Cosmic ray energy spectrum measurement with AMANDA

Ver. 0.2

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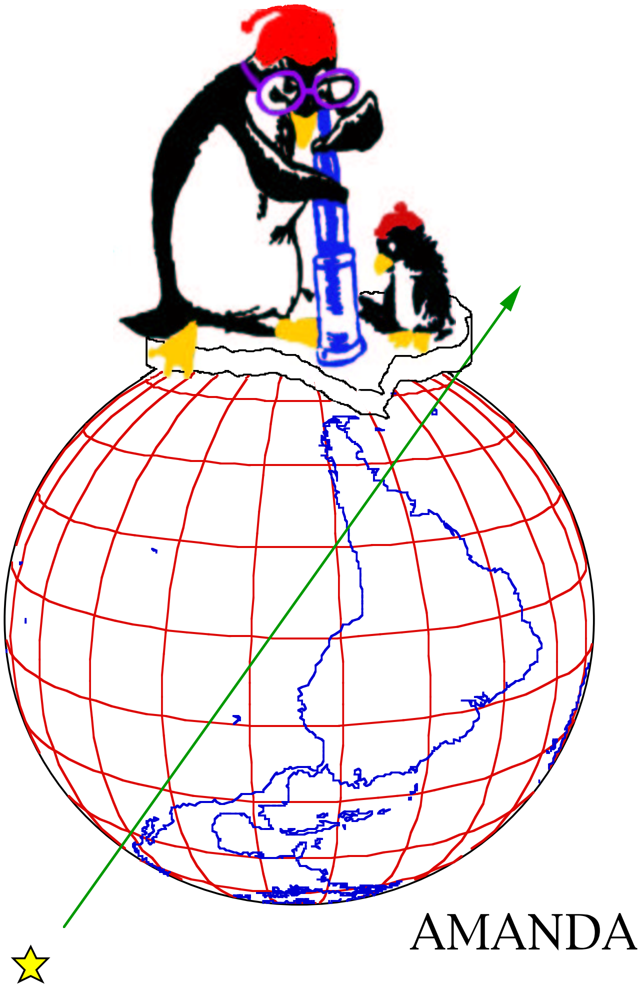
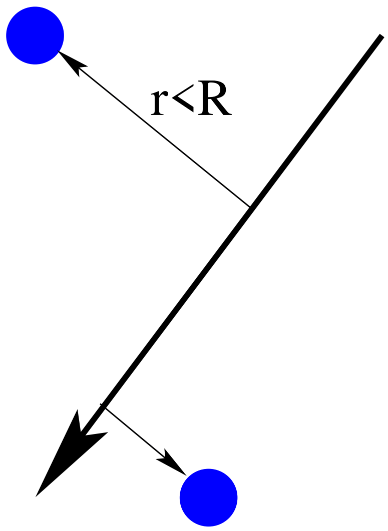
Talk outline



1. A model of N_{ch} distribution
2. Noise cleaning and OM efficiencies
3. Results

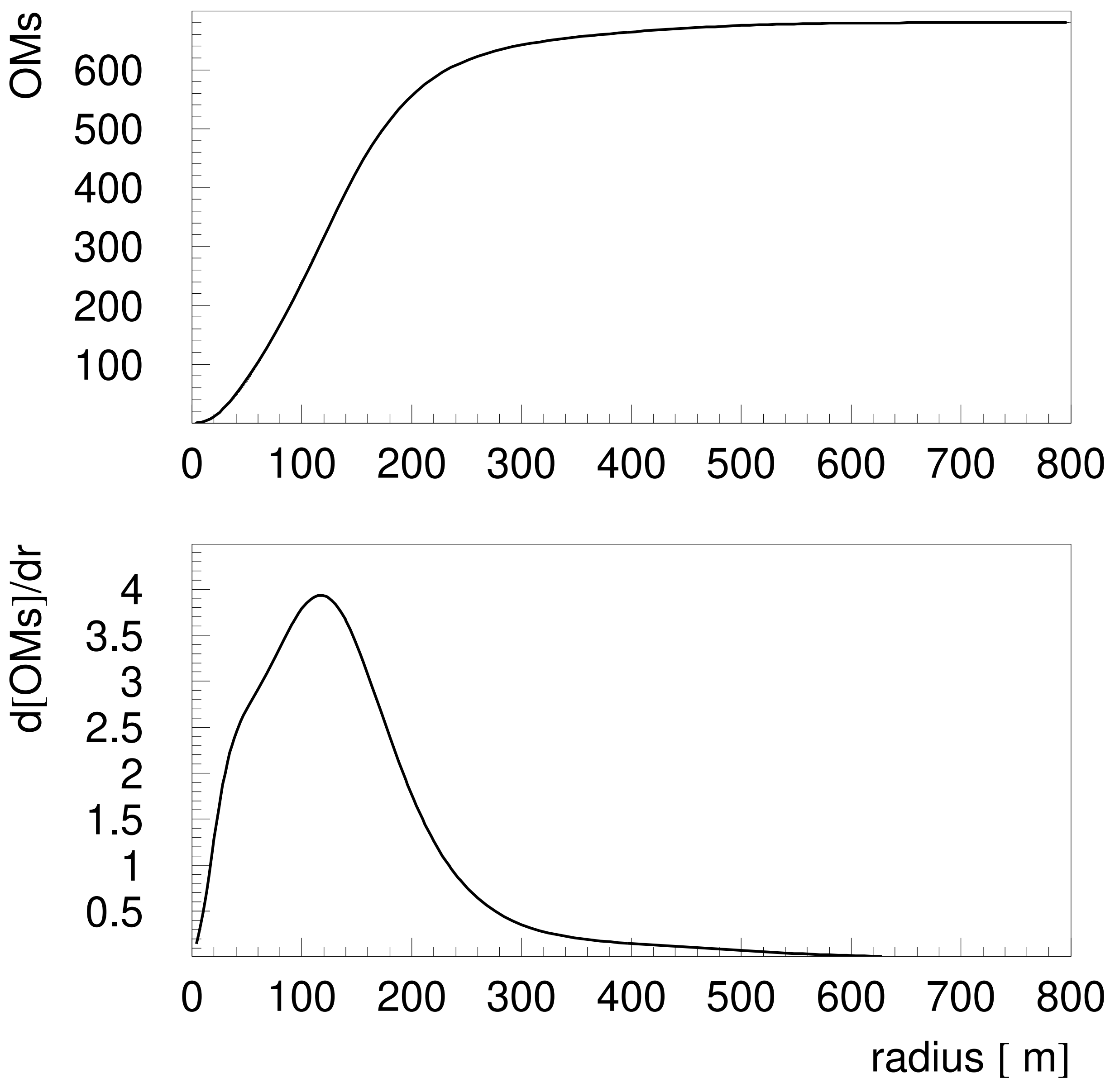
A model of N_{ch} distribution

For a muon track going through the detector a number of OMs within cylinder of radius R centered on the track is counted:



The resulting number of OMs distance R away from the track is averaged over several thousand "representative" muon tracks going through a grid of points with -100 m $< x, y < 100$ m in the $z = 0$ plane with zenith angles $10 \leq \theta < 70$ and azimuth angles $0 \leq \phi < 360$.

The resulting $N(R)$ and $\frac{dN(R)}{dr}$ distributions are shown below:



An "average" muon has most of the detector OMs within 200 m of its track.

The probability to see a photon distance r away from its origin at time t is

$$p(r,t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left\{-\frac{r^2}{4Dt} - \frac{c_i t}{\lambda_a}\right\}$$

Here $D = \dfrac{c_i \lambda_{eff}}{3}$ with $\lambda_{eff} = \dfrac{\lambda_{scat}}{1-\tau}$ and $\tau =< \cos(\theta_{scat}) >$

λ_a is absorption length and c_i is speed of light in the ice.

