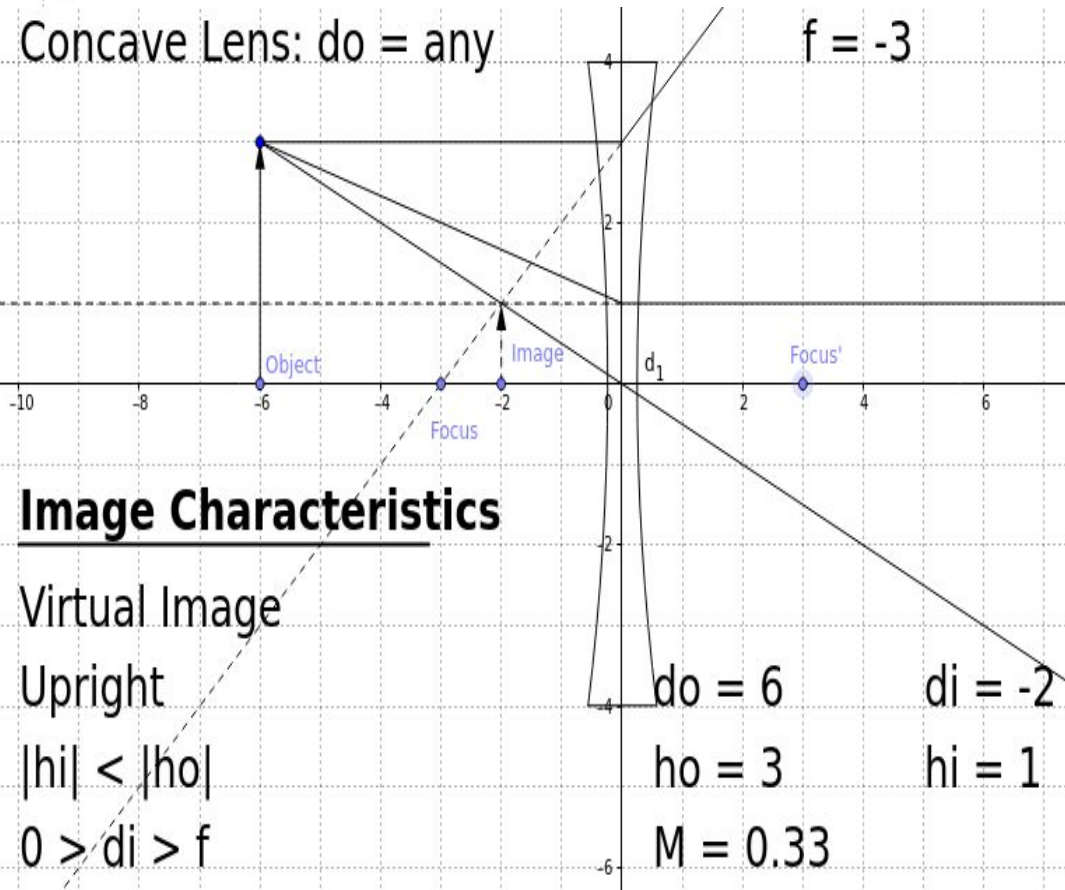
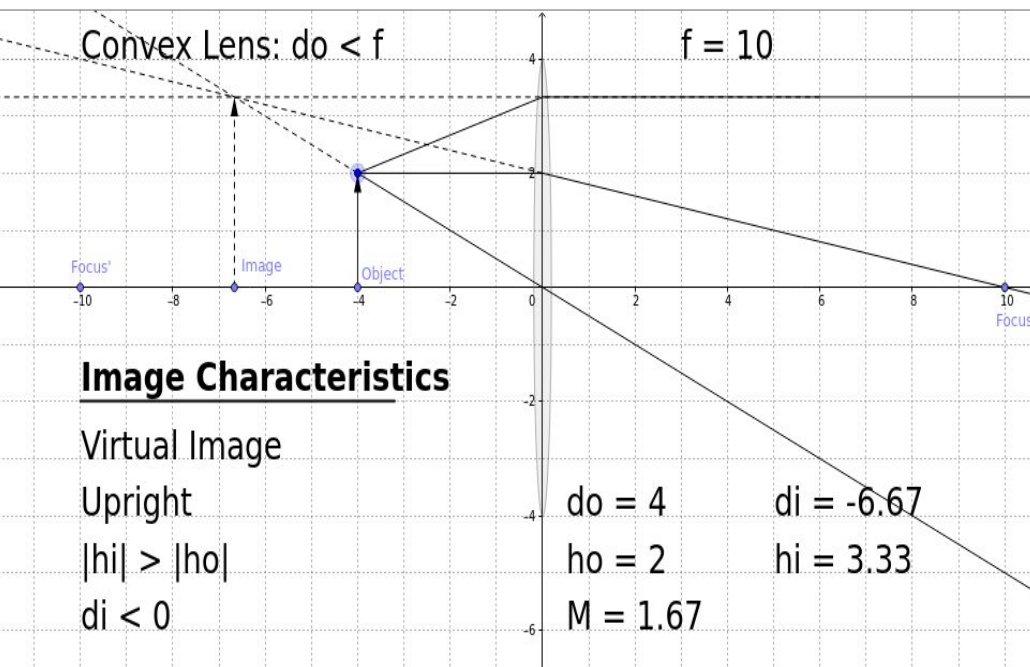




$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

$$\text{magnification} = -\frac{d_i}{d_o}$$

(<https://ophysics.com/>)



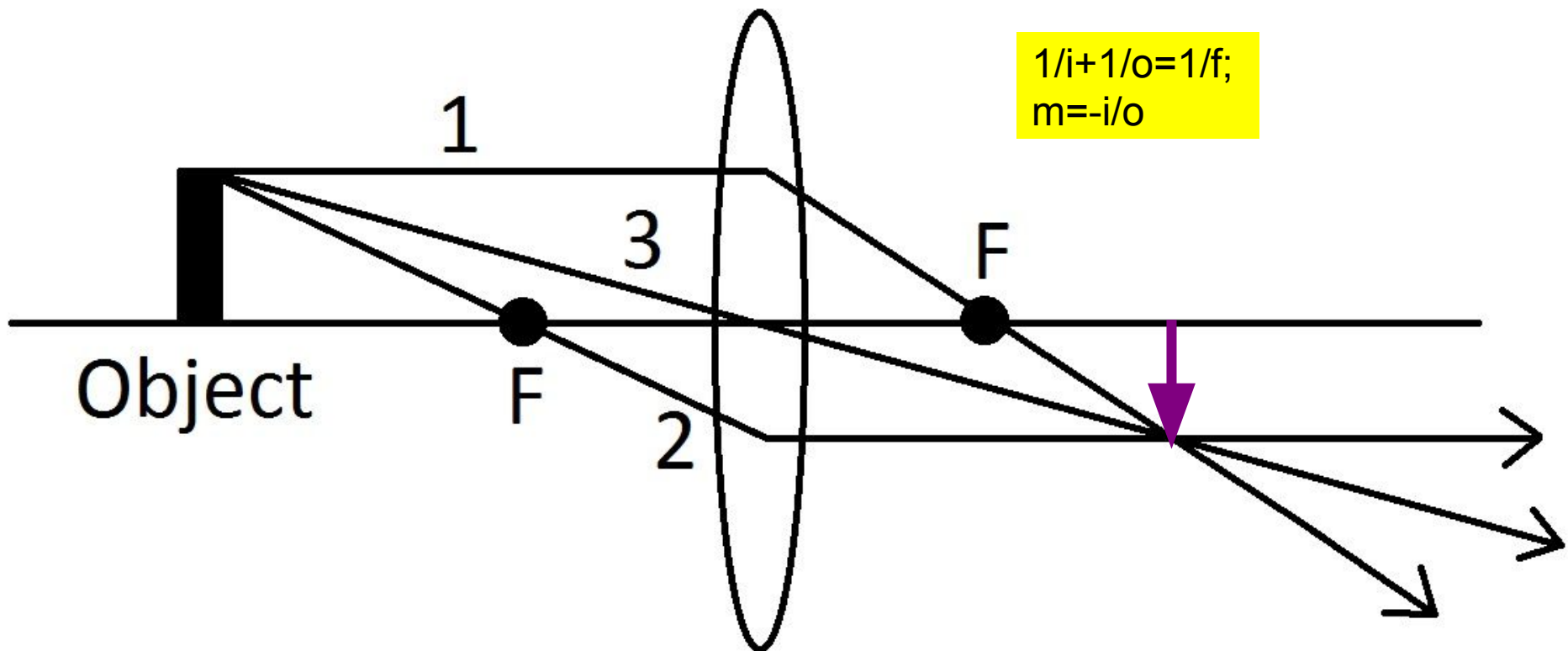
**Thin lens formula:  $1/i + 1/o = 1/f$**

**Note human eye has fixed  $f$  (ergo, difference b/w diverging and converging corrective lenses).**

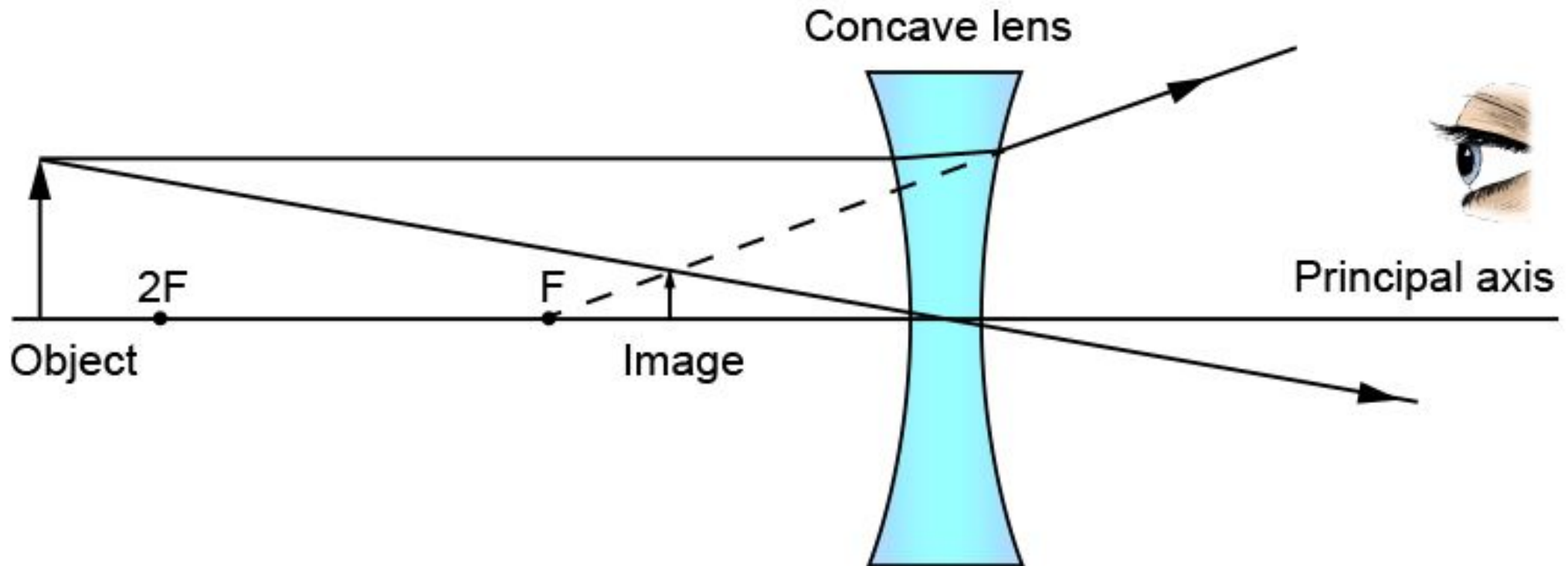
**(Lord of the Flies intermezzo)**

**Double lenses can have either real or virtual second objects (e.g., we see virtual images with our eyes!)**

A 'lens' is a device which, taking into account the index-of-refraction of the lens and, therefore, how much bending rays entering from air will undergo in the lens itself, has the 'right' curvature so that light rays coming from an object will focus at some other point, resulting in an 'image'. Here, rays 1, 2 and 3 all originate at the top of the object, and all meet (are 'focused') at the crossing point ("I" or " $d_i$ ") shown to the right of F.

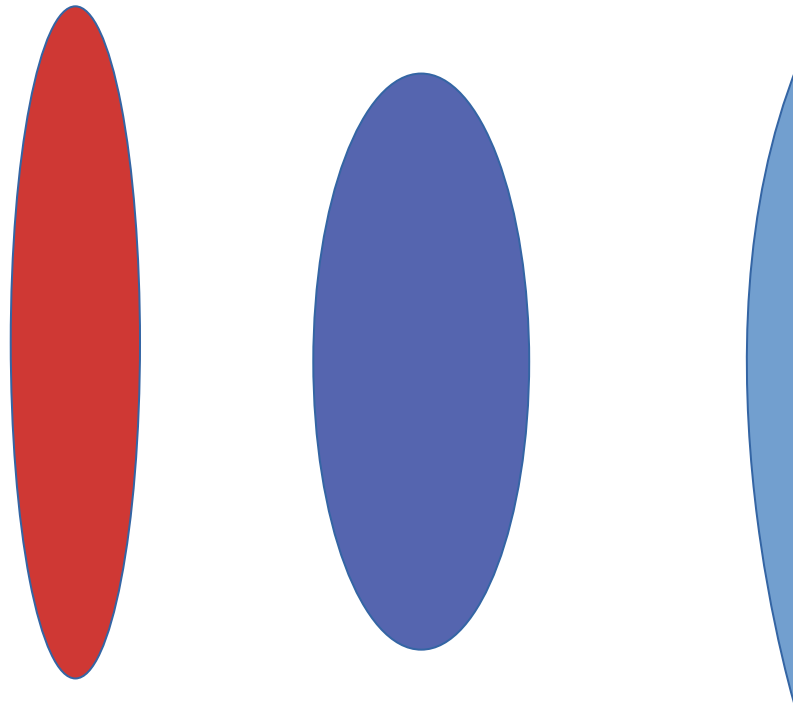


Concave (diverging lens): focal length **negative**!



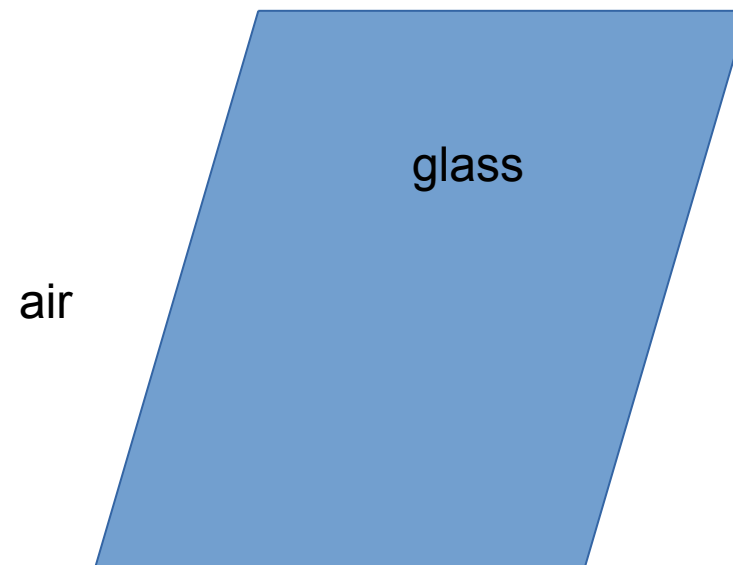
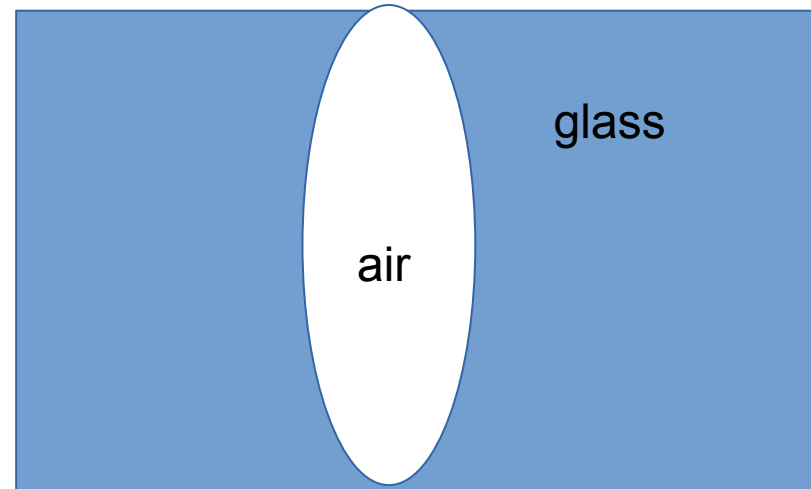
**Rank the lenses shown from smallest to largest focal length.**

- A) a, b, c**
- B) c, b, a**
- C) b, a, c**
- D) c, a, b**
- E) NOTA**



**Given the two lenses shown. Which statement is true?**

- A) Lens 1 has a negative focal length
- B) Lens 2 has an infinite focal length
- C) Both are true
- D) NOTA



**A 4 cm tall object is 12 cm from a converging lens with  $|f|=6$  cm. The image formed is:**

- A) 12 cm tall, upright
- B) 12 cm tall, inverted
- C) 4 cm tall, upright
- D) 4 cm tall, inverted
- E) NOTA

$$\begin{aligned} 1/i + 1/o &= 1/f; \\ m &= -i/o \end{aligned}$$

**If the object is now moved to a distance of 4 cm from the lens, the image formed is:**

- A) 12 cm tall, upright
- B) 12 cm tall, inverted
- C) 4 cm tall, upright
- D) 4 cm tall, inverted
- E) NOTA



**As the object moves closer to the lens, the image gets**

- A) Smaller
- B) Bigger
- C) Is unchanged

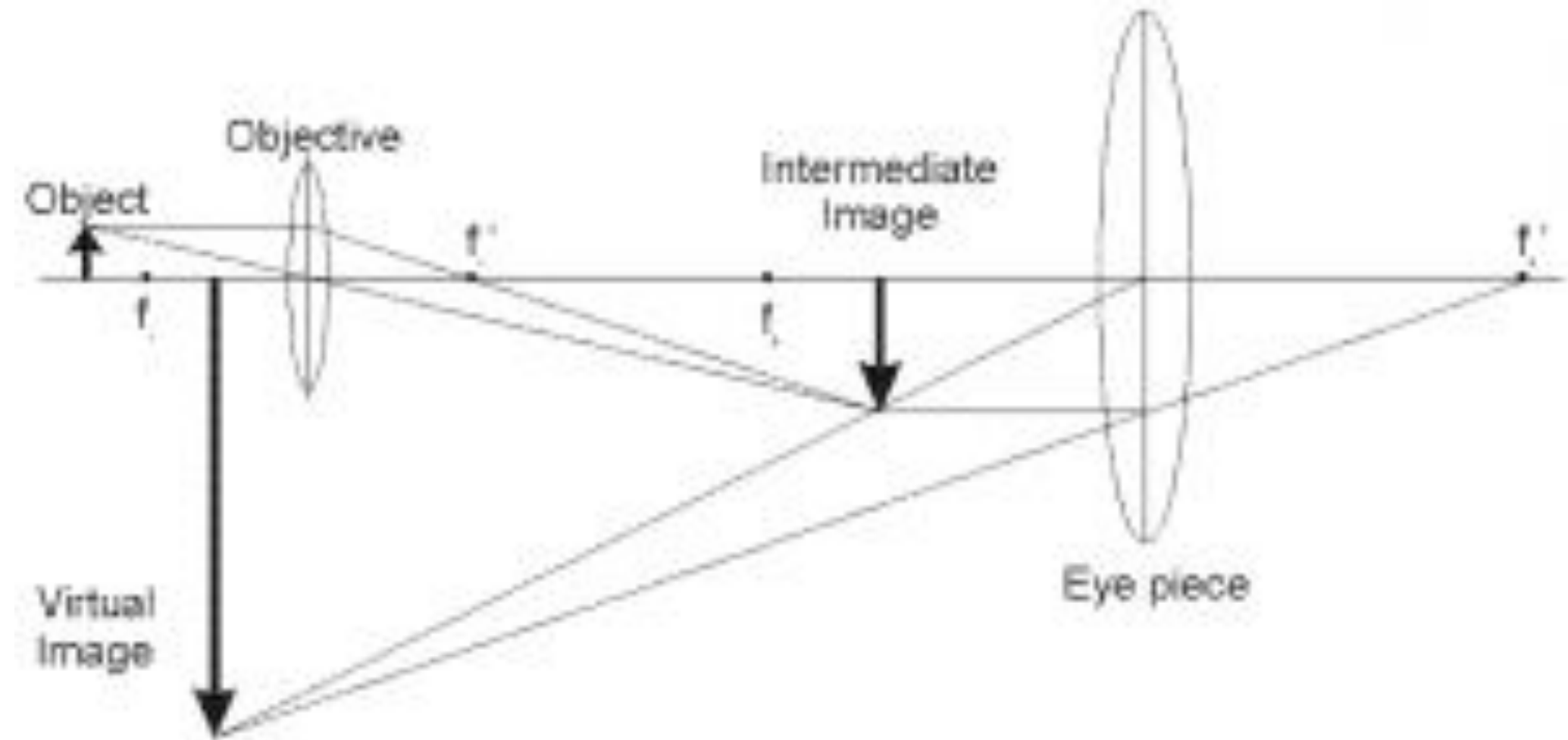
**A 4 cm tall object is placed 6 cm from a diverging lens with  $|f|=4$  cm. How large is the image formed?**

- A) 4 cm
- B) 2.4 cm
- C) 1.6 cm
- D) 0.8 cm
- E) NOTA

**As the object moves closer to the lens, the image gets**

- A) Smaller
- B) Bigger
- C) Is unchanged

## Example of double-lens system with virtual image

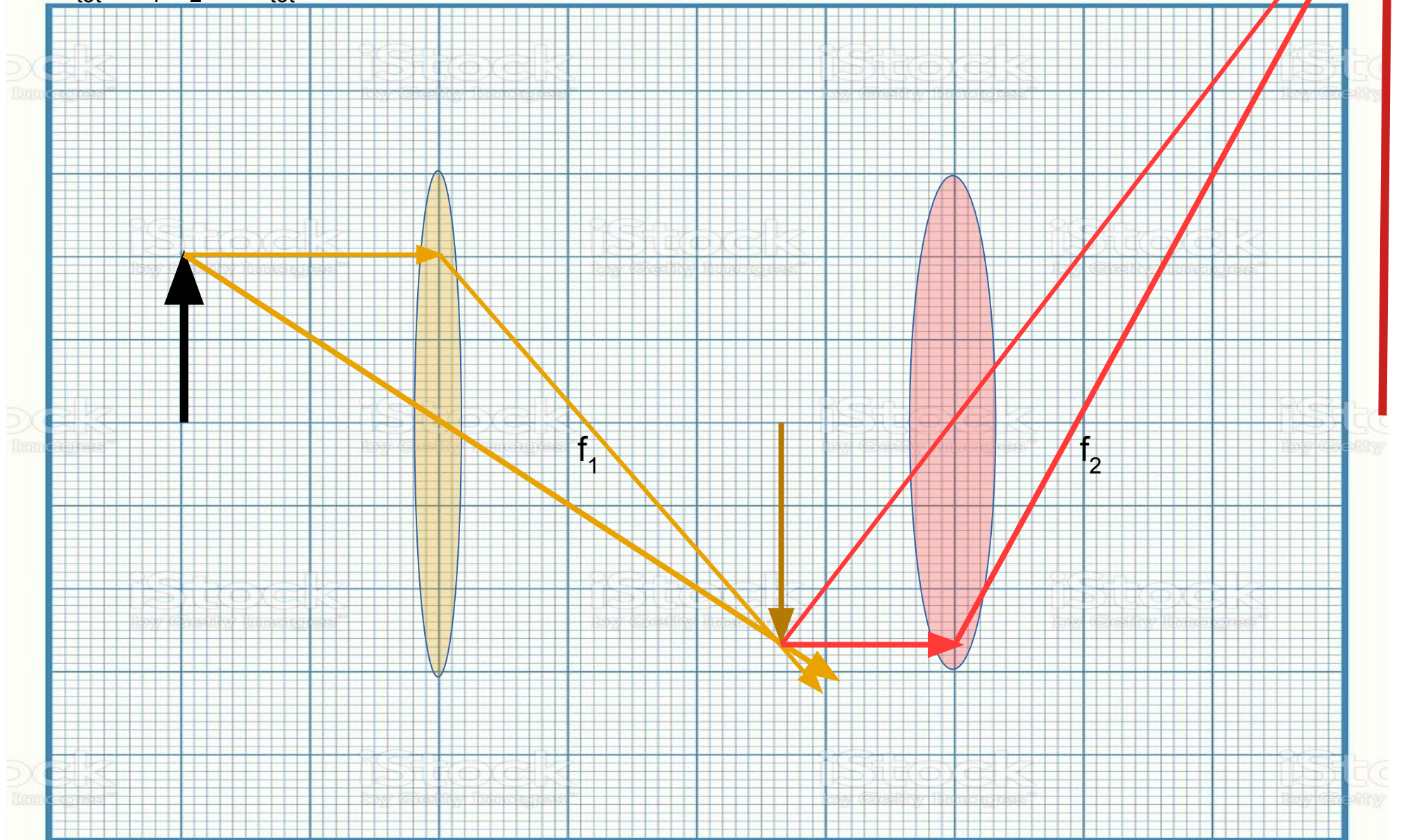


Given:  $h_1=20$ ;  $|f_1|=11$ ;  $o_1=19$ ;  $f_2=10$ , lenses 40 cm distant.

