Atmospheric & Solar Oscillations

Atmospheric

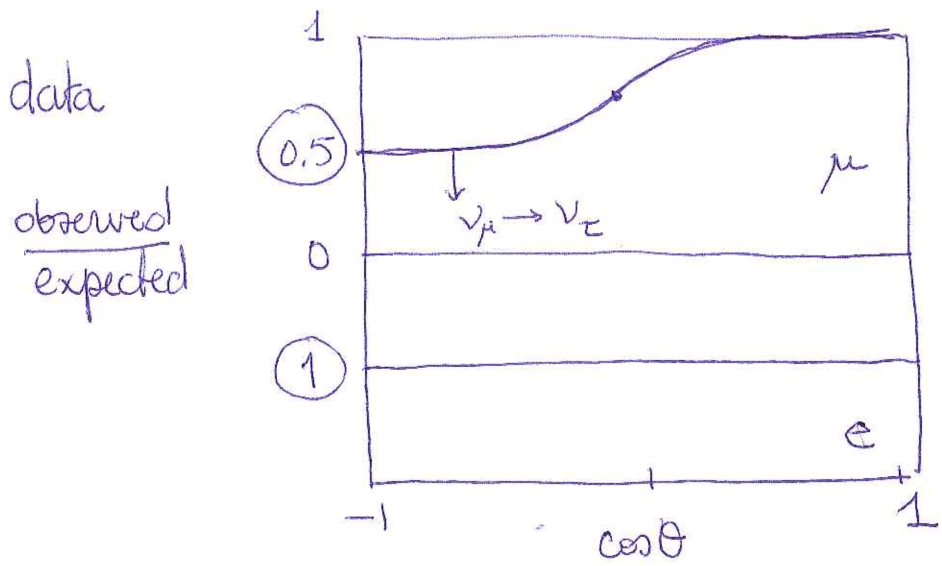
$$\langle \nu_\mu | \nu_\mu \rangle = \left[1 - \sin^2 2\theta \sin^2\left\{\left[E_2 - E_1\right]\frac{t}{2}\right\}\right] \left[\langle \nu_\mu(0)\right] \nu_\mu(0)\rangle$$

$$E \simeq p + \frac{m^2}{2p} \qquad E = (m^2 + p^2)^{1/2}$$

$$\{\,\} = \frac{\Delta m^2}{4p} t = \frac{\Delta m^2}{4(pc)}(ct) = \frac{\Delta m^2}{4E} L = 1{,}27\, \Delta m^2_{eV} \frac{L_{km}}{E_{GeV}}$$

$$P_{\nu_\mu \to \nu_\tau} = \frac{|\,|\nu_\mu\rangle|^2}{|\,|\nu_\mu(0)\rangle|^2} = 1 - \sin^2 2\theta \sin^2 \{\,\}$$

$$\frac{\nu_\mu}{\nu_e} = \frac{2}{1} \longrightarrow \frac{1}{1}$$



12500km 500km 20km = L

Solar

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}p & 0\\ 0 & p\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} + \begin{pmatrix}\frac{m_1^2}{2p} & 0\\ 0 & \frac{m_2^2}{2p}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

affects both ν's in the same way $\rightarrow$
does not contribute to the oscillation

$$i\frac{d}{dt}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\frac{m_1^2}{2p} & 0\\ 0 & \frac{m_2^2}{2p}\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

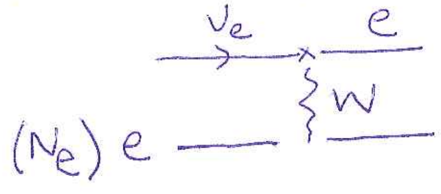
$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = M_\nu\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$M_\nu = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\frac{m_1^2}{2p} & 0\\ 0 & \frac{m_2^2}{2p}\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

vacuum

$$= \frac{(m_1^2+m_2^2)}{4p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{\Delta m^2}{4p}\begin{pmatrix}-\cos2\theta & \sin2\theta\\ \sin2\theta & \cos2\theta\end{pmatrix}$$

$\boxed{\text{Matter}}$: electron feels add. potential. (effect. mass from forward scattering)



$$m^2 = E^2 - p^2 \longrightarrow (E + V_e)^2 - p^2$$

$$= \left(E + \sqrt{2}\, G_F N_e\right)^2 - p^2$$

$$= (E^2 - p^2) + 2\sqrt{2}\, G_F N_e\, E + \ldots$$

$$\frac{1}{2}(m_1^2 + m_2^2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \frac{1}{2}(m_1^2 + m_2^2) + 2\sqrt{2}\, G_F N_e p \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \left[\frac{1}{2}(m_1^2 + m_2^2) + \sqrt{2}\, G_F N_e p\right]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$+ \sqrt{2}\, G_F N_e p \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_M = \begin{bmatrix} \text{overall} \\ \text{phase} \end{bmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \left[\frac{\Delta m^2}{4p}\right]\begin{pmatrix} -\cos 2\theta + A & \sin 2\theta \\ \sin 2\theta & \cos 2\theta - A \end{pmatrix}$$

$$A = 2\sqrt{2}\, G_F N_e \frac{p}{\Delta m^2}$$

$$\equiv \left[\frac{\Delta m_m^2}{4p}\right]\begin{bmatrix} -\cos 2\theta_m & \sin 2\theta_m \\ \sin 2\theta_m & \cos 2\theta_m \end{bmatrix}$$

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - A} = \frac{\tan 2\theta}{1 - \frac{L_V}{L_M}\sec 2\theta}$$

$$L_V = \frac{4\pi p}{\Delta m^2} \qquad L_M = \frac{4\pi}{2\sqrt{2}\, G_F N_e} \qquad A = \frac{L_V}{L_M}$$

matter resonance

$$L_V = L_M \cos 2\theta \qquad N_e(res) = \Delta m^2 \frac{\cos 2\theta}{2\sqrt{2}\, G_F p}$$