Plan of Lectures

I. Standard Neutrino Properties and Mass Terms (Beyond Standard)

II. Effects of ν Mass: Neutrino Oscillations (Vacuum)

III. Matter Effects in Neutrino Oscillations

IV. The Emerging Picture and Some Lessons

Summary I+II

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  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
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  $\longrightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\longrightarrow$ Majorana ν: $\nu = \nu^C$
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- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$
- Neutrino masses and mixing $\Rightarrow$ Flavour oscillations
- Atmospheric, K2K and MINOS (+ negative SBL searches)

  $\Rightarrow \nu_\mu \rightarrow \nu_\tau$ with $\Delta m^2 \sim 2 \times 10^{-3}$ eV^2 and $\tan^2\theta \sim 1$

Plan of Lecture III

Matter Effects in Neutrino Oscillations

Solar Neutrinos: Fluxes and Data

Matter Potentials

Neutrino Oscillations in Matter: MSW Effect

Oscillation Solutions to Solar Neutrino Data

Learning How the Sun Shines

Solar Neutrinos: Fluxes

Solar Neutrinos: Fluxes

- The Sun shines converting protons into α, e^+ and $\nu' s$

  $4\,p \;\rightarrow\; {}^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$

  $4m_p - m_{^4He} - 2m_e \simeq 26$ MeV Thermal energy mostly in γ
- Two major chains of nuclear reactions

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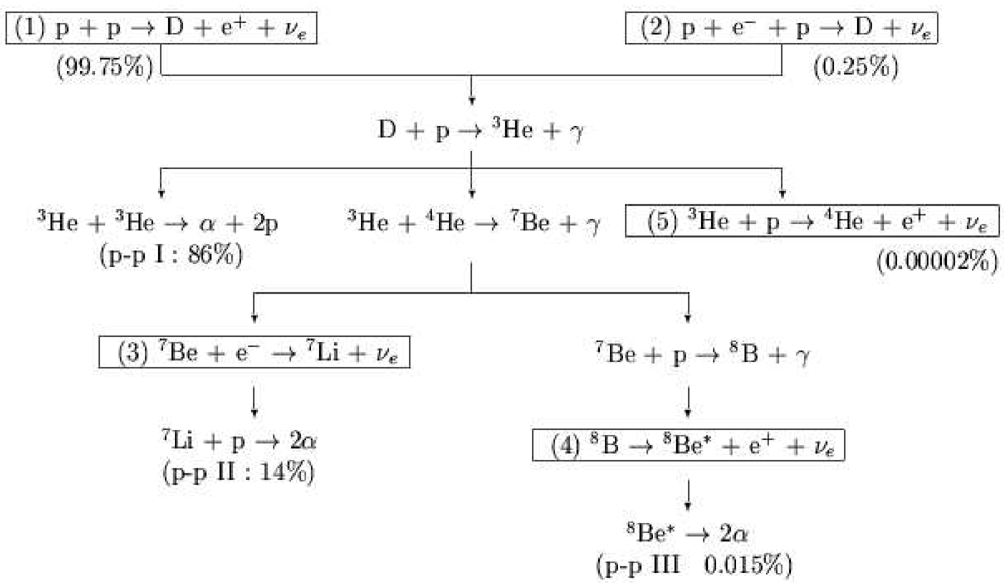
  pp chain:



