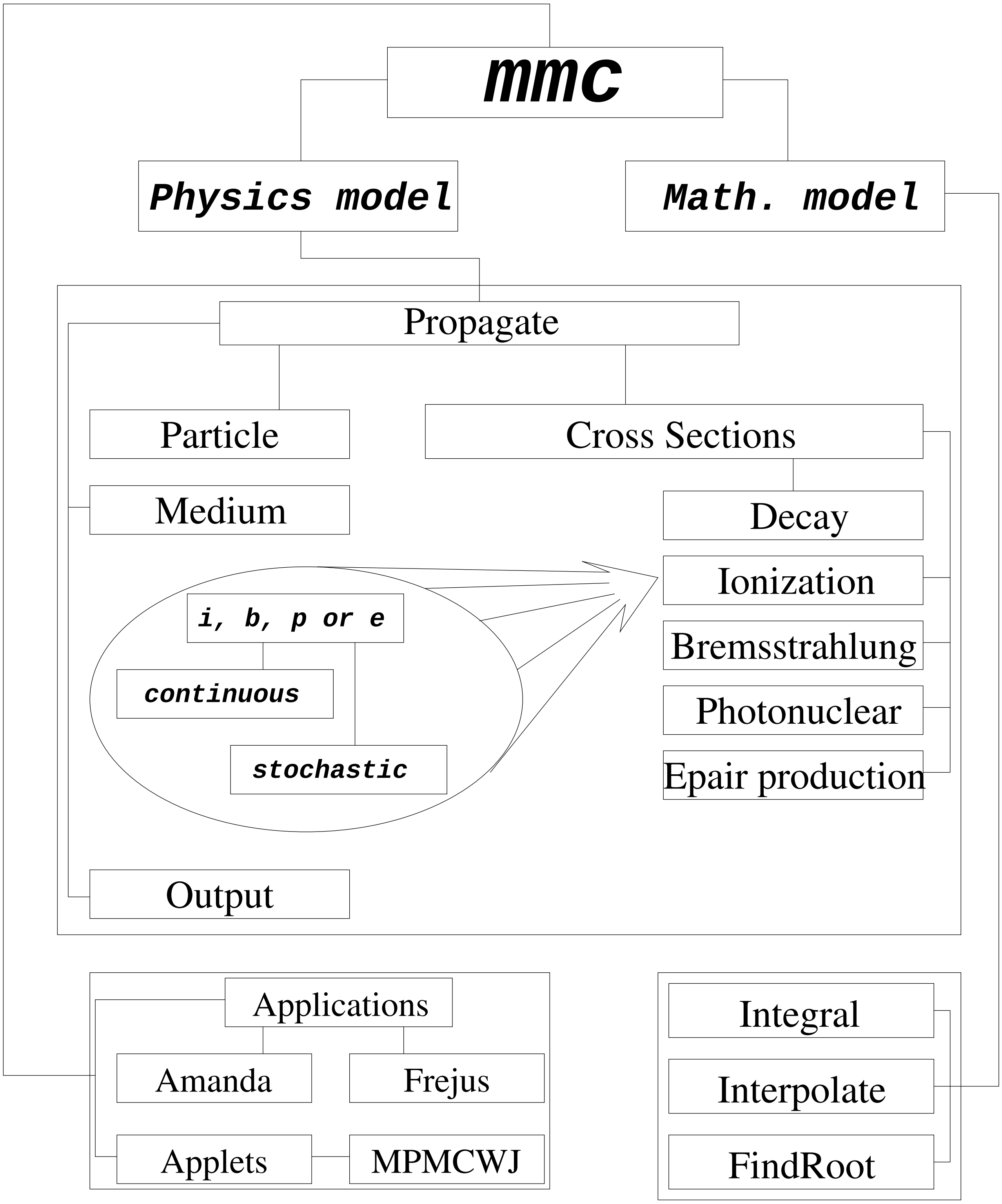
Muon Monte Carlo: a new high-precision tool for tracking of muons in medium

Wolfgang Rhode[1], **Dmitry Chirkin**[1,2]
rhode@wpos7.physik.uni-wuppertal.de
chirkin@physics.berkeley.edu

[1]*Bergische Universitaet Gesamthochschuele Wuppertal, Germany*
[2]*University of California at Berkeley, USA*

mmc code homepage is
http://area51.berkeley.edu/ dima/work/MUONPR/muonPropagation.html

mmc code available at
http://area51.berkeley.edu/ dima/work/MUONPR/BKP/mmc.tgz



mmc
Physics model
Math. model
Propagate
Particle
Cross Sections
Medium
Decay
Ionization
i, b, p or e
Bremsstrahlung
continuous
Photonuclear
stochastic
Epair production
Output
Applications
Integral
Amanda
Frejus
Interpolate
Applets
MPMCWJ
FindRoot

Hierarchy For All Packages

Class Hierarchy

- class java.lang.Object
  - class mmc.**MathModel**
    - class mmc.**FindRoot**
    - class mmc.**Integral**
    - class mmc.**Interpolate** (implements mmc.FunctionInt)
  - class mmc.**Output**
  - class mmc.**PhysicsModel** (implements mmc.FunctionInt, mmc.FunctionInt2, mmc.FunctionOfx)
    - class mmc.**CrossSections**
      - class mmc.**Bremsstrahlung**
        - class mmc.**BremsContinuous**
        - class mmc.**BremsStochastic**
      - class mmc.**Decay**
      - class mmc.**Epairproduction**
        - class mmc.**EpairContinuous**
        - class mmc.**EpairStochastic**
      - class mmc.**Ionizationloss**
        - class mmc.**IonizContinuous**
        - class mmc.**IonizStochastic**
      - class mmc.**Photonuclear**
        - class mmc.**PhotoContinuous**
        - class mmc.**PhotoStochastic**
    - class mmc.**Medium**
    - class mmc.**Particle**
    - class mmc.**Propagate**

Interface Hierarchy

- interface mmc.**FunctionInt**
- interface mmc.**FunctionInt2**
- interface mmc.**FunctionOfx**
  - interface mmc.**DFunctionOfx**

Ionization

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A\beta^2}\left[\frac{1}{2}ln\left(\frac{2m_e\beta^2\gamma^2\nu_{\text{upper}}}{I(Z)^2}\right) - \frac{\beta^2}{2}\left(1+\frac{\nu_{\text{upper}}}{\nu_{\text{max}}}\right)\right.$$
$$\left.+\frac{1}{2}\left(\frac{\nu_{\text{upper}}}{2E(1+1/\gamma)}\right)^2 - \frac{\delta}{2}\right]$$
$$\text{where}\quad \nu_{\text{max}} = \frac{2m_e(\gamma^2-1)}{1+2\gamma\frac{m_e}{\mu}+\left(\frac{m_e}{\mu}\right)^2}$$
$$\nu_{\text{upper}} = min(\nu_{\text{cut}}, \nu_{\text{max}})$$
$$I(Z) = 75\text{eV (water)}, \; K = 0.307075MeVg^{-1}cm^2.$$

$$\begin{aligned}\delta &= 0, \quad \text{if } X < X_0\\ \delta &= 4.606X + C + a(X_1 - X)^m, \quad \text{if } X_0 \le X < X_1\\ \delta &= 4.606X + C, \quad \text{if } X \ge X_1\end{aligned}$$
$$\text{where}\quad X = log_{10}(\beta\gamma)$$

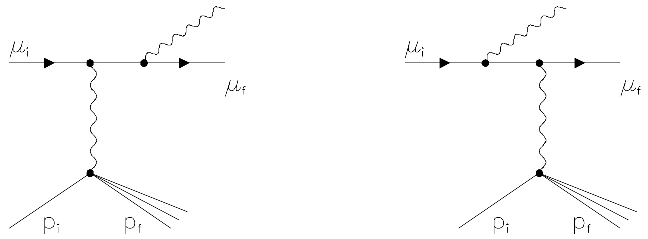
$C = -3.5017,\ a = 0.09116,\ m = 3.477,\ X_0 = 0.240,\ X_1 = 2.8004$ (water).

$$\frac{d^2N}{d\nu dx} = \frac{1}{2}Kz^2\frac{Z}{A}\frac{1}{\beta^2}\frac{1}{\nu^2}\left[1\ -\beta^2\frac{\nu}{\nu_{\text{max}}} + \frac{1}{2}\left(\frac{\nu}{E(1+1/\gamma)}\right)^2\right]$$

Bremsstrahlung

$$\sigma = \sigma_{el} + \Delta\sigma_a^{in} + \Delta\sigma_n^{in}$$

Elastic Bremsstrahlung



$$\sigma_{el}(E, v) = \frac{\alpha}{v}\left(2Z\frac{m}{\mu}r_e\right)^2\left(\frac{4}{3} - \frac{4}{3}v + v^2\right)\left[\ln\left[\frac{\mu}{\delta}\right] - \frac{1}{2} - \Delta_a^{el} - \Delta_n^{el}\right]$$

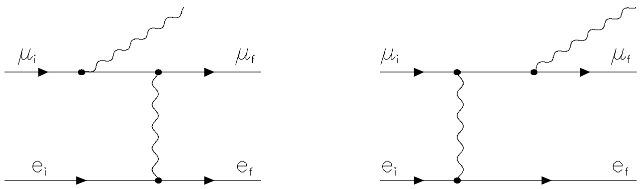
where $\quad \delta \approx \dfrac{\mu^2\omega}{2E(E-\omega)}$

$$\begin{aligned} \Delta_a^{el}(\delta) &= \ln\left[1 + \frac{1}{\delta\sqrt{e}BZ^{-1/3}/m}\right] \\ \Delta_n^{el}(\delta) &= \ln\left[\frac{D_n}{1 + \delta(D_n\sqrt{e} - 2)/\mu}\right]; \qquad D_n = 1.54A^{0.27} \end{aligned}$$

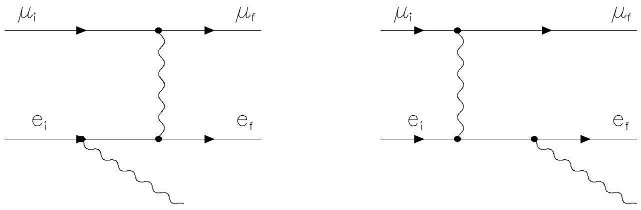
B=182.7 for $Z \geq 2$ and B=202.4 for Z=1.

