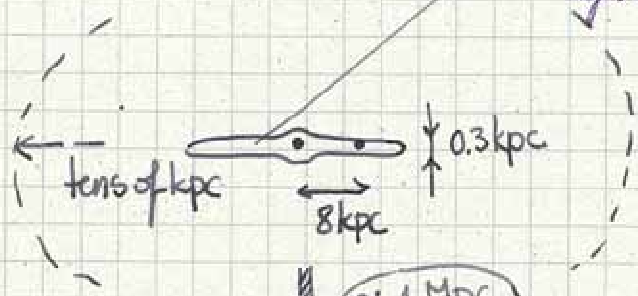
Scales of the Universe

1pc = 3.26 ly 0
≅ $3.0 \cdot 10^{18}$ cm



sun
$M = 2 \times 10^{33}\ g$
$L_\odot = 4 \times 10^{33}\ \frac{erg}{s}$
$L_{Edd} \simeq 10^{38}\ \frac{erg}{s}$

⇓ ~ 1 Mpc

satell. gal: LMC (55 kpc), Fornax .....
local cluster: Andromeda (2Mpc), M33 . . . .

Cepheids (< 5 Mpc for Hubble)
$\tau \sim L^{0.8}$
↳ oscill of envelope determined by crossing time of sound waves

⇓ ~ 10 Mpc

local supercl.: Virgo (20Mpc), M_{81}, Coma, Ursa Major

⇓ ~ 100 Mpc (nearest quasars)
$L\ 10^{45} \sim 10^{48} \ldots \frac{erg}{sec}$

Hubble radius: 14 G years ≃ 4500 Mpc

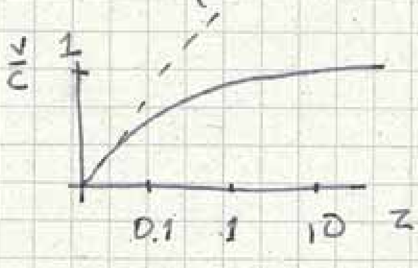
10^{11} galaxies = $(\underbrace{\lesssim 10^3}_{\text{4Mpc out of 6000Mpc}})^3 \times 10^3$ galaxies per cluster

redshift → $\boxed{\text{Hubble flow}}$ $\boxed{\text{Nucleosynthesis}}$ $\boxed{\text{Microwave background}}$ pillars of cosmology

$\lambda_{obs} > \lambda_{rest}$

$\nu_{rest} > \lambda_{obs}$

$$1+z = \frac{\lambda_{obs}}{\lambda_{rest}} = \frac{\nu_{rest}}{\nu_{obs}} = \underbrace{\frac{c+v}{c}}_{\text{Doppler shift}} \cdot \underbrace{\frac{1}{\sqrt{1-v^2/c^2}}}_{\text{clock slows down}} = \sqrt{\frac{c+v}{c-v}} = 1+z$$



e.g. $z = 1000$

$z \simeq \frac{v}{c}$ for small v

2.25

1

from Perkins Particle Astrophysics

Table 2.1 Approximate sizes and masses in the universe (1 parsec = 1 pc = 3.09×10^{16} m = 3.26 lightyears)

	Radius	Mass
Sun	7×10^8 m	2×10^{30} kg $= M_\odot$
Galaxy	15 kpc	$10^{11}\ M_\odot$
Cluster	5 Mpc	$10^{14}\ M_\odot$
Supercluster	50 Mpc	$10^{15}\ M_\odot$
Universe	4500 Mpc	$10^{23}\ M_\odot$

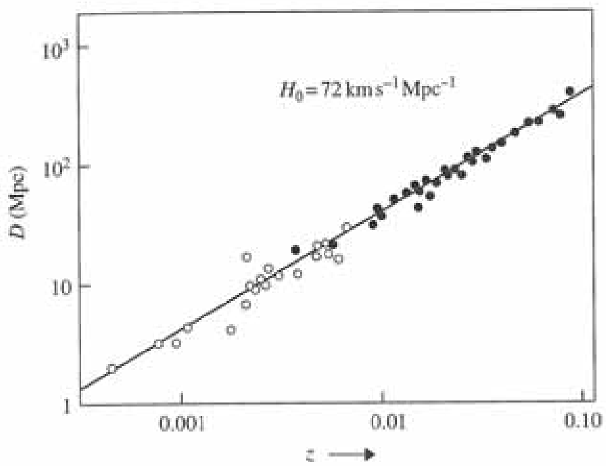


Fig. 2.3 Log–log plot of distance versus redshift, for small redshifts, $z < 0.1$. The points for $z < 0.01$ are from Cepheid variables (open circles), and those of higher z (full circles) include results from Type Ia and Type II supernovae. The straight line is that for the Hubble parameter $H_0 = 72$ km s^{-1} Mpc^{-1} (after Freedman *et al.* 2001). The absolute distance scale has been established by parallax measurements of nearby sources, working out to larger distances, eventually reaching the nearest Cepheids.