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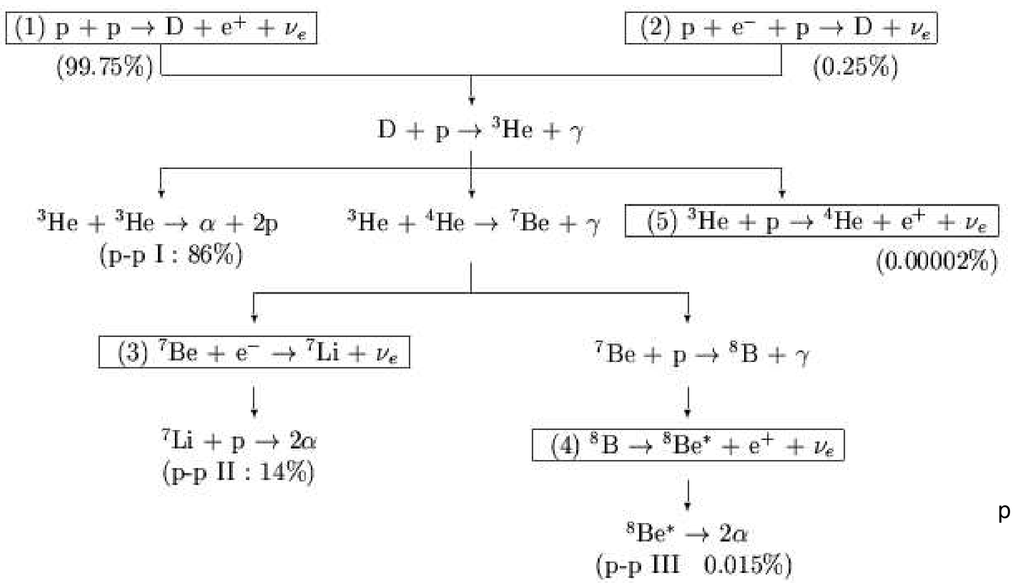
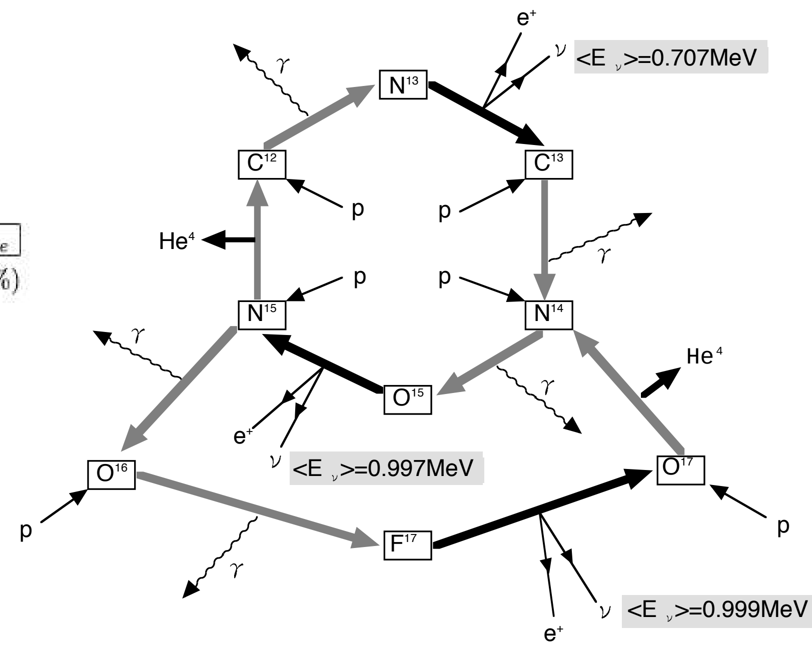
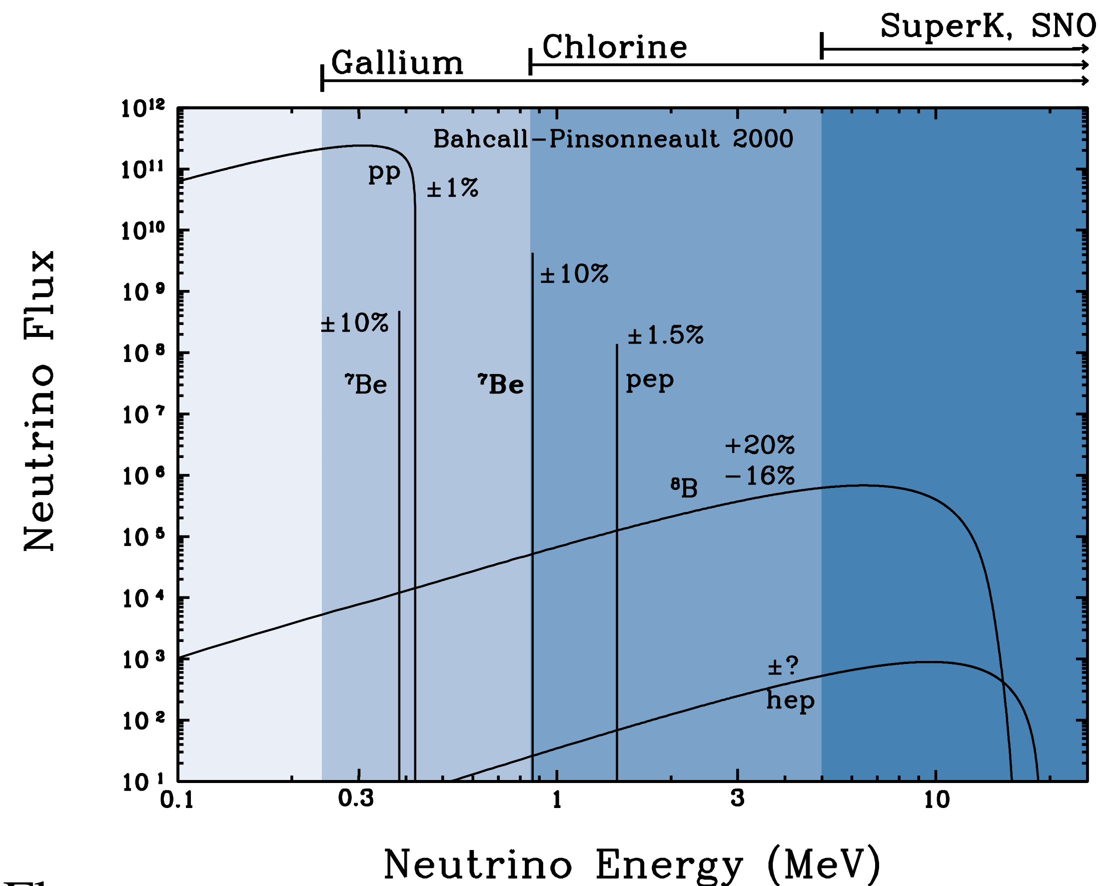
pp chain: CNO cycle:





- Present Solar Model ⇒ *pp*-chain dominates by 99%

Solar Neutrinos: Fluxes



- Most Relevant Fluxes :

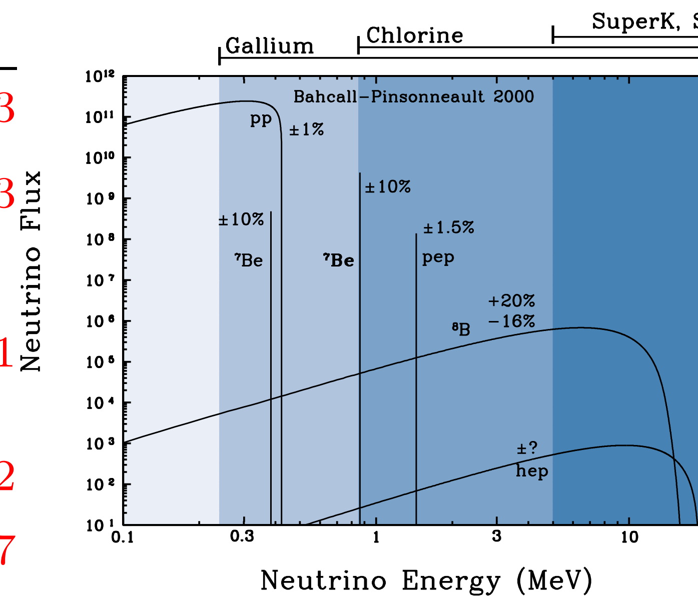
  - At SK, SNO and Chlorine, ^{8}B neutrinos: 20% accuracy in total flux
    At $1/10^5$ spectrum independent of solar physics

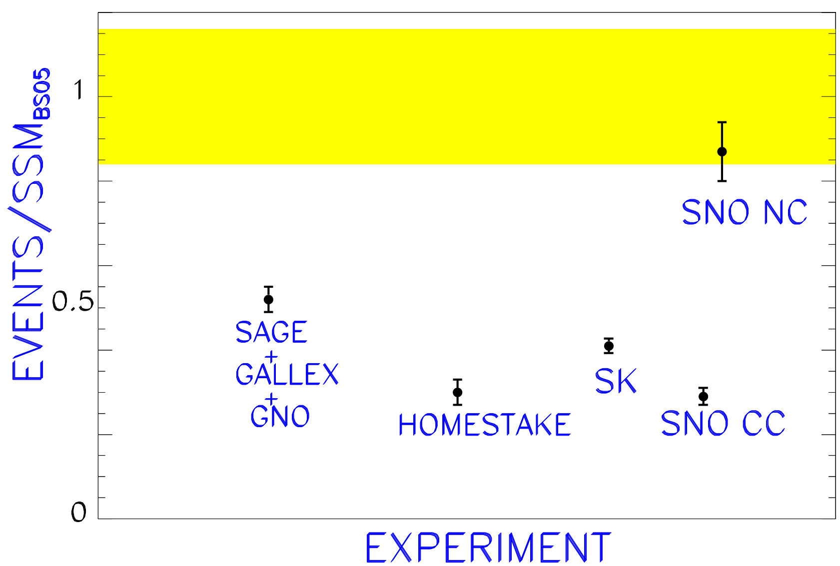
  - At Ga, pp neutrinos : Best determined by SSM (1%)

  - At Chlorine, also ^{7}Be neutrinos

Solar Neutrinos: Data

	Experiment	Detection	Flavour	E_{th} (MeV)	$\frac{\text{Data}}{\text{BS05}}$
radio-chemical	Homestake	$^{37}\text{Cl}(\nu, e^-)^{37}\text{Ar}$	ν_e	$E_\nu > 0.81$	0.30 ± 0.03
	Sage + Gallex+GNO	$^{71}\text{Ga}(\nu, e^-)^{71}\text{Ge}$	ν_e	$E_\nu > 0.23$	0.52 ± 0.03
real time	Kam ⇒ SK	ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$ $\left(\frac{\sigma_{\mu\tau}}{\sigma_e} \simeq \frac{1}{6}\right)$	$E_e > 5$	0.41 ± 0.01
	SNO	CC $\nu_e d \to p p e^-$	ν_e	$T_e > 5$	0.29 ± 0.02
		NC $\nu_x d \to \nu_x p\, n$	$\nu_e, \nu_{\mu/\tau}$	$T_\gamma > 5$	0.87 ± 0.07
		ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$T_e > 5$	0.41 ± 0.05





All experiments measuring mostly ν_e observed a deficit

Deficit is energy dependent

Deficit disappears in NC

Solar Neutrinos: Flavour Conversion Evidence

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SK and SNO measure $\Phi_{^8\mathrm{B}}$ in different reactions

ES $\nu_x e^- \to \nu_x e^-$ $\Phi_{^8\mathrm{B}}^{\mathrm{SK,ES}} = (2.35 \pm 0.08) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

CC $\nu_e d \to p p e^-$ $\Phi_{^8\mathrm{B}}^{\mathrm{SNO,CC}} = (1.68 \pm 0.1) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

NC $\nu_x d \to \nu_x d$ $\Phi_{^8\mathrm{B}}^{\mathrm{SNO,NC}} = (4.94 \pm 0.42) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