Inelastic Bremsstrahlung

$$\Delta_n^{in} = \frac{1}{Z}\Delta_n^{el};\ (Z \neq 1)$$



μ diagrams



e diagrams

e-diagram, included with ionization losses:

$$\Delta\frac{d^2N}{d\nu dx} = \left(\frac{d^2N}{d\nu dx}\right)_{I_0} \cdot \frac{\alpha}{2\pi}(a(2b+c) - b^2)$$

$$a = \log(1 + 2\nu/m_e), \quad b = \log((1 - \nu/\nu_{max})/(1 - \nu/E)),$$

$$c = \log((2\gamma(1 - \nu/E)m_e)/(m_\mu \nu/E))$$

Maximum energy lost by a muon is the same as in the pure ionization (knock-on) energy losses. Minimum energy is given by the following formula:

$$\nu_{min} = \frac{1}{2m_e} \cdot \left(\frac{I(Z)}{\beta\gamma}\right)^2$$

μ-diagram, included with bremsstrahlung:

$$\Delta\sigma_a^{in}(E,v) \approx \frac{\alpha}{v}\left(2Z\frac{m}{\mu}r_e\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\Delta_a^{in}$$

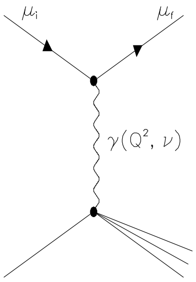
$$\Delta_a^{in} \approx \frac{1}{Z}\tilde{\Phi}_a^{in}(\delta)$$

$$\tilde{\Phi}_a^{in}(\delta) = \ln\left[\frac{\mu/\delta}{\delta\mu/m^2+\sqrt{e}}\right] - \ln\left[1+\frac{m}{\delta\sqrt{e}B'Z^{-2/3}}\right]$$

B'=1429 for $Z \geq 2$ and B'=446 for Z=1.

$$v_{max} = \frac{m(E-\mu)}{E(E-p+m)}$$

Photonuclear Interactions



$$\begin{aligned}
\frac{d\sigma}{dv} &= \frac{\alpha}{2\pi} A \sigma_{\gamma N} v \left\{ 0.75 G(x) \left[\kappa \ln\left(1+\frac{m_1^2}{t}\right) - \frac{\kappa m_1^2}{m_1^2+t} - \frac{2\mu^2}{t} \right] + \right. \\
&+ 0.25 \left[\kappa \ln\left(1+\frac{m_2^2}{t}\right) - \frac{2\mu^2}{t} \right] + \\
&+ \left. \frac{\mu^2}{2t} \left[0.75 G(x) \frac{m_1^2}{m_1^2+t} + 0.25 \frac{m_2^2}{t} \ln\left(1+\frac{t}{m_2^2}\right) \right] \right\} \qquad (1) \\
\kappa &= 1 - \frac{2}{v} + \frac{2}{v^2} \\
m_1^2 &= 0.54\ \mathrm{GeV}^2 \quad \text{and} \quad m_2^2 = 1.8\ \mathrm{GeV}^2
\end{aligned}$$

$$\begin{aligned}
\sigma_{\gamma A}(\nu) &= A \sigma_{\gamma N}(\nu) \{0.75 G(x) + 0.25\} \\
\text{with } G(x) &= \frac{3}{x^3}\left(\frac{x^2}{2} - 1 + e^{-x}(1+x)\right), \quad \text{for } Z \neq 1 \text{, and 1 for Z=1} \\
x &= Rn\sigma_{\rho N} \simeq 0.00282 A^{\frac{1}{3}} \sigma_{\gamma N}(\nu)
\end{aligned}$$

$$\begin{aligned}
\sigma_{\gamma N}(\nu) &= 114.3 + 1.647 \ln^2[0.0213\nu]\ \mu\mathrm{b}, \quad \text{for } \nu \in [17, 10^6 \mathrm{GeV}] \\
\sigma_{\gamma N}(\nu) &= 96.1 + \frac{82}{\sqrt{\nu}}, \quad \text{for } \nu \leq 17\ \mathrm{GeV}
\end{aligned}$$

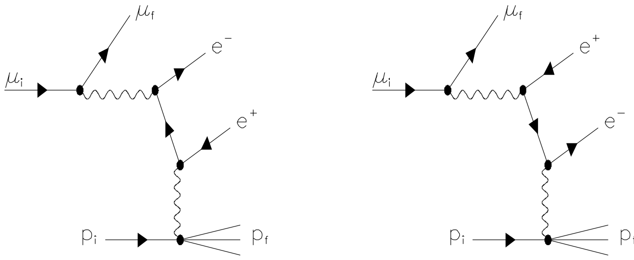
$\sigma_{\gamma N}(\nu) = 49.2 + 11.1 \ln[\nu] + 151.8/\sqrt{\nu}\ \mu$b, above 200 GeV.

$$t = Q^2_{max} = \frac{\mu^2 v^2}{1-v}$$

Integration limits:

$$\begin{aligned}
m_\pi + \frac{m_\pi^2}{2M} \ &< \ \nu \ < E - \frac{M}{2} \cdot \left(1 + \frac{m_\mu^2}{M^2}\right) \\
\frac{m_\mu^2 \nu^2}{EE'} - \frac{m_\mu^4}{2EE'} \ &< \ Q^2 < 2M(\nu - m_\pi) - m_\pi^2 \qquad (2)
\end{aligned}$$

Pair Production



$$\frac{d\sigma(E,v,\rho)}{dv d\rho} = \frac{2}{3\pi} Z(Z+\zeta)(\alpha r_e)^2 \frac{1-v}{v}\left(\Phi_e + \frac{m^2}{\mu^2}\Phi_\mu\right) \tag{3}$$

$$v = (\epsilon_+ + \epsilon_-)/E, \quad \rho = (\epsilon_+ - \epsilon_-)/E$$

$$\begin{aligned}
\Phi_e &= \left\{[(2+\rho^2)(1+\beta)+\xi(3+\rho^2)]\ln\left(1+\frac{1}{\xi}\right)+\frac{1-\rho^2-\beta}{1+\xi}-(3+\rho^2)\right\}L_e \\
\Phi_\mu &= \left\{\left[(1+\rho^2)\left(1+\frac{3}{2}\beta\right)-\frac{1}{\xi}(1+2\beta)(1-\rho^2)\right]\ln(1+\xi)+\right. \\
&+ \left.\frac{\xi(1-\rho^2-\beta)}{1+\xi}+(1+2\beta)(1-\rho^2)\right\}L_\mu \\
L_e &= \ln\left(\frac{BZ^{-1/3}\sqrt{(1+\xi)(1+Y_e)}}{1+\frac{2m\sqrt{e}BZ^{-1/3}(1+\xi)(1+Y_e)}{Ev(1-\rho^2)}}\right)-\frac{1}{2}\ln\left[1+\left(\frac{3m}{2\mu}Z^{1/3}\right)^2(1+\xi)(1+Y_e)\right] \\
L_\mu &= \ln\left(\frac{\frac{2}{3}\frac{\mu}{m}BZ^{-2/3}}{1+\frac{2m\sqrt{e}BZ^{-1/3}(1+\xi)(1+Y_\mu)}{Ev(1-\rho^2)}}\right) \\
Y_e &= \frac{5-\rho^2+4\beta(1+\rho^2)}{2(1+3\beta)\ln(3+1/\xi)-\rho^2-2\beta(2-\rho^2)} \\
Y_\mu &= \frac{4+\rho^2+3\beta(1+\rho^2)}{(1+\rho^2)(3/2+2\beta)\ln(3+\xi)+1-\frac{3}{2}\rho^2} \\
\beta &= \frac{v^2}{2(1-v)}, \quad \xi = \left(\frac{\mu v}{2m}\right)^2\frac{1-\rho^2}{1-v}
\end{aligned} \tag{4}$$

$$\zeta_{loss}^{pair}(E,Z) \sim \frac{0.073 \ln\left(\frac{E/\mu}{1+\gamma_1 Z^{2/3} E/\mu}\right) - 0.26}{0.058 \ln\left(\frac{E/\mu}{1+\gamma_2 Z^{1/3} E/\mu}\right) - 0.14}$$

here $\gamma_1 = 1.95\ 10^{-5}$ and $\gamma_2 = 5.3\ 10^{-5}$,
respectively $\gamma_1 = 4.4\ 10^{-5}$ and $\gamma_2 = 4.8\ 10^{-5}$ for hydrogen.