Probability to see the photon pass through the sphere of radius R_{OM} located at distance r, integrated over time is

$p(r) = \pi R_{OM}^2 \cdot \dfrac{c_i}{4} \cdot \dfrac{1}{4\pi D r} e^{-\frac{r}{\lambda_{prop}}}$, where $\lambda_{prop} = \sqrt{\dfrac{\lambda_a \lambda_{eff}}{3}}$

Introducing OM sensitivity $s = \frac{1}{2} \cdot$ *efficiency*, probability to record the photon at distance r is

$$P(r) = s \cdot \frac{3R_{OM}^2}{16\lambda_{eff}} \cdot \frac{1}{r} e^{-\frac{r}{\lambda_{prop}}}$$

This is only valid for $r \gg \lambda_{eff}$. In the opposite case of very small r, we obviously have

$$P(r) = s \cdot \frac{\pi R_{OM}^2}{4\pi r^2} e^{-\frac{r}{\lambda_a}}$$

In the following both functions are stitched together and λ_a is assumed $\sim \lambda_{prop}$.

The muon passing through the detector deposits an amount of energy proportional to its average energy while inside the detector. Most of this energy is eventually passed to the photons which are then detected with the probability derived above. In the following it is assumed that the number of photons produced by the muon is proportional to its energy: $N_{phot} = E/E_0$.

A muon with energy E produces an event with on average $N(E)$ hit OMs:

$$\begin{aligned} N(E) &= \int_{+0}^{\infty} \left(1 - (1 - P(r))^{E/E_0}\right) \frac{dN(r)}{dr} dr \\ &\approx \int_{+0}^{\infty} \left(1 - e^{-P(r)\cdot E/E_0}\right) \frac{dN(r)}{dr} dr \end{aligned}$$

The N_{ch} distribution is then given by

$$\begin{aligned} d\Phi \approx \Phi_0 \left(\frac{E}{E_0}\right)^{-(\gamma+1)} dE &= \Phi_0 \left(\frac{E(N)}{E_0}\right)^{-(\gamma+1)} \frac{dN}{dN/dE} \\ &= P(N) dN \end{aligned}$$

$$\text{where} \quad \frac{dN}{dE} = \int_{+0}^{\infty} \frac{P(r)}{E_0} e^{-P(r)\cdot E/E_0} \frac{dN(r)}{dr} dr$$

In the above the muon flux is assumed to have a power law spectrum with exponent $-(\gamma+1)$. For high enough energy muons obey this energy spectrum at the surface, according to the Gaisser formula:

$$\frac{dN}{dE} \propto E^{-\gamma}(\frac{1}{1+\frac{1.1E_\mu \cos\vartheta}{115\ \mathrm{GeV}}} + \frac{0.054}{1+\frac{1.1E_\mu \cos\vartheta}{850\ \mathrm{GeV}}})$$

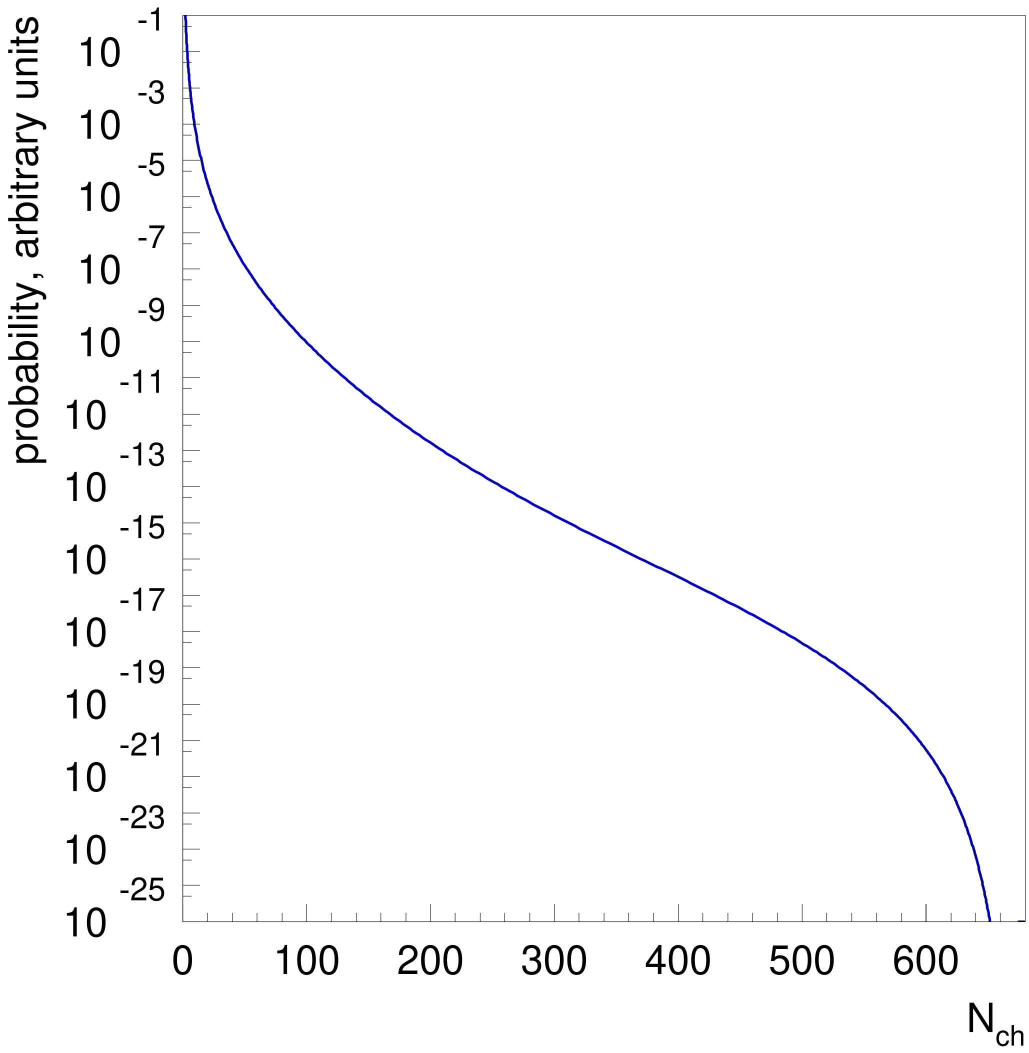
According to the muon energy loss approximation

$$\frac{dE}{dx} = a + b \cdot E \quad \text{with } a = 0.259\frac{\text{GeV}}{\text{mwe}},\ b = 0.357\frac{10^{-3}}{\text{mwe}}$$