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$*$ In the SSM with SM interaction all results should be equal

$$\Phi_{^8\text{B}}^{\text{ES,SK}} = \Phi_{^8\text{B}}^{\text{CC,SNO}} \Rightarrow 3.2\sigma \text{ out}$$

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$$\left(r = \frac{\sigma_{\mathrm{ES}}(\nu_e)}{\sigma_{\mathrm{ES}}(\nu_\mu)} \simeq \frac{1}{6}\right)$$

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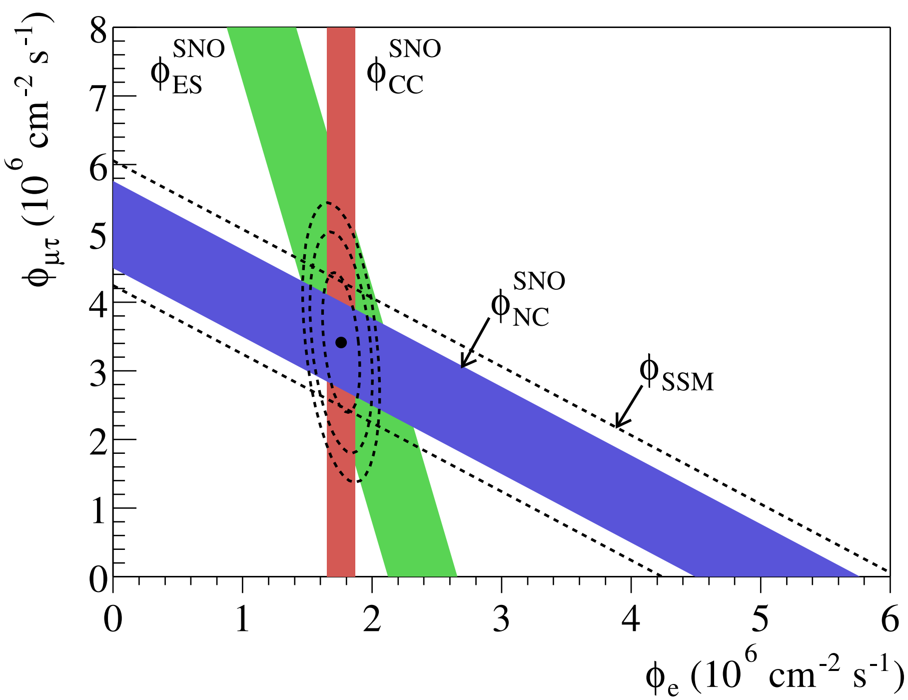

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- Fitting the observed rates:

		$\frac{\text{Data}}{\text{SSM}}$	R_{th}
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Ga		0.52 ± 0.03	$0.1\ f_B\ \langle P_{ee}\rangle_H + 0.36\ \langle P_{ee}\rangle_I + 0.54\ \langle P_{ee}\rangle_L$
SK		0.41 ± 0.01	$f_B[\langle P_{ee}\rangle_H + \frac{1}{6}(1-\langle P_{ee}\rangle_H)]$
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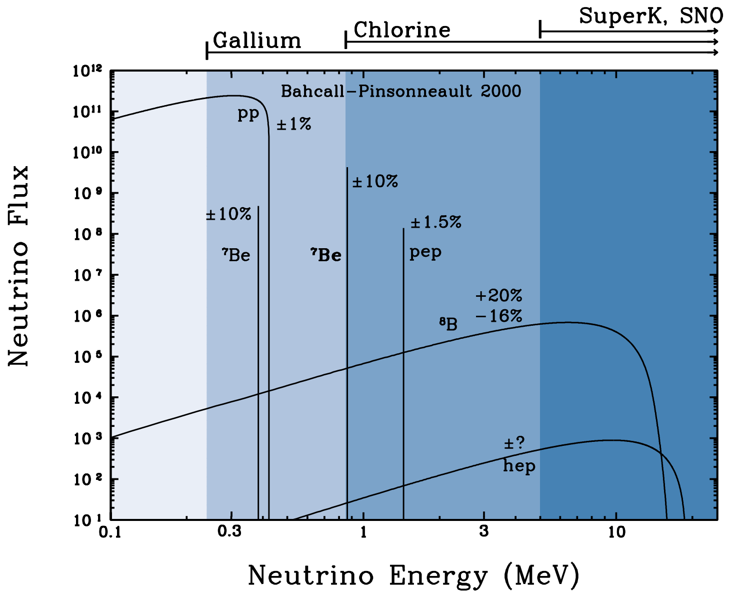