Integration limits:

$$\begin{aligned}
\frac{4m}{E} &= v_{min} \leq v \leq v_{max} = 1 - \frac{3\sqrt{e}}{4}\frac{\mu}{E}Z^{1/3} \\
0 &\leq |\rho| \leq \rho_{max} = \sqrt{1 - \frac{4m}{Ev}}\left[1 - \frac{6\mu^2}{E^2(1-v)}\right]
\end{aligned}$$

Decay

Decay probability is calculated according to

$$\frac{dN}{dx} = \frac{1}{\gamma\beta c\tau}$$

Physics of the calculation

1. **Energy and Tracking integrals**

   (a) All energy losses are devided into 2 parts: stochastic and continuous. Let the sum of the continuous part be described by f(E):

   $$-\frac{dE}{dx} = f(E)$$

   (b) Then $\sigma(E)$ describes the stochastic part,

$$(1 - dP(E(x_i))) \cdot ... \cdot (1 - dP(E(x_f)))$$

$$= \exp(-\sigma(E(x_i)) \cdot dx) \cdot ... \cdot \exp(-\sigma(E(x_f)) \cdot dx)$$

$$= \exp(-\int \sigma(E(x))) \cdot dx$$

$$= \exp(-\int \frac{\sigma(E)}{-f(E)} \cdot dE) = \xi$$

   where ξ is a random number from 0 to 1.

To find final energy on each step we need to solve equation (b):

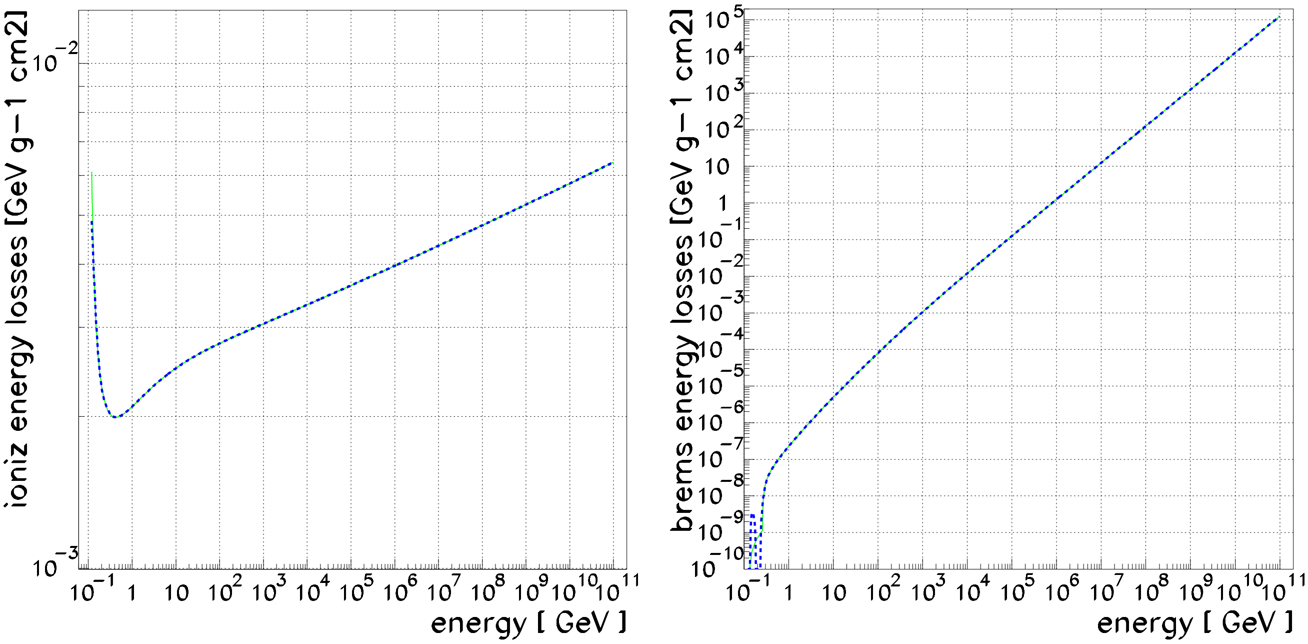
$$\int^{E_f} \frac{\sigma(E)}{-f(E)} \cdot dE = -\log(\xi) \qquad \text{(energy integral)}$$

and then to find corresponding displacement we solve integral (a):

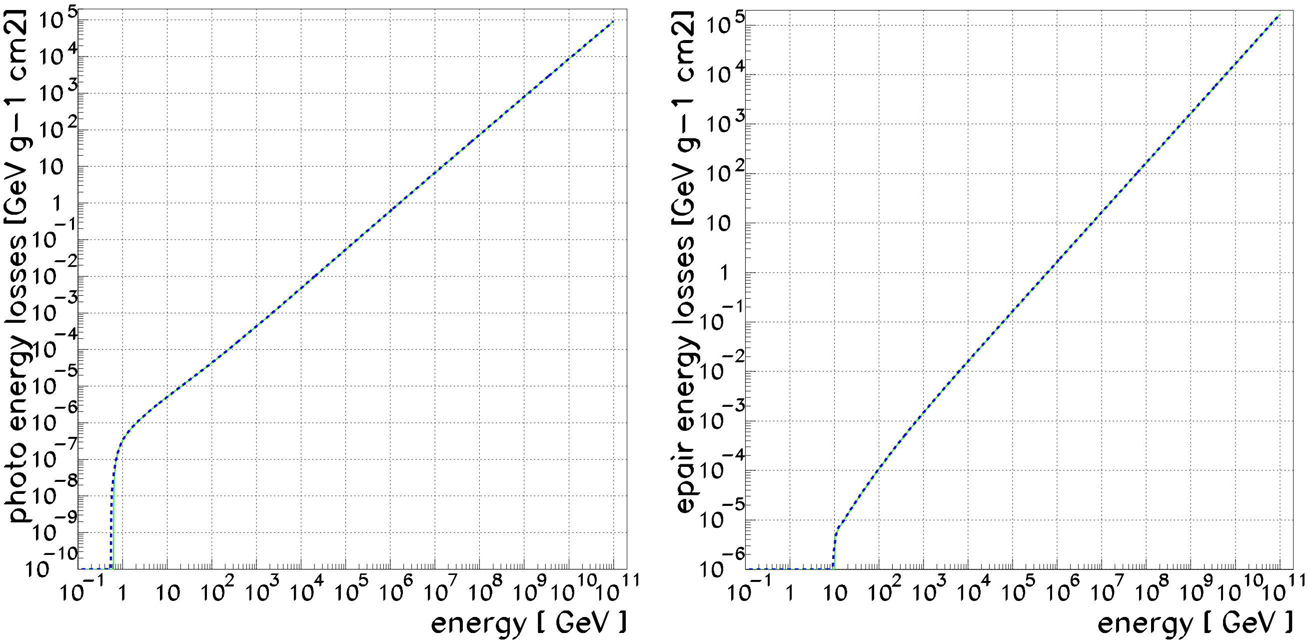
$$x = -\int^{E_f} \frac{dE}{f(E)} \qquad \text{(tracking integral)}$$

Status of the code

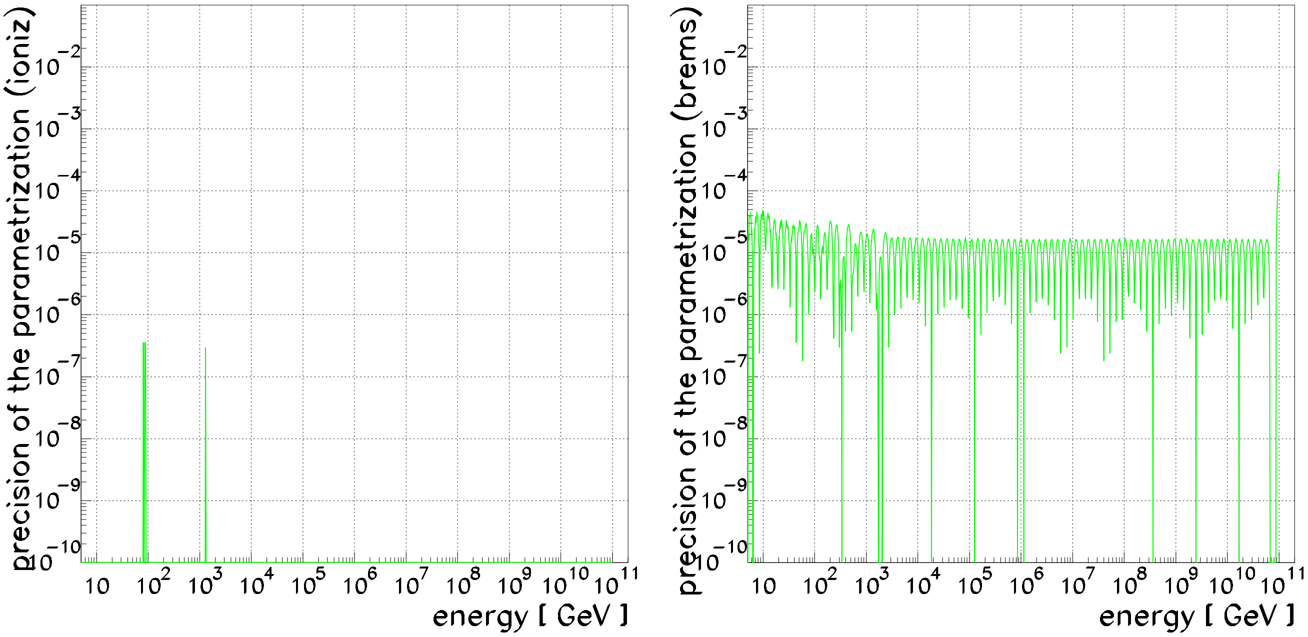
1. Program is complete for use: all cross sections are in the code
2. Interpolation formulae and calculation of integrals for all cross sections, energy and tracking integrals are optimized
3. Integrated in the AMANDA mass production monte carlo and Frejus data analysis chain
4. The output of the program was extensively examined against various sources and the agreement was found satisfactory or on paar with the best of the competition
5. Code can be used for both muons and taus
6. Code is 3-dimension ready, with the angular dependence of the cross sections to be implemented
7. Code is available for download (current version 0.08.5), it includes README files and java generated documentation pages