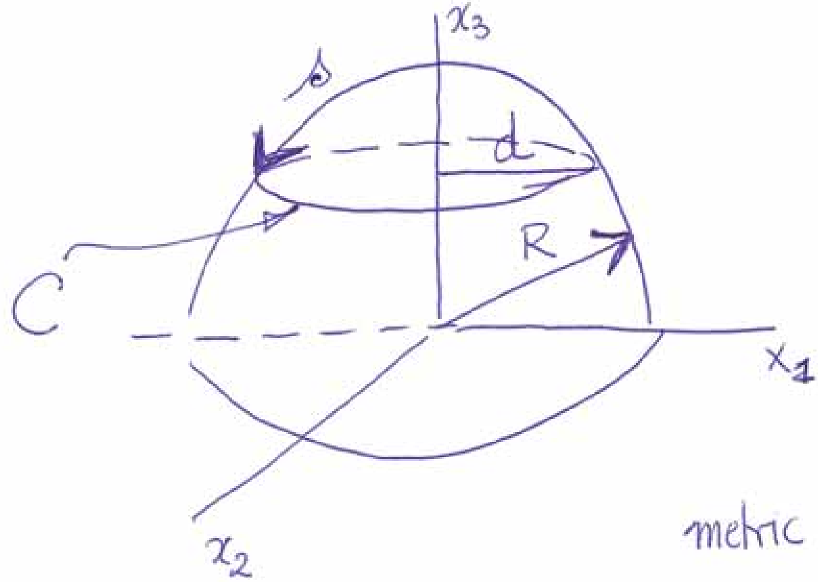
In fact astronomers use a logarithmic scale of luminosity, called **magnitude**, running (perversely) from small values of magnitude for the brightest stars to large values for the faintest. The defining relation between the apparent magnitude $m(z)$ at redshift z, the so-called absolute magnitude M (equal to the value that m would have at $D_L = 10$ pc) and the distance D_L in Mpc, is given by the **distance modulus**

$$m(z) - M = 5\log_{10} D_L(z) + 25 \tag{2.3}$$

In the Hubble diagram, $(m - M)$ or $\log_{10} D_L$ is plotted against $\log_{10} z$.

A modern version of the Hubble plot at small redshifts is shown in Fig. 2.3, for events of $z < 0.1$. The various sources in this plot include, for example, Cepheid variable stars for $z < 0.01$, and Type Ia and Type II supernovae for higher redshifts. Cepheid variables can be used as 'standard candles', since they vary in luminosity due to oscillations of the envelope, the period τ being determined by the time for sound waves to cross the stellar material—$\tau \propto L^{0.8}$. Supernovae, discussed in Chapter 7, signal the death throes of stars in the final stages of evolution, and when they occur, their light output for a time—typically weeks or even months—can completely dominate that from the local galaxy. So, in principle, they are useful for probing out to large distances and redshifts, or equivalently, back to earlier times. The absolute distance scale to the thirty or so nearest spiral galaxies where a few Type Ia or Type II supernovae have occurred, has been established by observations on Cepheid variables, and this provides a means of calibrating supernova luminosity.

- Universe as the surface of a balloon or FLRW in 2D space



$$x_1^2 + x_2^2 + \frac{x_3^2}{k} = \frac{R^2}{k}$$

$k = 1$ is balloon

$$ds^2 = (d\theta \; d\varphi) \, g \begin{pmatrix} d\theta \\ d\varphi \end{pmatrix}$$

metric $\longrightarrow$ $g = \begin{pmatrix} R^2 & 0 \\ 0 & R^2 \sin^2\theta \end{pmatrix}$

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta \, d\varphi^2 = R^2 d\Omega$$

- Volume V and curvature K

$$\boxed{V = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sqrt{\det g}} = 2\pi R^2 \int_0^{\pi} \sin\theta \, d\theta = 4\pi R^2$$

(area of balloon)