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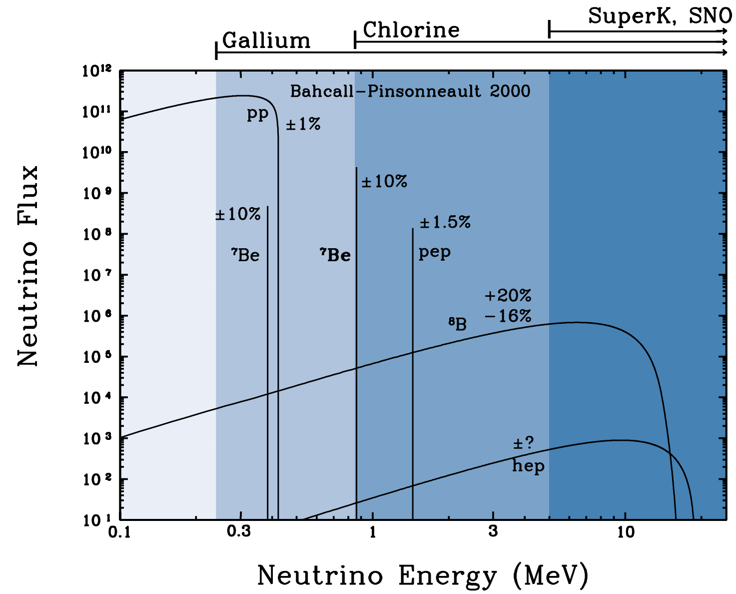
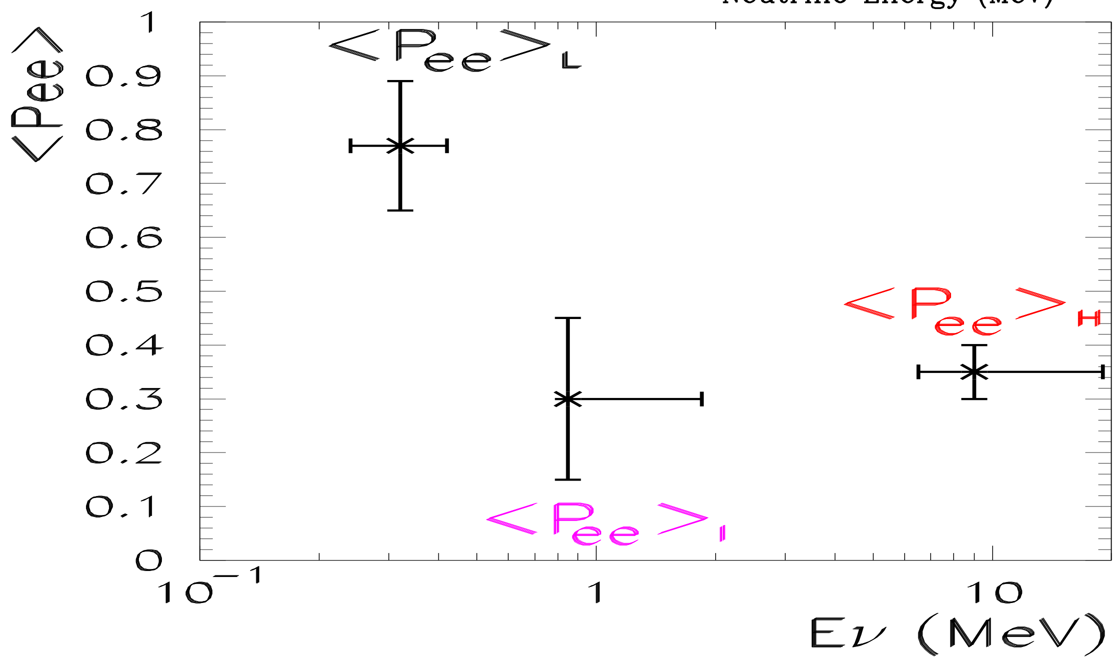


- The ν_e survival probability :



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• Coherence $\Rightarrow$ decoupling of ν evolution equation from eqs of medium.

• The effect of the medium is described by an effective potential depending on density and composition of matter

- Lets consider ν_e in a medium with e, p, and n. The effective low-energy Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}}[J^{(+)\alpha}(x)J_\alpha^{(-)}(x) + \frac{1}{4}J^{(N)\alpha}(x)J_\alpha^{(N)}(x)]$$

CC Int $\quad J_\alpha^{(+)}(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x) \qquad J_\alpha^{(-)}(x) = \overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)$

NC Int $\quad J_\alpha^{(N)}(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x) - \overline{e}(x)[\gamma_\alpha(1-\gamma_5) - s_W^2\gamma_\alpha]e(x)$

$$+\overline{p}(x)[\gamma_\alpha(1-g_A^{(p)}\gamma_5) - 4s_W^2\gamma_\alpha]p(x) - \overline{n}(x)[\gamma_\alpha(1-g_A^{(n)}\gamma_5) - 4s_W^2\gamma_\alpha]n(x)$$

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• Example: The effect of CC with the e medium. The effective CC Hamiltonian:

$$H_C^{(e)} = \frac{G_F}{\sqrt{2}}\int d^3p_e f(E_e,T)\Big\langle\langle e(s,p_e)|\overline{e}\gamma^\alpha(1-\gamma_5)\nu_e\overline{\nu_e}\gamma_\alpha(1-\gamma_5)|e(s,p_e)\rangle\Big\rangle$$

Fierz rearrange

$$= \frac{G_F}{\sqrt{2}}\overline{\nu_e}\gamma_\alpha(1-\gamma_5)\nu_e\int d^3p_e f(E_e,T)\Big\langle\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle\Big\rangle$$