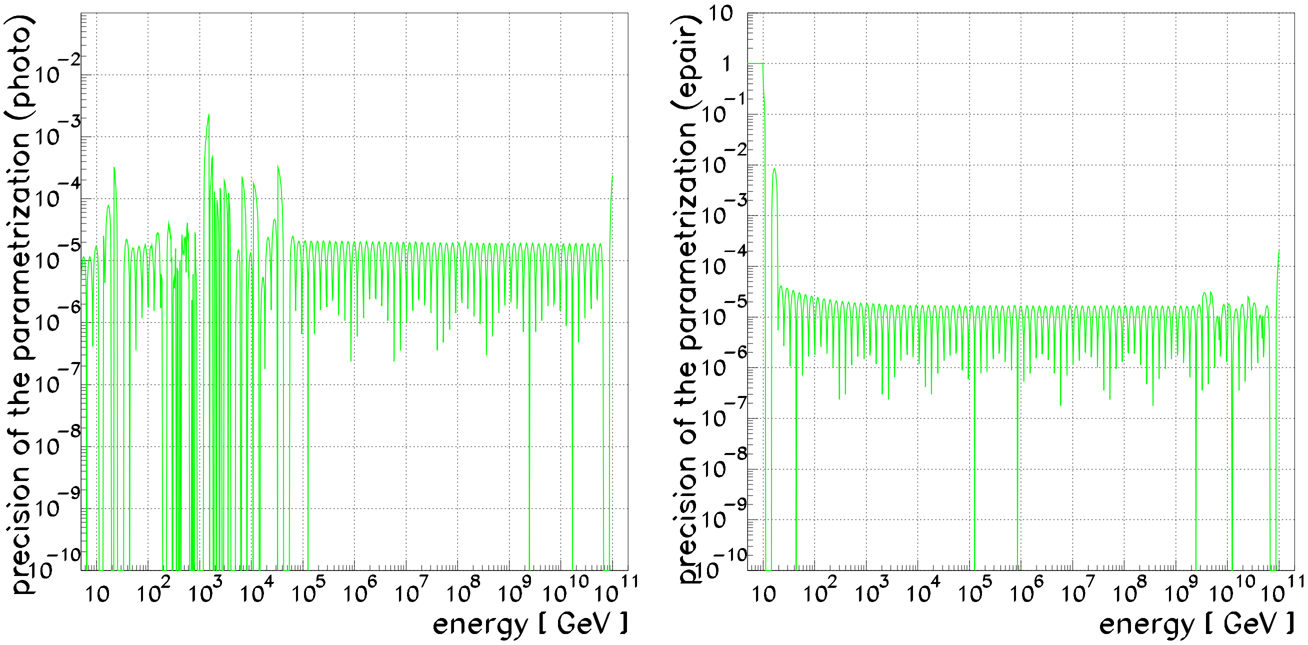
Comparison of parametrized with non-parametrized calculations



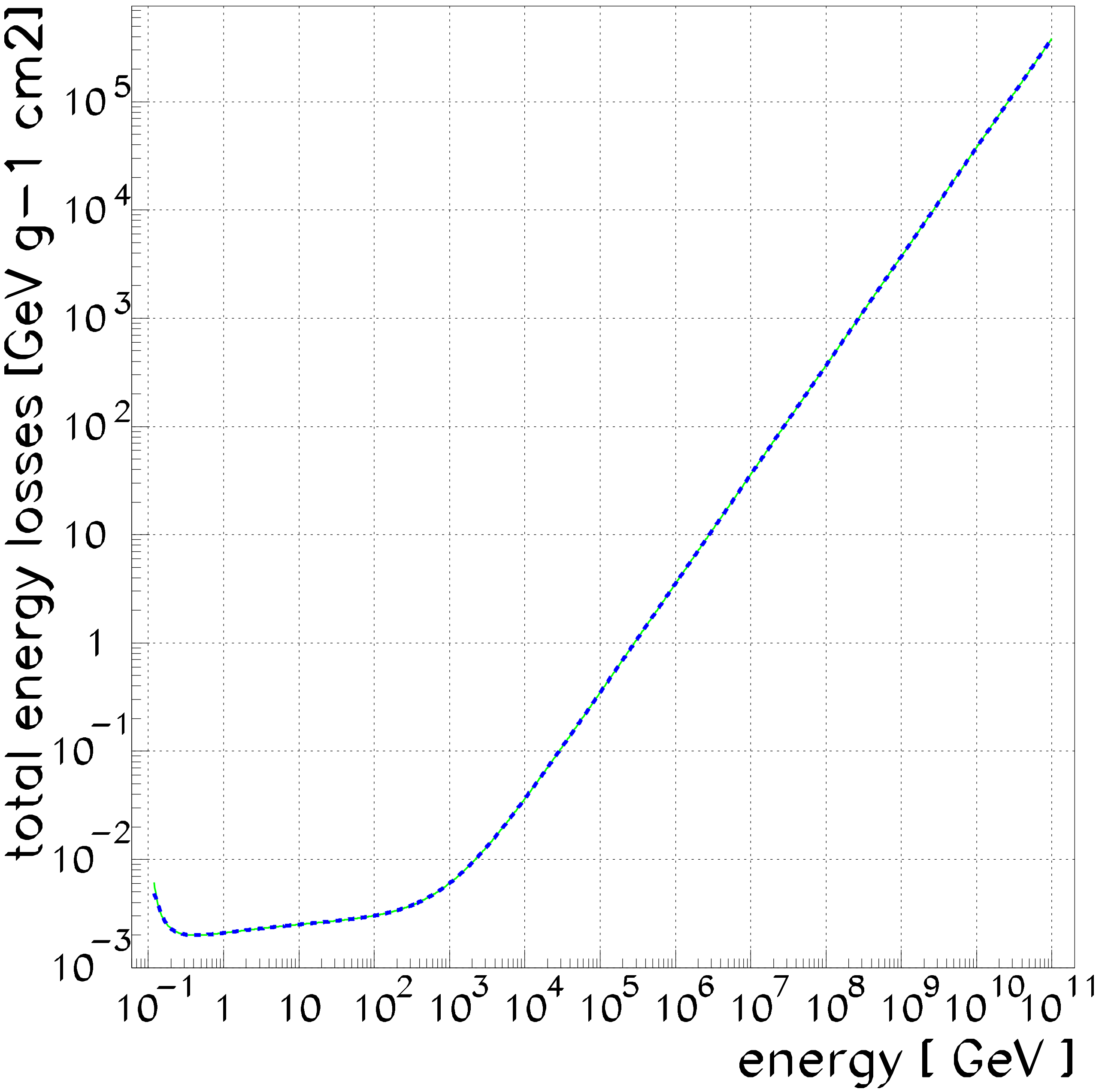
Comparison of parametrized with non-parametrized calculations



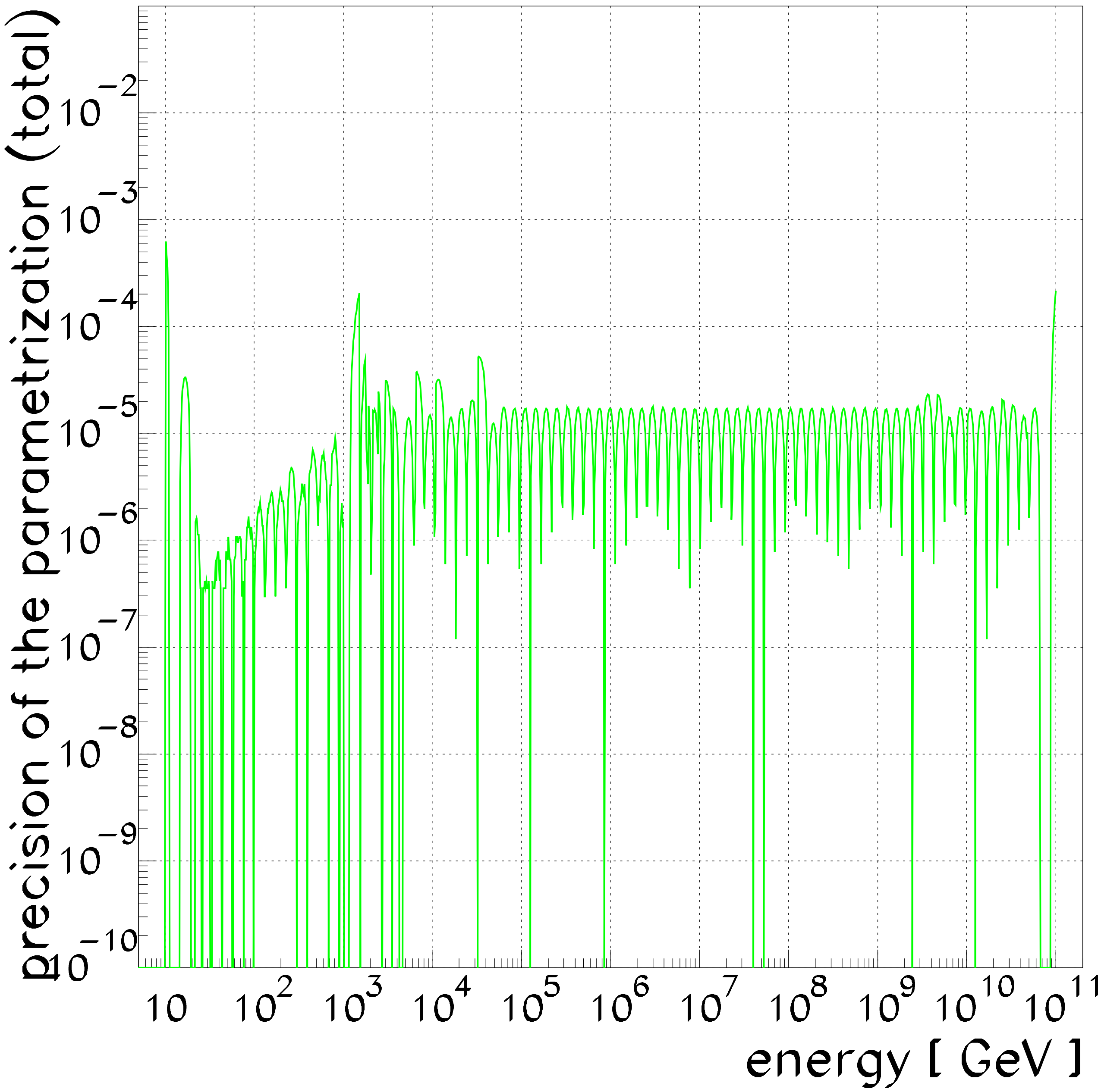
Precision $(e_{pa} - e_{np})/e_{pa}$ of parametrized relative to non-parametrized version