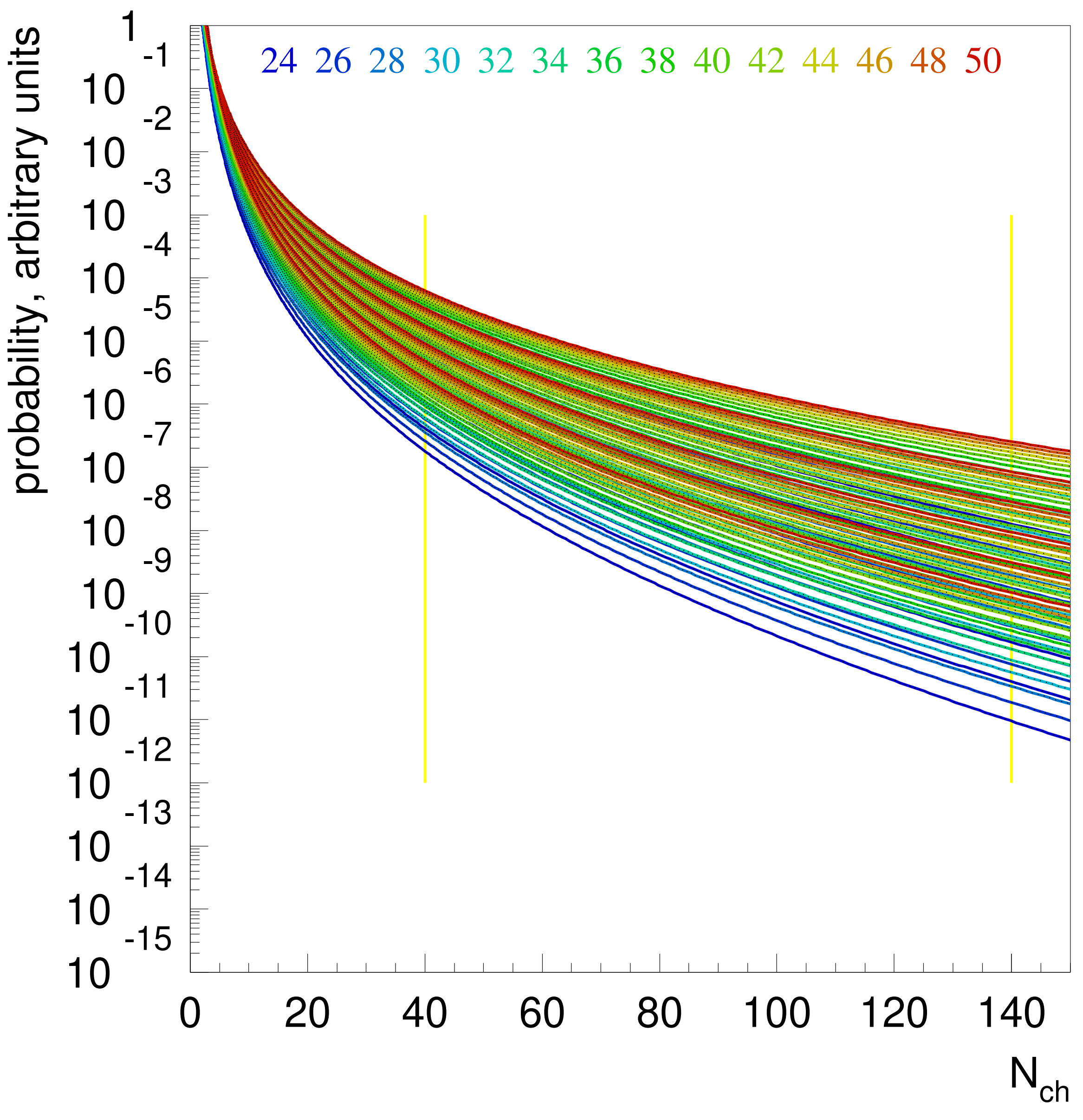
$$\Rightarrow E_f = E_i \cdot e^{-bx} - \frac{a}{b}(1 - e^{-bx})$$

i.e. $E_f \sim E_i$ for high enough initial energy. As the energy of the muon drops below 1 TeV, the muon energy spectrum is no longer a power law with $-(\gamma+1)$ exponent. However, for the above treatment it is only important that the energy spectrum of muons at the location of the detector is falling with the slope which depends on γ.

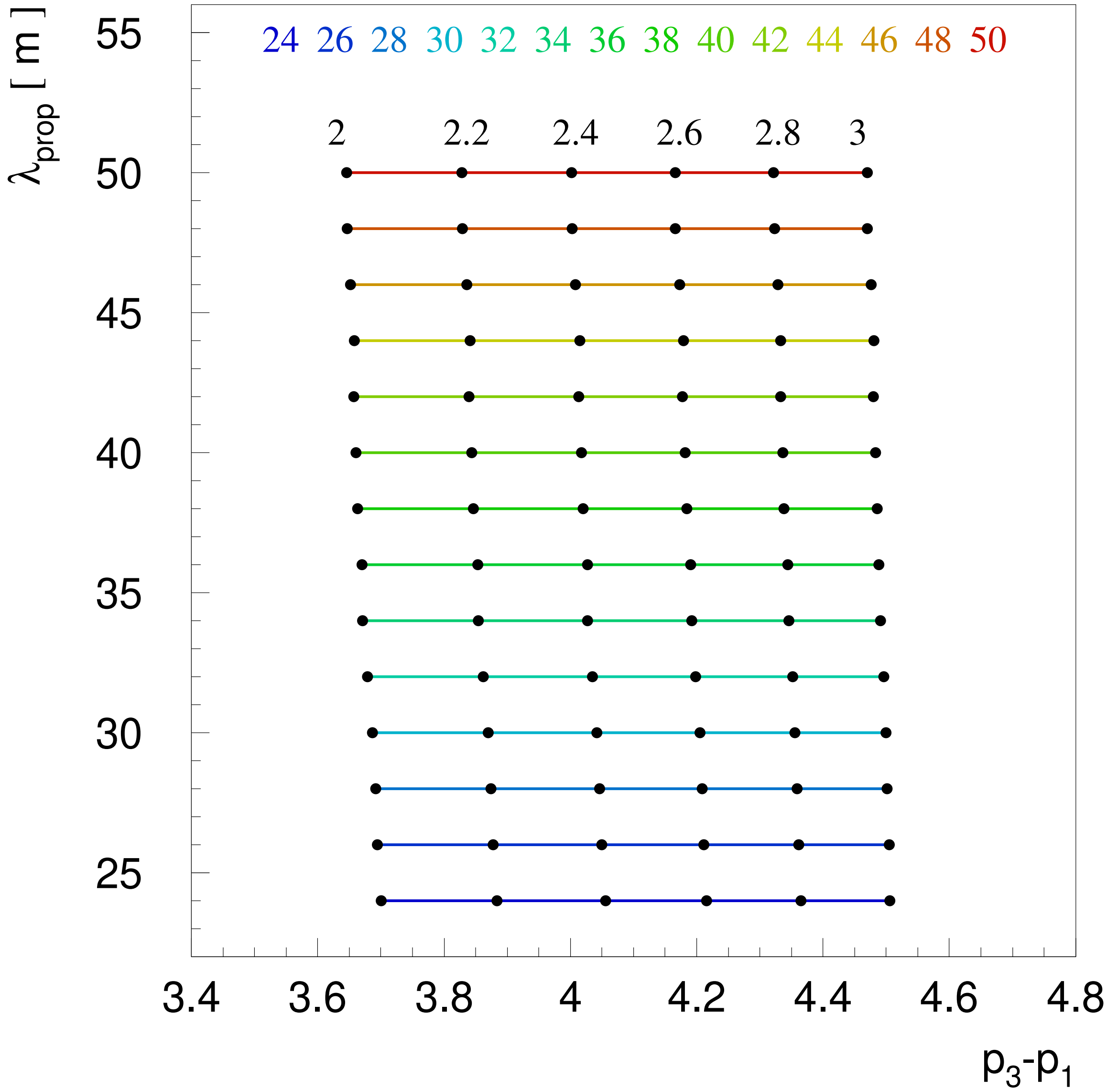
Since $P(r)$ comes in a combination $P(r)/E_0$, the only effect of the sensitivity s of the OMs is on the overall normalization of the N_{ch} distribution. By changing E_0 to compensate for changes in s we simply change region of energies which the detector can see. The resulting N_{ch} distribution for $s = 0.17$, $\lambda_{eff} = 100$ m, $\lambda_{prop} = 35$ m, $\gamma = 2.76$ and $E_0 = 200$ MeV is shown below:



Since λ_{prop} is known with limited precision, it would be interesting to find a quantity which depends strongly on γ, but less so on λ_{prop}. Below is a plot with N_{ch} distributions for 6 values of γ from 2.0 to 3.0. For each γ 14 plots for λ_{prop} varying from 24 to 50 m are shown in different colors.



All curves were fitted between yellow lines with $\exp(p_1 + p_2 \cdot N_{ch}) + \exp(p_3 + p_4 \cdot N_{ch})$ and a quantity $\log(\exp(p_3)/\exp(p_1)) = p_3 - p_1$ was found to depend most weakly on the λ_{prop} while changing significantly with γ:



Noise cleaning and OM efficiency

In the above several simplifications were made:

1. Number of OMs $N(R)$ within distance R of the muon track was assumed the same for all OMs.
2. Scattering and absorption lengths and OM sensitivities were assumed to be constant throughout the detector.
3. Muon is assumed to go through the detector without changing its energy.
4. Only average N_{ch} was evaluated for each muon energy.