$f(E_e,T)$ statistical energy distribution of e in *homogeneous and isotropic* medium.

$$\int d^3p_e f(E_e,T) = 1$$

$\Big\langle\ldots\Big\rangle \equiv$ averaging over electron spinors and summing over all e.

coherence $\Rightarrow$ s, p_e same for initial and final e

- Expanding the electron fields e in plane waves

$$\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \frac{1}{V}\langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

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$$\frac{1}{V}\Big\langle \langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle \Big\rangle \equiv N_e(p_e)\frac{1}{2}\sum_s$$

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$$\Big\langle \langle e(s,p_e)|\bar{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \Big\rangle = \frac{N_e(p_e)}{2}\sum_s \overline{u_{(s)}}(p_e)\gamma_\alpha(1-\gamma_5)u_{(s)}(p_e)$$

$$= \frac{N_e(p_e)}{2}\sum_s Tr\left[\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)\right] = \frac{N_e(p_e)}{2}\sum_s Tr\left[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\right]$$

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• Expanding the electron fields e in plane waves

$$\langle e(s,p_e)|\bar{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \frac{1}{V}\langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

• Since $a_s^\dagger(p_e)a_s(p_e) = \mathcal{N}_e^{(s)}(p_e)$ (number operator) and assuming that there are the same number of electrons with spin 1/2 and -1/2

$$\frac{1}{V}\left\langle \langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle \right\rangle \equiv N_e(p_e)\frac{1}{2}\sum_s$$

where $N_e(p_e)$ number density of electrons with momentum p_e

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• Isotropy $\Rightarrow \int d^3p_e \vec{p_e} f(E_e,T) = 0$

• Also $\int d^3p_e f(E_e,T)N_e(p_e) = N_e$ electron number density

- The effective charged current Hamiltonian due to electrons in matter is then:

$$H_C^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x) \gamma_0 (1 - \gamma_5) \nu_e(x)$$

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$$\begin{aligned} V_C &= \langle \nu_e | \int d^3x H_C^{(e)} | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \langle \nu_e | \int d^3x \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x) | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \frac{1}{V} 2 \int d^3x\, u_\nu^\dagger u_\nu = \sqrt{2} G_F N_e \end{aligned}$$

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$$V_C = \sqrt{2} G_F N_e$$

• for $\overline{\nu_e}$ the sign of V is reversed

• Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}} N_n$

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• Estimating typical values:

$$V_C = \sqrt{2}G_F N_e \simeq 7.6\, Y_e \frac{\rho}{10^{14}\text{g/cm}^3}\text{ eV}$$

$Y_e = \frac{N_e}{N_p+N_n} \equiv$ relative number density

$\rho \equiv$ matter density

– At the solar core $\rho \sim 100$ g/cm$^3 \Rightarrow V \sim 10^{-12}$ eV

– At supernova $\rho \sim 10^{14}$ g/cm$^3 \Rightarrow V \sim$ eV

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$$E\,\Phi_1 = \Big[-i\,\alpha_x \frac{\partial}{\partial x} + \beta\, m_1\Big]\Phi_1$$
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- We decompose $\Phi_i(x) = \nu_i(x)\phi_i$ ϕ_i is the Dirac spinor part satisfying:

$$\left(\alpha_x \left\{E^2 - m_i^2\right\}^{1/2} + \beta\, m_i\right)\phi_i = E\phi_i \qquad (1)$$

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- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial\nu_1(x)}{\partial x} = \left\{E^2 - m_1^2\right\}^{1/2}\nu_1(x)$$
$$-i\frac{\partial\nu_2(x)}{\partial x} = \left\{E^2 - m_2^2\right\}^{1/2}\nu_2(x)$$

- In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

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- In weak ($\equiv$ flavour) basis $\nu_\alpha = U_{\alpha i}(\theta)\nu_i$

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- And the flavour transition probability

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \text{sterile})$

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(a) In vacuum in the mass basis: $$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = E - \begin{pmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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(c) In matter (e, p, n) in weak basis

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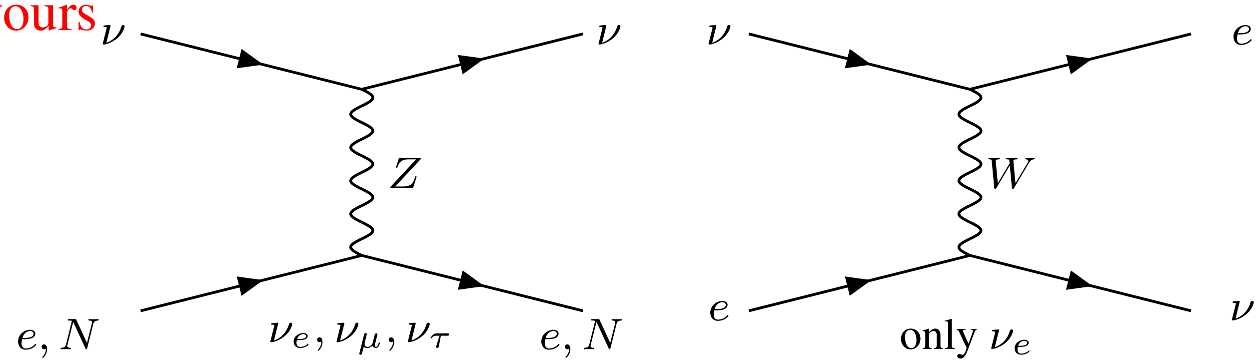
(c)$\neq$ (b) because different flavours have different interactions