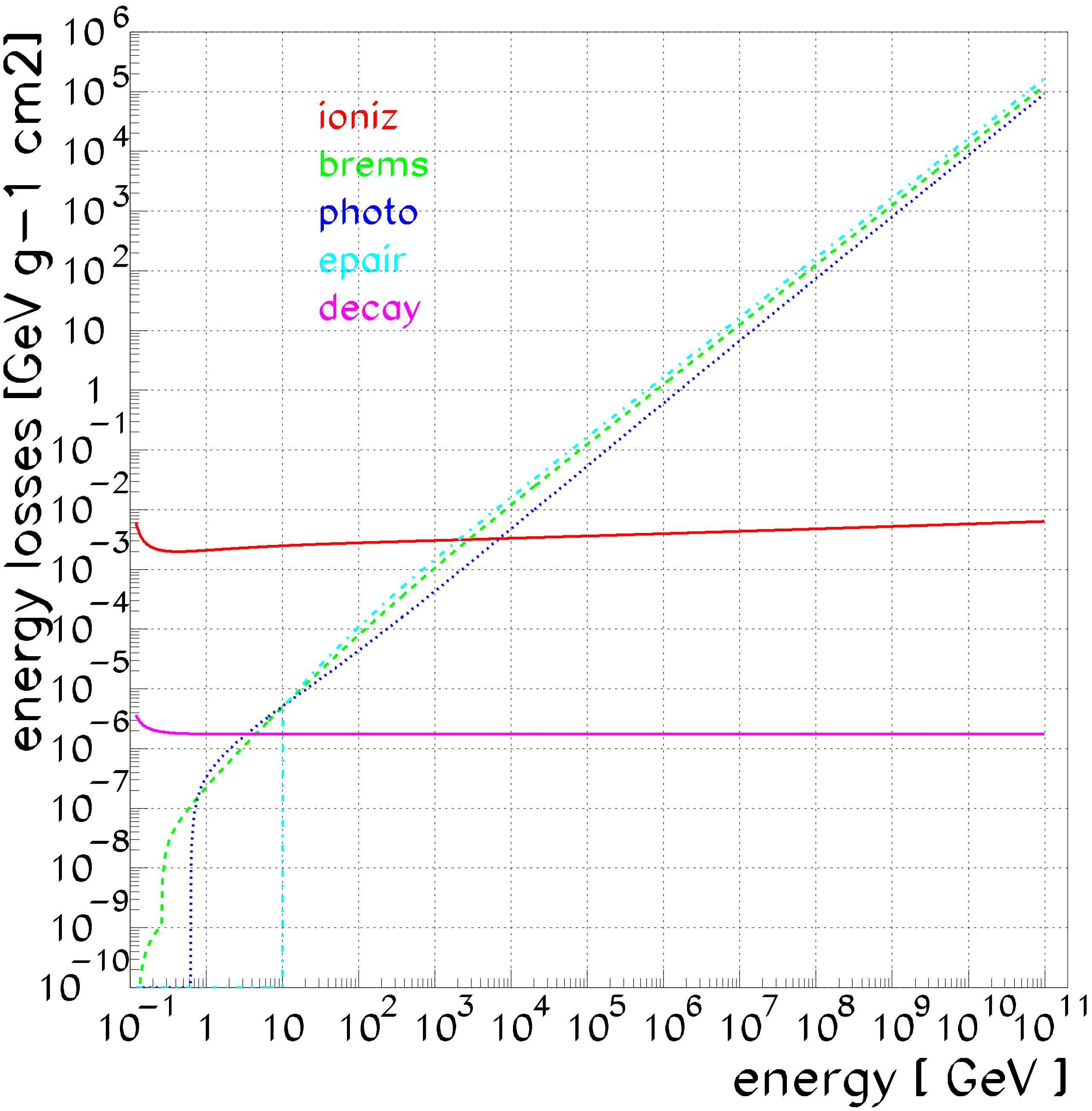
Precision $(e_{pa} - e_{np})/e_{pa}$ of parametrized relative to non-parametrized version



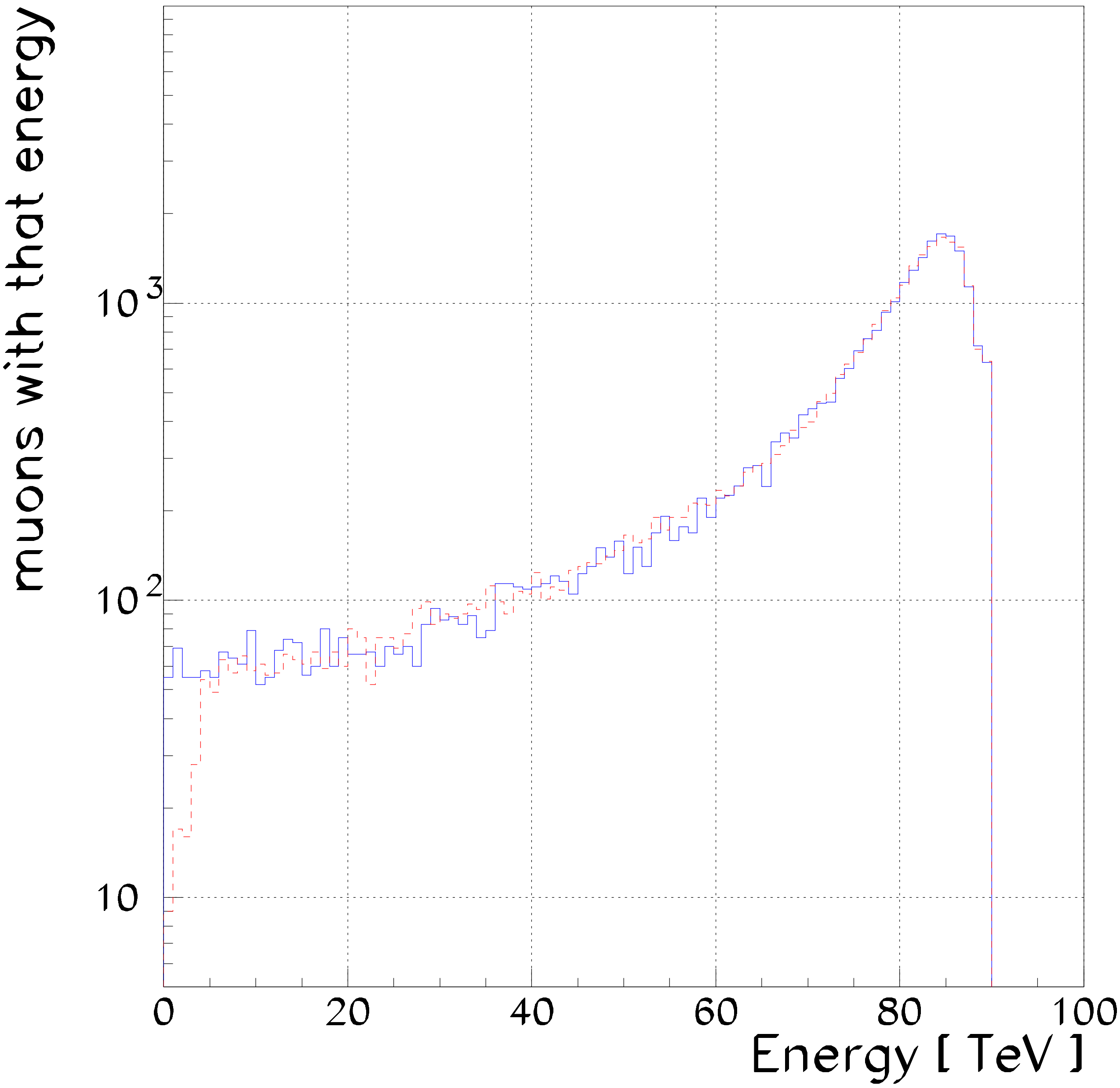
Comparison of the total parametrized with non-parametrized energy losses