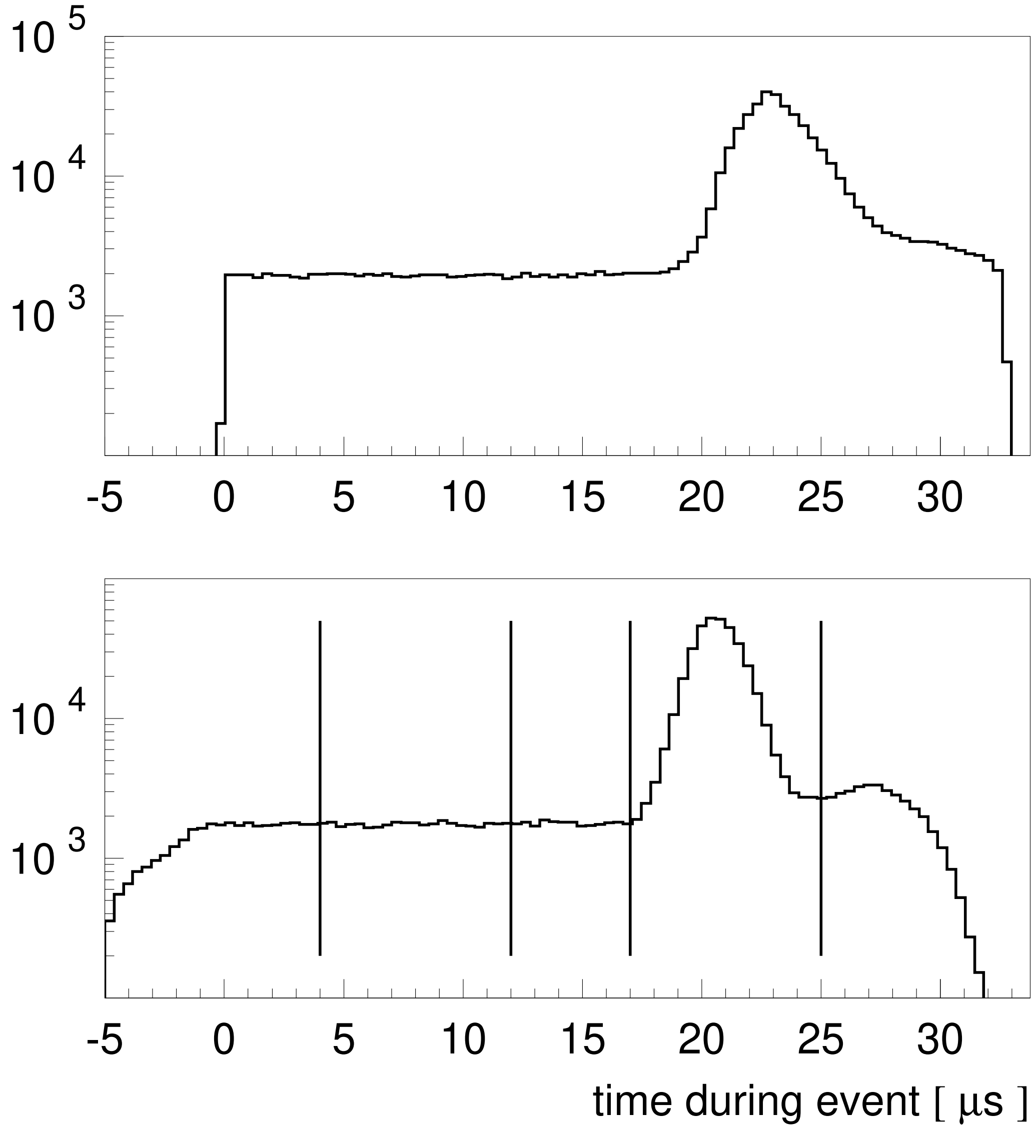
All these simplifications can be removed in the detailed detector simulation (montecarlo using amasim).

Before the above algorithm is applied to data or montecarlo, signals from all OMs are corrected in the following way:

1. noise is subtracted from signal
2. muons passing through the depth of each OM and not recorded by it are added to the signal

I.e. "muon signal" $=$ signal $-$ noise $+$ missed signal.

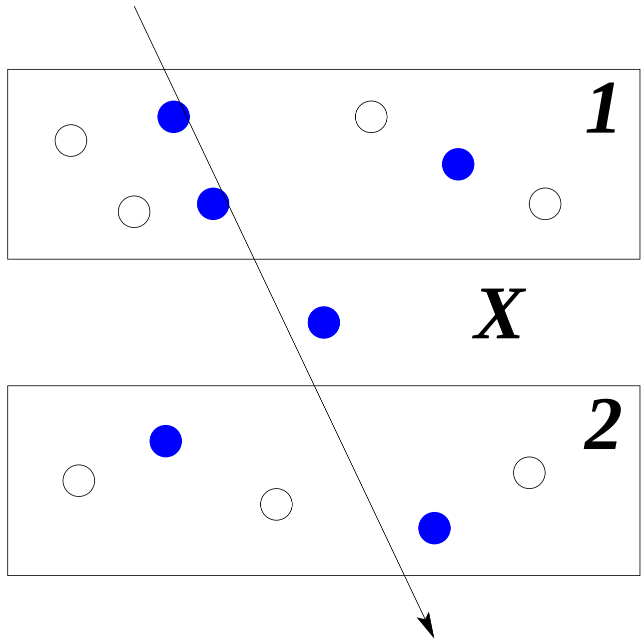
With the current data taking setup in AMANDA it is possible to evaluate noise independently from the signal. One simply needs to select hits during some time window before the trigger fires (and the majority of muon-related hits come in). In the plot below two windows are selected: one for noise and another for “total signal” = signal + noise.



The width of the window is chosen such as to contain most of the signal. For the following analysis afterpulses coming from the signal are as good as signal, coming from noise are as good as noise. Therefore, the window can be selected as large as possible. However, with the larger window noise hits will occur in almost every event, so the noise subtraction algorithm will not work. On the above plot two time distributions are shown: one for uncalibrated file and another for time-calibrated. It appears that the muon hits form a peak, which is more compact when the file is calibrated. The acceptable choice of the signal and noise windows was found to be 8000 ns when used with calibrated file.

In the following events are counted as 1 if there was at least one hit in the corresponding window. For each OM probability to have noise during the noise window in N_{all} events is evaluated as $P_n = \frac{N_{noise}}{N_{all}}$. Probability to have any signal (muon signal or noise) during the signal window is $P_t = \frac{N_{tot.\ sig.}}{N_{all}}$. Probability to have at least one muon signal during the signal window is then $P_s = \frac{N_{muon\ sig.}}{N_{all}}$. Probability to have neither noise nor muon signal during the signal window is equal to (since these events are uncorrelated) $1 - P_t = (1 - P_s) \cdot (1 - P_n)$, i.e. $P_s = 1 - \frac{1-P_t}{1-P_n}$, and $N_{muon\ sig.} = P_s \cdot N_{all}$.

Let's call the ratio of the number of muon signals an OM recorded to the total number of muons which crossed the depth at which this OM is located and caused detector to trigger, "efficiency" of this OM. If the efficiency of the OM at $z = h$ were known the signal from this OM could be used to compute the number of muons which passed the depth $z = h$ and were recorded by the detector.
To determine efficiency of OM "X", the following simple construction is used:

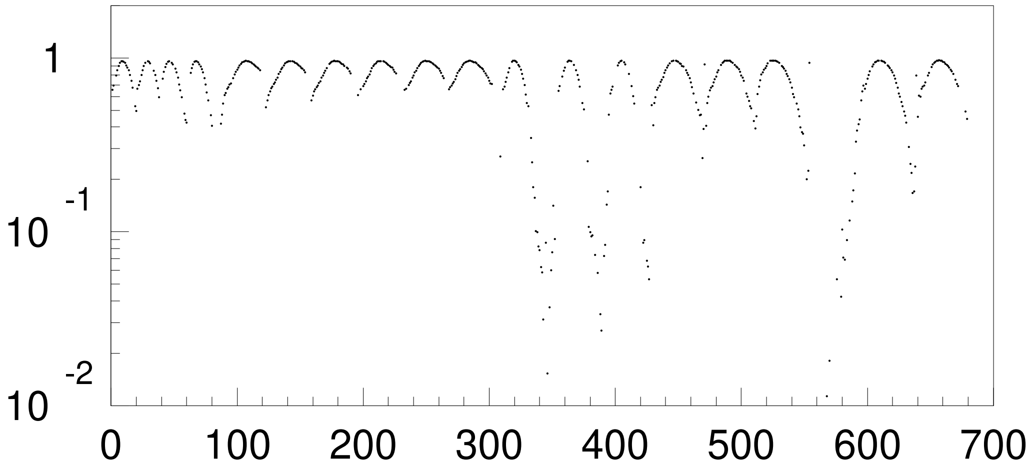


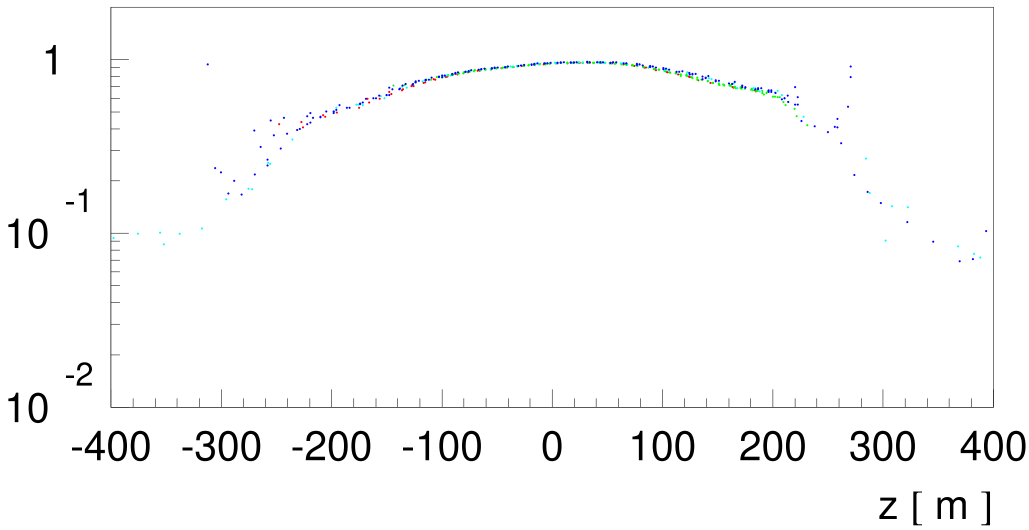
If the muon was seen in a layer above OM "X" and in a layer below, then it must have passed through the depth of the OM "X". Let's denote "1" and "2" the number of events in which there was muon signal in the layer above (1) and below (2). Correspondingly, "X" denotes the number of events in which OM "X" have had muon signal. Then the efficiency of OM "X" is determined as

$$\frac{X \cap (1 \cap 2)}{1 \cap 2}$$

and the number of muons which passed through the depth of OM "X" can be estimated as

$$X \cdot \frac{1 \cap 2}{X \cap (1 \cap 2)}$$





and is shown here vs. OM number and vs. depth.

To estimate $1 \cap 2$ and $X \cap (1 \cap 2)$ quantities, the following relations were used:

$$1 \cup 2 = 1 + 2 - 1 \cap 2 \quad \Rightarrow \quad 1 \cap 2 = 1 + 2 - 1 \cup 2$$

$$X \cap (1 \cap 2) = 1 + 2 + X - 1 \cup 2 - 1 \cup X - 2 \cup X + 1 \cup 2 \cup X$$

Since the noise subtraction algorithm works for not only single OMs, but also for unions of OMs, quantities 1, 2, X, $1 \cup 2$, $1 \cup X$, $2 \cup X$ and $1 \cup 2 \cup X$ are estimated for signal and noise time windows, and then subtracted from each other using the noise subtraction formula to get these quantities for "pure" muon signal.

In the above plot the numbers of muons show slight variations from one OM to the next. This is due to the fact that the efficiency, estimated with the above method, is valid for muons with energy high enough to be seen in both layers 1 and 2. In order to get the total muon counts one has to interpolate this value for the lower energy muons, which are seen in the layer 1, but not 2.
It is possible to get rid of this interpolation. Let's denote $P(S)$ the total number of muons which pass through the layer containing the set S of OMs. Then obviously,

$$P(X \cap (1 \cap 2)) = P(1 \cap 2) \quad \Rightarrow$$
$$P(1)+P(2)-P(1\cap 2) = P(1)+P(2)+P(X)-P(1\cup 2)$$
$$-P(1 \cup X) - P(2 \cup X) + P(1 \cup 2 \cup X) \quad \Rightarrow$$
$$P(X) = P(1 \cup X) + P(2 \cup X) - P(1 \cup 2 \cup X)$$

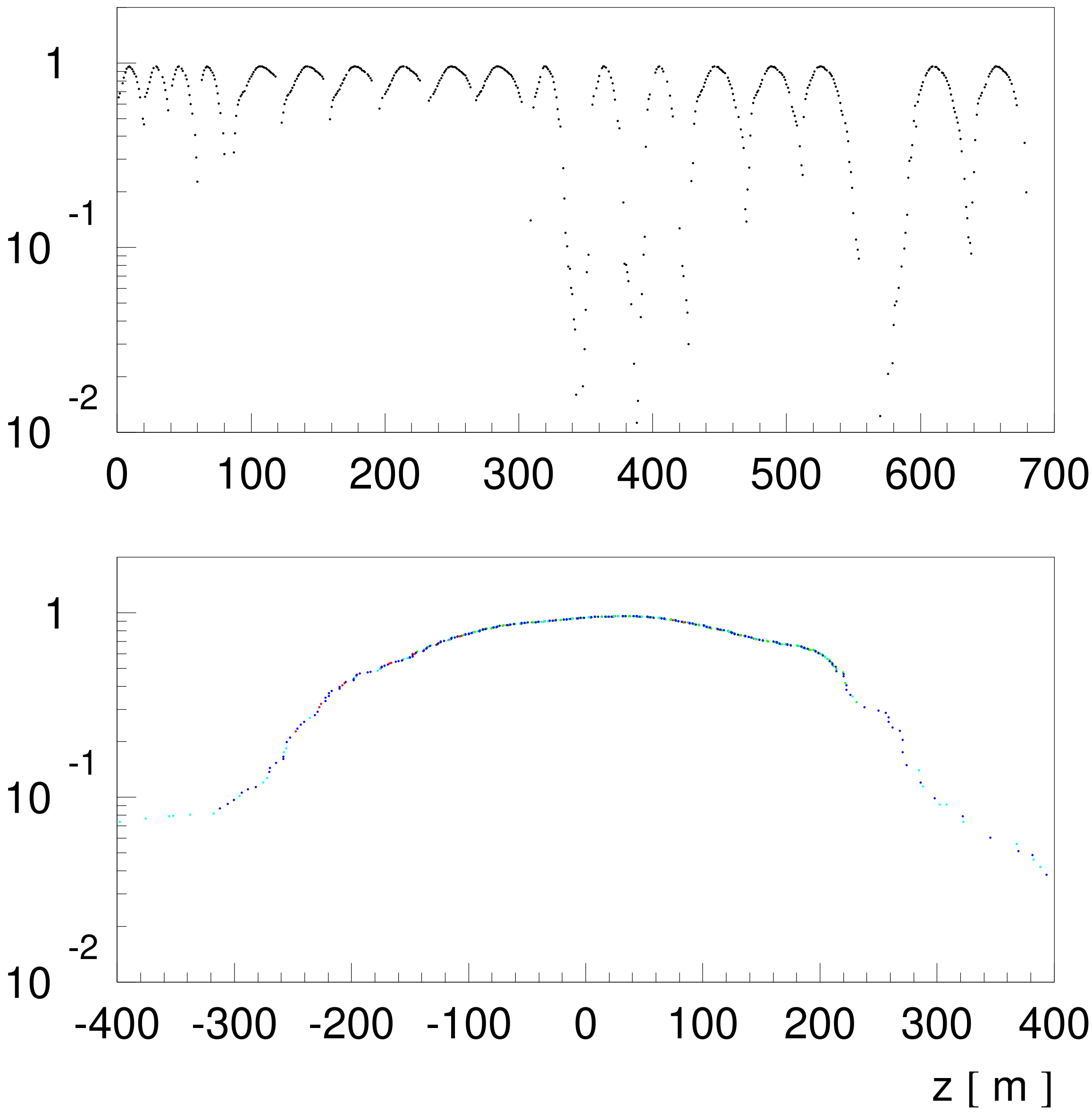
Sets $1 \cup X$, $2 \cup X$ and $1 \cup 2 \cup X$ can be chosen very large so that

$$P(1 \cup X) \approx 1 \cup X$$
$$P(2 \cup X) \approx 2 \cup X$$
$$P(1 \cup 2 \cup X) \approx 1 \cup 2 \cup X$$