For example $X = \mu, \tau$:

$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$

(opposite sign for $\overline{\nu}$)



$\Rightarrow$ Effective masses and mixing are different than in vacuum

$\Rightarrow$ If matter density varies along ν trajectory the effective masses and mixing vary too

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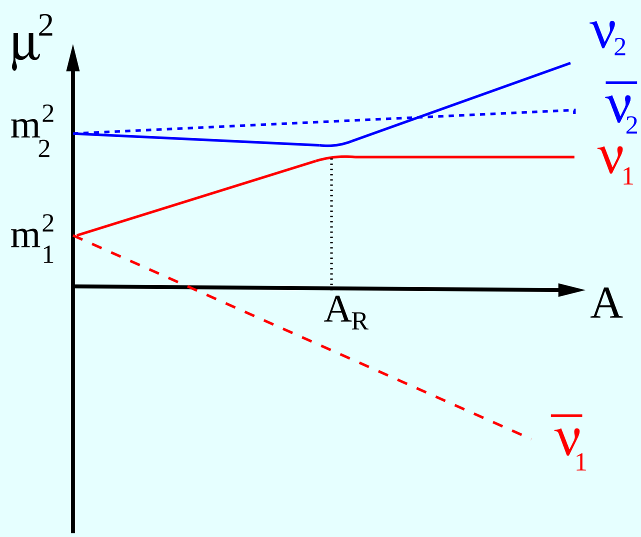
The effective masses: $(A = 2E(V_e - V_X))$

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm\frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$



At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

⇒ Effective masses and mixing are different than in vacuum

⇒ If matter density varies along ν trajectory the effective masses and mixing vary too

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$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
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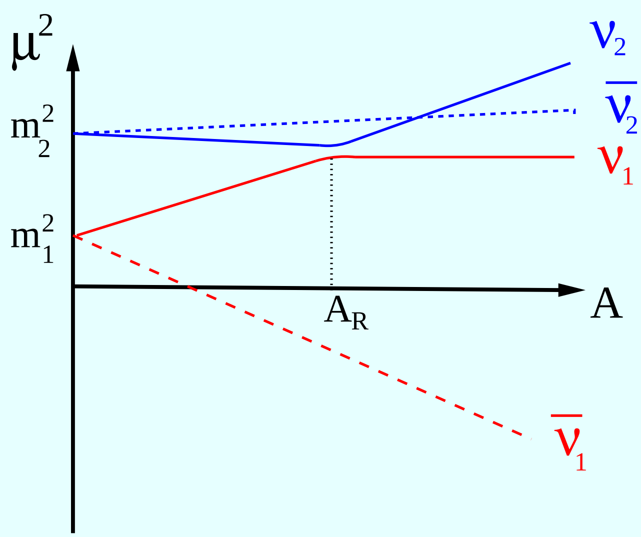
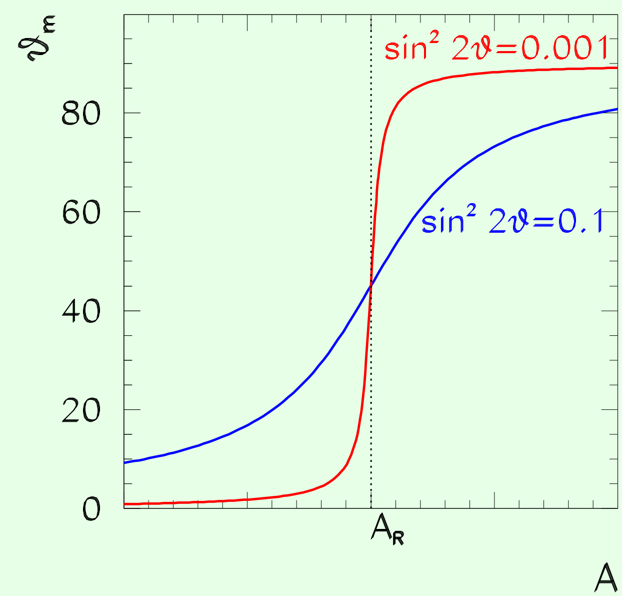


At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



* At $A = 0$ (vacuum) ⇒ $\theta_m = \theta$
* At $A = A_R$ ⇒ $\theta_m = \frac{\pi}{4}$
* At $A >> A_R$ ⇒ $\theta_m \to \frac{\pi}{2}$