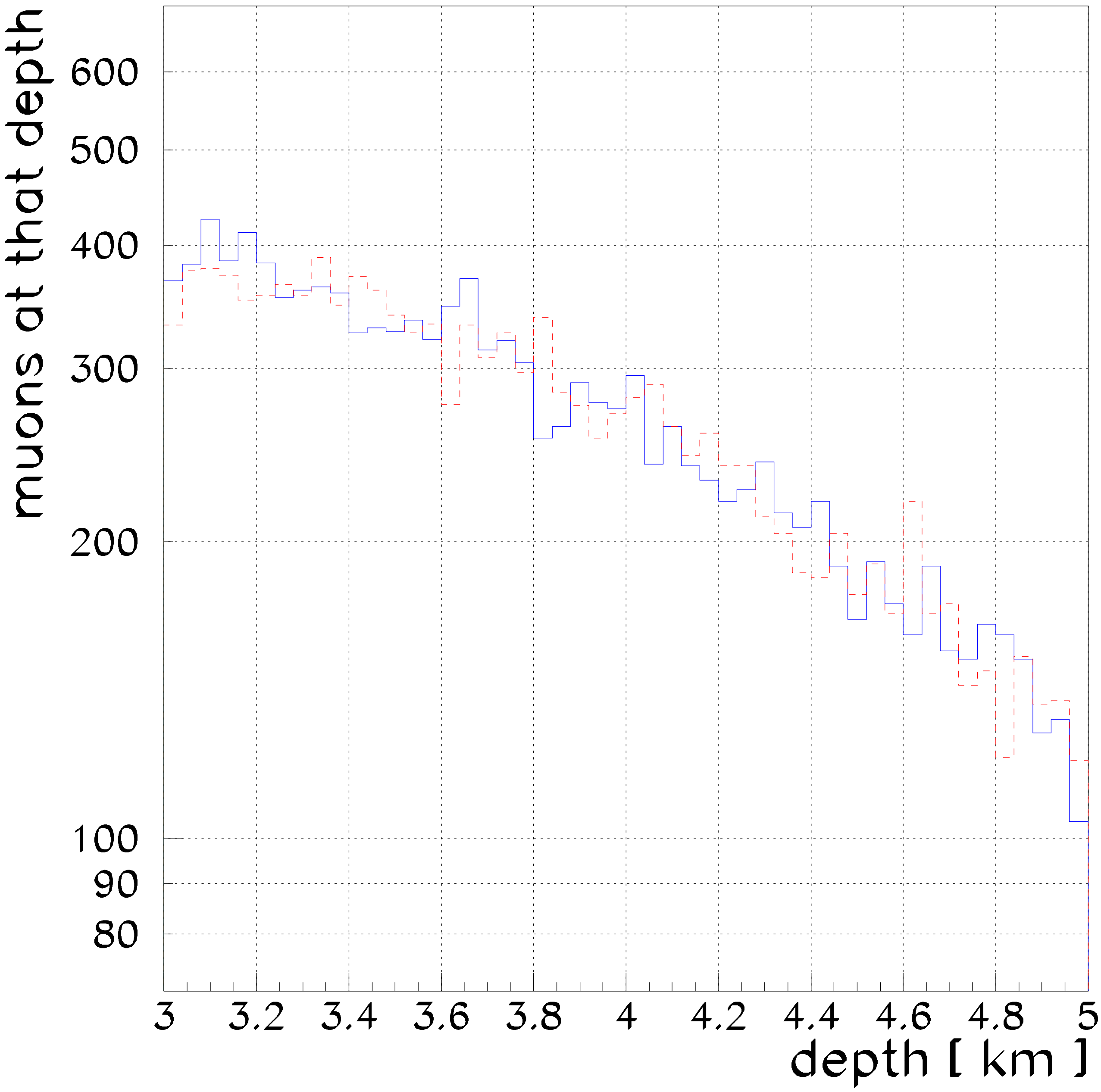
Precision $(e_{pa} - e_{np})/e_{pa}$ of parametrized relative to non-parametrized version for total energy losses



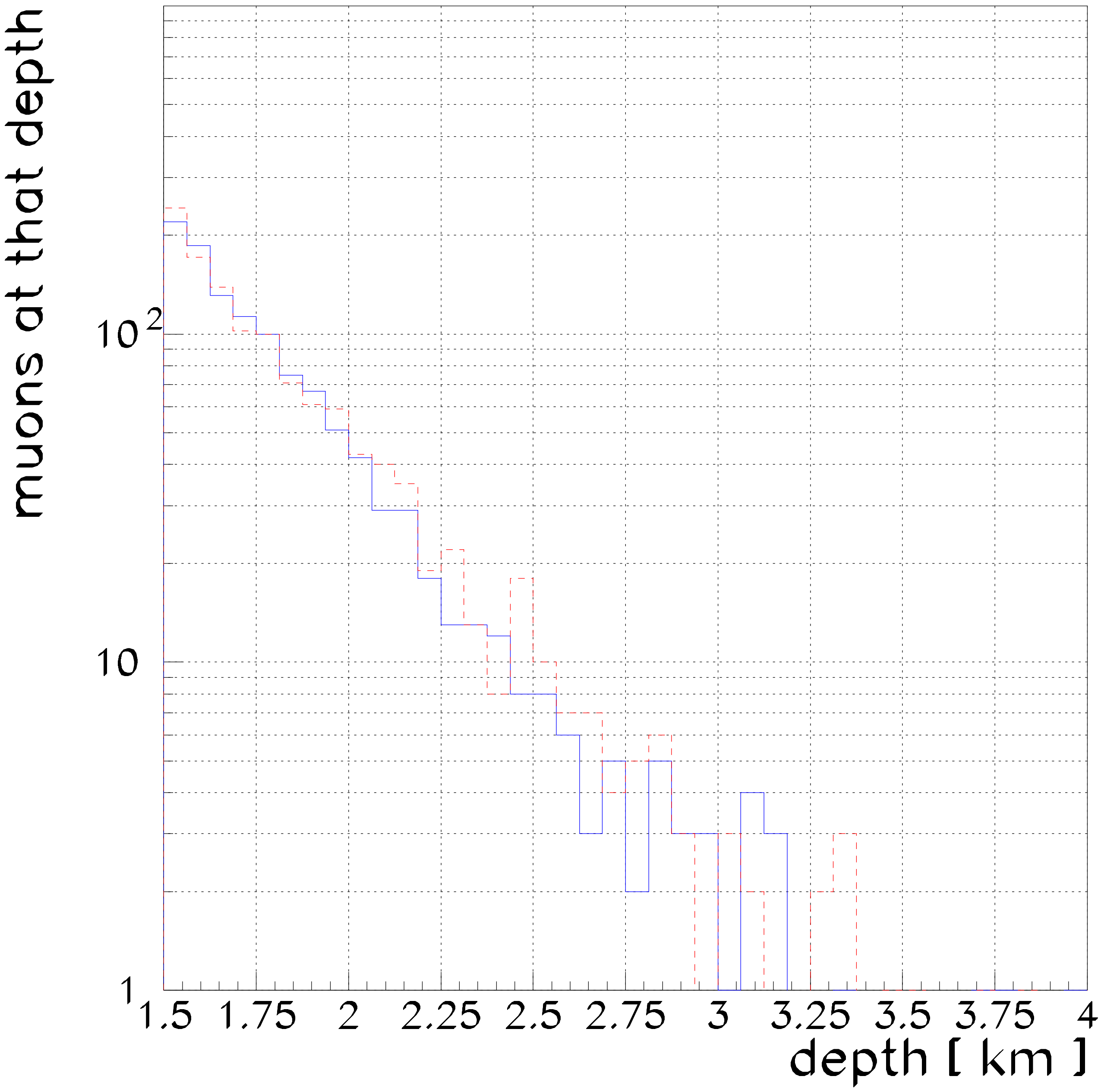
All energy losses: ioniz (upper solid curve), brems (dashed), photo (dotted), epair (dashed-dotted) and decay (lower solid curve)