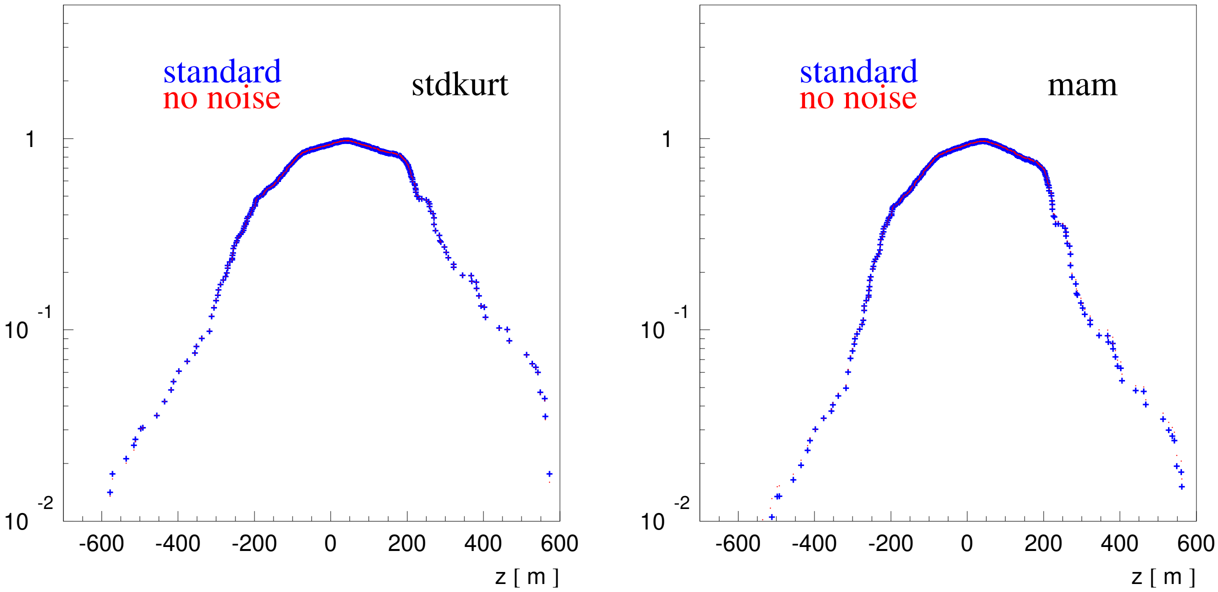
If the sets 1 and 2 are chosen such as to extend to the very top and bottom of the detector, then the last relation is exact: all muons which trigger the detector will show up in at least one of 1, 2 or X.

The quantity $P(X)$ is then the number of muons which pass through the depth of OM X and trigger the detector. OM X's sensitivity can now be evaluated as $X/P(X)$, but it is of secondary interest.

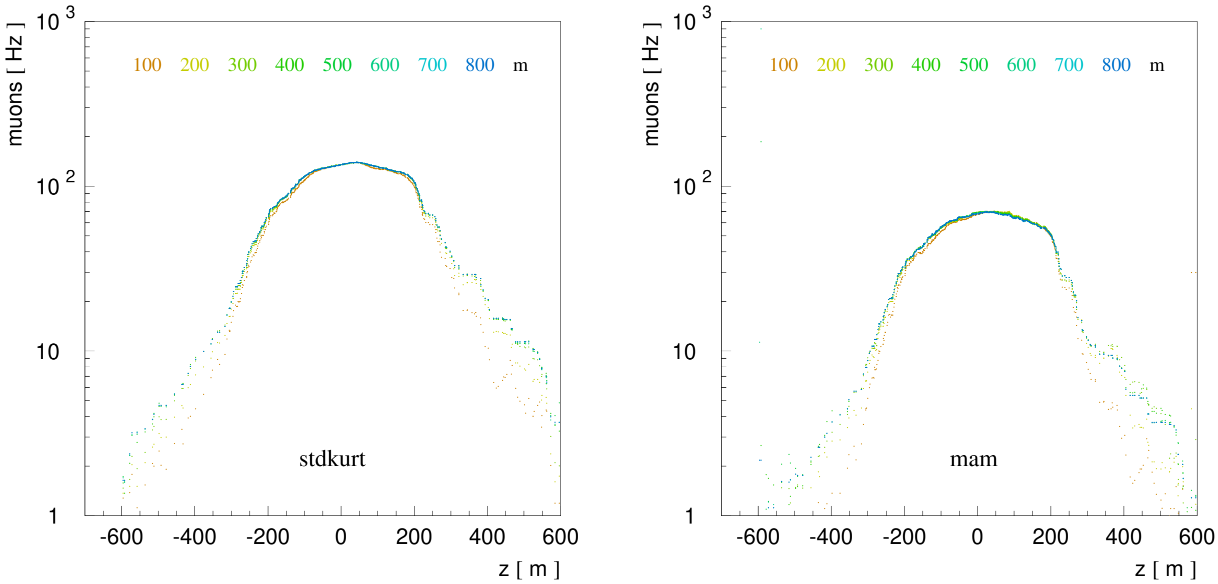
This plot shows the number of muons passing though the depths of detector OMs vs. OM number and vs. depth, evaluated using the second method:



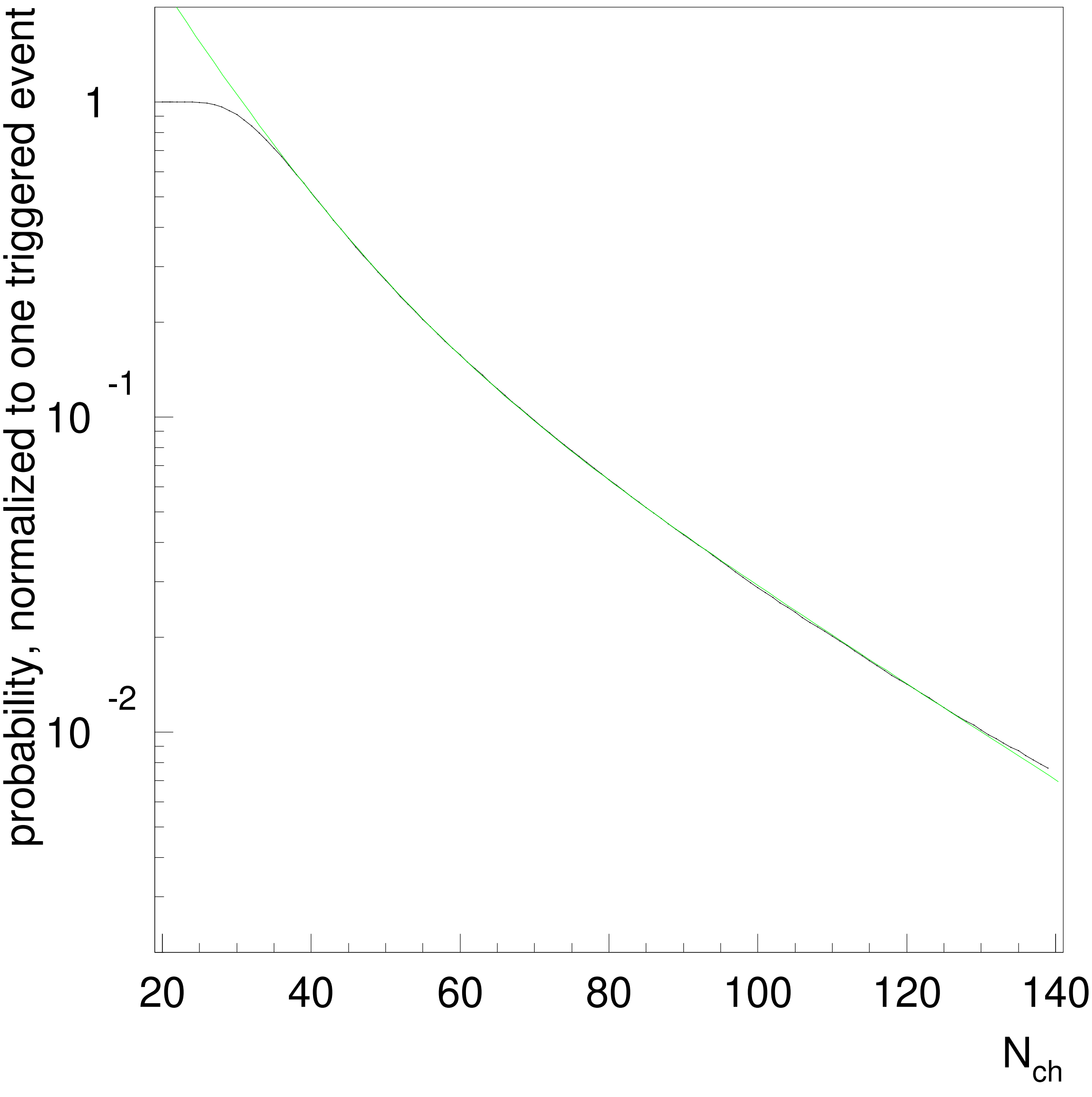
For montecarlo it is possible to get rid of noise by deleting it:



The bigger the layers 1 and 2, the more precise is the method:



N_{ch} distribution for data can be nicely fit with $\exp(p_1 + p_2 \cdot N_{ch}) + \exp(p_3 + p_4 \cdot N_{ch})$:



The N_{ch} distribution flattens at $N_{ch} \lesssim 30$ because the signal window is larger than the trigger window and due to some cleaning.

Signal window contains on average 4 noise hits. The exponential fits can be easily adjusted to account for that. Consider an exponential distribution $P(n) = e^{a+b\cdot n}$ to which a Poissonian noise $\frac{\lambda^n}{n!}e^{-\lambda}$ (with $\lambda = 4$) is added. Then the total distribution is

$$P_{tot}(n) = \sum_{k=0}^{n} e^{a+b\cdot(n-k)} \frac{\lambda^k}{k!} e^{-\lambda}$$

$$= e^{a+b\cdot n} \sum_{k=0}^{n} \frac{(\lambda \cdot e^{-b})^k}{k!} e^{-\lambda\cdot e^{-b}} \frac{e^{-\lambda}}{e^{-\lambda\cdot e^{-b}}}$$

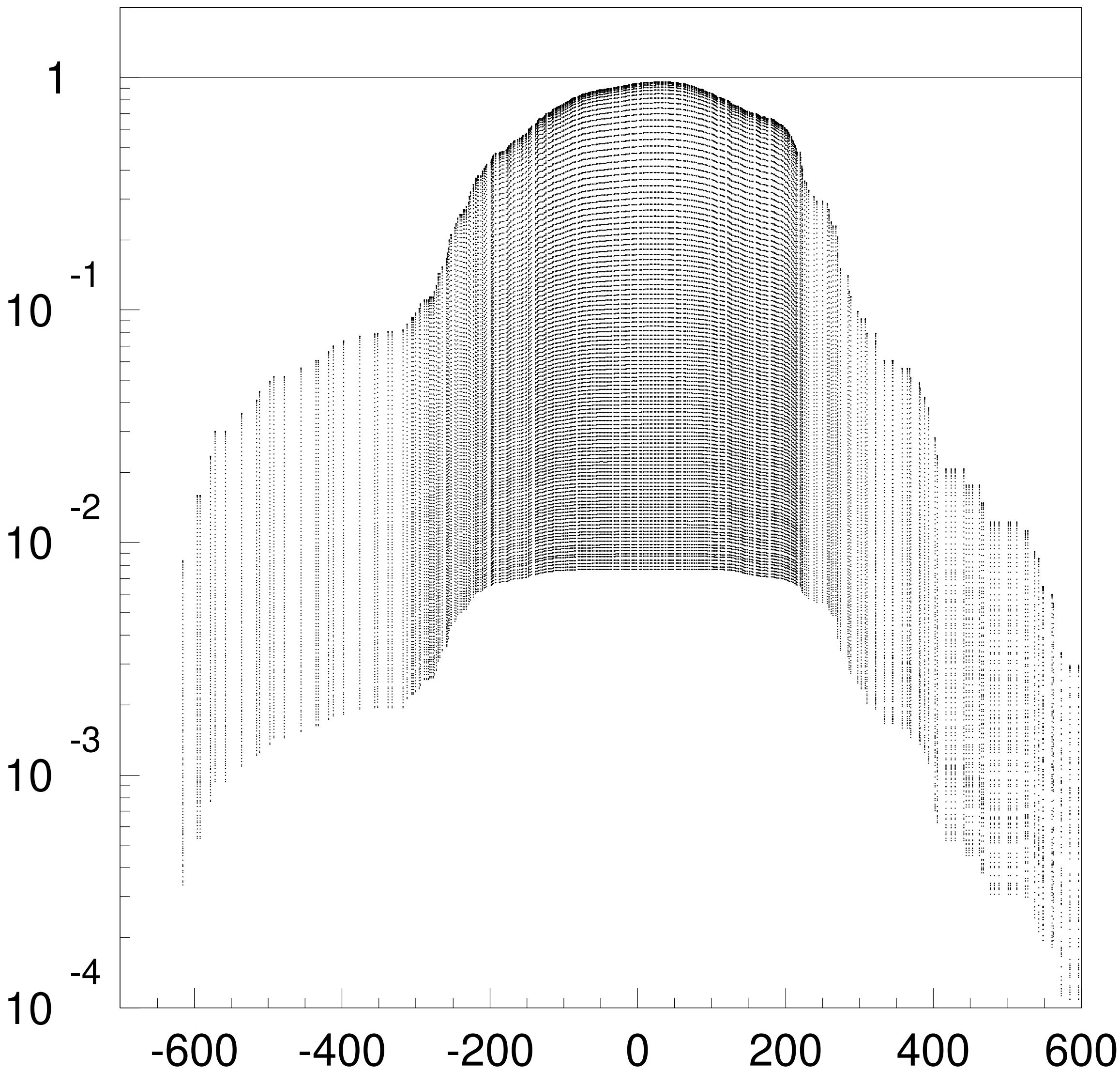
$$\approx e^{a+b\cdot n} e^{\lambda\cdot e^{-b}-\lambda} \sum_{k=0}^{\infty} \frac{\lambda'^k}{k!} e^{-\lambda'}$$