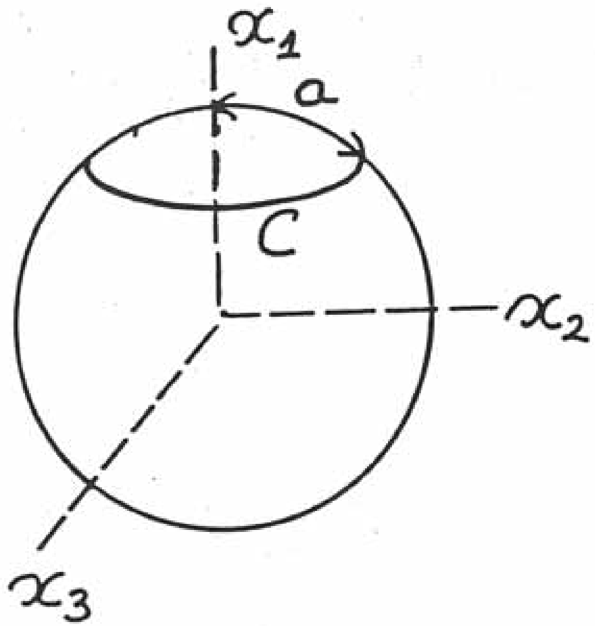
$$\boxed{K = \frac{3}{\pi} \lim_{s \to 0} \left(\frac{2\pi s - C}{s^3} \right)}$$

$$2\pi d = 2\pi R \sin\frac{s}{R} = 2\pi R \left(\frac{s}{R} - \frac{1}{3!}\frac{s^3}{R^3} \right)$$

$$\boxed{K = \frac{k}{R^2}} \qquad \left(\frac{1}{R^2} \text{ for balloon} \right)$$

Problem : Draw the[24] universe*

* assume 2D like a balloon



Closed surface

$C < 2\pi a$

$$x_1^2 + x_2^2 + x_3^2 = R^2$$

(1D circle)

Open surface

$C > 2\pi a$

$$x_1^2 + x_2^2 - x_3^2 = -R^2$$

(1D hyperbola)

Flat surface

$C = 2\pi a$

(1D line)

. no curvature
. R not defined
[illegible]

- FLRW metric of the real Universe
  balloon in 3D, not 2D
  space + time

cosmological principle : Universe is homogeneous & isotropic

$$\boxed{ds^2 = c\,d\tau^2 - dl^2} \qquad \boxed{dl^2 = R^2(t)\left[\frac{d\tau^2}{1-k\tau^2} + \tau^2 d\Omega\right]}$$

$$l^2 = x_1^2 + x_2^2 + x_3^2 + \frac{x_4^2}{k} = \frac{R^2(t)}{k}$$

$$x_1 = R(t)\tau \sin\theta \cos\varphi$$
$$x_2 = R(t)\tau \sin\theta \sin\varphi$$
$$x_3 = R(t)\tau \cos\theta$$
$$x_4 = \sqrt{1-k\tau^2}\; R$$

scale factor

co-moving coördinate (see later) time independent!

- radial coördinate is $R(t)\tau$ to express the fact that the time dependence is the same for all τ. Different time dependence at τ_1, τ_2 violates cosmological principle → would create anisotropy.

- FLRW revisited

using coördinates (cdt dr dθ dφ)