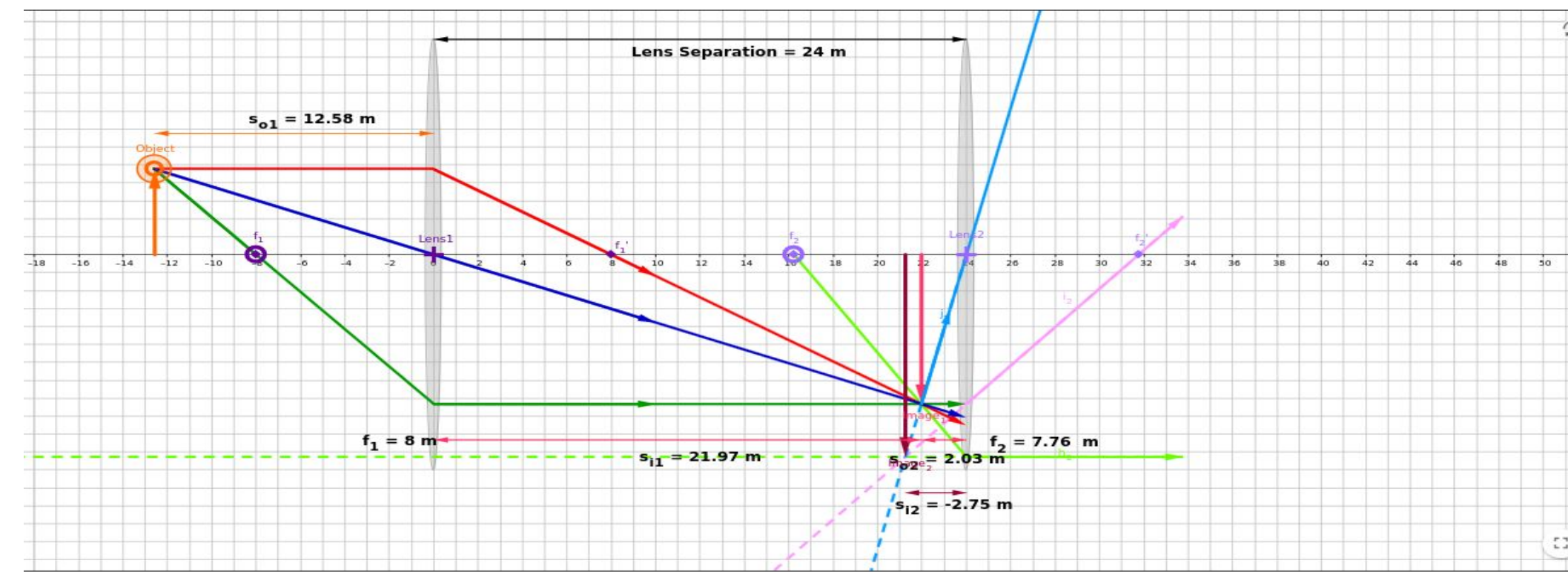
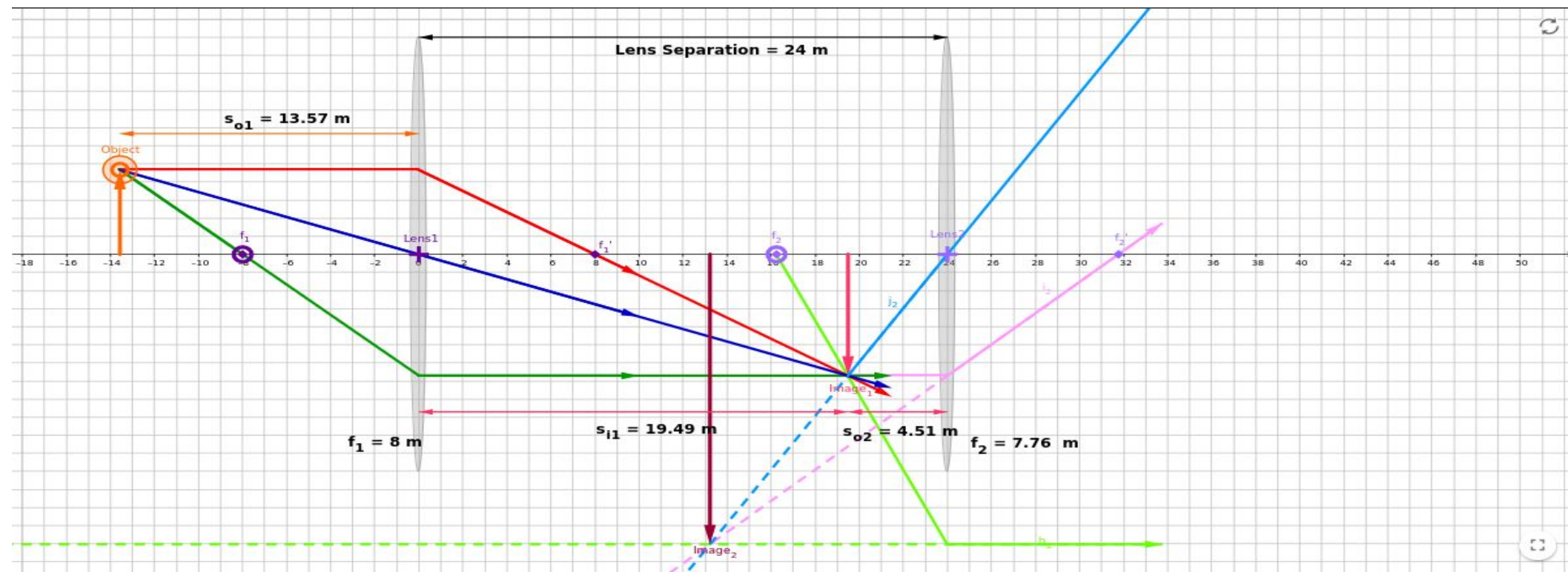
find final image by both ray tracing and calculation: Lens1:  $1/19+1/i=1/11 \Rightarrow i=26.125$  cm

$O_2=40-26.125=13.875$  cm, so  $1/13.875+1/i_2=1/10 \Rightarrow i_2=35.8$  cm

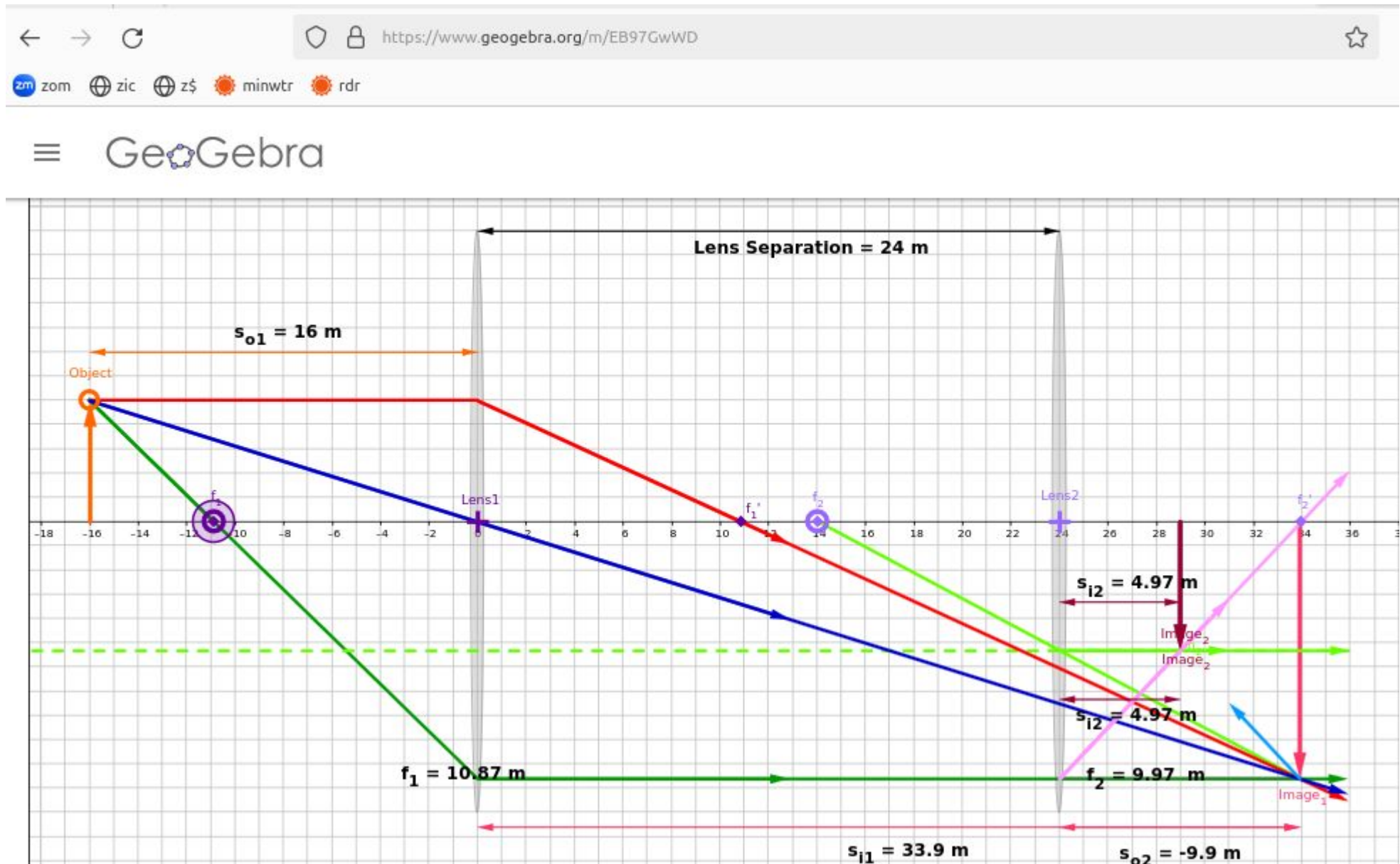
$m_{\text{tot}}=m_1 m_2 \Rightarrow m_{\text{tot}}=(-26.125/19.)(-35.8/13.875)=3.55$ , so final image is  $3.55 \times 20=67$  cm tall

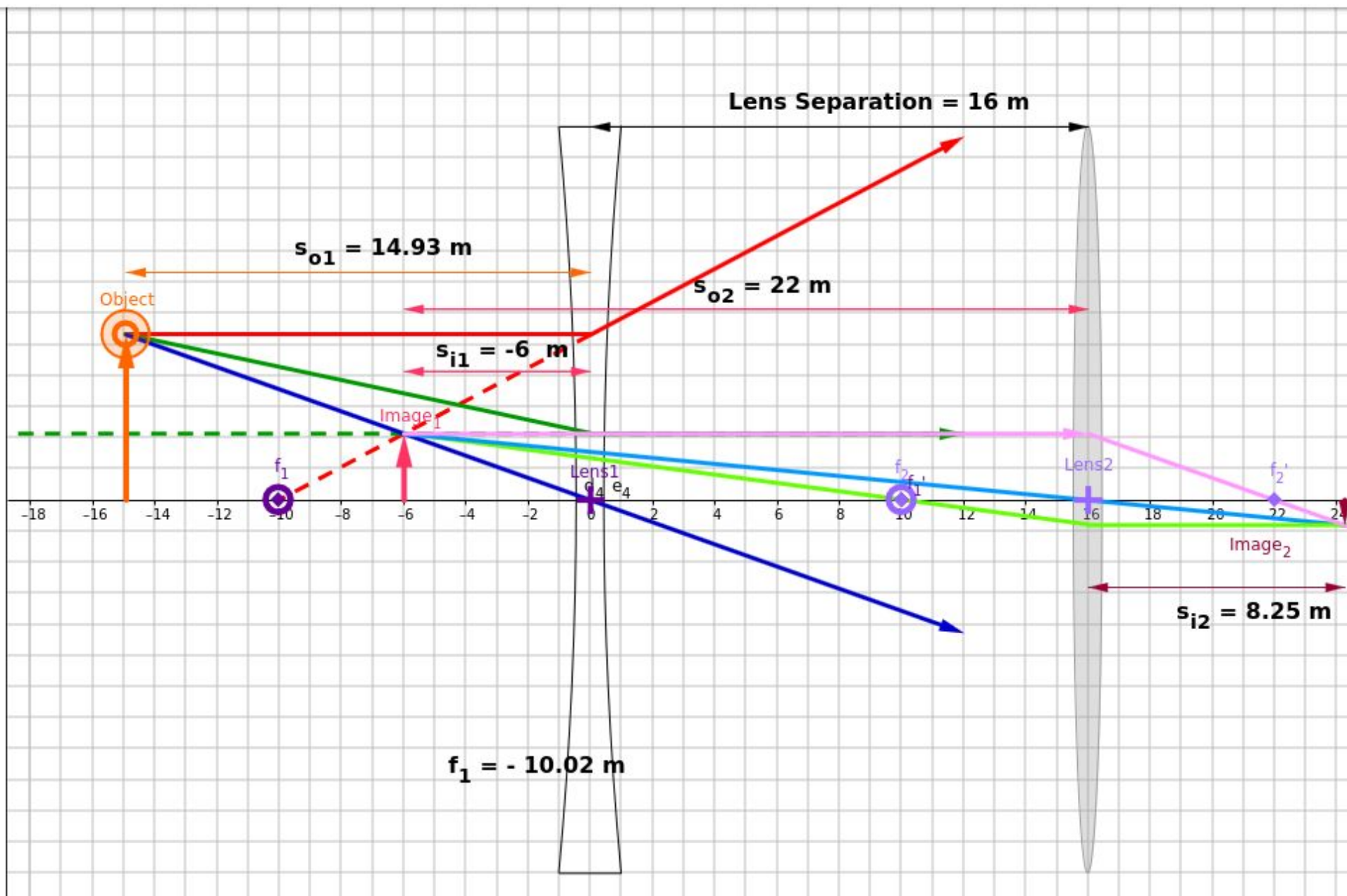






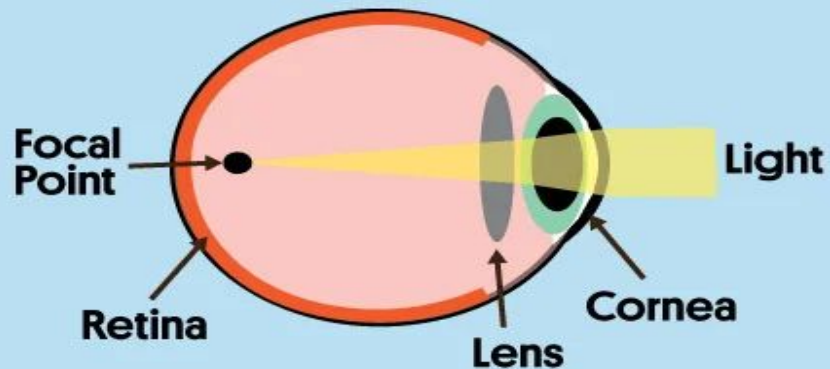
N.B. в этом моделировании 1) кажется, что о1 «проходит сквозь» вторую линзу, и 2) вторая линза использует отрицательное расстояние до объекта в формуле тонкой линзы. Возможное объяснение: изображение на самом деле создается волнами, а лучевая картина здесь нарушается? Вторая концептуальная проблема: почему  $i_2$  не становится о3 для L1?



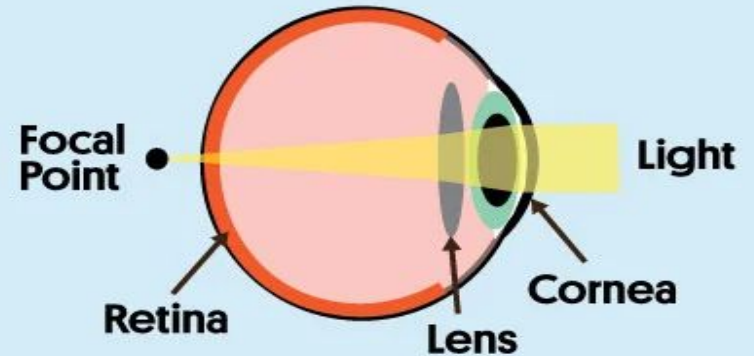




# Nearsighted



# Farsighted



Glasses for nearsighted/farsighted people are:

- A) converging/diverging lenses
- B) converging/converging lenses
- C) diverging/converging lenses
- D) diverging/diverging lenses

**A 20 cm high object is 50 cm from  $L_1$  ( $f=+30$  cm) and 185 cm from  $L_2$  ( $f=+20$  cm). How big is the final image?**

- A) 20 cm
- B) 6 cm
- C) 12 cm
- D) 15 cm
- E) NOTA

**If the object moves slightly closer to the first lens, which is true of the magnification magnitudes?**

- A)  $M_1$  increases,  $M_2$  increases
- B)  $M_1$  increases,  $M_2$  decreases
- C)  $M_1$  decreases,  $M_2$  increases
- D)  $M_1$  decreases,  $M_2$  decreases

If object is 1 cm high, given  $|f_1|=4$  cm,  $o=6$  cm,  $|f_2|=8$  cm,  $d(\text{conv. lens } 1 \rightarrow \text{conv. lens } 2)=18$  cm, find size of  $i_2$

- A) 12 cm
- B) 8 cm
- C) 6 cm
- D) 24 cm
- E) NOTA

**X stands in front of a mirror. Draw the rays coming from her head which leads to the image of her head in the mirror.**

**A 4 cm tall object is located 12 cm from a concave mirror having radius of curvature 8 cm. How large is the final image?**

- A) 4 cm
- B) 2 cm
- C) 8 cm
- D) 1 cm
- E) NOTA

**As the ends of the mirror are bent outwards, the image gets**

- A) Larger
- B) Smaller
- C) Is unchanged

**A 4 cm tall object is located 12 cm from a convex mirror having radius of curvature 8 cm. How large is the final image?**

- A) 4 cm
- B) 2 cm
- C) 8 cm
- D) 1 cm
- E) NOTA



**An object is 8 cm in front of an  $R=12$  cm concave mirror. As the object moves from 8 cm away to 4 cm away, the image changes from:**

- A) Real  $\rightarrow$  virtual
- B) Inverted  $\rightarrow$  upright
- C) Virtual  $\rightarrow$  real
- D) Upright  $\rightarrow$  inverted
- E) Two of the above

**A 4 cm object is 18 cm to the right of an  $f=+8$  cm lens and 12 cm to the left of an 8 cm radius of curvature mirror. The ratio of the image heights**

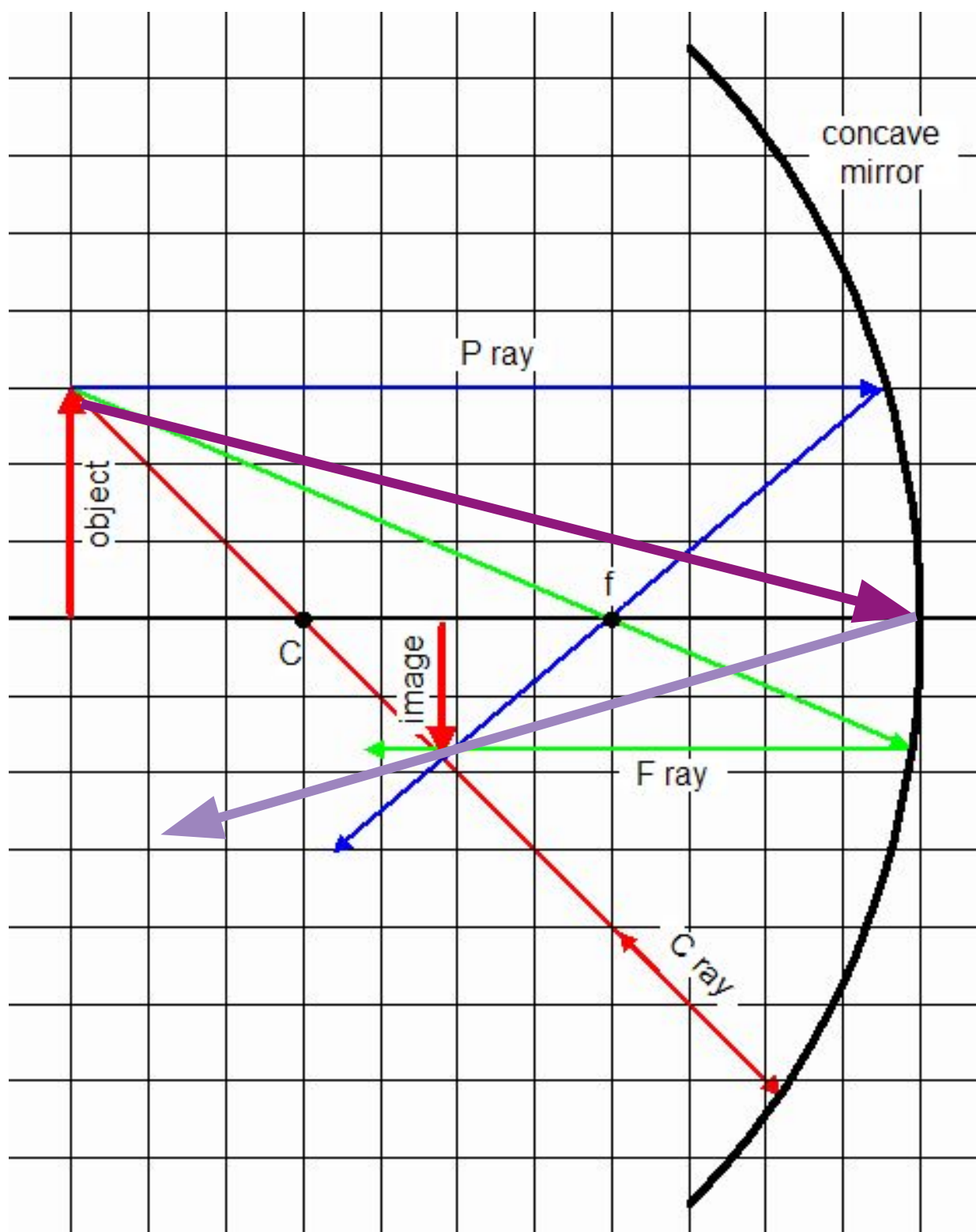
**$(i_{\text{lens only}}/i_{\text{lens + mirror}})$  is:**