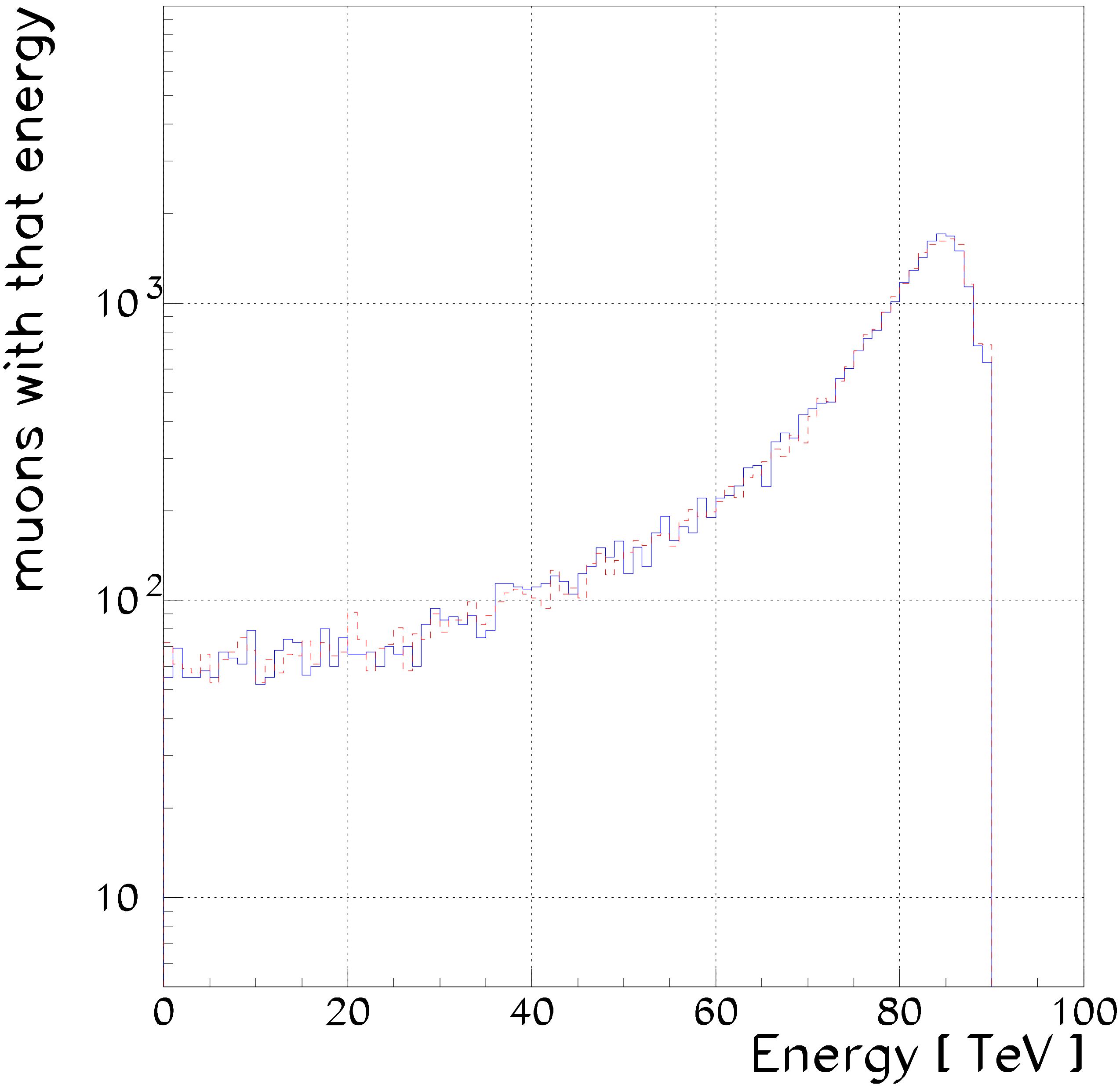
distribution of the final energy of the muons that crossed 300 m of Frejus Rock with initial energy 100 TeV, v 0.08.4



depth distribution of muons that reached that depth in a Frejus-type test. The initial energy of muons was taken at random according to the $E^{-}3.7$ from 100 TeV and up, and depth range was uniformly chosen between 3 and 5 km. v0.08.4

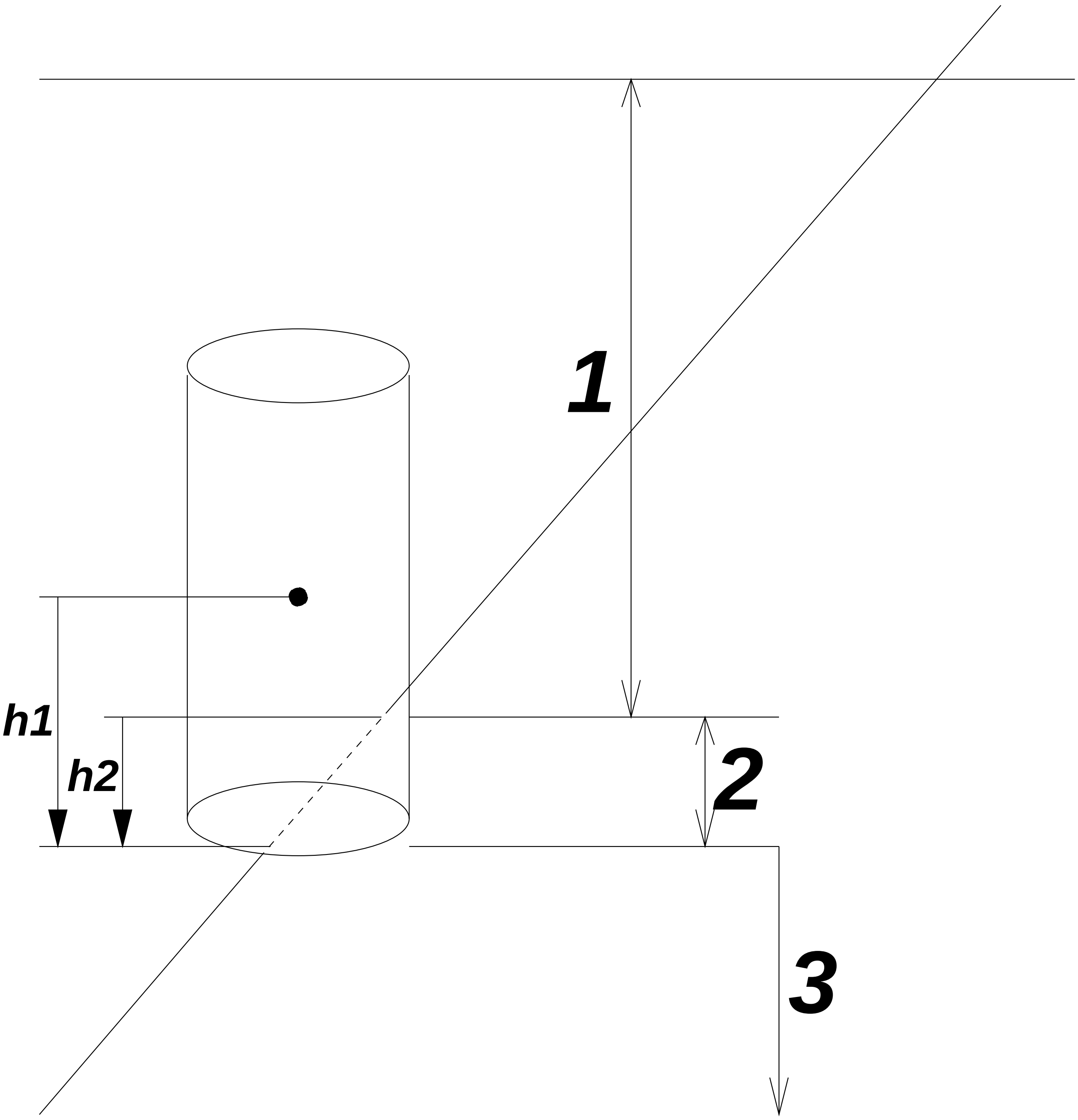


depth distribution of muons that reached that depth in a Frejus-type test. The initial energy of muons was taken at random according to the $E^{-}3.7$ from 1 TeV and up, and depth range was uniformly chosen between 1.5 and 4 km. Only 1.15+-.34% (exact) or 1.20+-.35% (parametrized) out of all muons reach the given depth. v0.08.4



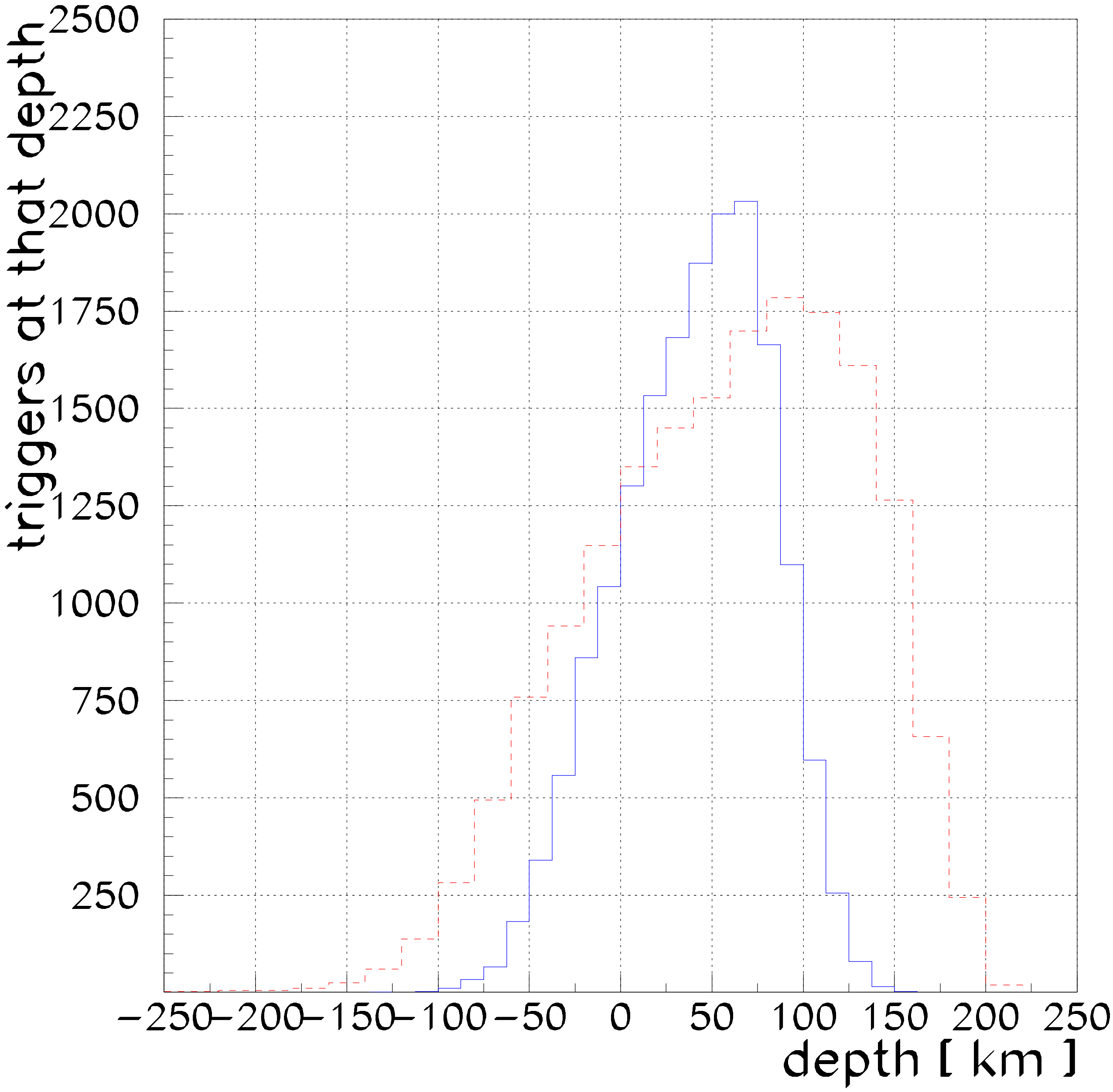
distribution of the final energy of the muons that crossed 300 m of Frejus Rock with initial energy 100 TeV done with the new version of the program. For a list of changes refer to the HISTORY file. v0.08.5