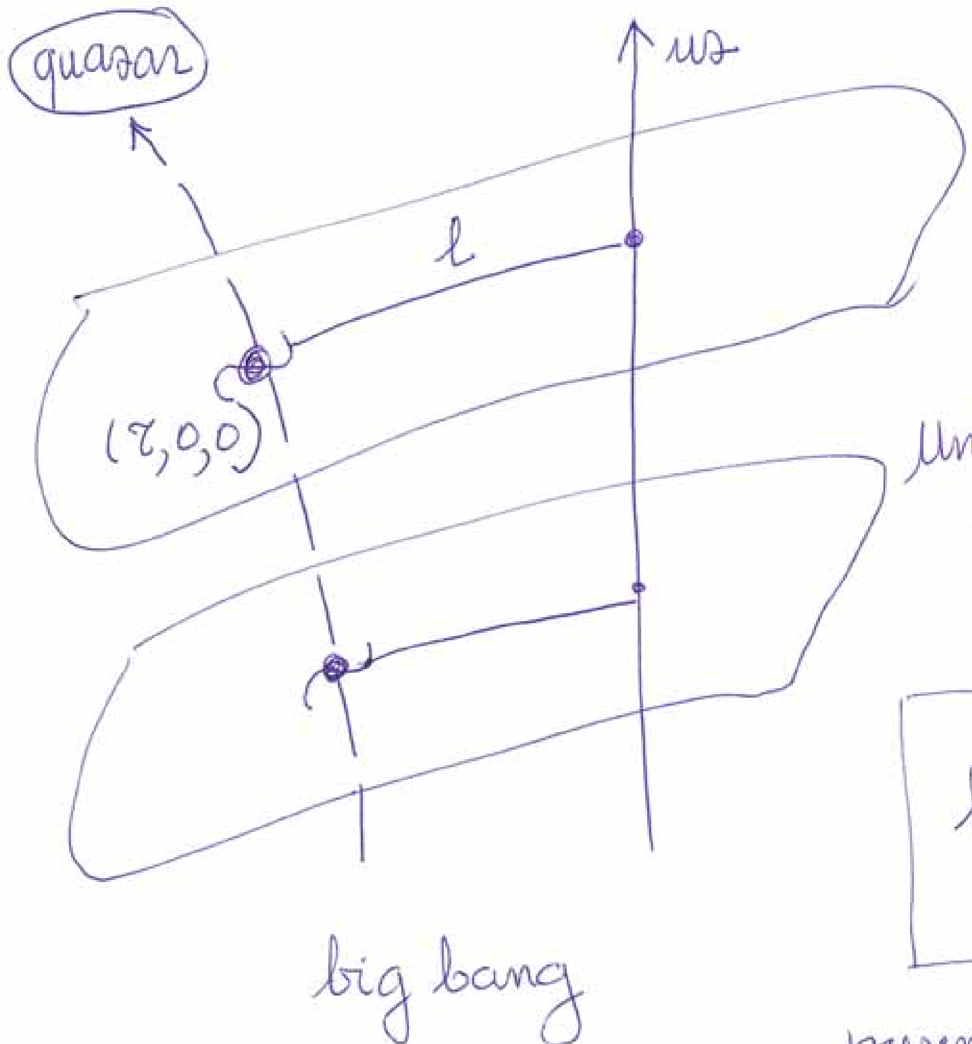
$$g = \begin{bmatrix} 1 & & & \\ & -\frac{R^2}{1-kr^2} & & \\ & & -R^2r^2 & \\ & & & -R^2r^2\sin^2\theta \end{bmatrix}$$

- Curvature at θ = φ = 0 from Riemann

$$K = \frac{1}{2g_{11}g_{22}}\left\{-\frac{\partial^2 g_{11}}{\partial x_2^2} + \frac{1}{2g_{11}}\left[\frac{\partial g_{11}}{\partial x_1}\cdot\frac{\partial g_{22}}{\partial x_2} + \left(\frac{\partial g_{11}}{\partial x_2}\right)^2\right] + (1\leftrightarrow 2)\right\}$$

$$= -\frac{\ddot{R}}{R}$$

• τ is the co-moving coördinate



$$\boxed{l = \int dl = \int_0^{\tau} \frac{R(t)}{\sqrt{1-k\tau^2}}\, d\tau}$$

present distance depends on how the Universe evolved since the light was emitted

flat $\}\ l = R(t)\tau$ $\quad k = 0$

$l = R \sin^{-1}\tau$ $\quad 1$

$l = 0$ and $l = 2\pi R$ are same τ
cyclic universe

$\tau = \sin \frac{l}{R}$

$\}\ l = R \sinh^{-1}\tau$ $\quad 1$

- Hubble flow : calculate recession velocity $\dot{l}$

$$v = \dot{l} = \dot{R} \underbrace{\int_0^r \frac{dr}{\sqrt{1-kr^2}}}_{\frac{l}{R(t)}} = \frac{\dot{R}(t)}{R(t)}\, l$$

$$\boxed{\dot{l} = H(t)\, l \qquad H(t) = \frac{\dot{R}(t)}{R(t)}}$$

- Einstein: curvature proportional to mass

$$\boxed{K \propto \rho}$$

or, from dimensional analysis:

$$K = -\frac{\ddot{R}}{R} = \alpha\, G^n c^m \rho + \underbrace{cte}_{\downarrow}$$