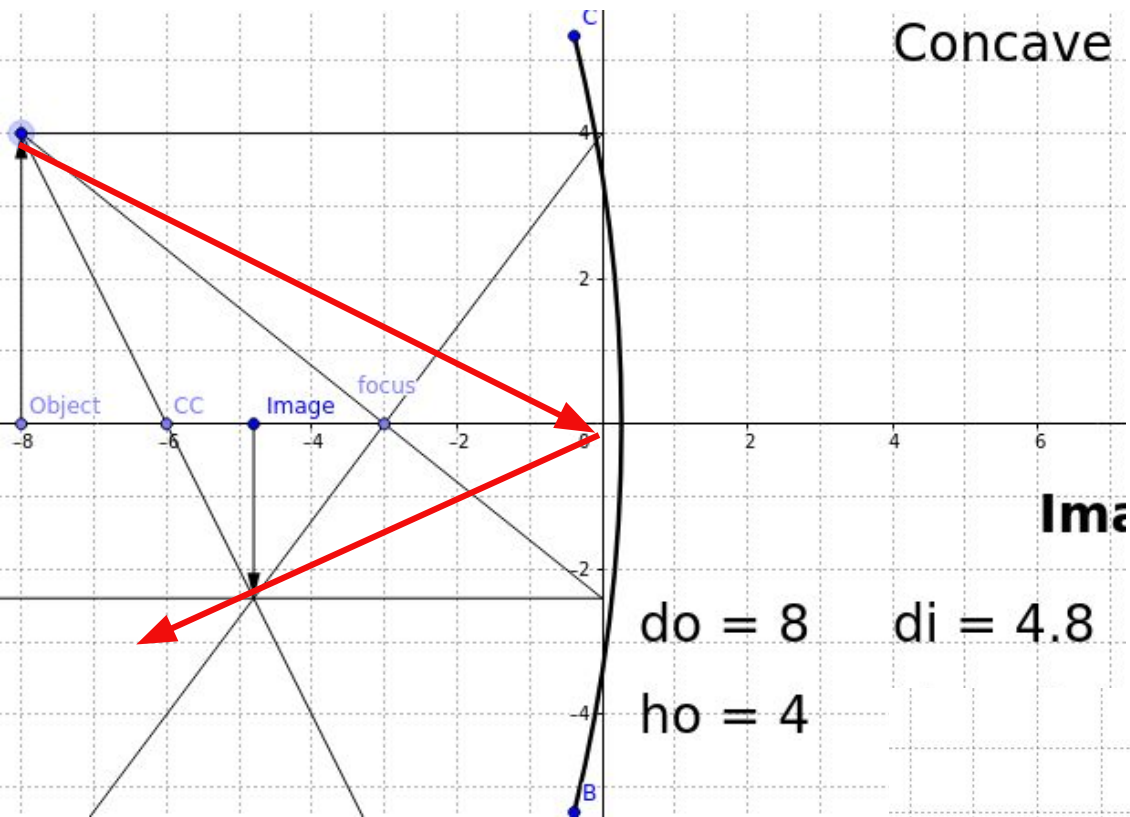
A)  $>1$

B)  $=1$

C)  $<1$

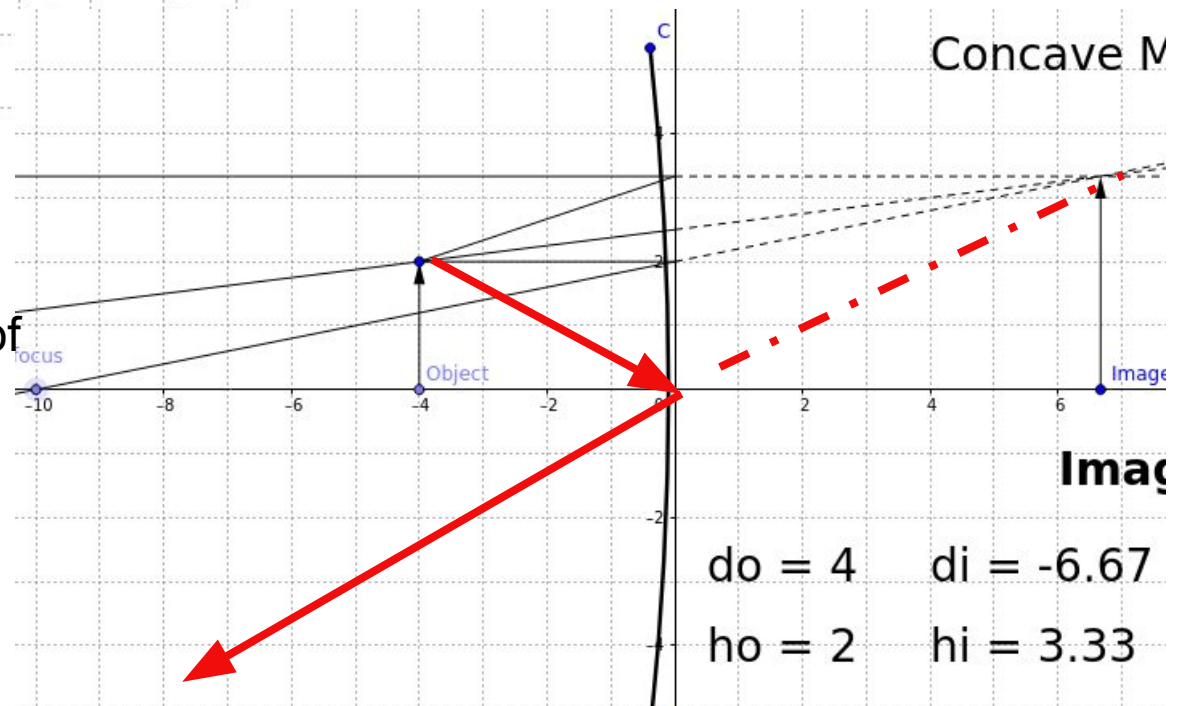
D)  $0$



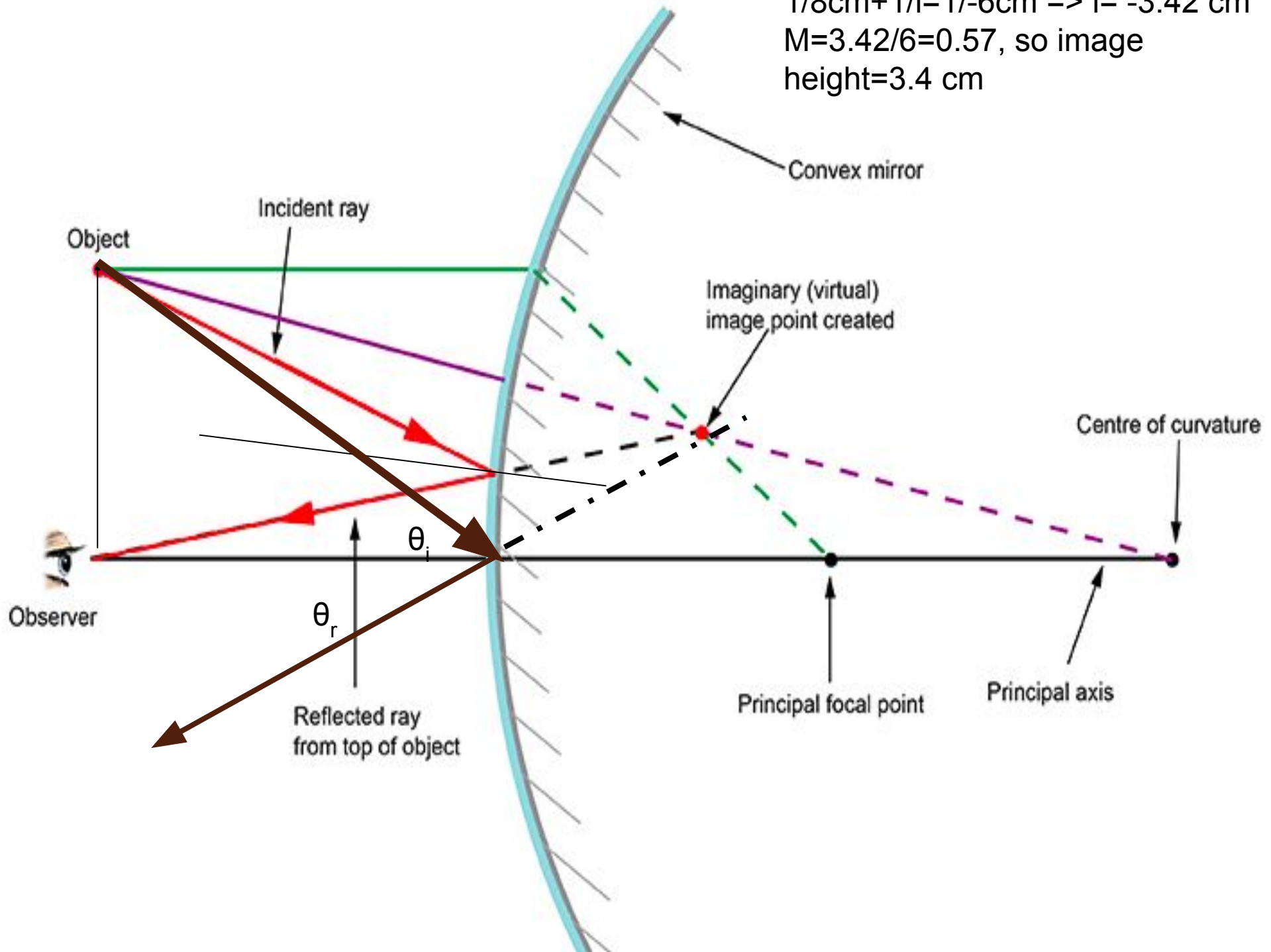


Top:  $|f|=3$ ;  $1/i+1/8=1/3$ ;  $m=-i/o$ ,  
Solve for  $i=4.8$ , so  $m=-4.8/8$ , so height of  
image  $=-(4.8/8)*4\text{cm}$ , or 2.4 in magnitude,  
but negative sign tells us inverted.

Bottom:  $1/i+1/4=1/10$ ;  $m=-i/o$ ,  
Solve for  $i=-6.67$ , so  $m=1.67$ , so height of  
image  $=1.67*2\text{cm}$ , or 3.33 in magnitude;  
positive sign tells us upright.



$H_o = 6 \text{ cm}$ ;  $|f| = 6 \text{ cm}$ ;  $d_o = o = 8 \text{ cm}$   
 $\frac{1}{8\text{cm}} + \frac{1}{i} = \frac{1}{-6\text{cm}} \Rightarrow i = -3.42 \text{ cm}$   
 $M = \frac{3.42}{6} = 0.57$ , so image height = 3.4 cm



**Hubble: spherical aberration – periphery too flat by  
0.0022 mm**

**As the ends of the mirror are bent outwards, the image from the lens+mirror system gets**

- A) Larger
- B) Smaller
- C) Is unchanged

# **Atomic Physics:**



## Wave-particle duality:

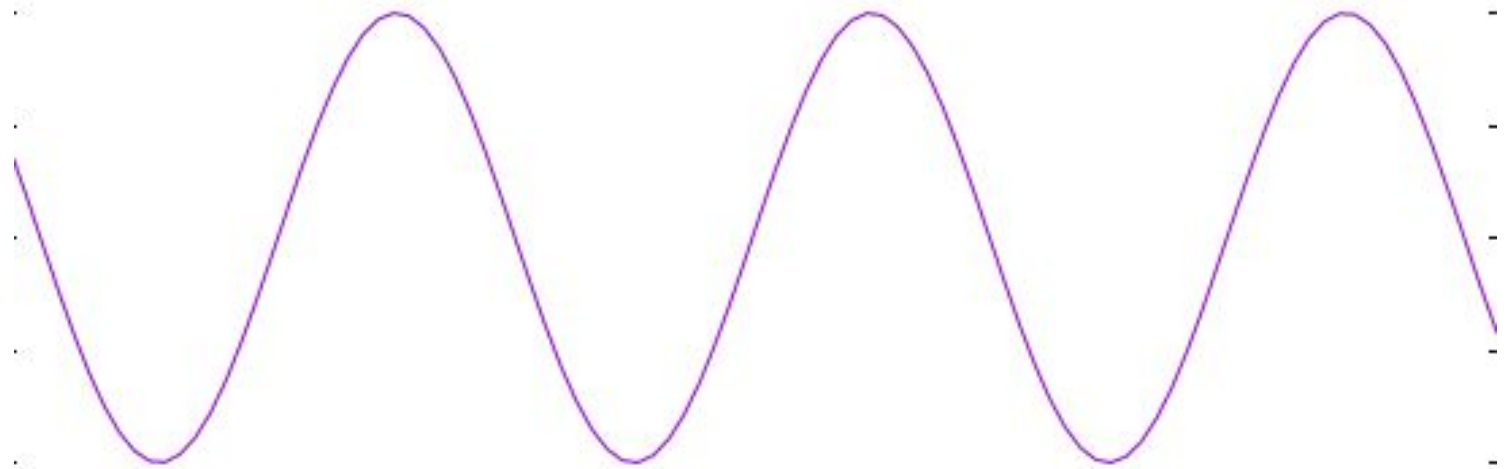
Newton: 'photon'/Huygens: 'wave' (1900: "Spectre haunting Europe"). 1) Michelson-Morley

2) Photoelectric puzzle: crank up intensity, but never get a photoelectron (same as in solar cell, or SiPM, or plant leaf) if  $\lambda$  large. But, small  $\lambda$  at low intensity gives P.E. Planck: Light energy delivered in bundles (quanta), each bundle has energy  $hc/\lambda = hf_{\text{light}}$ . So light wave has particle-like properties. DeBroglie: particles behave like light, as well: dimensionally (KE, e.g.):  $[E]:[p][v]$ , for photon:  $E=pc$ , so  $p=h/\lambda$ . Significance of  $\lambda$  for Bohr model of Hydrogen and resolution of stability of electron orbital. Physical significance vis-a-vis: "Where is electron?" Electron simultaneously occupies entire region defined by one wavelength.... (EPR and reln to Heisenberg Uncertainty).

$$L = nh/2\pi$$

## Quantum Mechanics:

- Huygens/Newton: Light is a wave/particle.
- deBroglie: 'particles' (well-defined location) also can act like 'waves' (spread out).
- More generally, any particle property (position, energy, momentum, spin) is 'spread out' until the particle is acted on by another force, particle, etc.



## Einstein-Padolsky-Rosen

A particle with no spin decays into two electrons, at  $t=0$ , which recoil from each other. If each electron can be either + or – spin, the probability that graduate student

A (indpdt of B) measures the  $e_A$  with + spin is:

A) 0%

B) 50%

C) 100%



Suppose the graduate students agree that GS A will measure the spin of electron A at midnight, and GS B will measure the spin of electron B at midnight+1 nanosecond. They let the two electrons migrate to opposite ends of the Universe, then GS A measures electron A to be + spin. The probability that GS B subsequently measures electron B to be + spin is now:

**A spin-0 X-particle decays into an electron and a positron, which move away from each other at  $v=0.5c$ . After 3 seconds, how long does it take information from the first  $e^-$  to reach the 2<sup>nd</sup>?**

**A spinless, stationary particle X decays into a spin  $\frac{1}{2}$  electron and a spin  $\frac{1}{2}$  positron. Which is true?**

A) The probability of measuring the electron spin-up is always  $\frac{1}{2}$

B) The probability of measuring the positron in the spin-up state after the electron has been measured in the spin-down state is  $\frac{1}{2}$

C) The probability of measuring the positron in the spin-up state after the electron has been measured in the spin-down state is 1

D) NOTA

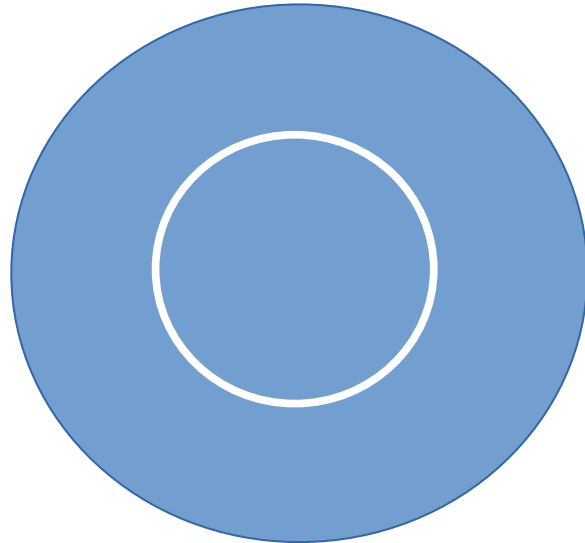
Find the number of yellow ( $\lambda=620$  nm) solar photons passing through a 10 cm x 10 cm sheet in 2 seconds, using  $S_{\text{solar}}=1367$  W/m<sup>2</sup>, the energy per photon  $E=hf=hc/\lambda$ , and using  $1.6 \times 10^{-19}$  Joules per eV ( $hc=1240$  eV-nm)

- A) 45.2
- B)  $9 \times 10^9$
- C)  $8.54 \times 10^{19}$
- D)  $1.71 \times 10^{20}$
- E) NOTA

A beam of electrons is shot out of a gun (in a physics classroom [legal at KU!]) with velocity  $v$ . Comparing the 'phase' at the two points (as with light), what is the relative intensity at  $(x_1, t_1)$  compared to  $(x_2, t_2)$ . **NB: 'wave' oscillates at velocity of light!**

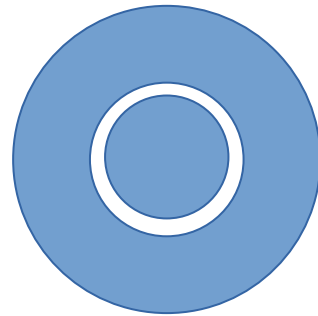
**A small circular aperture is cut into an otherwise solid sheet. If light is shone at the sheet, a point directly behind the aperture will be:**

- A) dark
- B) bright





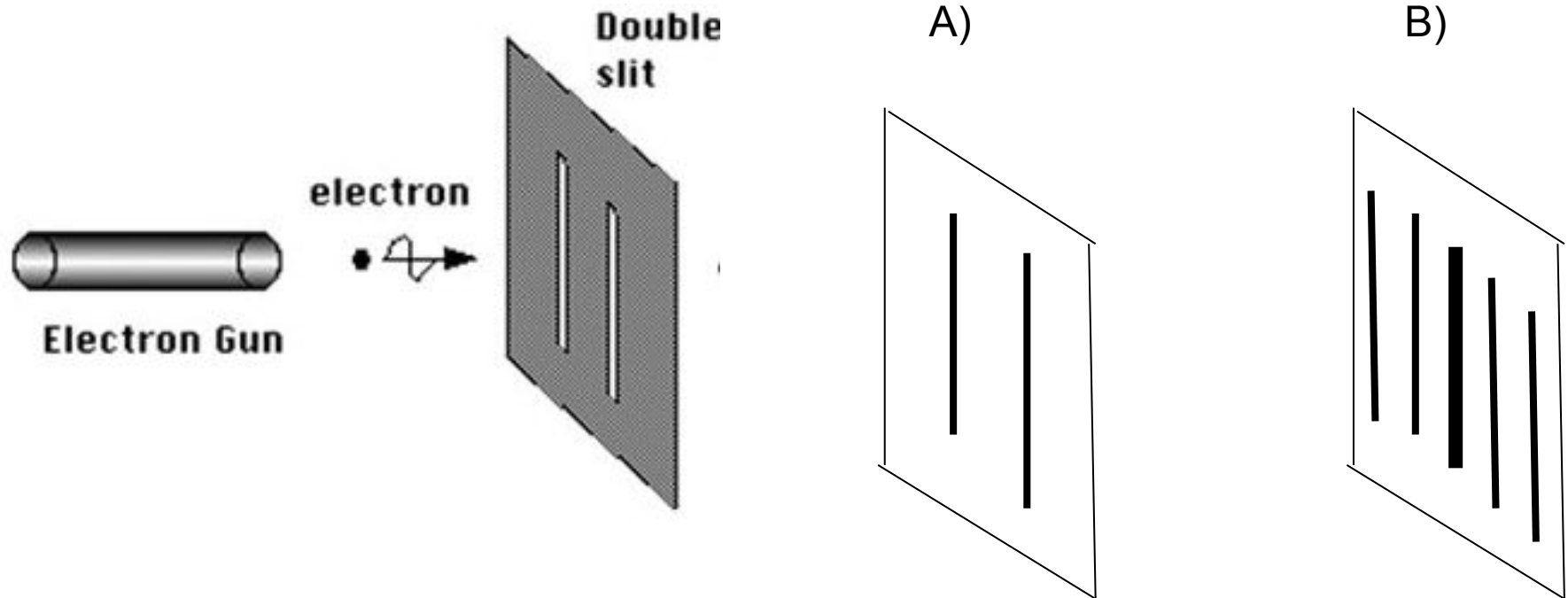
**A small circular aperture is cut into an otherwise solid sheet. A beam of slow electrons is shone at the sheet. A point directly behind the aperture is**



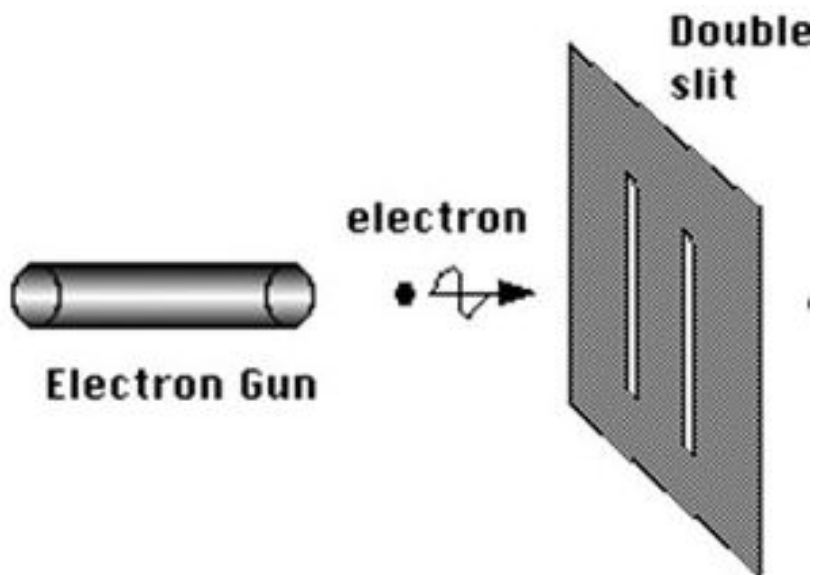
A) dark

B) bright

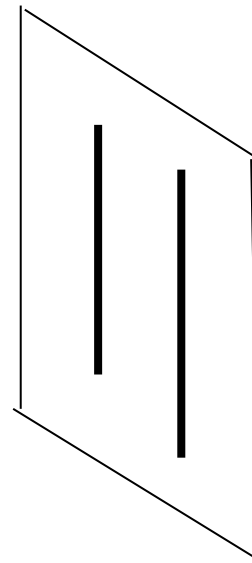
You have a 'gun' that fires high-energy single electrons (such that  $\lambda=h/p$  is comparable to the slit separation) at a double-slit apparatus. The 'gun' fires over a range of angles, so you don't know which slit the electrons go through. Assuming the electron gun is very far from the double-slit screen, which pattern of electron 'hits' would you expect to see on the screen? (Assume the slits are infinitely thin, so the electrons cannot deflect off the walls of the slits themselves)



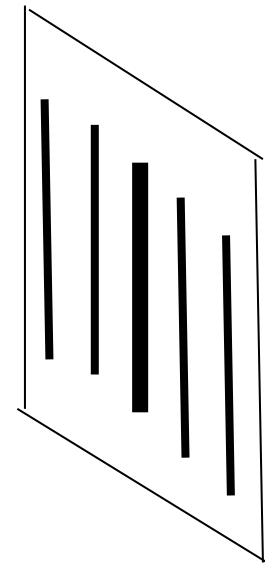
**Baffled, you now decide to illuminate the electron before it gets to the slits to 'see' which slit it went through. Which pattern do you observe now?**



A)



B)



## **Moral of the story:**

**In the first case, the electron exhibits wave characteristics. The single electron 'wave' simultaneously goes through both slits. In the second case, you have 'collapsed the wave function' and forced the electron to behave as a particle.**

**A particle having mass  $3.315 \times 10^{-31}$  kg is shot from a gun with velocity 2 cm/s. What is the wavelength of the wave associated with this particle?**

- A) 1 m
- B) 10 cm
- C) 1 cm
- D) 1 mm
- E) NOTA

**In the previous problem, the  
'frequency' associated with the  
particle 'wave' is:**

A)  $2 \times 10^{-24} \text{ /s}$

B)  $4 \times 10^{-5} \text{ /s}$

C)  $8 \times 10^{-12} \text{ /s}$

D)  $3 \times 10^9 \text{ /s}$

E) NOTA

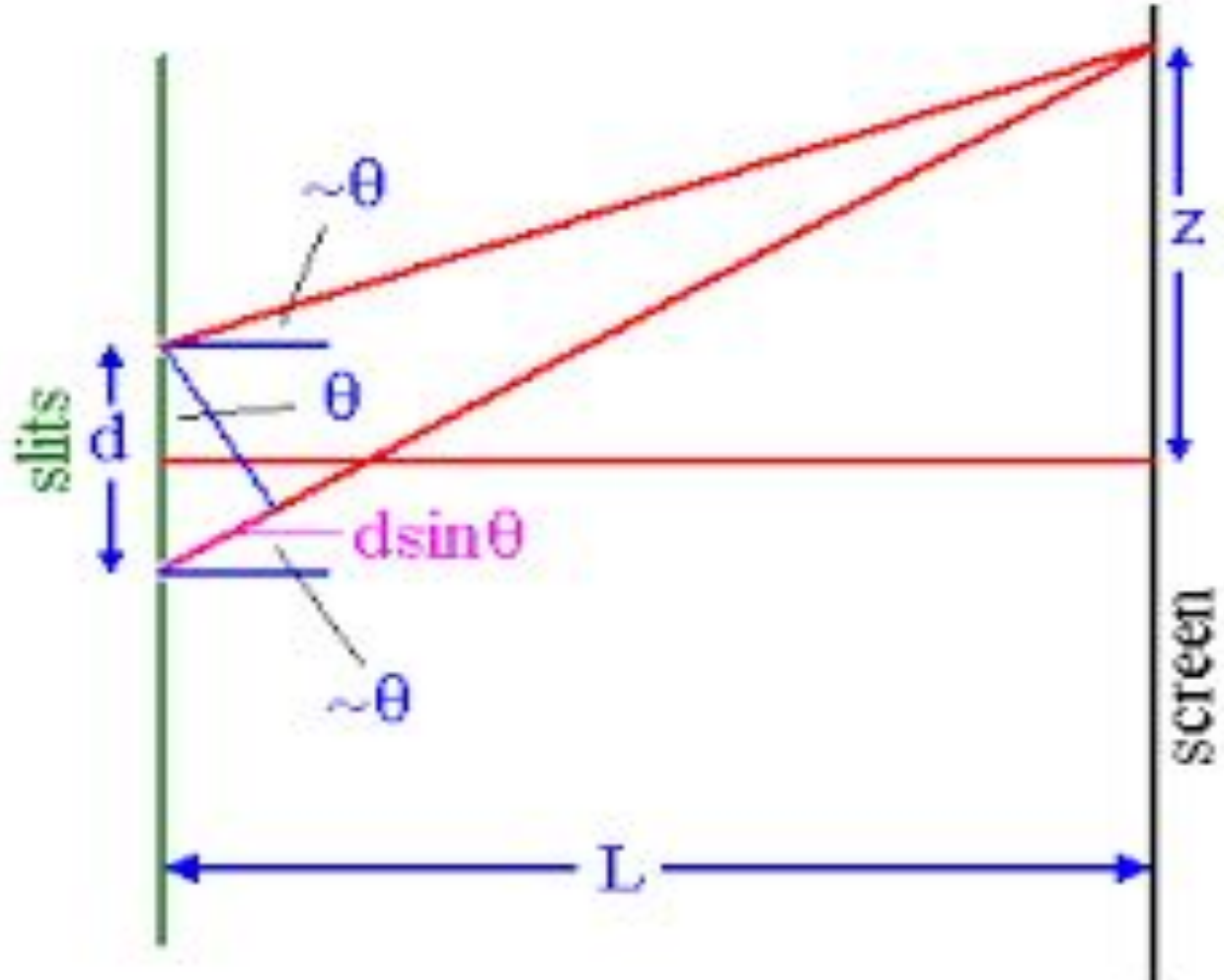
**Does  $f\lambda=v$  for this 'particle-wave'?**

A) No

B) Yes

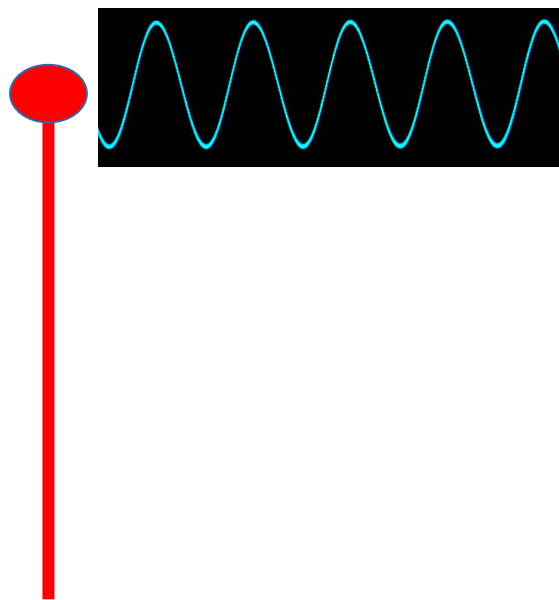
Electrons are “fired” at a double-slit apparatus. Which of the following has the effect of reducing the spacing between electron “intensity” maxima on the screen (recall that  $m\lambda = d\sin\theta$  is the constructive interference condition for waves in phase at the source)?

- A) Increasing electron velocity
- B) Increasing (if possible) electron mass
- C) Increasing spacing between slits
- D) Two of the above
- E) All of the above





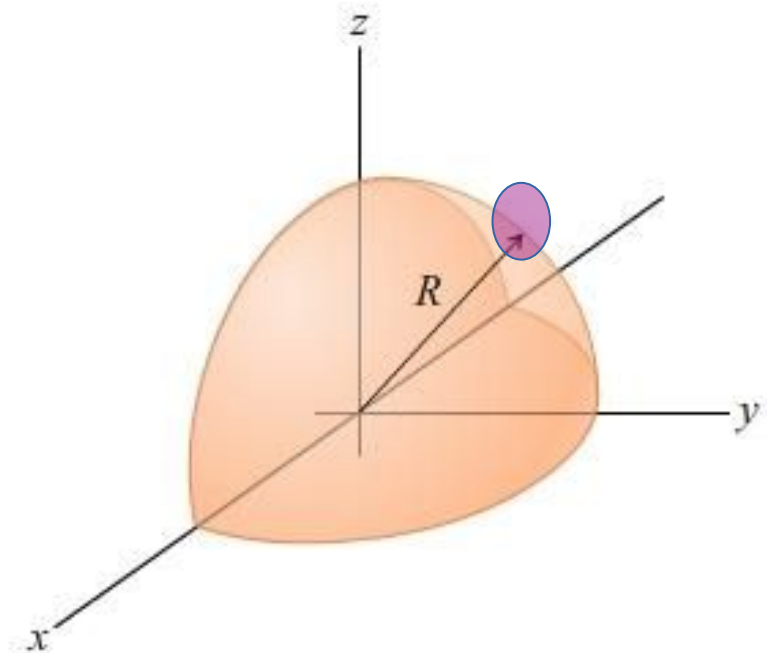
WDZB-FM broadcasts EM waves having  $\lambda = 1.5$  m, with  $P = 40$  kW. 1) (review) Determine the energy intensity ( $[J/s \cdot m^2]$ ) at a distance of 1000 meters from the source antenna, assuming the broadcast power is beamed hemispherically; from that value, now determine a)  $|E_{\text{rad}}|$  using  $S = E_{\text{rad}}^2 / 240\pi$ , and b) the energy passing through a  $0.25 m^2$  surface, over  $t = 8$  s. 2) (new) Now we transition to the "photon" picture: From  $\lambda$ , determine  $f$ , and, from that,  $E$  per photon. Now determine a) the total number of photons intercepted by your surface after 8 seconds, and b) if the surface is a perfect reflector, using the formula for the momentum of each photon, determine the recoil velocity of the surface, if it has a mass of 2 micrograms.