where $\lambda' = \lambda \cdot e^{-b}$ and n is sufficiently high. So,

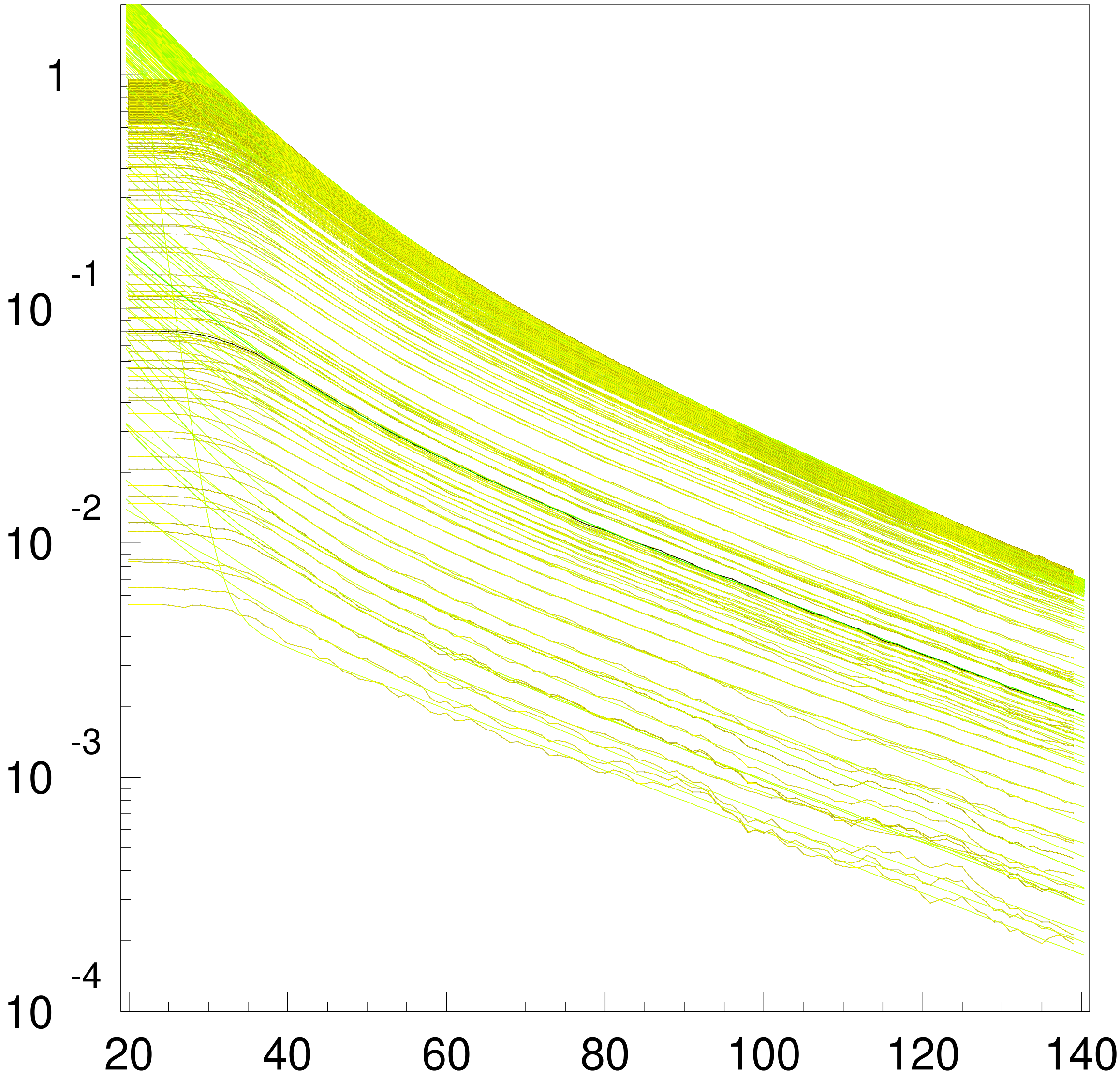
$$P_{tot}(n) = e^{a'+b\cdot n} \quad \text{with} \quad a' = a + \lambda(e^{-b} - 1)$$

i.e. for the exponential fit the noise correction includes adding $\lambda(1 - e^{-b})$ to the fitted value of a.

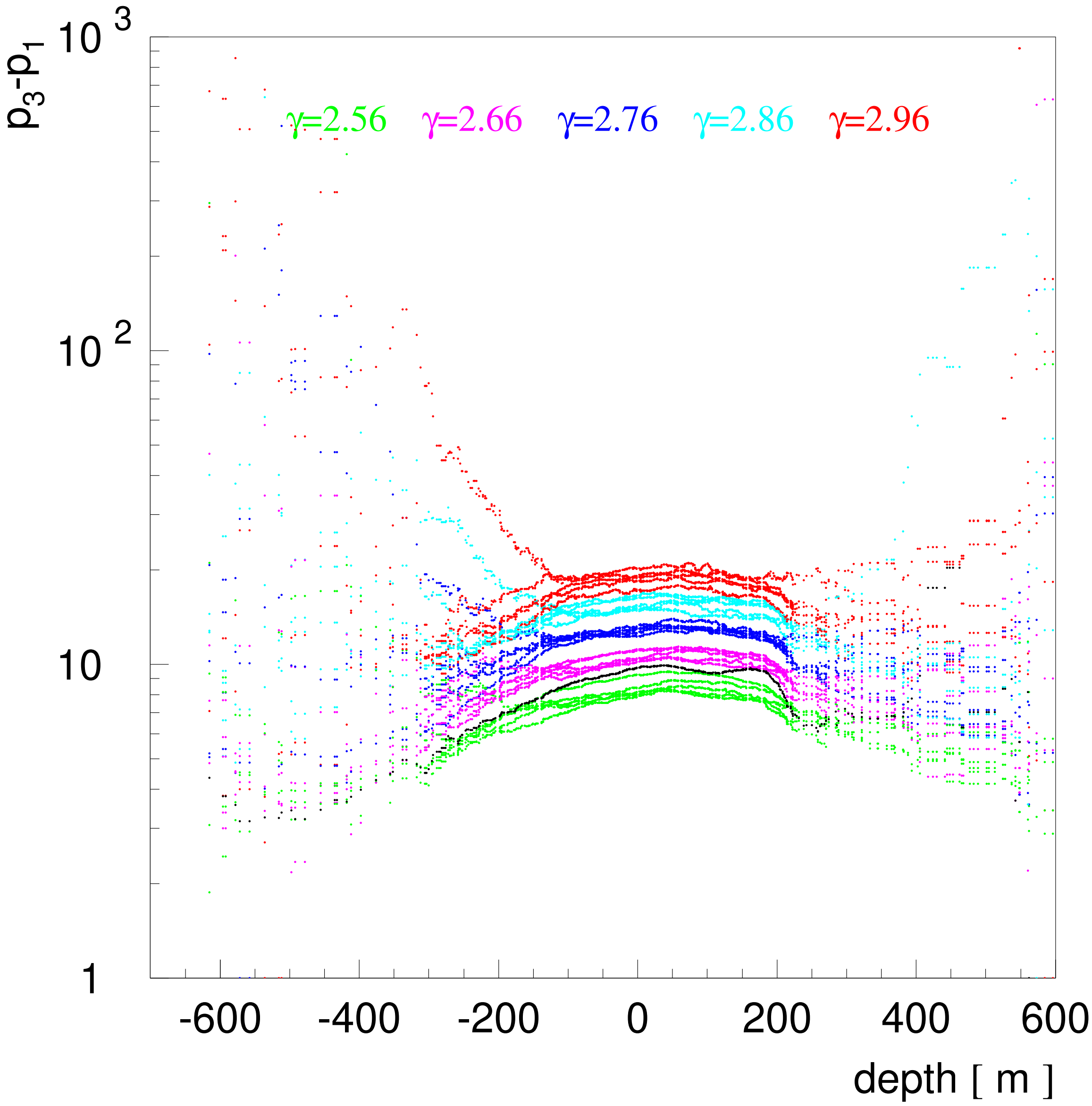
The number of muons at depths of the detector OMs with more than N =20 to 140 hits in the signal window for a sample of 2000 data is shown here:



For each OM the number of muons seen with number of signal hits in the signal window above $40 \leq N < 140$ is fit with $\exp(p_1 + p_2 \cdot N_{ch}) + \exp(p_3 + p_4 \cdot N_{ch})$:



Finally, data is compared to 5 montecarlos (*stdkurt*, *sudkurt*, *f125*, *mam*, and *bulk*) for 5 different spectral indices each:



data is represented by a black curve that goes between $\gamma = 2.56$ and $\gamma = 2.66$.

Conclusions

1. Developed method provides accuracy for measuring γ with systematical errors from insufficient knowledge of ice and OM sensitivities smaller than the deviation of γ from the used value 2.76 (for H, 2.63 for He, etc.)
2. Noise subtraction and sensitivity corrections work reliably and provide robust results.
3. N_{ch} model discussed in the first part explains most features of the N_{ch} distribution and demonstrates how the method for getting γ works

and further analysis

1. Evaluate influence of atmosphere on measurement of γ (seasonal changes)
2. If negligible - estimate charm contribution (existence of which is implied by smaller γ than predicted from CR measurements)
3. Use the found value of γ to re-evaluate background signal in AMANDA