curvature not related to mass: violates Mach

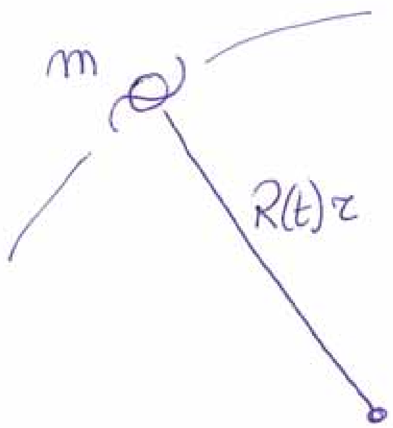
$$[K] = \frac{1}{T^2} \qquad [G] = \frac{L^3}{MT^2} \qquad [c] = \frac{L}{T} \qquad [\rho] = \frac{M}{L^3}$$

$$\boxed{K = -\frac{\ddot{R}}{R} = \alpha G \rho}$$

what is the proportionality constant α ?

answer: Newtonian limit

• α ?



Force on galaxy of mass m only depends on $\rho(t)$ inside the volume of radius $R(t)r$

Newton's law

$$m r \ddot{R} = -G \frac{m \frac{4\pi}{3} \rho (rR)^3}{(rR)^2}$$

$$\boxed{-\frac{\ddot{R}}{R} = \frac{4\pi}{3} G\rho}$$

Friedman I

therefore $\alpha = \frac{3\pi}{2}$

• energy of m ? multiply both sides with $\dot{R}$ and use mass conservation

$$M = \rho R^3 = \rho_0 R_0^3$$

$$m \ddot{R}\dot{R} = -G \frac{mM}{R^2} \dot{R}$$ integrate

$$\frac{1}{2} m\dot{R}^2 - \frac{mM}{R^2} = 0$$ $\longrightarrow$ total energy is zero

- previous derivation in flat universe $k=0$

for $k \neq 0$ particles and photons move on geodesics (because the scatter of gravitational fields)

$$\frac{1}{2} m \dot{R}^2 - \frac{mM}{R} = -k \frac{1}{2} mc^2 \quad \text{(GR result)}$$

$$\boxed{\frac{\dot{R}^2}{R} = \frac{8\pi}{3} \rho - \frac{kc^2}{R^2}}$$ Friedman II

- introduce cosmological constant (long range repulsive force proportional to R)

$$K = -\frac{\ddot{R}}{R} = \alpha G \rho(t) - \frac{\Lambda}{3}$$

Therefore the Friedman equations become

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi}{3} G\rho + \frac{\Lambda}{3}}$$

$$\boxed{\frac{\dot{R}^2}{R} = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3} R^2}$$

- Hubble flow revisited

we only measure the Universe now ($t=0$)

$$R(t) = R_0 + \dot{R}(t-t_0) + \frac{1}{2}\ddot{R}_0 (t-t_0)^2$$

$$H_0 = \frac{\dot{R}_0}{R_0} \qquad q_0 = -\frac{1}{H_0^2}\frac{\ddot{R}_0}{R_0}$$

$$\boxed{R(t) = R_0\left[1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2\right]}$$

definition of the observables H_0 (Hubble constant) and q_0 (acceleration parameter)

$$\Lambda = 4\pi G\rho_0 - 3q_0 H_0^2$$

$$k = \frac{R_0^2}{c^2}\left[4\pi G\rho_0 - H_0^2(q_0+1)\right]$$

for flat Universe with no cosmological constant $\Lambda = 0$; $k=0$

$$\boxed{\begin{aligned}\rho_0 &= \frac{3H_0^2}{8\pi G} \\ q_0 &= \frac{1}{2}\end{aligned}}$$

- Possible universes: $\Lambda = 0$!

$k = -1$ open

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{c^2}{R^2} \xrightarrow{R \text{ large}} \frac{c^2}{R^2} \qquad \left(\rho \sim \frac{1}{R^3}\right)$$

$$\dot{R} = c \qquad\qquad R = ct$$

expands at constant rate

$k = 0$ flat

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho \rightarrow \frac{1}{R^3} \qquad \left(\rho \sim \frac{1}{R^3}\right)$$

$$\dot{R} \propto R^{-\frac{1}{2}} \qquad\qquad R \propto R^{1/2}$$

expands, rate slows down

$k = -1$ cyclic

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{c^2}{R^2} \qquad\qquad \dot{R} = 0 \text{ for } R_{MAX}$$

$$R_{MAX} = \frac{2GM}{c^2}$$

$$\Downarrow$$

$$M = \frac{4}{3}\pi R^3 \rho$$