$S = 40000 / \text{Area} = 40000 / (2\pi \cdot 10^6) = 6.37e-3 = E_{\text{rad}}^2 / 240\pi$ ,  
 so  $E_{\text{rad}} = 2.19$  N/C; Energy thru  $0.25 m^2$  in  $8s = 0.0127$  J  
 Photon picture:  $f = 3e8 / 1.5m = 200e6$  Hz;  $E_{\text{photon}} = hf$ ;  
 $h = 6.63e-34$ , so  $E_{\text{photon}} = 1.33e-25$  J. To get to  $0.0127$  J,  
 we need  $0.0127 J / 1.33e-25 J = 9.6e22$  photons.  
 Momentum of each photon  $= E_{\text{photon}} / c$ , so total  
 momentum transferred to 2 microgram  
 surface  $= (0.0127 J / 3e8) \times 2$  (since perfect reflector) =  
 $4.23e-11$  kg-m/s  $= m_{\text{surface}} v_{\text{recoil, surface}}$ , using  $m_{\text{surface}} = 2e-6$   
 kg gives  $v_{\text{recoil}} = 4.23e-5$  m/s

WDZB-FM broadcasts EM waves at a  $f=1200$  MHz, with an output power of  $200$  W into one-quarter of a full sphere (as shown). At  $R=60$  m, the energy intensity  $[W/m^2]$ ,  $E_{\text{rad}}$  magnitude, and total energy passing through a disk of radius  $2$  m, in a time  $t=3$  seconds, are:

- A)  $0.22 \text{ W/m}^2$ ,  $4.77 \text{ N/C}$ ,  $0.667 \text{ J}$
- B)  $0.0044 \text{ W/m}^2$ ,  $4.77 \text{ N/C}$ ,  $0.333$
- C)  $0.055 \text{ W/m}^2$ ,  $4.77 \text{ N/C}$ ,  $0.30 \text{ J}$
- D)  $0.017 \text{ W/m}^2$ ,  $3.65 \text{ N/C}$ ,  $0.667 \text{ J}$
- E) NOTA



**The total number of photons  
passing through the surface, in the  
same time, is:**

A)  $8.38 \times 10^{23}$

B)  $4.1 \times 10^{24}$

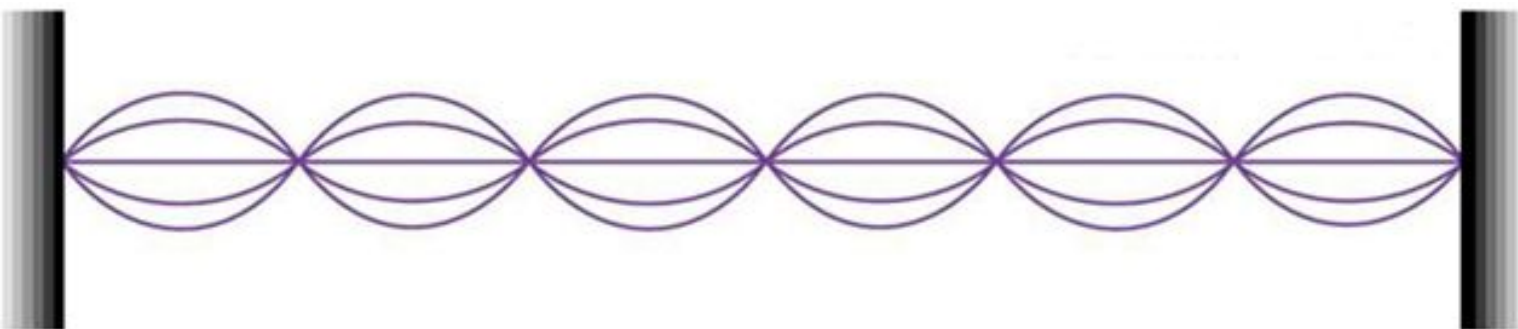
C)  $4.1 \times 10^7$

D) 589.1

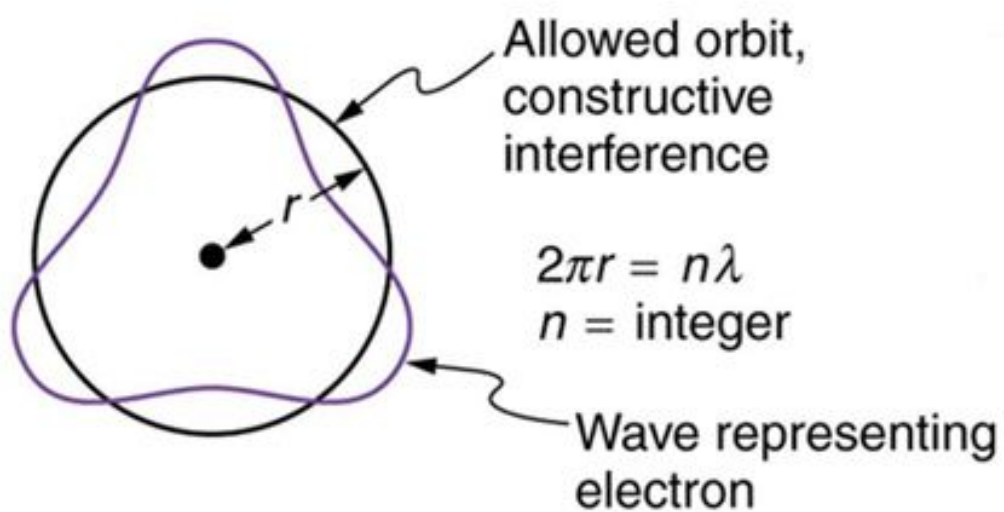
E) NOTA

**Assuming the disk is a perfect reflector having mass 10 nanograms, the recoil velocity of the disk is:**

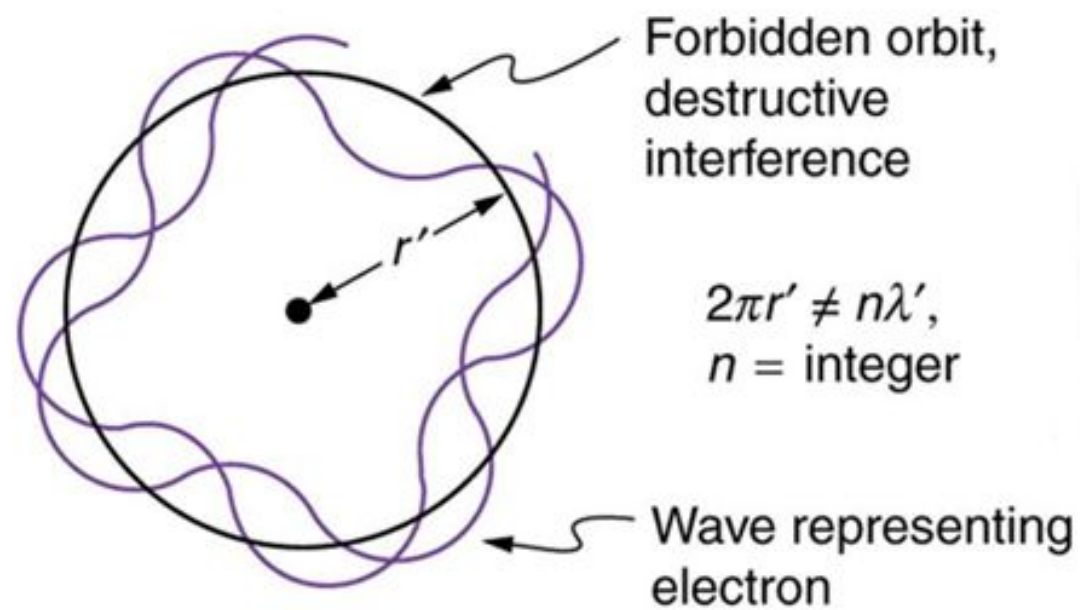
- A) 44.43 m/s
- B) 4.443 m/s
- C) 0.444 m/s
- D) 0.044 m/s
- E) NOTA



(a)

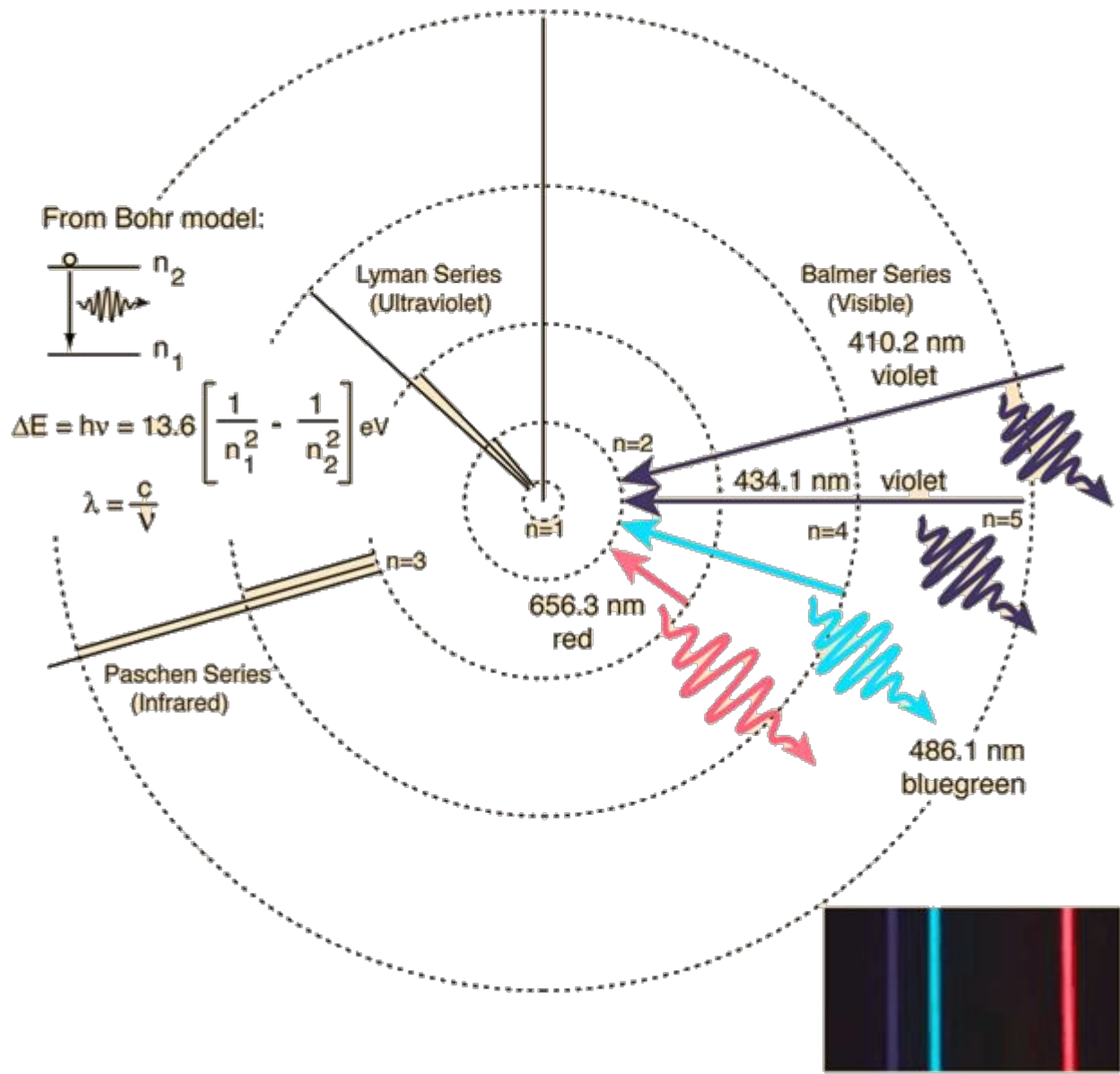


(b)



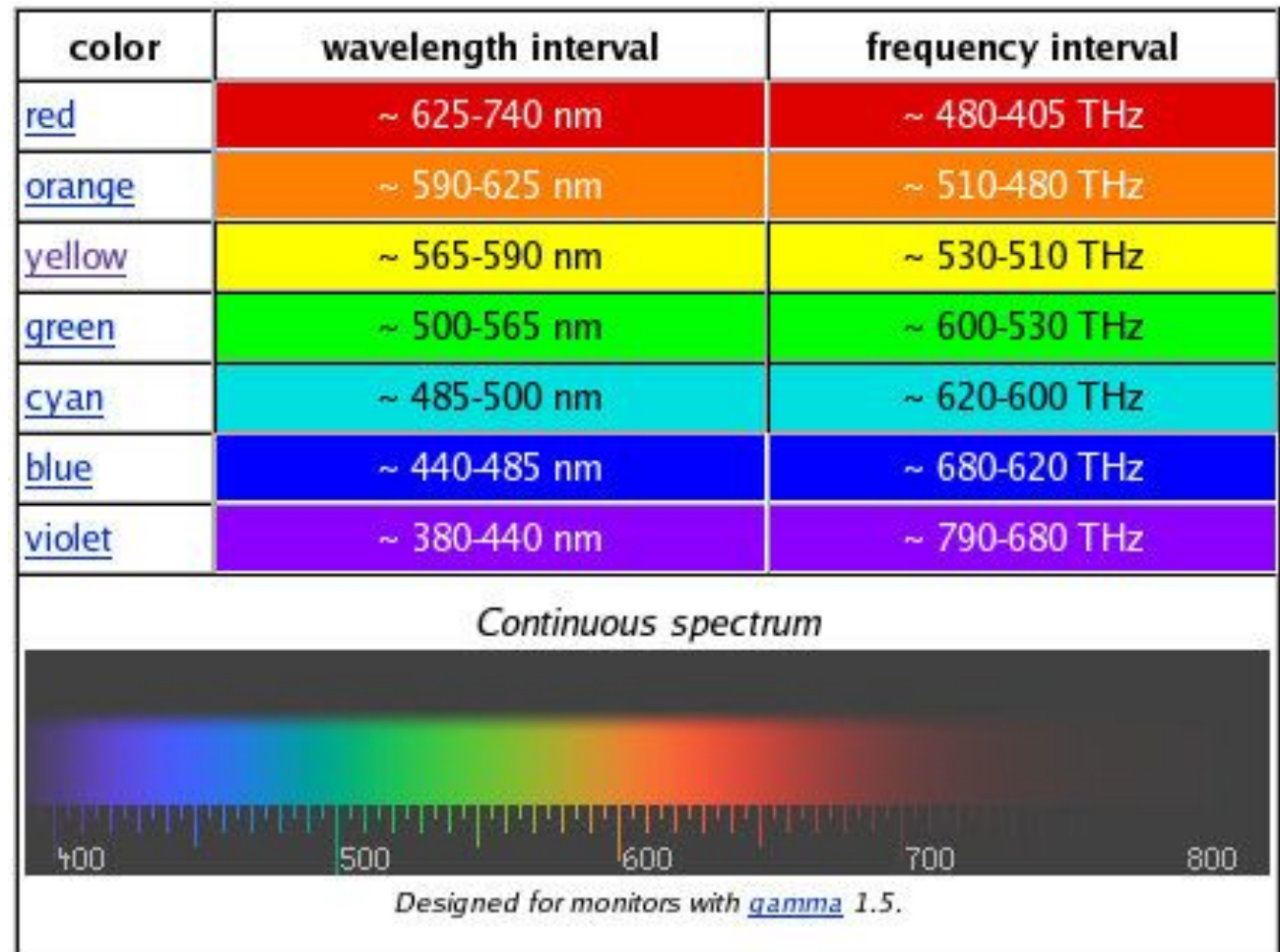
(c)

# Pictorial summary of Bohr model



**The wavelength of light emitted in the  $n=3 \rightarrow n=2$  Hydrogen transition is:**

- A) 121 nm
- B) 656 nm
- C) 102 nm
- D) 456 nm
- E) NOTA



**Virial Theorem: For a particle  
stuck in an attractive, circular  
orbit:**

$$kq^2/r^2 = mv^2/r \Rightarrow$$

$$kq^2/r = mv^2 \Rightarrow (V = kq/r):$$

$$qV = mv^2$$

$$|PE| = 2KE$$



**A particle with  $m=1.657 \times 10^{-26}$  kg and  $v=4 \times 10^4$  m/s moves in a circular orbit. The smallest radius  $r$  such that the orbit is 'stable' is:**

- A) 0.01 cm
- B)  $1.59 \times 10^{-13}$  m
- C)  $2.4 \times 10^{-31}$  m
- D)  $1.6 \times 10^{-19}$  m
- E) NOTA

If the particle's mass were increased (but velocity unchanged), then the smallest stable orbital radius, relative to the previous case, would be:

- A) larger
- B) smaller
- C) unchanged

**Light of wavelength 248 nm strikes a metal plate with  $W_0=3$  eV. The KE with which the ejected  $e^-$  strikes a metal bar held at  $V=+8V$  is:**

- A) 5 eV
- B) 3 eV
- C) 10 eV
- D) 13 eV
- E) NOTA

**In the previous problem, which would increase the KE of the  $e^-$  striking the bar?**

- A)  $W_o$  increased
- B) Increasing  $\lambda$
- C) Increasing  $V_o$
- D) Two of the above
- E) NOTA

**From the velocity solved for (and the implied acceleration) in the previous problem, what is  $E_{\text{rad}}$  at 1 meter?**

**From  $E_{\text{rad}}$  in the previous problem,  
what is the energy intercepted by a 1  
cm x 1 cm square at 1 m?**

**From the solution to the previous problem, what is the total power radiated by the electron?**

**From the output power in the previous problem, what is the lifetime of an electron orbiting Hydrogen?**

**Alternately, calculate the total power directly from the Larmor formula for a non-relativistic charge accelerating at**  
**a:  $P = (2/3)kq^2a^2/c^3$**



In this CAP, we'll work our way backwards from an "observation" of a spectral line due to electron transitions from the  $n=2 \rightarrow n=1$  state to deduce the nuclear charge of the atoms, whose transitions are being observed, using the "virial theorem", combined with several equations from 114 & 115. Here goes:

In your capacity as the Astronomer Extraordinaire of the Royal Astronomical Association of Kansas, and desperate for a sensational observation to save your organization from the chopping block of fiscal responsibility (" "), you have devoted yourself to searching in the skies for something/anything to grab headlines. One night, you observe, in a scan of the sky, a spectral line having wavelength  $X$  nanometers ( $X < 200$  nm). You realize that you are observing a transition from the 2S state to the 1S state of electrons in a possibly unobserved atom; and set yourself to the task of identifying the  $Z$ -value of that atom, as follows:

- a) You first convert the observed wavelength into an energy, in electron Volts of the photon you've observed.
- b) Now you use the fact that, since the energy of your observed photon is  $3/4$  the magnitude of the energy of the "ground state" of the electron, you determine the magnitude of the ground state energy.
- c) Now you invoke the virial theorem, which tells you that the kinetic energy of the electron is the same as the ground state energy magnitude you just determined, which allows you, knowing the velocity of the electron in the ground state.
- d) knowing the velocity, you use Bohr's principle that the ground-state angular momentum is equal to  $nh$ , with  $n=1$  in the ground state, to determine the distance between the electron and the nucleus  $r$ .
- e) Finally, using the virial theorem once more, and knowing that the magnitude of the potential energy is twice the magnitude of the kinetic energy, you determine the charge of the nucleus.

Pick 3S energy level for Beryllium ( $Z=4$ ): need:  $mv=h/\lambda$ ,  $2\pi r=3\lambda=3(h/mv)$  and  $kq(4q)/r^2=mv^2/r$   
 Solve for  $r=3h/(2\pi mv)$ ; rewrite  $4kq^2/r=mv^2$  and now plug  $r=3h/(2\pi mv)$  and solve for  
 $Mv^2=(4kq^2)*(2\pi mv)/3h$ ;  $m$  cancels, so  $v=(4kq^2)(2\pi)/3h$ ;  $q=1.6e-19$  C,  $h=6.634e-34$  J-s,  $k=9e9$ ,  
 so  $v=2.91 \times 10^6$  m/s. This is about 1% of the speed of light,  $r=3h/(2\pi mv)=1.19e-10$  m

Can now calculate KE ( $\frac{1}{2}mv^2$ ), with  $m=9.11 \times 10^{-31}$  kg for electron (NOT proton!) and PE  
 $(kq_e * 4q_p / r)$ ;

Note that PE is negative since  $q_e < 0$  and  $4q_p > 0$ , find:

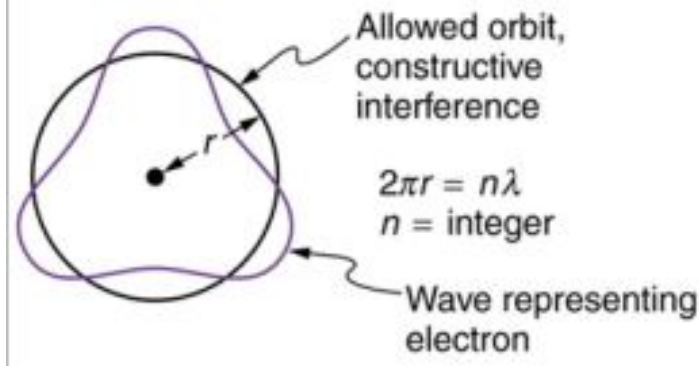
PE(e-) =  $-7.74e-18$  J (using  $1eV=1.6e-19$  J, this is equal to  $-47.62$  eV)

KE(e-) =  $3.87e-18$  J or  $23.81$  eV; total  $E=PE+KE=-23.81$  eV

Note that Virial Theorem checks out!  $|PE|=2KE$

Now consider transitions – we'll take  $n=3$  state to  $n=1$  state. We know that the energy levels go as  
 $E_n = E_{n=1}/n^2$ ; here we have  $-23.81$  eV =  $E_{n=1}/9$ , so  $E_{n=1} = -214.29$  eV.  $\Delta E$  magnitude =  $190.48$  eV. What  
 is the wavelength of this photon transition?  $hc/\lambda = E_y = 190.48$  eV;  $hc=1240$  nm-eV, so  $\lambda=6.51$  nm  
 (ultra-violet)

Note that for hydrogen,  $E_{n=1} = -13.6$  eV. If we've increased the nuclear charge by a factor of 4, why  
 is the  $n=1$  energy so much larger (in magnitude) than that factor of 4? Reason is that the electron  
 is pulled closer to the nucleus, so i  
 ger magnitude of (negative)  
 PE.



**Which of the following are the equations needed to solve for  $r$ ,  $v$ , and  $E$  of an electron (charge= $q$ ) in the 2<sup>nd</sup> energy level of Helium**

- A)  $m_e v^2/r = kq^2/r^2$ ;  $2\pi r = \lambda$ ;  $mv = h/\lambda$ ;
- B)  $m_e v^2/r = k(2q^2)/r^2$ ;  $2\pi r = 2\lambda$ ;  $mv = h/\lambda$ ;
- C)  $m_e v^2/r = k(4q^2)/r^2$ ;  $2\pi r = \lambda$ ;  $mv = h/\lambda$ ;
- D) NOTA

**An electron in the  $n=2$  level of Hydrogen absorbs a photon of wavelength 200 nm. Determine the electron's KE after ejection from the Hydrogen, using  $E_n = -13.6 \text{ eV}/n^2$  and  $E_y = hc/\lambda$ , with  $hc = 1240 \text{ nm-eV}$**

- A) 6.044 eV
- B) 3.4 eV
- C) 13.6 eV
- D) 2.8 eV
- E) NOTA

**If the same photon were incident on an electron in the 2S state of Helium, the KE of the electron would be**

- A) Smaller
- B) Higher
- C) The same

**The wavelength of a photon emitted in an  $n=3 \rightarrow n=2$  transition in Hydrogen is:**

- A) 82 nm
- B) 164 nm
- C) 328 nm
- D) 656 nm
- E) NOTA

**You are told that the total energy (KE+PE) of an electron in the 3S level of Helium is -6.04 eV. Given that, the potential energy of the electron in the 1S level is**

- A) -54.4 eV
- B) -108.8 eV
- C) -27.2 eV
- D) 54.4 eV
- E) NOTA

**Given that the ground state energy of Hydrogen has  $E_{\text{tot}} = -13.6 \text{ eV}$ . Using the standard  $E_n = -13.6 \text{ eV} \cdot Z^2/n^2$ , the  $n=2$  potential energy of Lithium is:**

- A)  $-13.6 \text{ eV}$
- B)  $-27.2 \text{ eV}$
- C)  $-54.4 \text{ eV}$
- D)  $-61.2 \text{ eV}$
- E) NOTA



**Given that  $KE = p^2/2m$ ; and  $p = h/\lambda$ , comparing the  $n=1S$  state of Lithium to the  $n=2S$  state implies that the radius of the  $n=1$  state is**

- A) The same
- B) Twice as large
- C) Twice as small
- D) Four times as small
- E) NOTA

**An electron in hydrogen orbits the proton at  $r=0.529 \times 10^{-10}$  m. Using the fact that  $E_n = -13.6\text{eV}/n^2$ , what is its velocity?**

- A)  $3 \times 10^8$  m/s
- B)  $2.19 \times 10^8$  m/s
- C)  $2.19 \times 10^6$  m/s
- D)  $1.4 \times 10^4$  m/s
- E) NOTA

**An electron in the  $n=6$  level of Helium absorbs a photon of wavelength 100 nm. Determine the electron's KE after ejection from the Helium, after it accelerates through a voltage of -2 V**

A)

**Setting centripetal force ( $mv^2/r$ ) equal to the Coulomb (electrostatic) force between two charges, and realizing that  $PE=q(\Delta V)$ , you can derive that the PE of a 1s electron is related to the KE of that electron via:**

- A)  $PE=KE$
- B)  $PE=-KE$
- C)  $PE=-2KE$
- D)  $2PE=KE$
- E) NOTA

**A 1s electron ( $m=9.11 \times 10^{-31}$  kg,  $q=1.6 \times 10^{-19}$  C) in hydrogen orbits the proton at  $r=0.529 \times 10^{-10}$  m, with a velocity of  $2.19 \times 10^6$  m/s.  
What is its KE and PE, respectively?  
(recall that  $\Delta PE=q\Delta V$  and  $1.6 \times 10^{-19}$  J/eV)**

- A) 13.6 eV, 13.6 eV
- B) 13.6 eV, -27.2 eV
- C) 27.2 eV, -27.2 eV
- D) 27.2 eV, 13.6 eV
- E) NOTA

**The ratio of velocity of an electron in the 3s orbital, compared to the 1s orbital is:**

- A) 1
- B)  $1/3$
- C)  $1/9$
- D) 3
- E) NOTA

**In going from Hydrogen to Helium,  
the 1s electron has:**

- A) Higher velocity
- B) Higher PE
- C) Higher KE
- D) Smaller radius
- E) Two of the above

**Given that the mass of Venus is about the same as the mass of Earth, and based on the similarity b/w gravitational and electrical systems, you can conclude that:**

- A)  $PE_{\text{Venus}} < PE_{\text{Earth}}$
- B)  $KE_{\text{Venus}} > KE_{\text{Earth}}$
- C)  $E_{\text{tot,Venus}} < E_{\text{tot,Earth}}$
- D) Two of the above are true
- E) All of the above are true

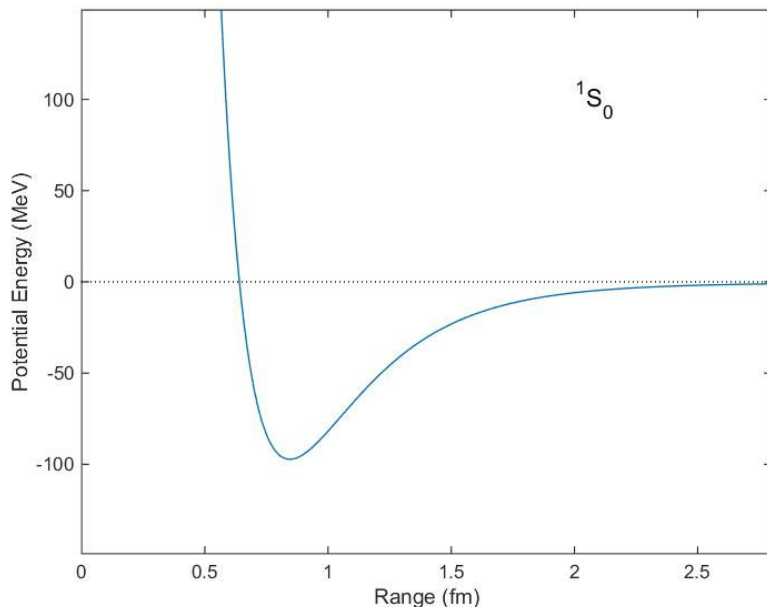


**Which of the following have the effect of increasing the energy of a photon emitted in the  $n=2 \rightarrow n=1$  transition in Hydrogen?**

- A) Increasing the electron mass
- B) Increasing the proton mass
- C) Increasing the electron charge
- D) Increasing the proton charge
- E) Two (or more) of the above

## Nuclear Physics:

System moving from high PE to low PE releases energy: Einstein-KE is frame-dependent (consider two putty pieces colliding wrt 'fixed' observer vs. observer on a train.) Fix it by stating that some small part of energy is equal to mass: Particles with smaller PE also have smaller mass!