Amanda.java

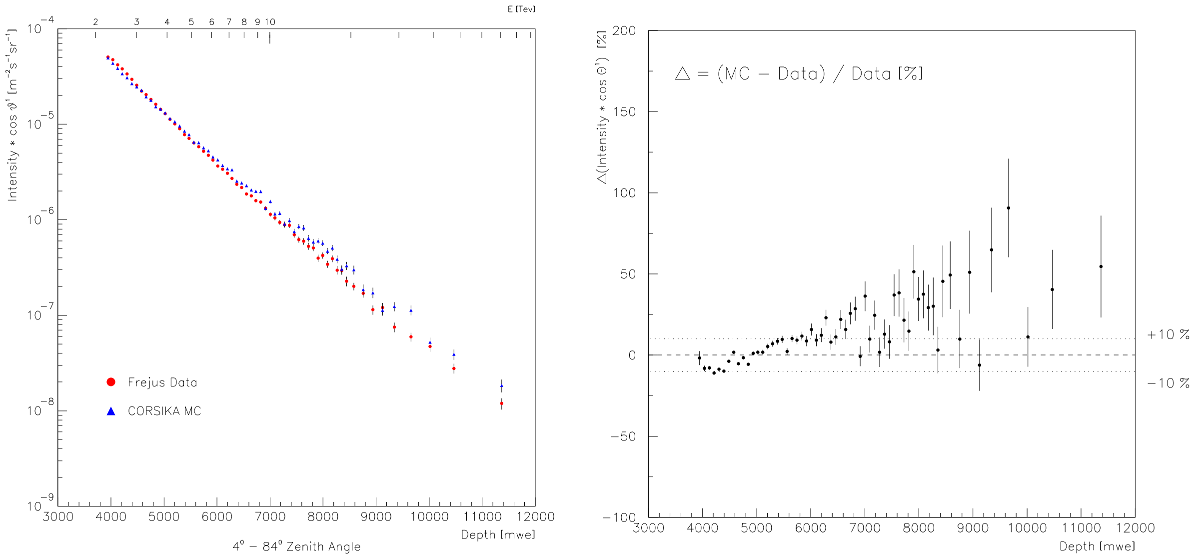


1. $v_{cut} = 0.1$
2. $e_{cut} = 500$ MeV
3. $v_{cut} = 1.$

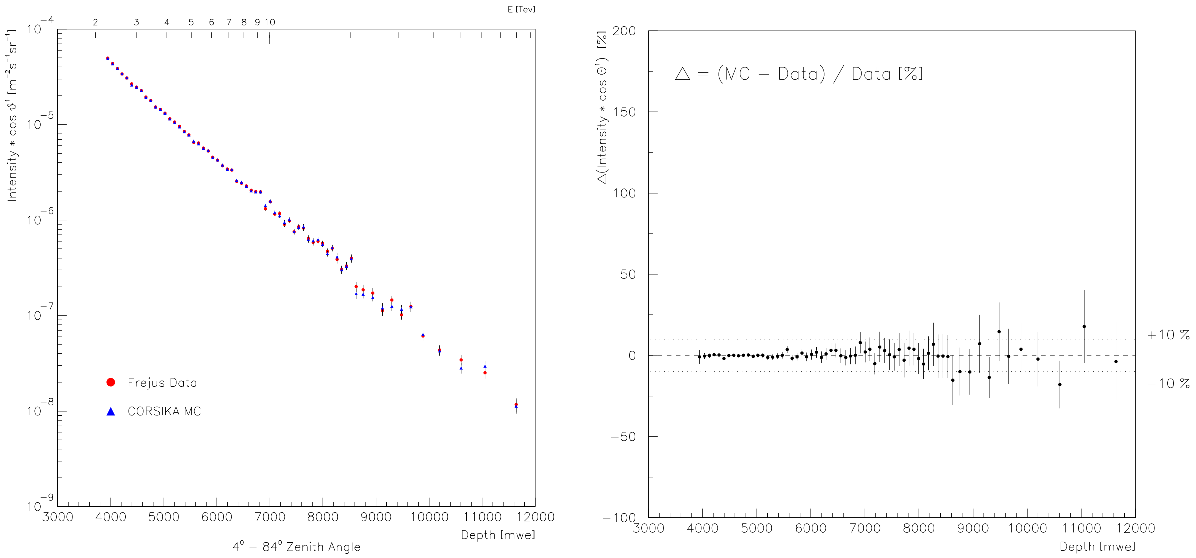
Interactions are only saved for output in region 2.



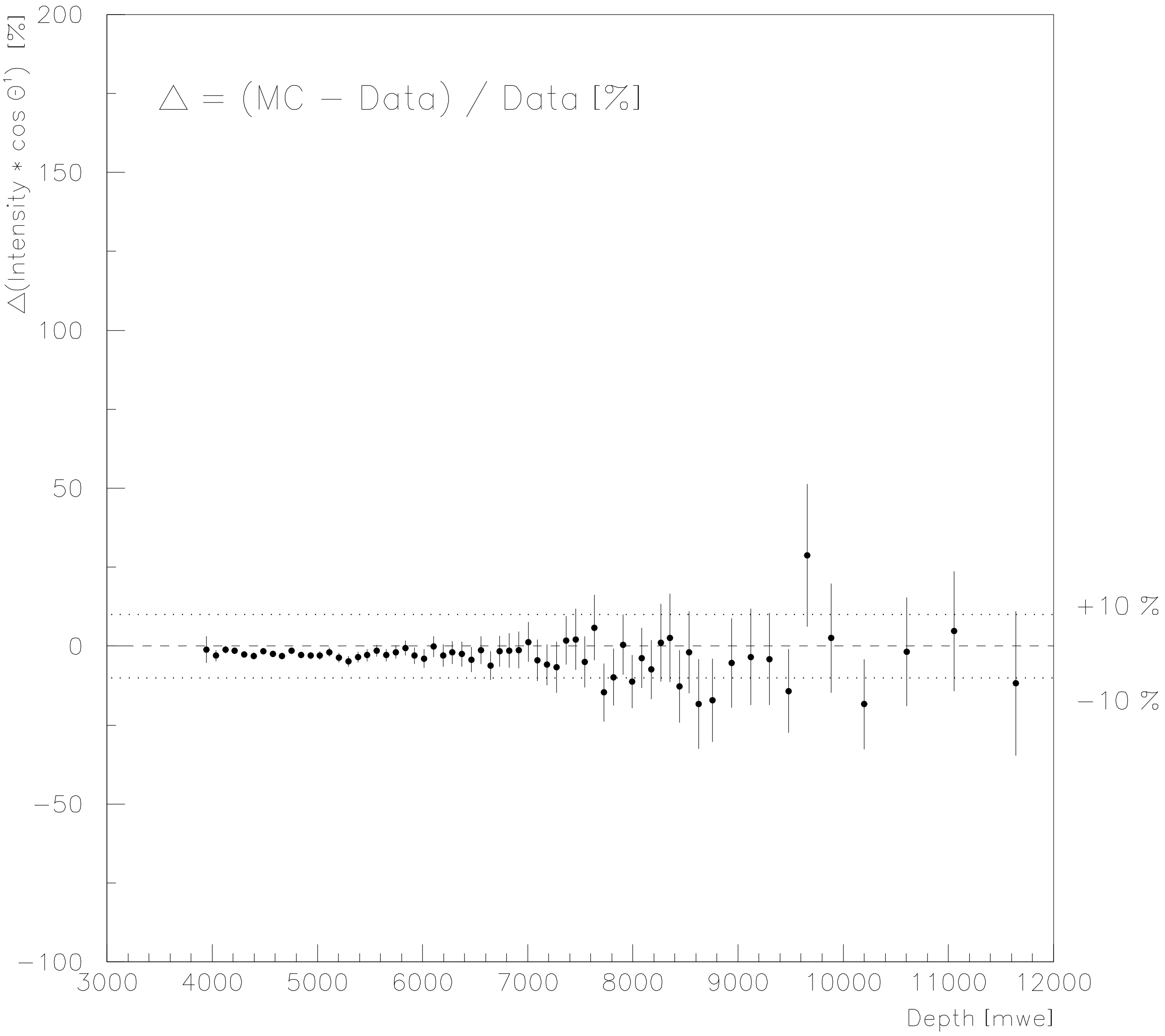
1 and pe weighted trigger depths



Comparison of mmc (vcut=0.1) with data



Comparison of mmc with vcut=0.1 and 0.2



Comparison of mmc with vcut=0.05, elow=120 MeV, and vcut=0.2, elow=240 MeV