Suppose two masses start at rest and fall into a 'well'. Phsx114:  $KE(0) + PE(0) = \text{heat}(+) + PE(-)$

$$E(\text{total}) = m(\text{total})c^2 = m_0c^2 + PE + KE;$$

$m_0c^2$  = 'rest-mass energy' of isolated particle

**Two puzzles: nuclei are stable despite proton-proton repulsion, and free neutron decays in 10 minutes, while neutron inside nucleus is stable.**

Significance of equivalence of mass and energy: MASS changes when an object is in a potential field!

$$E(\text{total}) = \text{rest-mass energy} + \text{PE} + \text{KE} = m_0 c^2$$

E.g., deuteron, at infity, starting/ending at rest:

$$m_{p,0} c^2 + m_{n,0} c^2 + \text{PE}_i + \text{KE}_i = m_{p,\text{in nucleus}} c^2 + m_{n,\text{in nucleus}} c^2 + \text{PE} + \text{heat} + m_{pn} c^2 + \text{heat}_{\text{Ext}};$$

$$\text{PE}_i = \text{KE}_i = \text{KE}_f = 0; \text{ i.e. } \Delta \text{PE}_f = \Delta(m c^2)$$

Alternately, can consider from the standpoint of starting with two particles, very far apart:

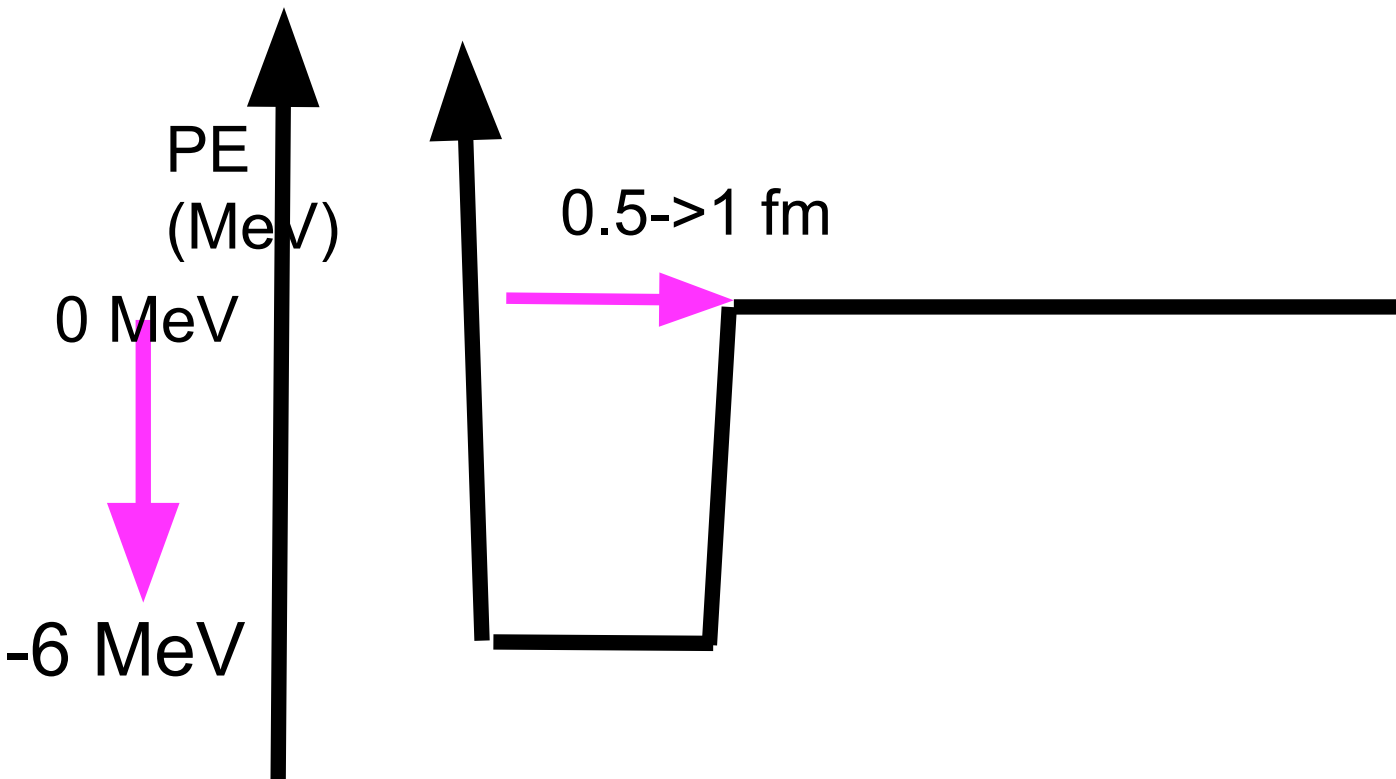
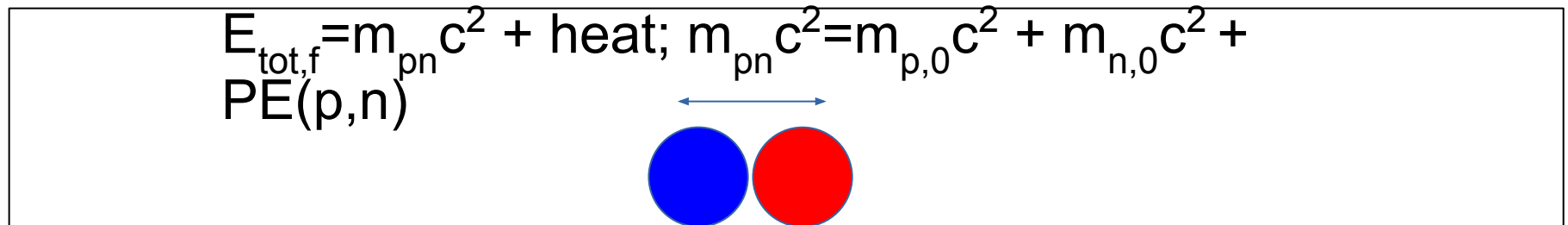
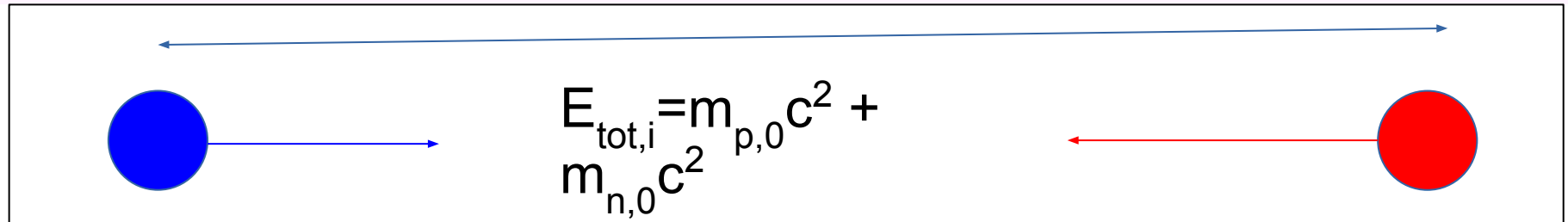
$m_p + m_n = m_{pn} + \text{heat}$ ; the heat produced is equivalently the depth of the potential energy well, as well as the amount of heat produced in the interaction.

Caution 1: the pn-potential is more than just strong force;

caution 2:  $m_{\text{deuteron}}$  is not equal to  $m_{\text{diatomic hydrogen}}$

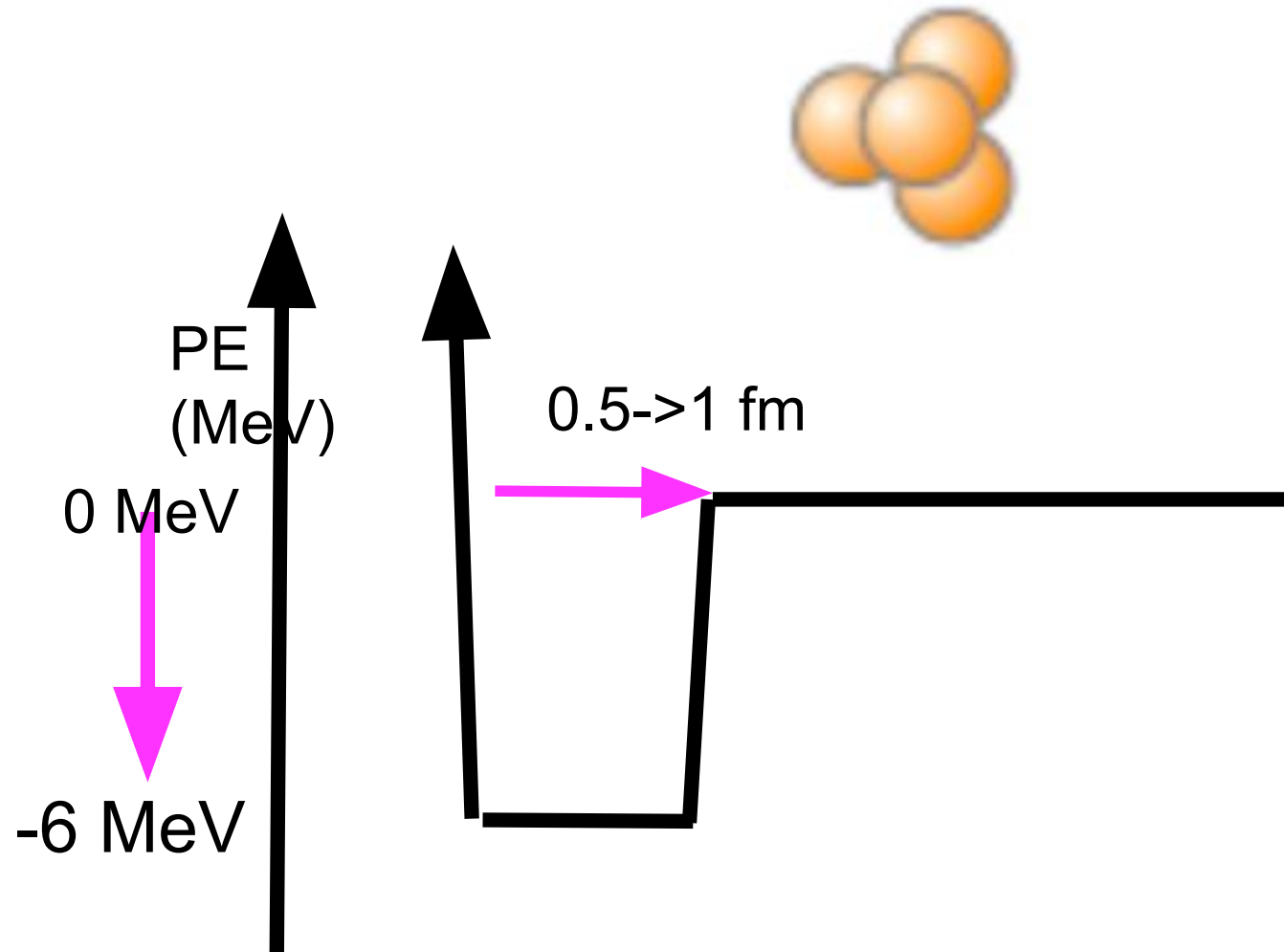
The more negative the PE: 1) the more tightly bound the nucleus, and 2) the smaller the mass per nucleon

# Step-by-step formation of a deuteron (pn)



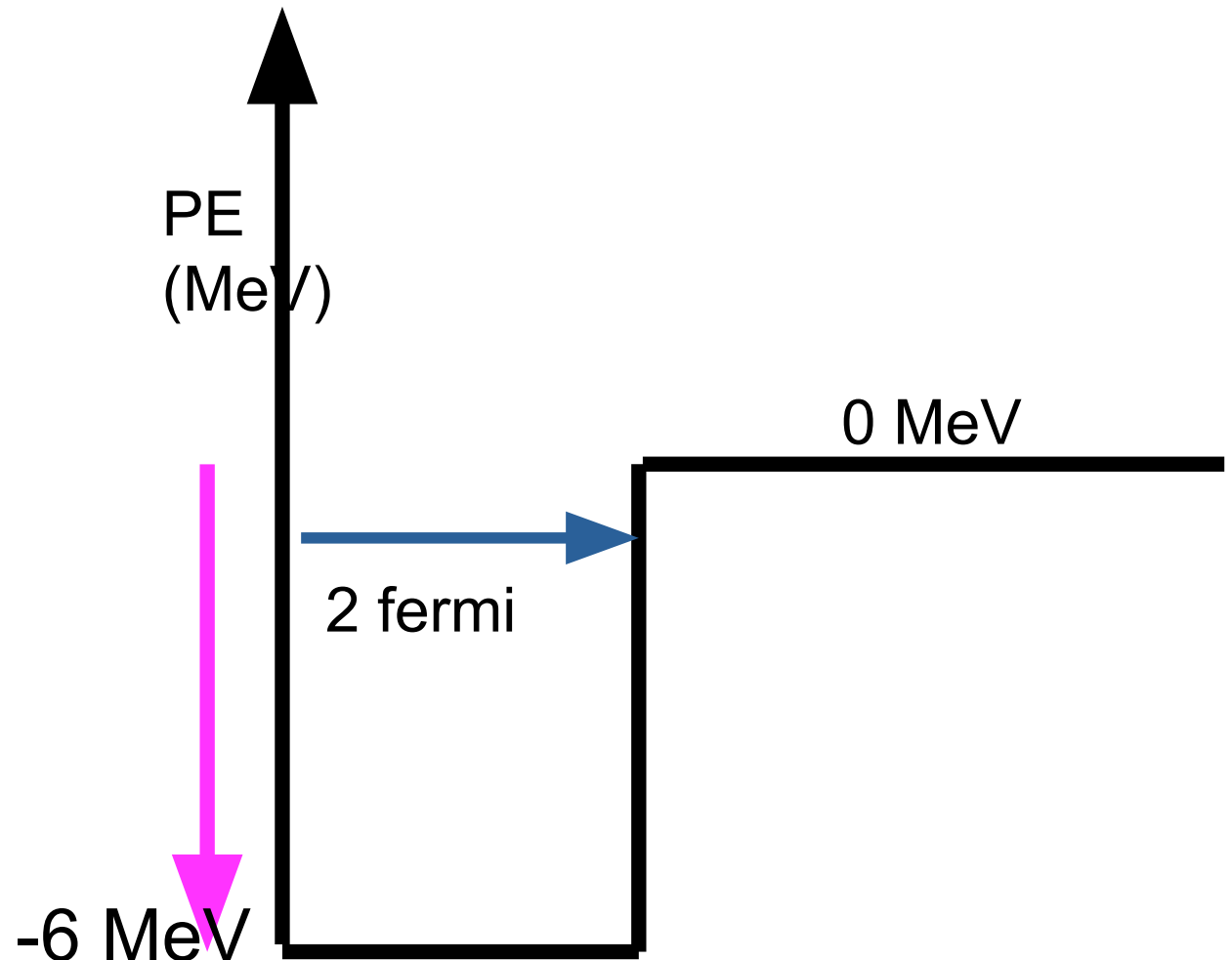
Given the PE “well” shown, and using  $m_{\text{proton}} = 938 \text{ MeV}/c^2$ ,  $m_{\text{neutron}} = 939 \text{ MeV}/c^2$ , with each having a diameter just less than 1 fm, what is the mass of a Helium nucleus (neglecting the electromagnetic force, and using a 3-dimensional model)

- A)  $3718 \text{ MeV}/c^2$
- B)  $3736 \text{ MeV}/c^2$
- C)  $3754 \text{ MeV}/c^2$
- D)  $3770 \text{ MeV}/c^2$
- E) NOTA



Given the PE curve shown, if the 'range' of the strong force is 2 fm, then the maximum number of 1 fm diameter nucleons such that they all interact in 3d is:

- A) 4
- B) 10
- C) 22
- D) 34
- E) NOTA

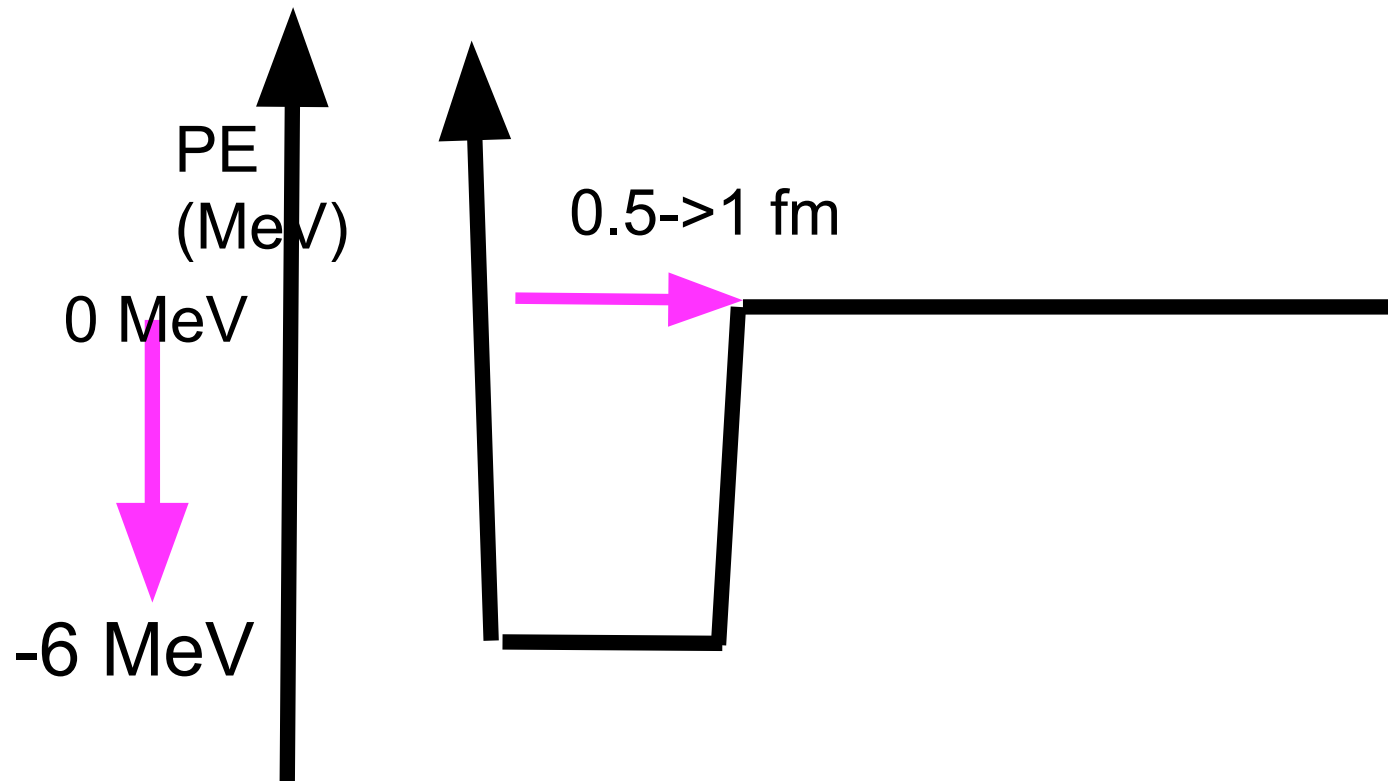


**If we now add an electromagnetic force,  
with  $PE=+1 \text{ MeV/r[fm]}$ , then the most  
likely 1-dim Li configuration (in 1d) is:**

- A) pppnnnn
- B) pnpnnpn
- C) pnpnnp
- D) NOTA

If the range were 1.5 fm, then, in a simplified, one-dimensional model, accounting for the strong nuclear force only, with diameter(p)=1 fm,  $m_p=938 \text{ MeV}/c^2$  and  $m_n=939 \text{ MeV}/c^2$ , the mass of  ${}^6_3\text{Li}$  would be:

- A)  $5631 \text{ MeV}/c^2$
- B)  $5583 \text{ MeV}/c^2$
- C)  $5577 \text{ MeV}/c^2$
- D)  $5541 \text{ MeV}/c^2$
- E) NOTA





**If we now add an electromagnetic force, with  $PE=+1.44 \text{ MeV}/r[\text{fm}]$ , then the previous answer becomes:**

- A) Smaller by 0.7 MeV
- B) Larger by 0.7 MeV
- C) Larger by 1.5 MeV
- D) Smaller by 1.5 MeV
- E) NOTA

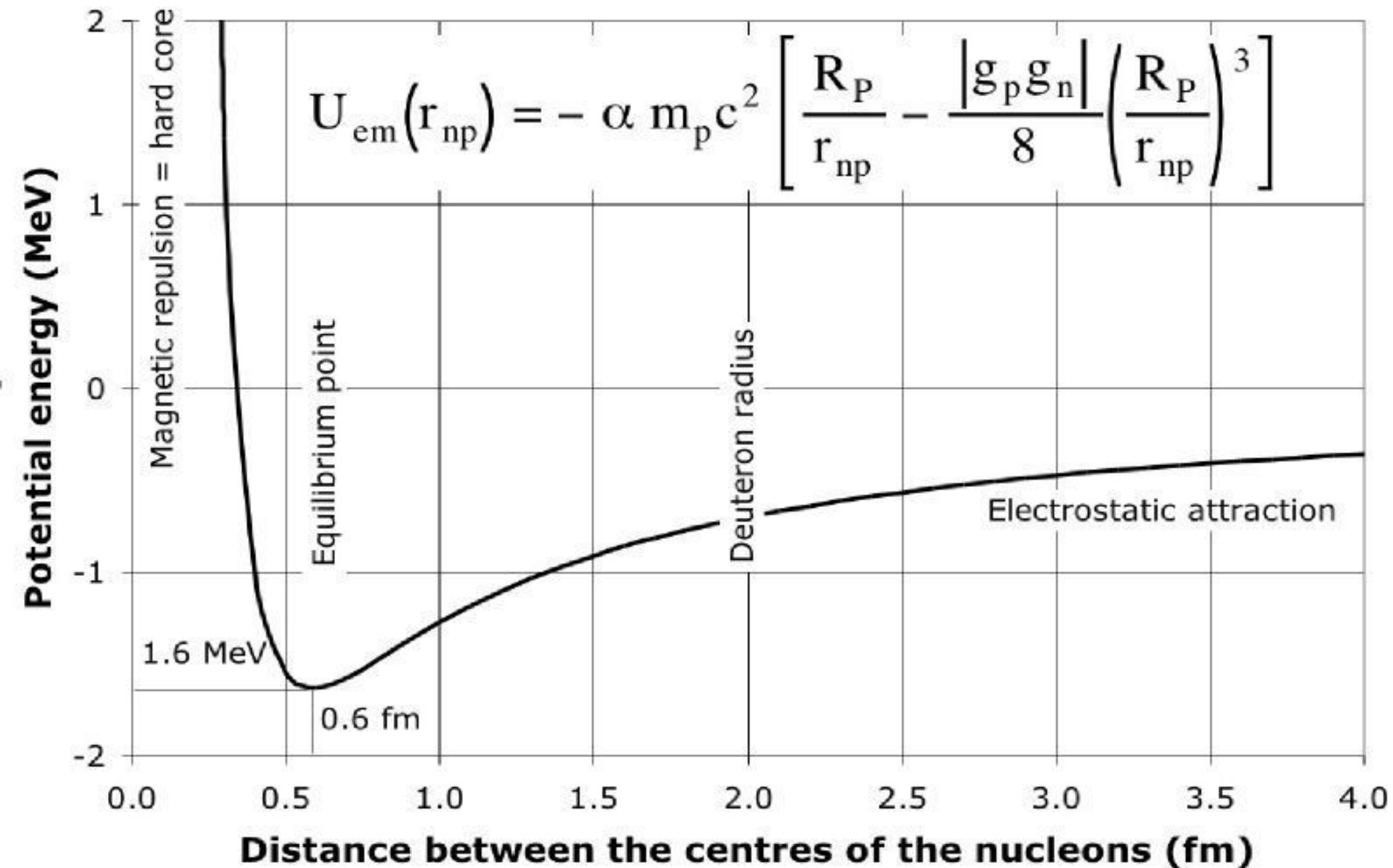
**If the PE range were doubled, the number of nucleons that would fit in 3-dimensions would grow by a factor:**

- A) 1
- B) 2
- C) 4
- D) 8
- E) NOTA

**The true answer to the previous problem for the entire Helium atom, taking into account all the electrical force effects, is:**

- A) Smaller
- B) Larger
- C) The same

# Deuteron electromagnetic nuclear potential

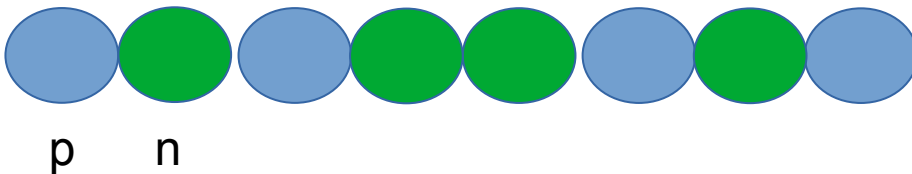
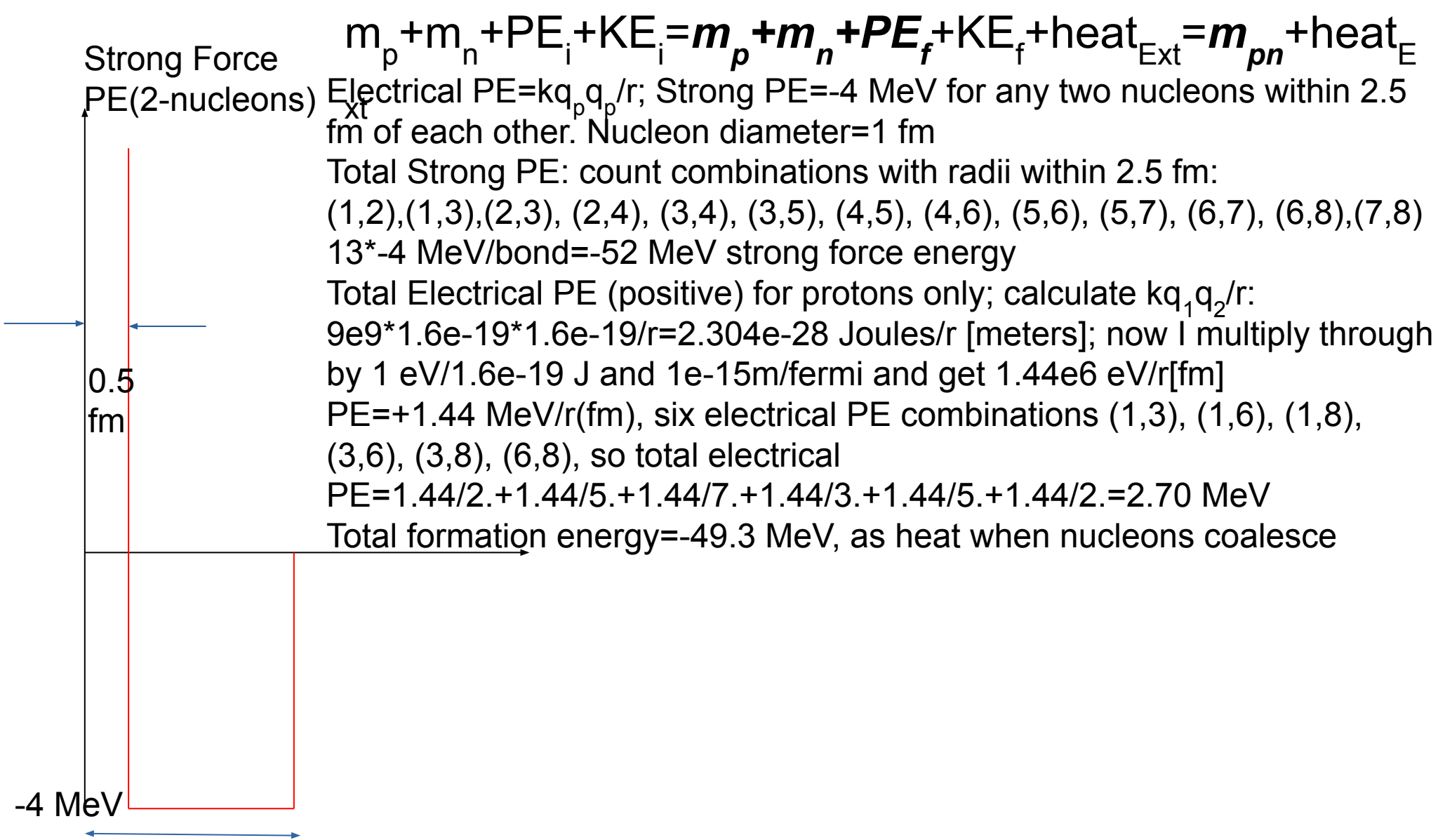


Given that the 'range' of the strong nuclear force=2.1 fm, the PE “depth” is -3 MeV, and the diameter of one nucleon is 1 fm, how much E is released (neglecting the electrical force) when 3 protons and 3 neutrons fuse, in one dimension to form Lithium (assume **pnpnpn** )?

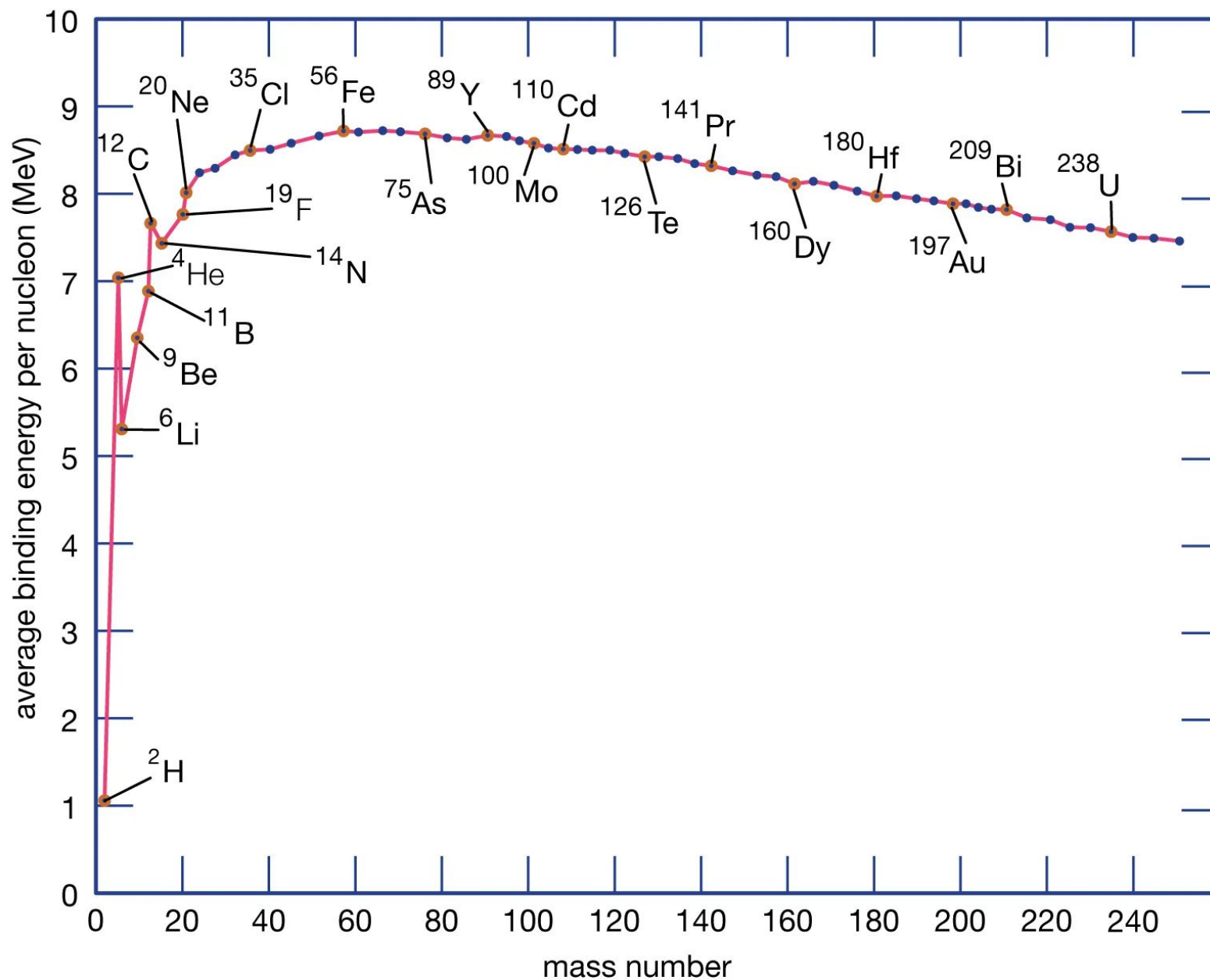
- A) 25.25 MeV
- B) 27 MeV
- C) 36 MeV
- D) 25.5 MeV
- E) NOTA

**The binding energy, in 3 dimensions  
would be:**

- A) Smaller  
than in 1-d
- B) Larger than  
in 1-d
- C) The same

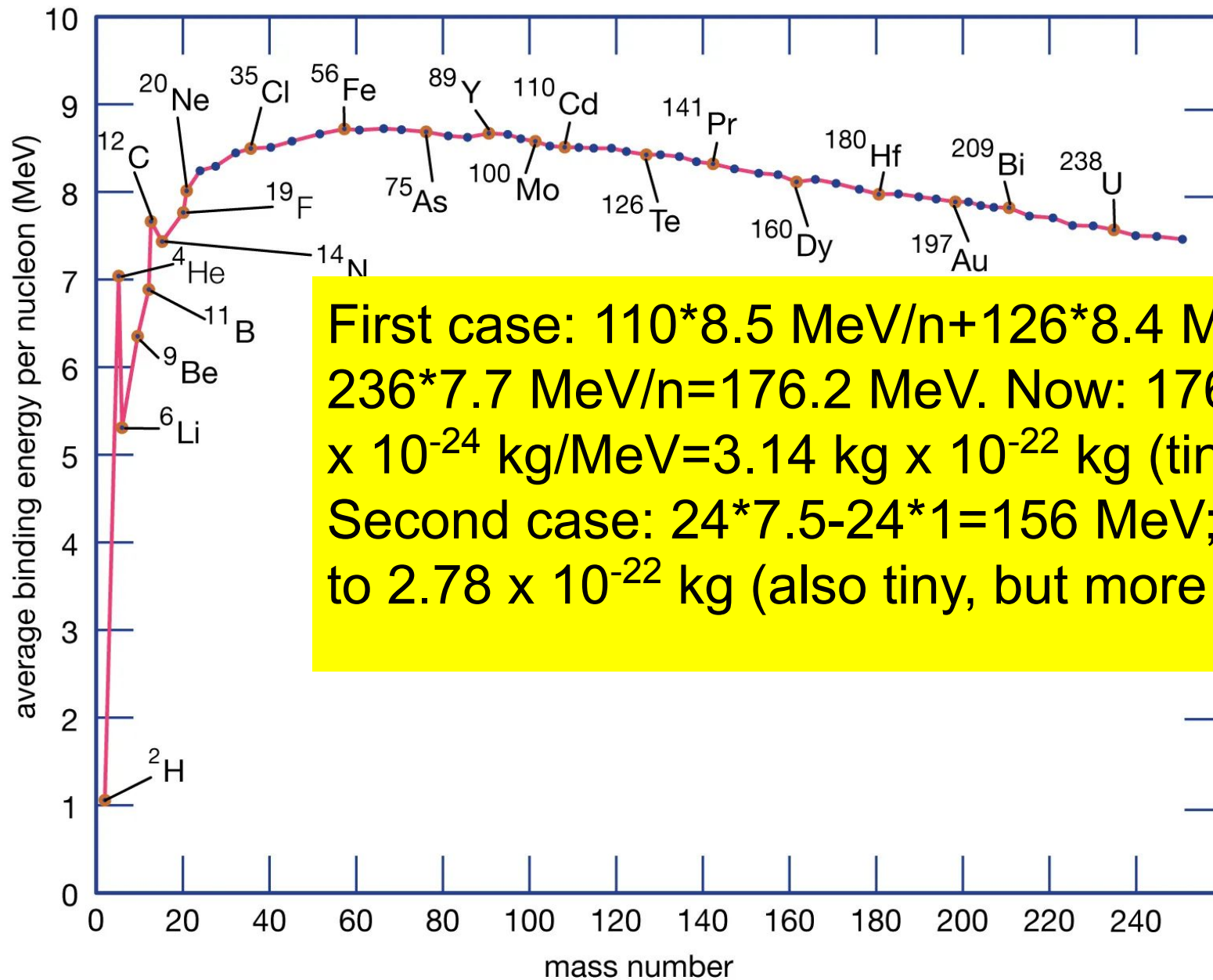


determine the mass (in kg) of  $^{236}\text{U}$  lost via  $^{238}\text{U} \rightarrow ^{126}\text{Te} + ^{110}\text{Cd}$ , and also  $12^2\text{H} \rightarrow 2^{12}\text{C}$ .

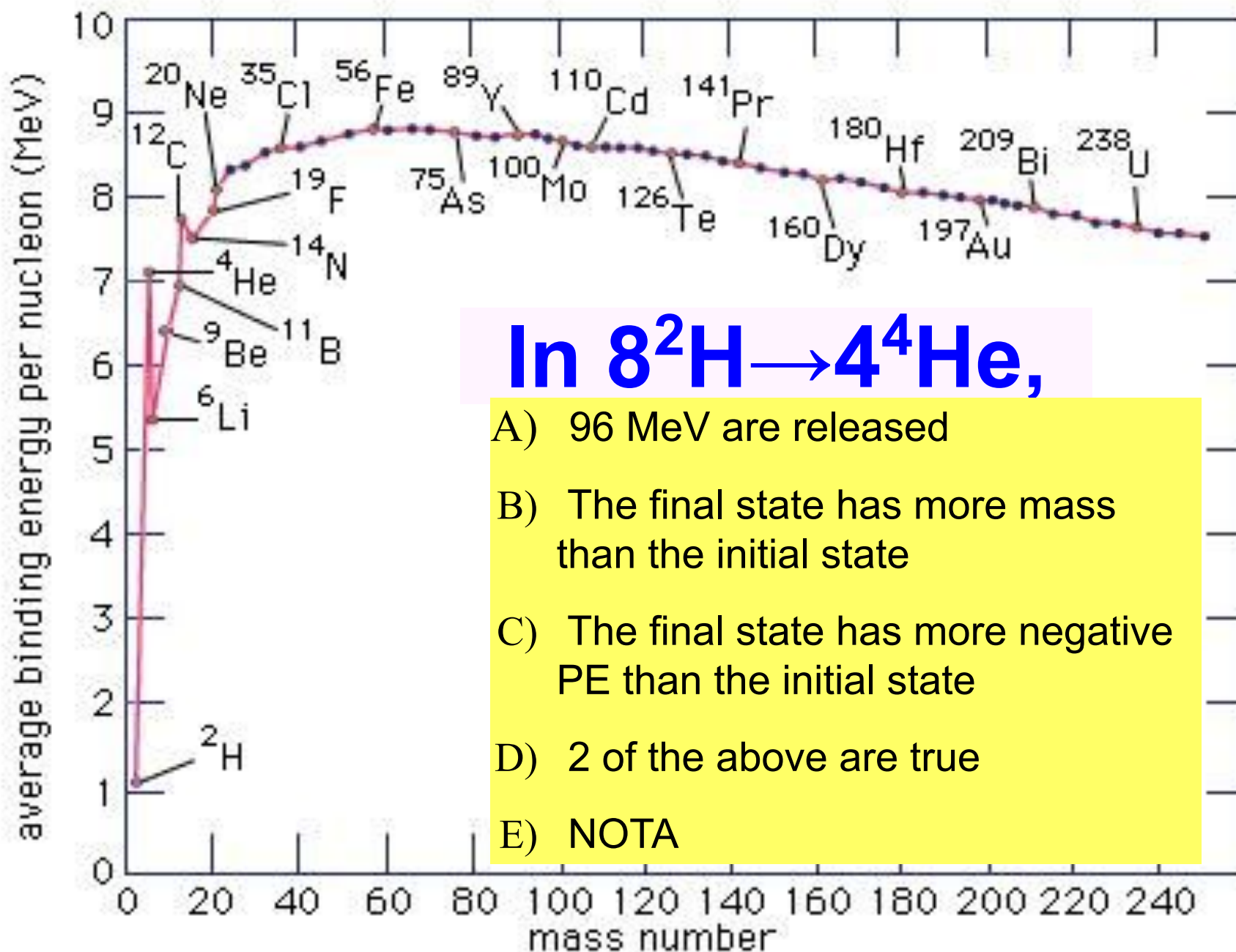




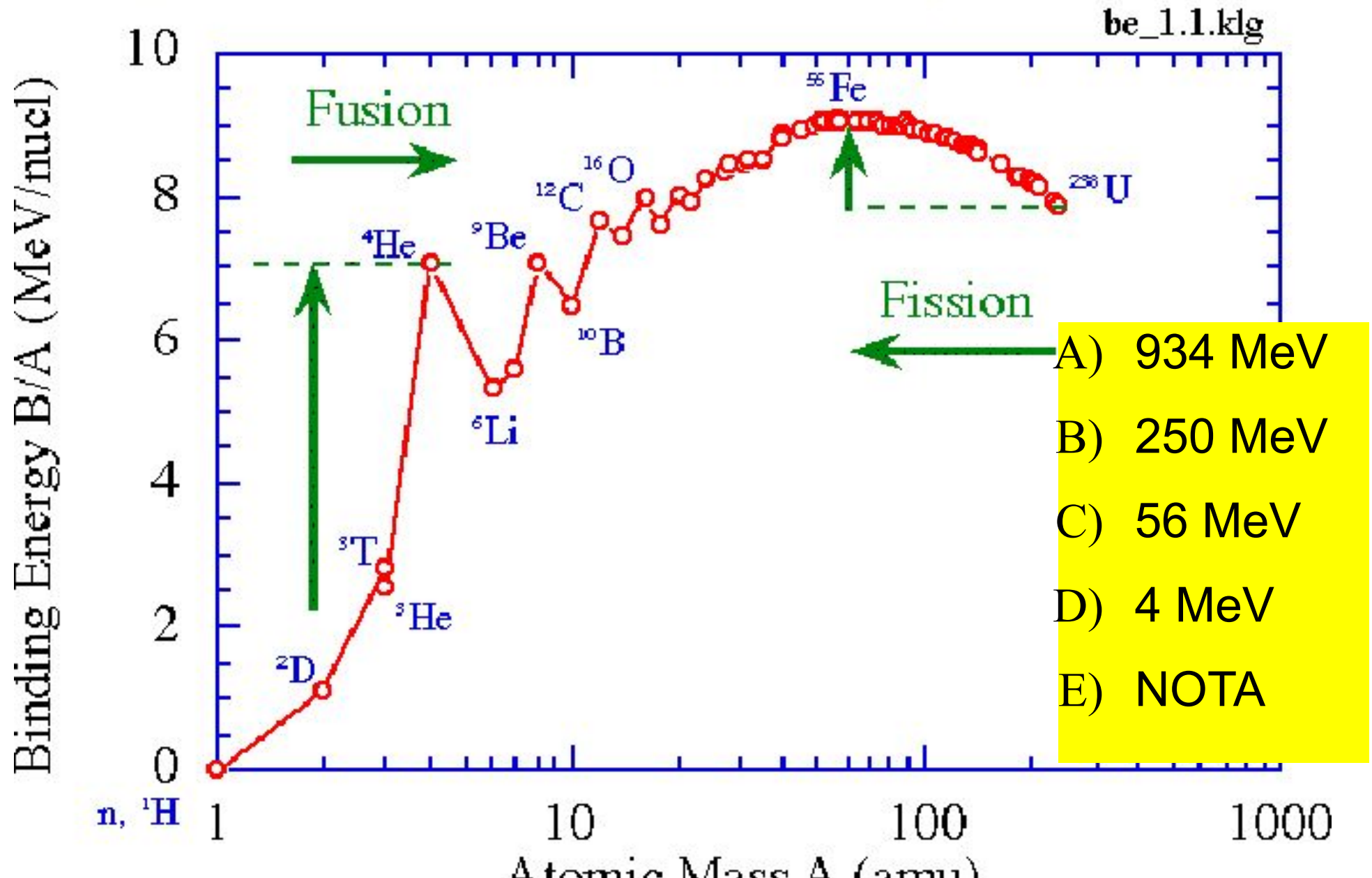
determine the mass (in kg) of  $^{236}\text{U}$  lost via  $^{238}\text{U} \rightarrow ^{126}\text{Te} + ^{110}\text{Cd}$ , and also  $12^2\text{H} \rightarrow 2^{12}\text{C}$ .



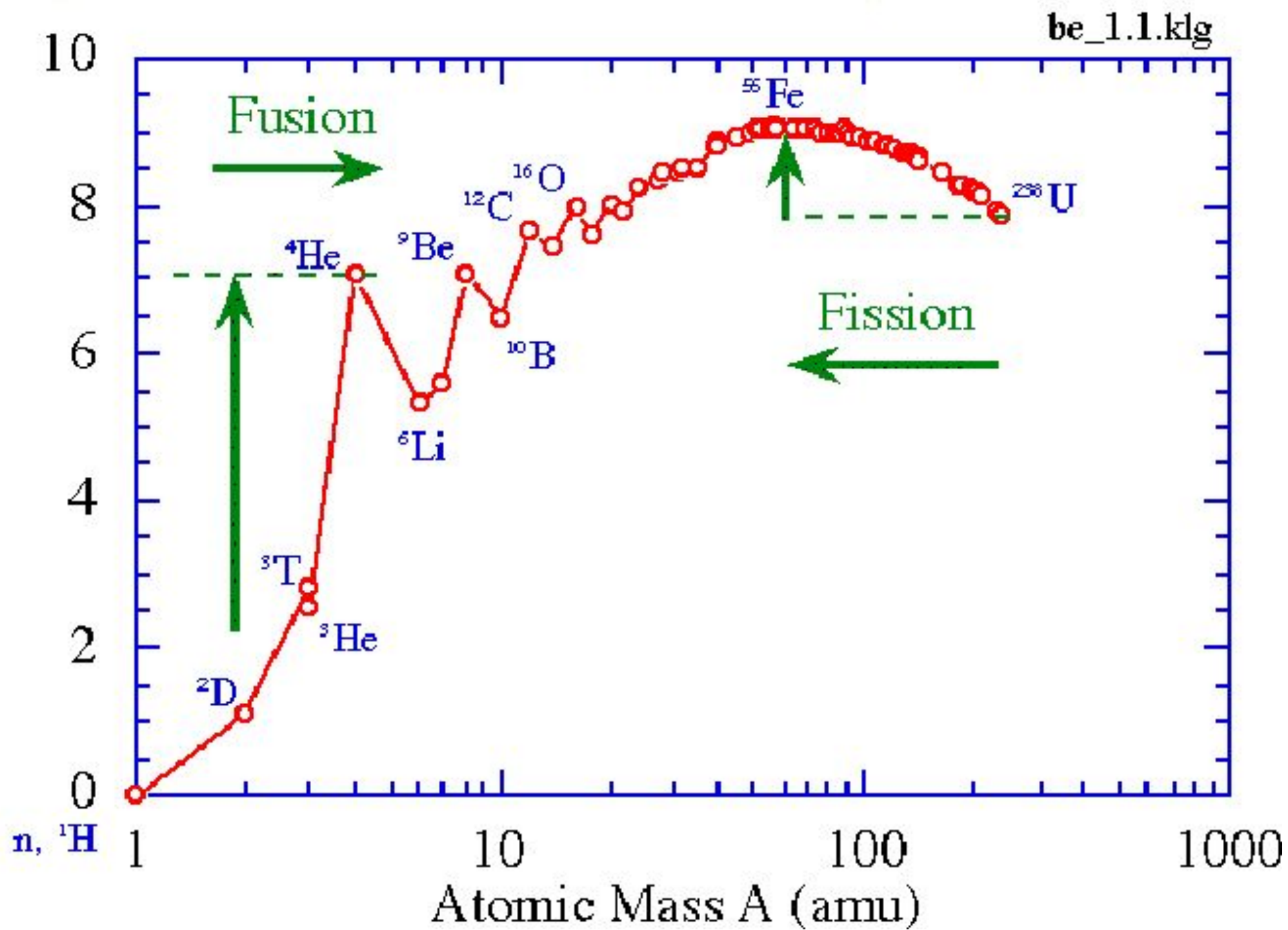
First case:  $110 \times 8.5 \text{ MeV/n} + 126 \times 8.4 \text{ MeV/n} - 236 \times 7.7 \text{ MeV/n} = 176.2 \text{ MeV}$ . Now:  $176.2 \text{ MeV} \times 1.783 \times 10^{-24} \text{ kg/MeV} = 3.14 \text{ kg} \times 10^{-22} \text{ kg}$  (tiny!!);  
 Second case:  $24 \times 7.5 - 24 \times 1 = 156 \text{ MeV}$ ; corresponds to  $2.78 \times 10^{-22} \text{ kg}$  (also tiny, but more explosive!)



In  $^{238}_{92}\text{U} \rightarrow 3(^{58}_{26}\text{Fe}) + \text{X}$ , the heat generated is about



Binding Energy B/A (MeV/nucleon)



**Which would have the effect of increasing the energy released in a Uranium fission reaction?**

- A) Increasing the strength of the electric force
- B) Increasing the range of the nuclear force
- C) Decreasing the strength of the nuclear force
- D) Two of the above
- E) All of the above

**Which would have the effect of increasing the energy released in a Hydrogen fusion reaction?**

- A) Increasing the strength of the electric force
- B) Increasing the range of the nuclear force
- C) Decreasing the strength of the nuclear force
- D) Two of the above
- E) All of the above

**Sketch the NBE curve if the range of the strong force were increased**

# Wave Functions

The uncertainty in position associated with a 40 kg person moving at 10 m/s (Sydney McGlaughlin-Levrone, of Dunellen, NJ) is about

- A)  $10^{-10}$  m
- B)  $10^{-34}$  m
- C)  $10^{-36}$  m
- D)  $10^{-32}$  m
- E) NOTA



An electron is in the 1s state in Hydrogen. What is the probability of measuring the electron with  $x > 0$ ,  $y > 0$  and  $z > 0$ ?

A)  $\frac{1}{2}$

B)  $\frac{1}{4}$

C)  $\frac{1}{8}$

D)  $\frac{1}{16}$

E) NOTA

**An electron has a wave function  $\Psi(x)=\sin(2x)$  ( $0 < x < 2\pi$ ). The probability that the electron is found at  $x=1$  m relative to  $x=2$  m is:**

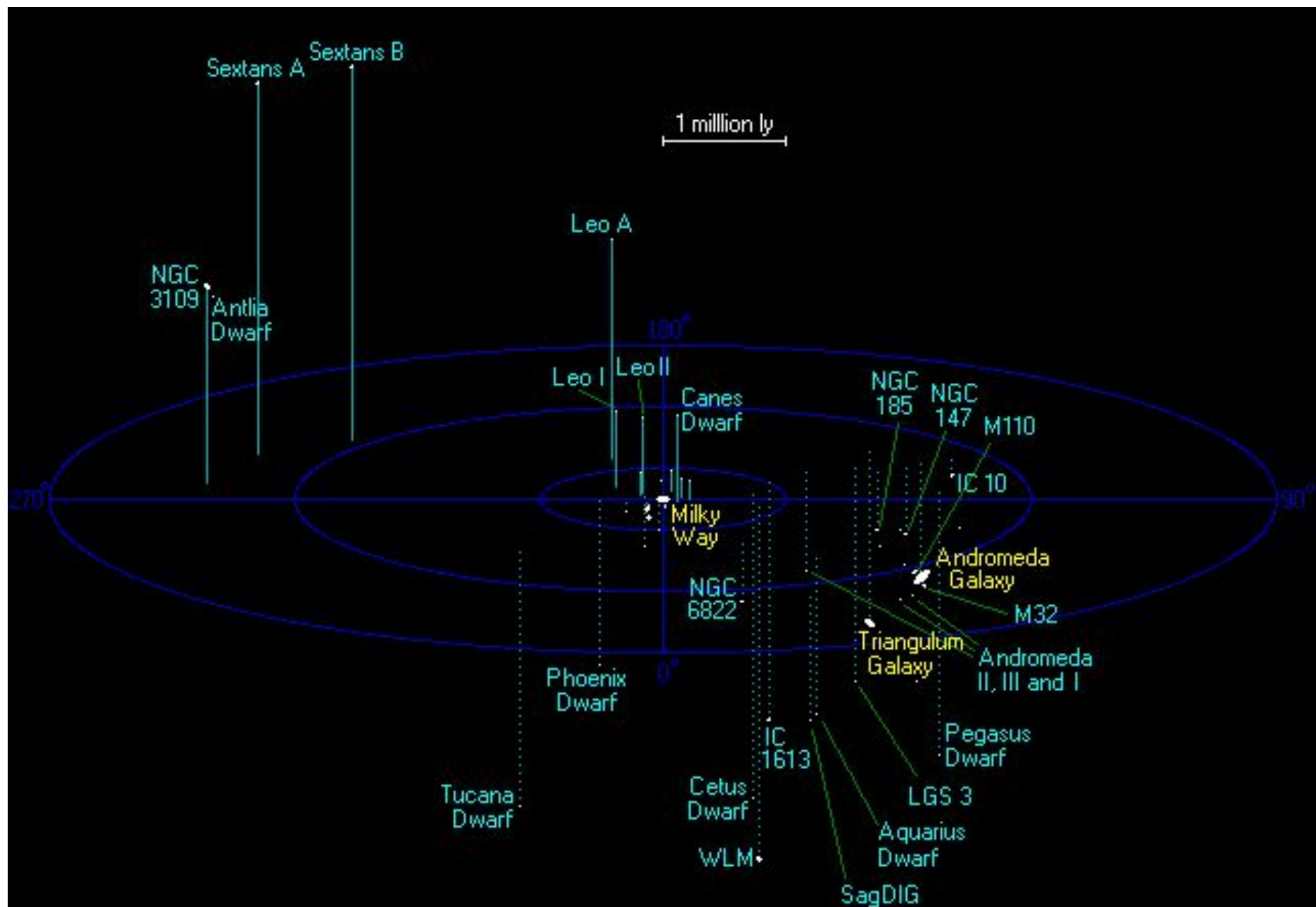
- A) 1.44
- B) 1.0
- C) 0.598
- D) 0
- E) NOTA

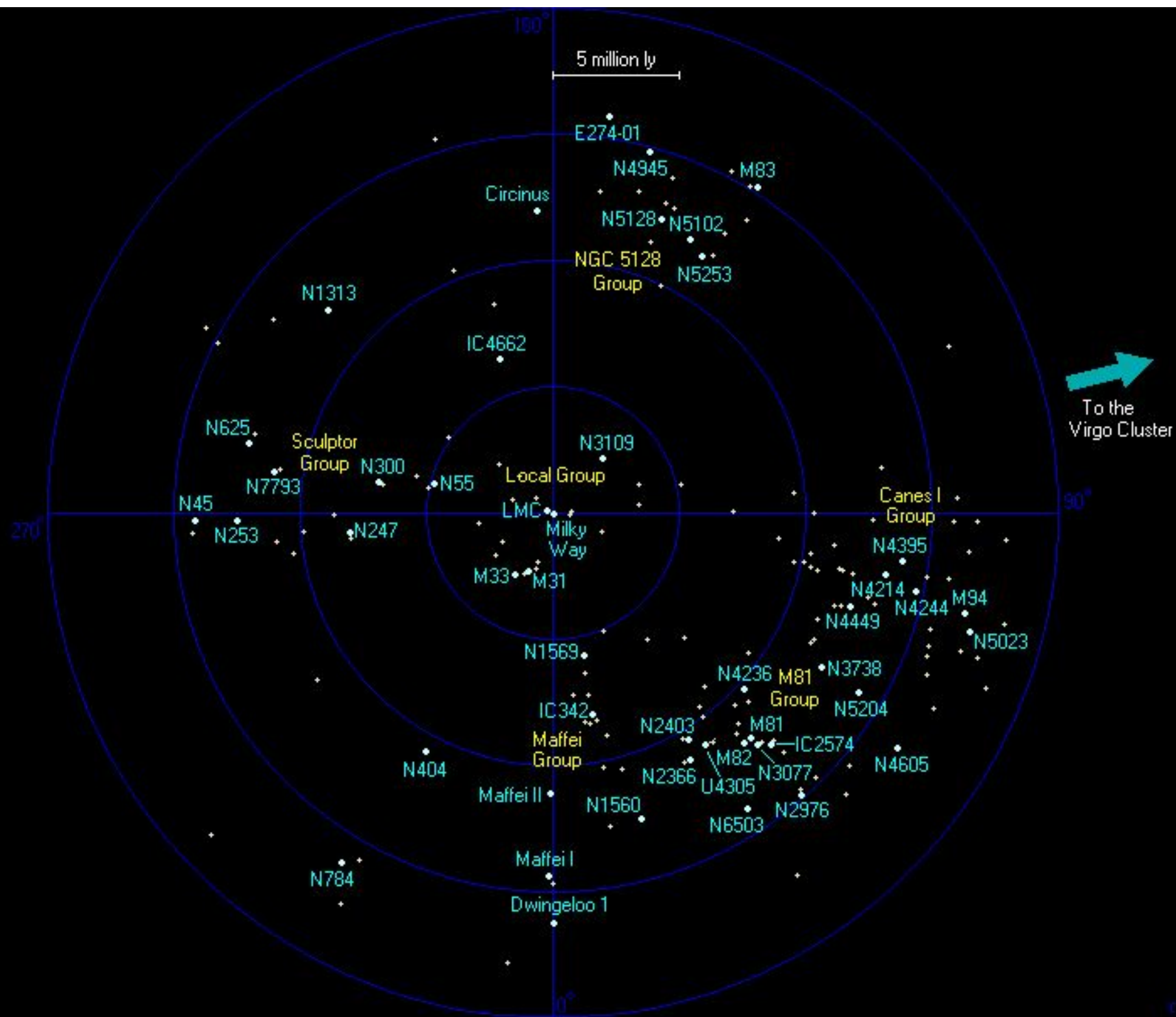
**Two protons and one neutron coalesce, in a 6 MeV deep, 3 fm wide potential, to form  $^3\text{He}$ . Which is true?**

- A) The probability of measuring the neutron in the interval 1 fm  $\rightarrow$  2 fm is **always**  $1/3$ , independent of any other measurements
- B) The probability of measuring the neutron in the interval 1 fm  $\rightarrow$  2 fm after one of the protons has been measured in the range 2 fm  $\rightarrow$  3 fm is  $1/2$
- C) The probability of measuring the neutron in the interval 1 fm  $\rightarrow$  2 fm after one of the protons has been measured in the range 2 fm  $\rightarrow$  3 fm is  $1/3$
- D) NOTA

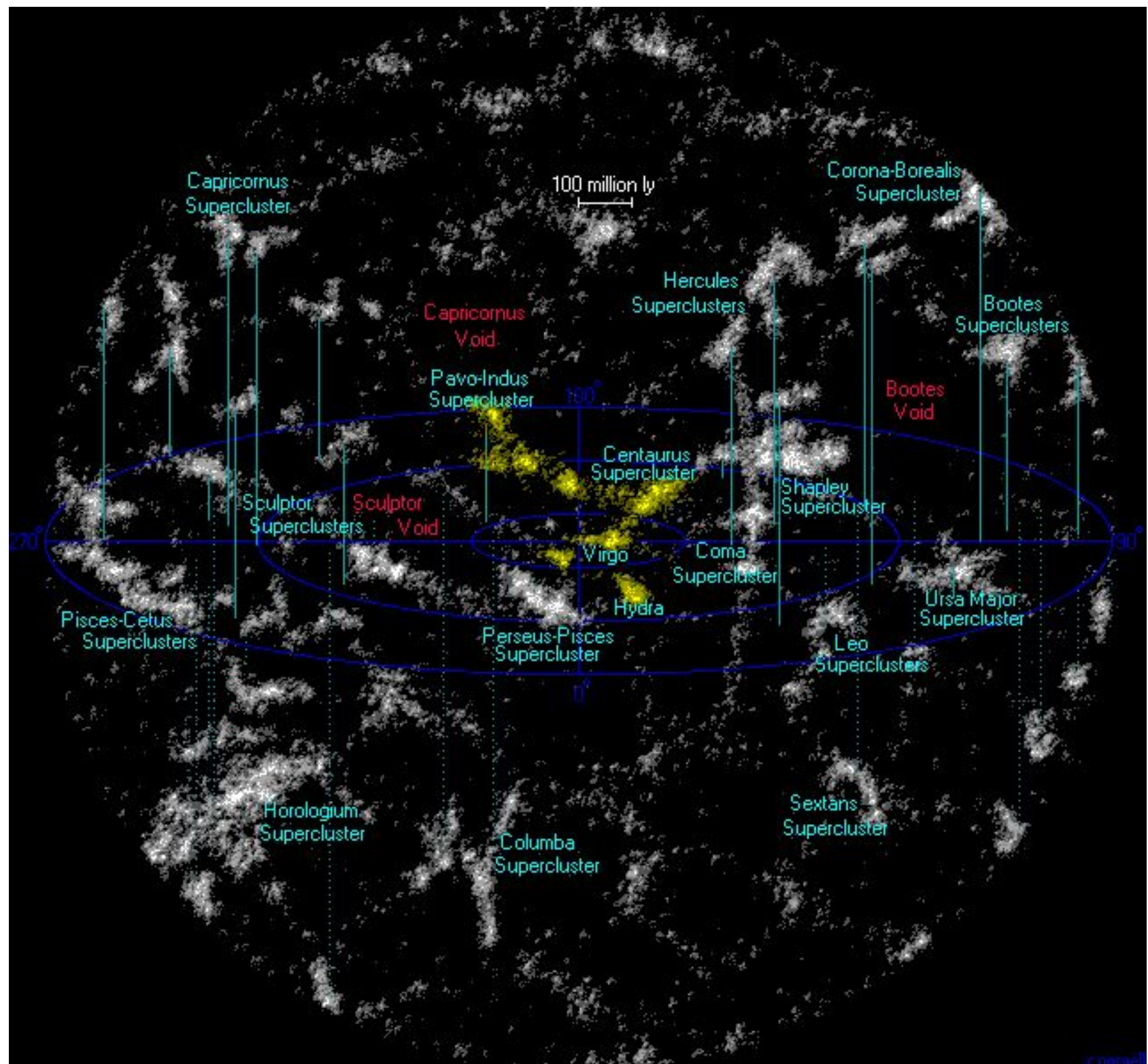
## Cosmology-

recall Hydrogen electron: if  $KE+PE<0$ ; electron is 'bound'; if  $KE+PE>0 \Rightarrow$  unbound









# Blitzkrieg Cosmology

Cosmology, Friday April 29, 1921:

a) the Universe==the Milky Way (lots of stars)

-nebulae = fuzzy things in the sky?

-Static, eternal

(St. Augustine's dilemma)

(Einstein's dilemma=>'cosmological constant')

b) Olber's Paradox argues against an infinite Universe

1929: Hubble looks at nebulae: "Universe is expanding!"

=>Gamow: play "The Universe, the movie" backwards=>'Big Bang'

Q: does expansion continue forever or does it reach a limit and then  
implode? (the Big Crunch)

Is  $PE(\text{total}) + KE(\text{total}) <, =, > 0$ ?

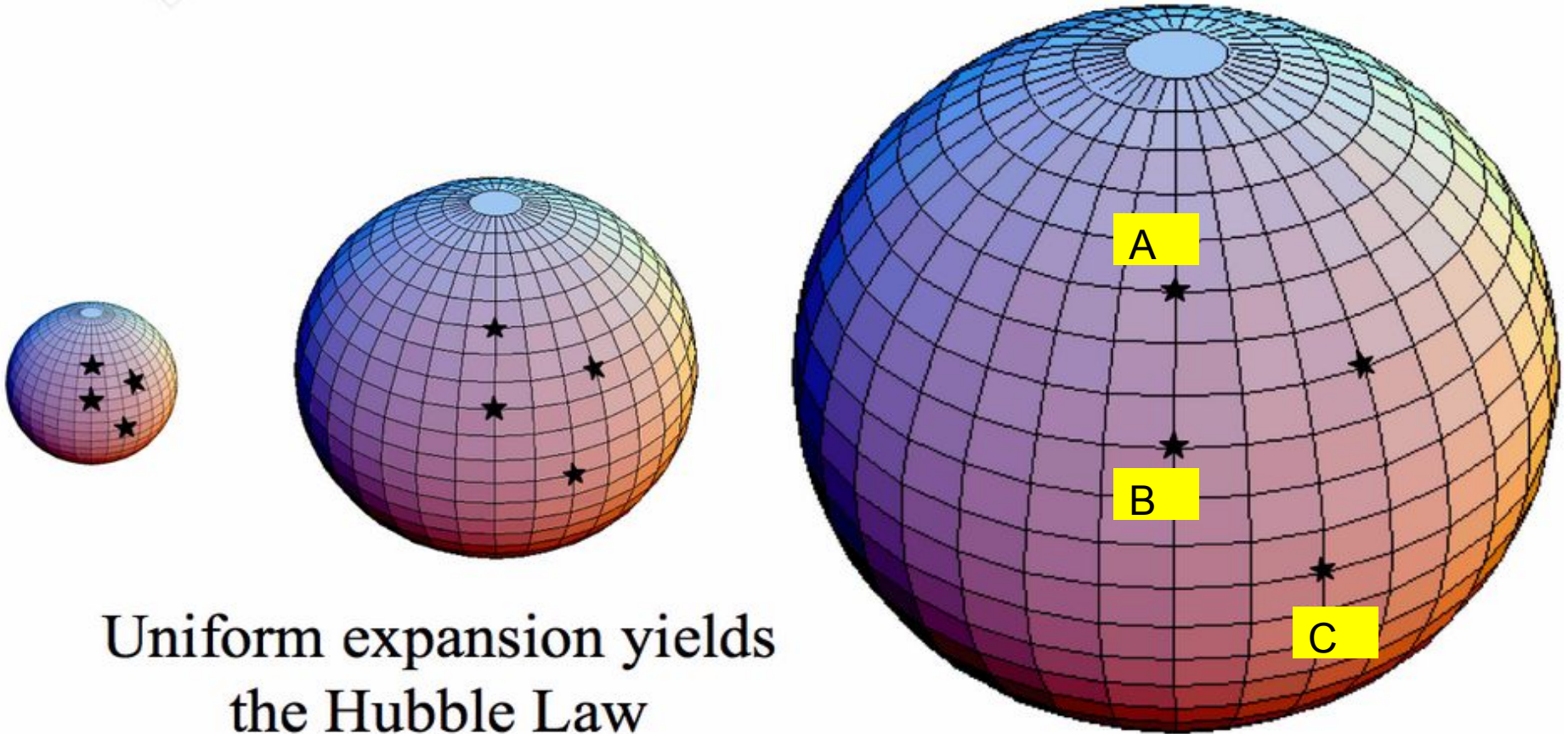
(recall PE for attractive forces (+/-, gravity) negative!

Cosmological task: calculate total (negative) gravitational PE and compare  
with KE. Complication: "Dark Matter"

Comment on length scales: 1 Mpc~d(Andromeda,Milky Way); 1 pc=3.26 light-yr  
size of visible Universe~28,519 Mpc



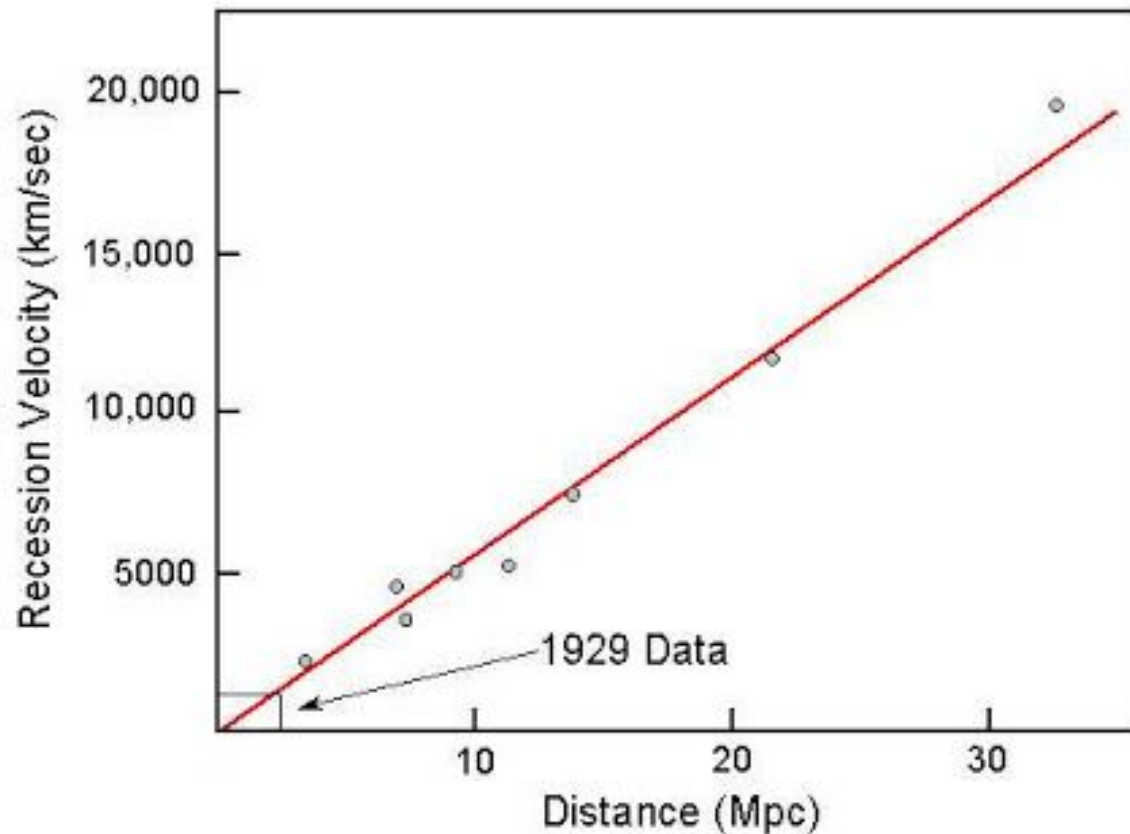
If, at  $t=0$ ,  $d(A \rightarrow B)=1$  Mpc and  $d(A \rightarrow C)=2$  Mpc, and A and B separate by 138.8 km in two seconds, then in four seconds, A and C separate by: A) 138.8 km B) 277.6 km C) 555.2 km D) NOTA



'Finite and unbounded'

# From these data, $H_0$ is closest to:

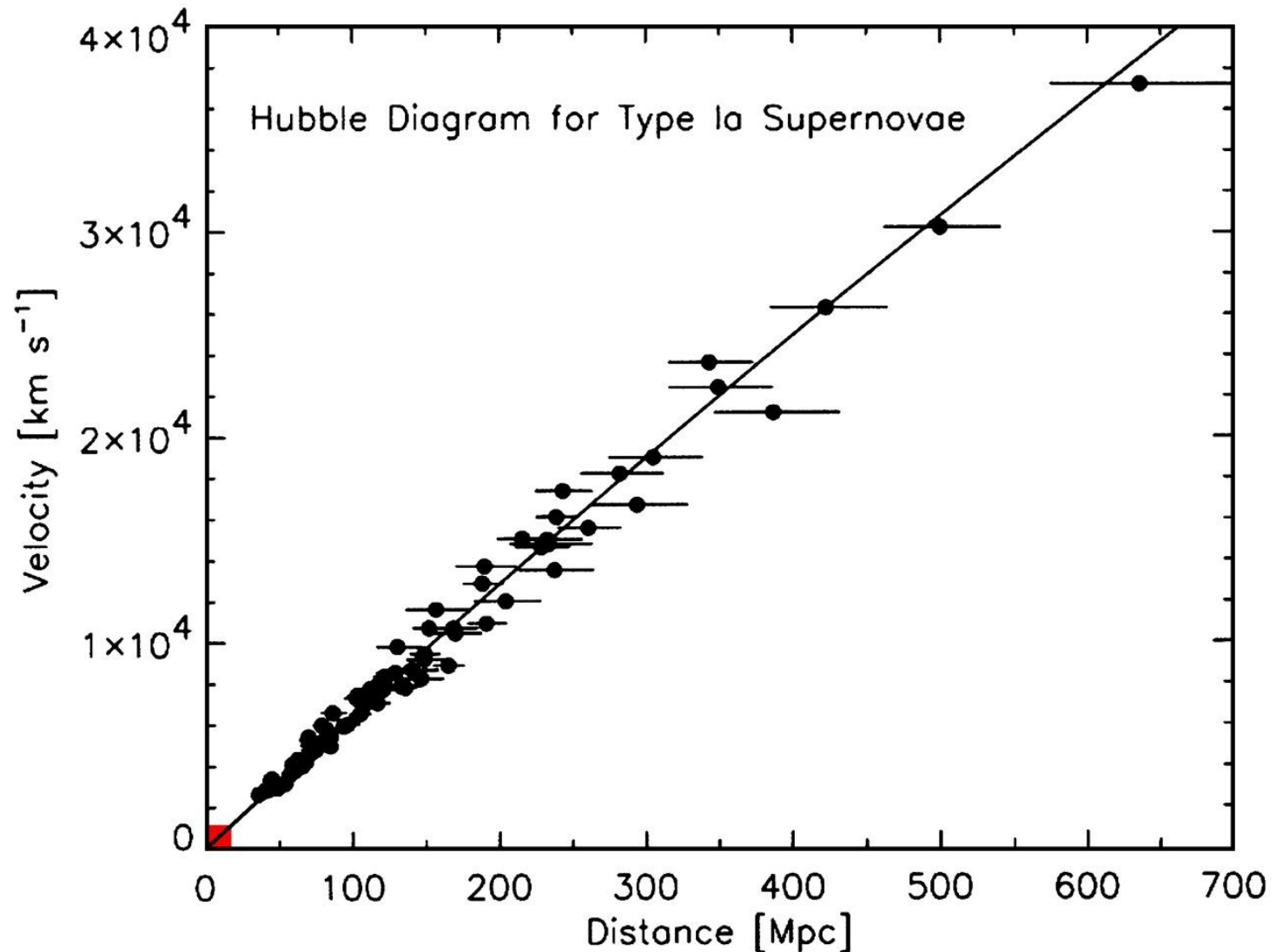
Hubble & Humason (1931)



- A) 1929 km/s/Mpc
- B) 67 km/s/Mpc
- C) 20000 km/s/Mpc
- D) 600 km/s/Mpc

Given that it takes 8 minutes for light from the sun to reach Earth...

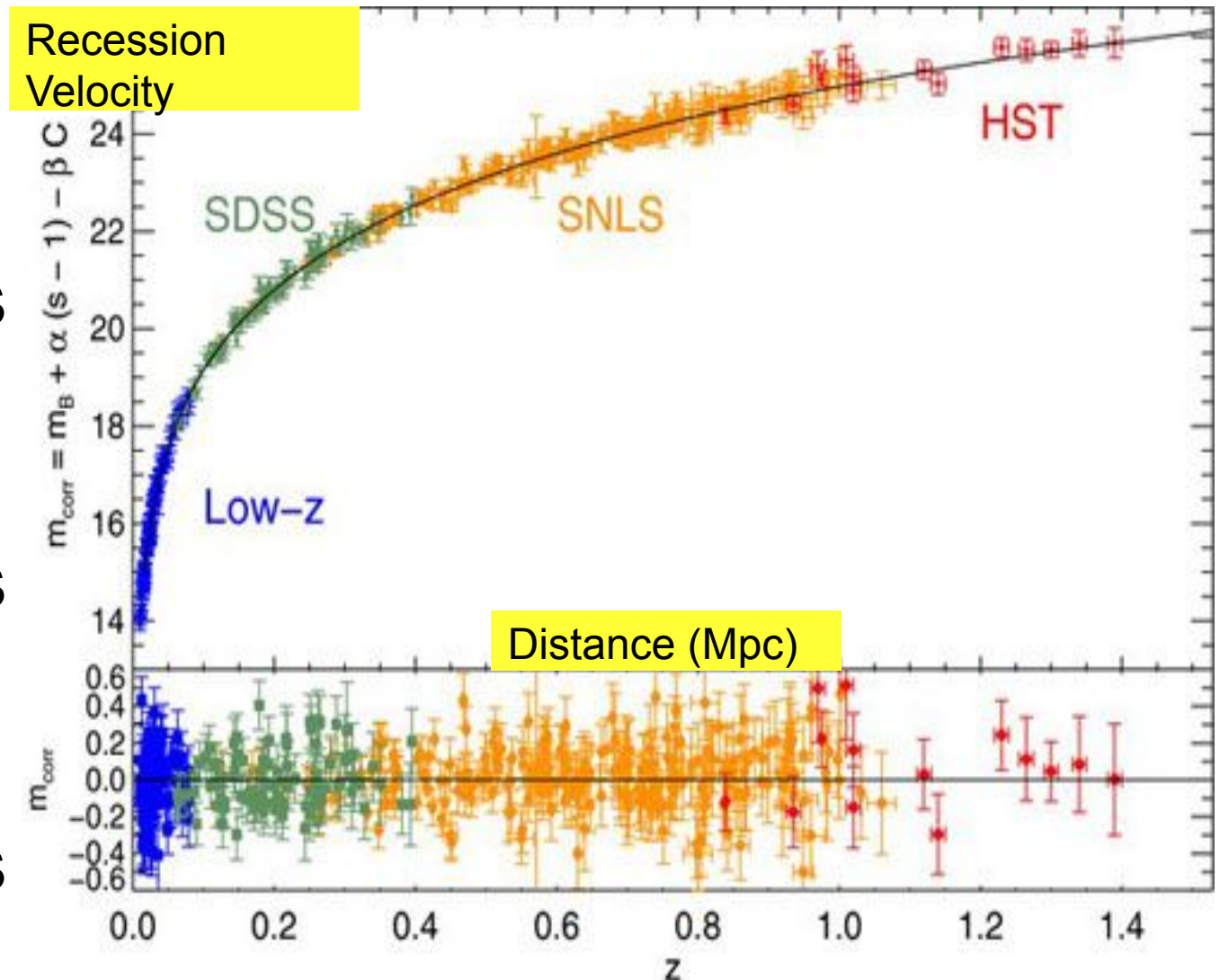
From the curve shown, in one minute, two galaxies in neighboring super-clusters ( $d=100$  Mpc) would separate by a distance:



- A) 500 km
- B) 370000 km
- C) 100 km
- D) NOTA

**Based on the curve shown, you would conclude that:**

- A) The U expansion is constant
- B) The expansion is decelerating
- C) The expansion is accelerating

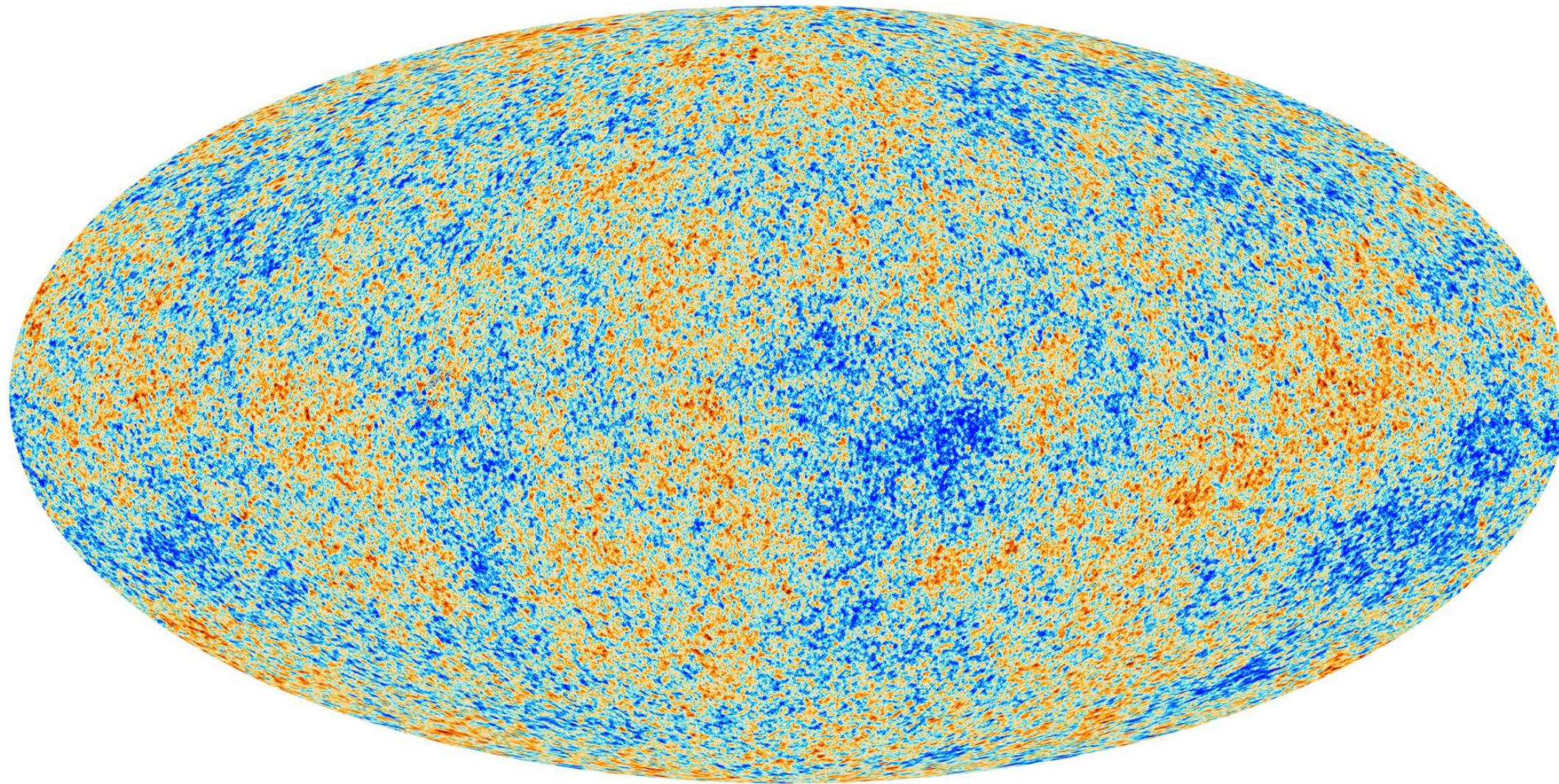


**If the Hubble constant were smaller,**

- A) The night sky would be brighter
- B) The Universe would be more likely to undergo a 'Big Crunch'
- C) Both of A) and B)
- D) Neither of A) or B)



# The Cosmic Microwave Background



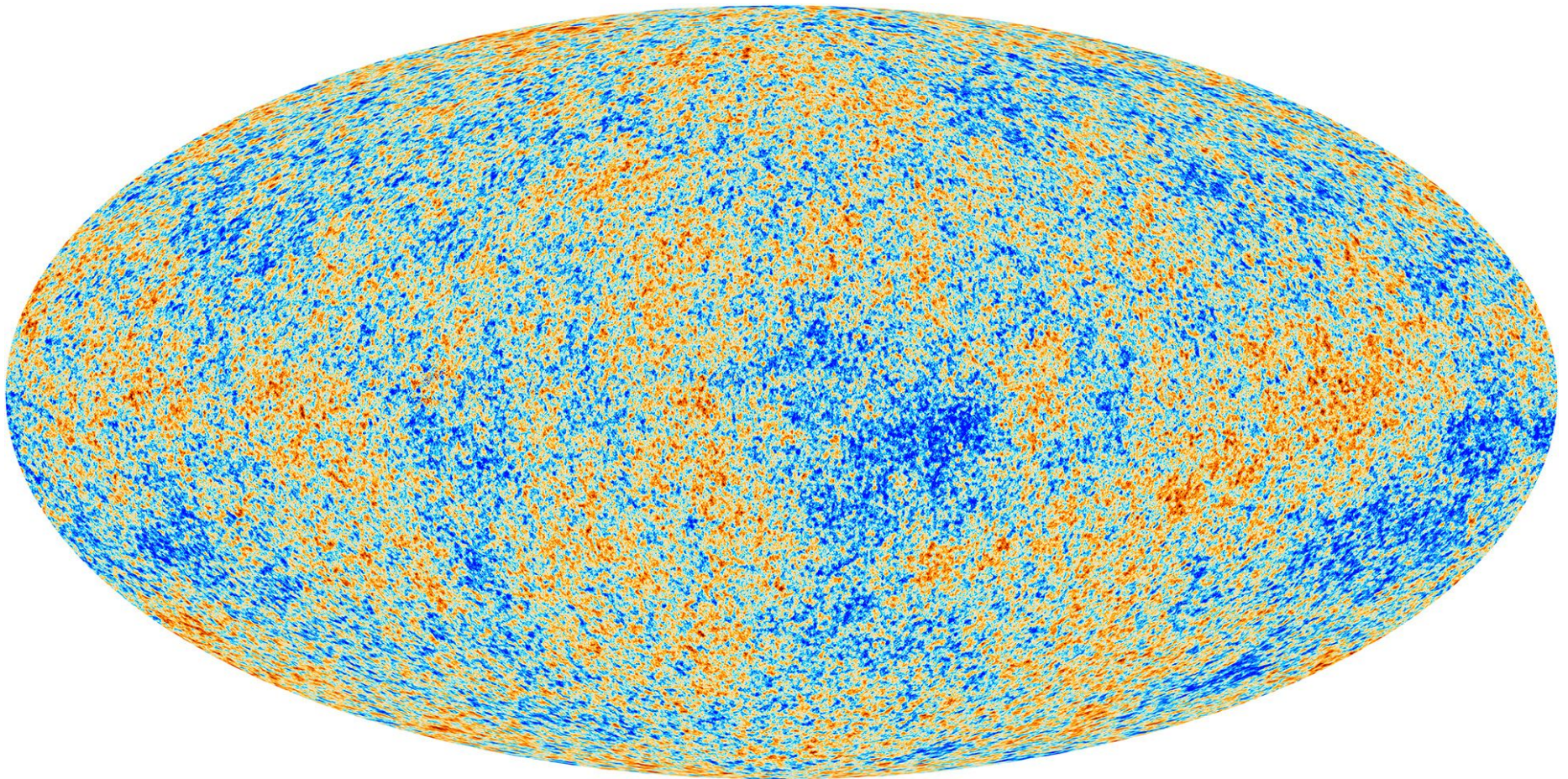


**From Lawrence, KS, you look into the night sky and see the cosmic microwave background coming at you from all directions. From that, you conclude that:**

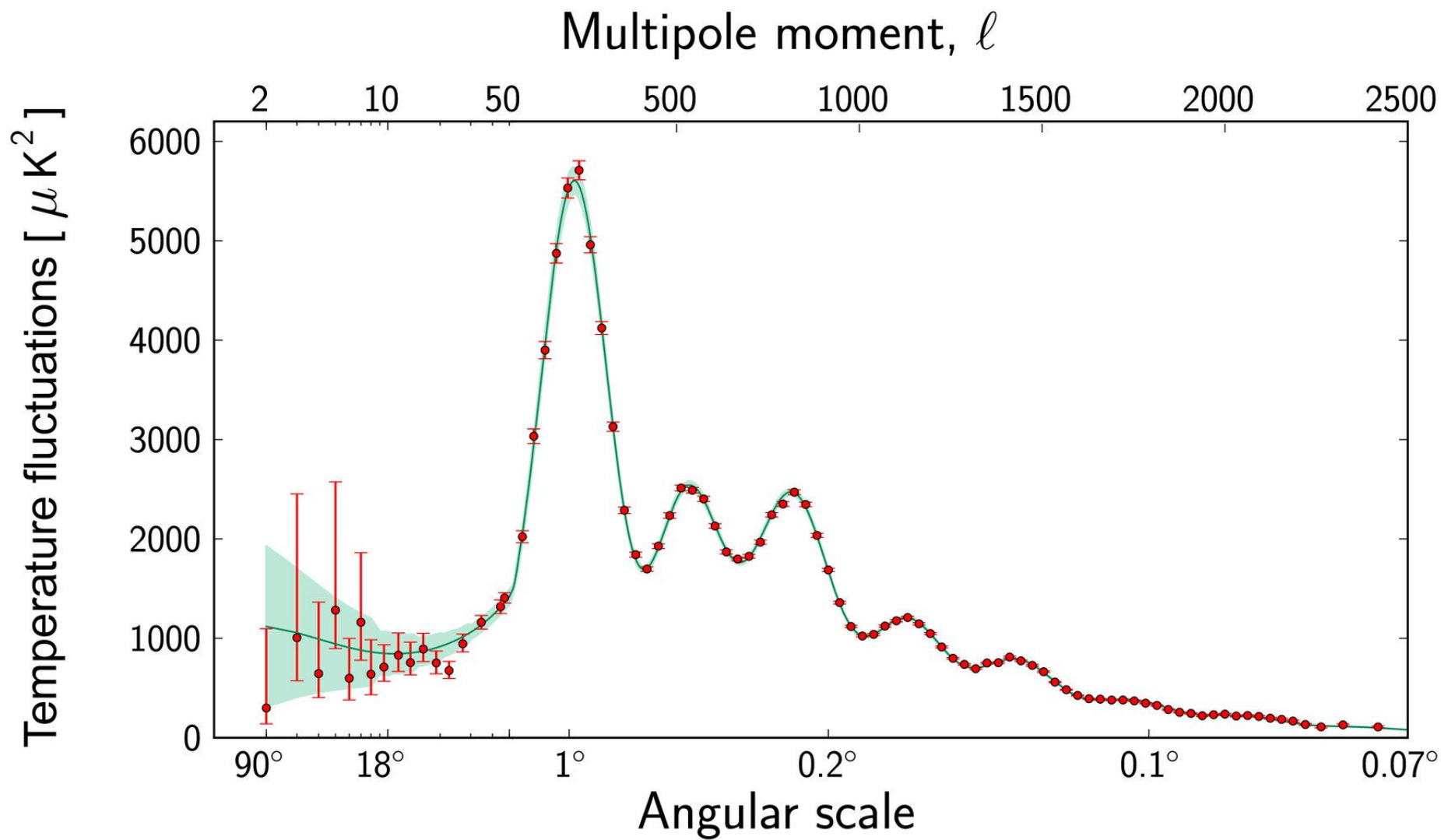
- A) The Earth is the center of the Universe
- B) The Universe is infinite
- C) Either A) or B)
- D) Neither A) or B)

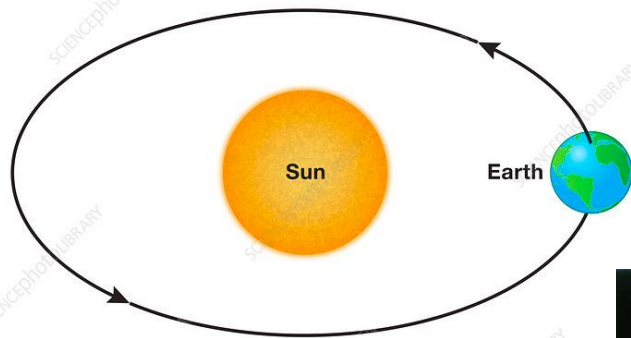
Based on the “map” shown, the greatest amount of “power” occurs at angular scales of:

A)  $180^\circ$ , B)  $90^\circ$ , C)  $22.5^\circ$



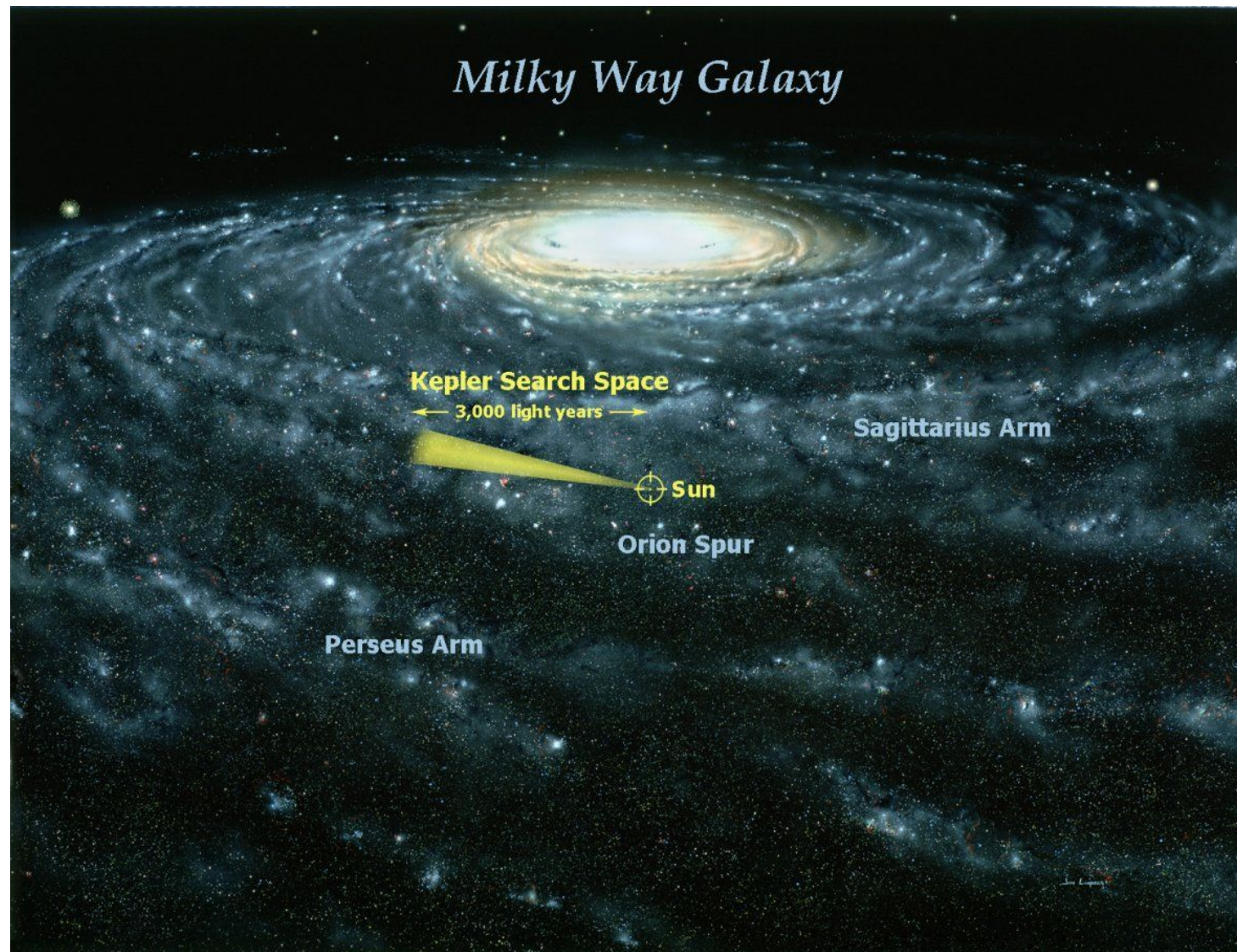






$$M_e v_E^2 / r_E = G M_E M_{\text{Sun}} / r^2$$

$$\text{Mass of Galaxy} = N_{\text{stars}} \times M_{\text{Sun}}$$



**A spiral galaxy consists of  $5 \times 10^4$  stars identical to our sun, which has a mass of  $2 \times 10^{30}$  kg. Given that, you conclude that the spiral galaxy is**

- A) 0% dark matter
- B) ~20% dark matter
- C) ~40% dark matter
- D) >60% dark matter





Light is measured at Earth from stars, at the periphery ( $10^{12}$  m) and also at the center of a rotating spiral galaxy, with wavelengths of 644, 656 and 668 nm. Infer the total mass of the galaxy, if  $r=10^{12}$  m.

- A)  $M_{\text{galaxy}} = 4.515 \times 10^{35}$  kg
- B)  $M_{\text{galaxy}} = 6.10 \times 10^{23}$  kg
- C)  $M_{\text{galaxy}} = 2.76 \times 10^{87}$  kg
- D)  $M_{\text{galaxy}} = 8.16 \times 10^{58}$  kg
- E) NOTA



**Two identical 5 kg masses are separated by 4 m. If their gravitational  $PE = -Gm_1m_2/r$ , what is their escape velocity?  
( $G = 6.67 \times 10^{-11} \text{ N/kg}^2\text{-m}^2$ )**

- A) About 10 m/s
- B) About 10 cm/s
- C) About 1 cm/s
- D) About 0.01 mm/s
- E)  $>10 \text{ m/s}$  or  $<0.001 \text{ mm/s}$

T=0: First, we use the orbital period of our planet (take earth) to measure one typical star:  $mv_E^2/r_E = GmM_{\text{Sun}}/r_E^2$ . The mass of the Earth  $m$  cancels,  $r_E = 150.23 \times 10^9$  m; from knowing that it takes one year to orbit a distance  $2\pi r_E$ ,  $v_E = 2\pi r_E / 3.154 \times 10^7$  seconds  $= 30 \times 10^3$  m/s, so  $M_{\text{Sun}} = 1.989 \times 10^{30}$  kg. We multiply this by the total number of 'Suns' in a typical galaxy ( $1 \times 10^{11}$ ), to get the 'visible' mass of the galaxy as  $1.989 \times 10^{41}$  kg.

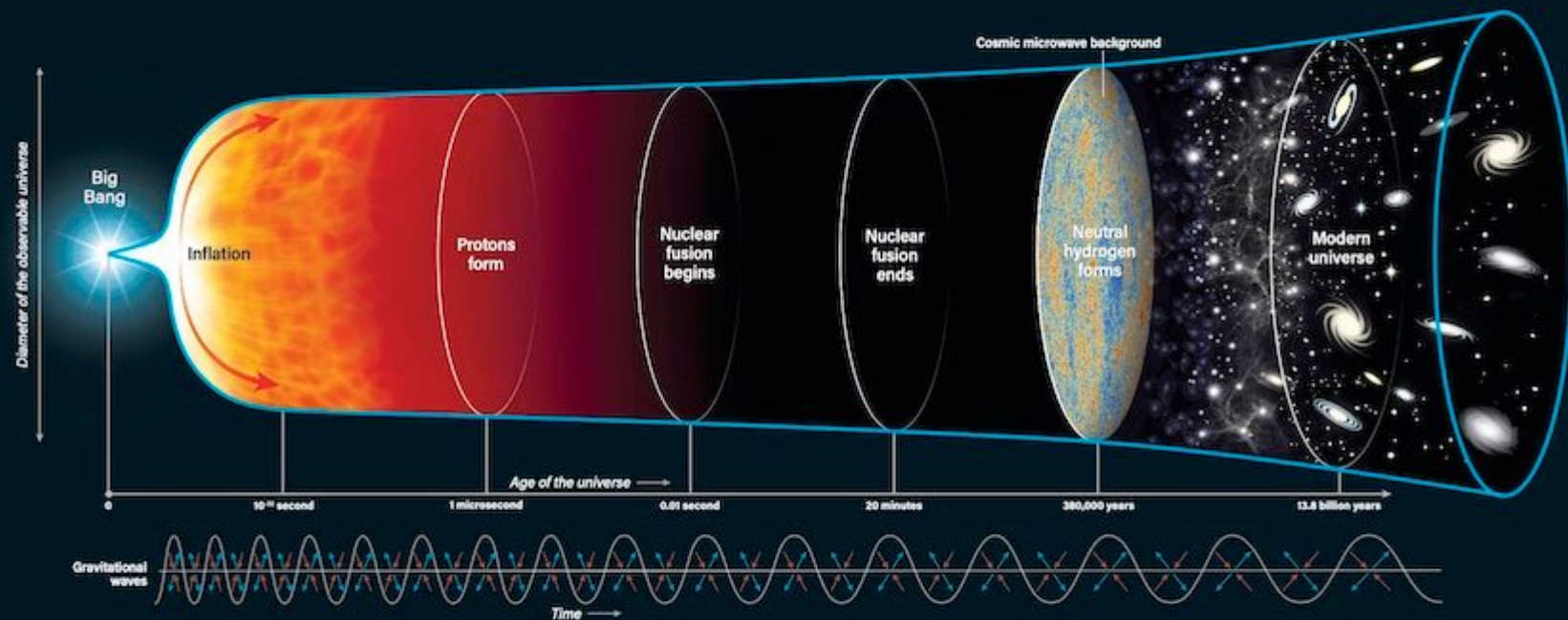
The typical distance between nearest 'big' galaxies (Andromeda to Milky Way, e.g.) is 0.89 Mpc (one Mpc  $= 3.086 \times 10^{22}$  m); the size of the visible Universe (not the 'true' Universe) is about 28,520 Mpc. Let's take two galaxies that are separated by 16 Mpc (the size of our local 'supercluster'). We'll take a Hubble constant of 80 km/s/Mpc, so after 5 billion years ( $5 \times 10^9 \times 3.154 \times 10^7 = 1.577 \times 10^{17}$  seconds), the two galaxies have separated through an additional  $16 \text{ Mpc} \times 1.577 \times 10^{17} \text{ s} \times 80000 \text{ m/s/Mpc}$  further away, or 6.54 Mpc. If we wait another 5 billion years, the two galaxies are now  $(16 + 6.54) \times 1.577 \times 10^{17} \times 80000 = 9.21$  Mpc additionally separated.

In reality, the separation, since it is exponential (sorta like a random virus spreading), the increase is much faster than this.

At what distance are the two galaxies separating at the velocity of light ( $3 \times 10^8$  m/s)?  
 $3 \times 10^8 \text{ m/s} / (80000 \text{ m/s/Mpc}) = 375$  Mpc!

What is ratio of PE between galaxies:  $-Gm_1m_2/r$  to relative KE:  $\frac{1}{2}m_1v_1^2$ ?  
 $PE = 6.67 \times 10^{-11} \times 1.989 \times 10^{41} \times 1.989 \times 10^{41} / (16 \times 3.086 \times 10^{22})$  vs.  $\frac{1}{2} \times 1.989 \times 10^{41} \times (16 \times 80 \times 10^3)^2 / (16 \times 3.086 \times 10^{22})$   
 $PE \ll KE \Rightarrow$  OPEN Universe!

(corrections: i) we only considered interaction of galaxy A with galaxy B; in principle, galaxy A interacts with all other galaxies in the Universe, so  $|PE|$  increases, ii) we ignored dark matter, which will increase PE relative to KE by about a factor of 5



Stuff you can't see that permeates all of space:

Big Bang Photons (CMB, 400/cc)

Big Bang Neutrinos (330/cc)

Other neutrinos (40 billion/cm<sup>2</sup>)

Dark Energy

Dark Matter

Higgs Field

Occasional CR (1/m<sup>2</sup>/sec)



Between the idea    And the reality    Between the  
motion    And the act    Falls the Shadow

Between the conception    And the creation  
Between the emotion    And the response    Falls  
the Shadow

Between the desire    And the spasm    Between  
the potency    And the existence    Between the  
essence    And the descent    Falls the Shadow

*This is the way the world ends  
This is the way the world ends  
This is the way the world ends  
Not with a bang but a whimper.* -T.S. Eliot, 1915