The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

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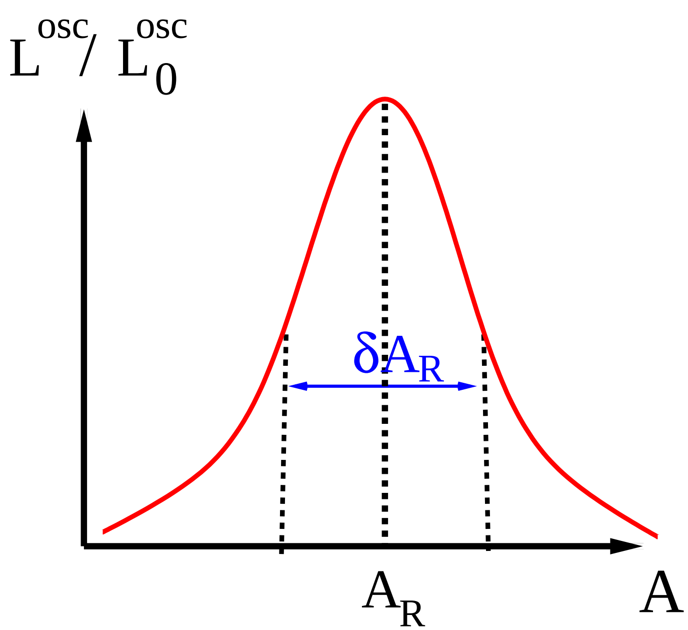
L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{\left|\frac{dV}{dr}\right|_R}$$

- In terms of the mass eigenstates in matter: $$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

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- For constant potential θ_m and μ_i are constant along ν evolution

  $\Rightarrow$ the evolution is determined by masses and mixing in matter as

$$P_{osc} = \sin^2(2\theta_m)\,\sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

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$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i\begin{pmatrix} \dot\nu_e \\ \dot\nu_X \end{pmatrix} = \frac{1}{2E}\begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

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$\Rightarrow$ the evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{2E} U^\dagger(\theta_m) \begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i\, U^\dagger \dot U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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⇒ the evolution is determined by masses and mixing in matter as

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$$\Rightarrow \; i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\, i\, E\, \dot\theta_m(x) \\ 4\, i\, E\, \dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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$\Rightarrow$ It is not diagonal $\Rightarrow$ Instantaneous mass eigenstates $\neq$ eigenstates of evolution

$\Rightarrow$ Transitions $\nu_1^m \to \nu_2^m$ can occur $\equiv$ *Non adiabaticity*

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• For $\Delta\mu^2(x) \gg 4\,E\,\dot{\theta}_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

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The adiabaticity condition

$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\, \delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

Neutrinos in The Sun : MSW Effect

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- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

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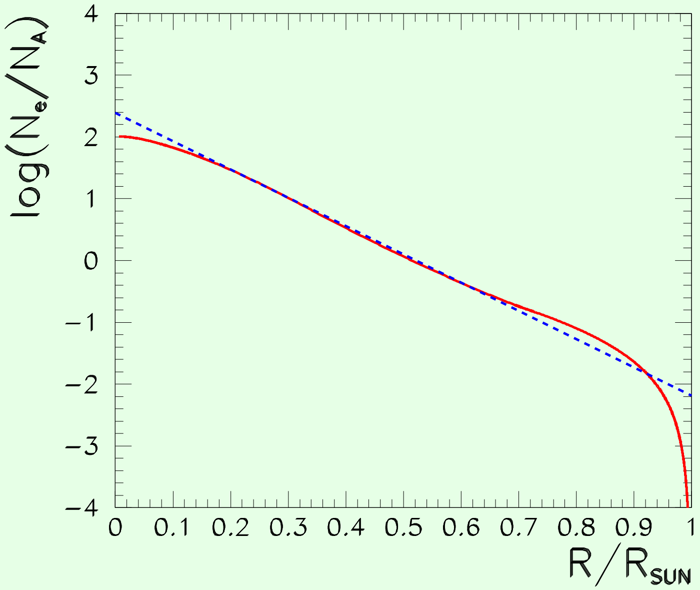
The solar matter density



$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A}$ eV

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

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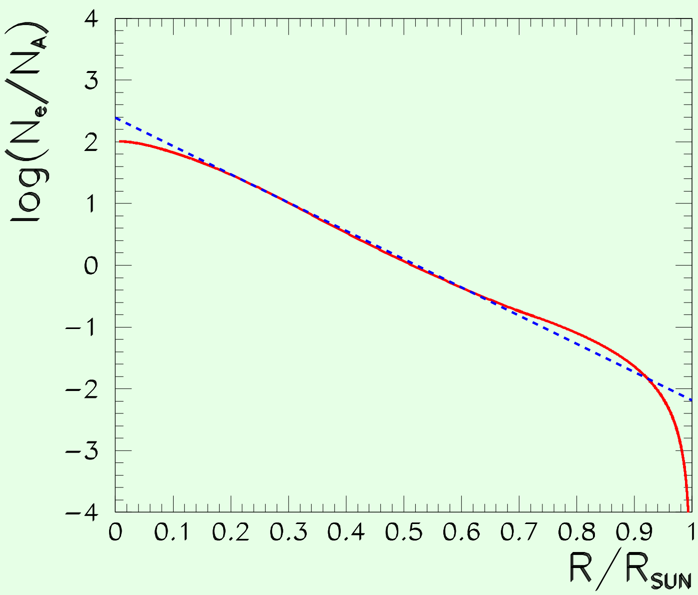

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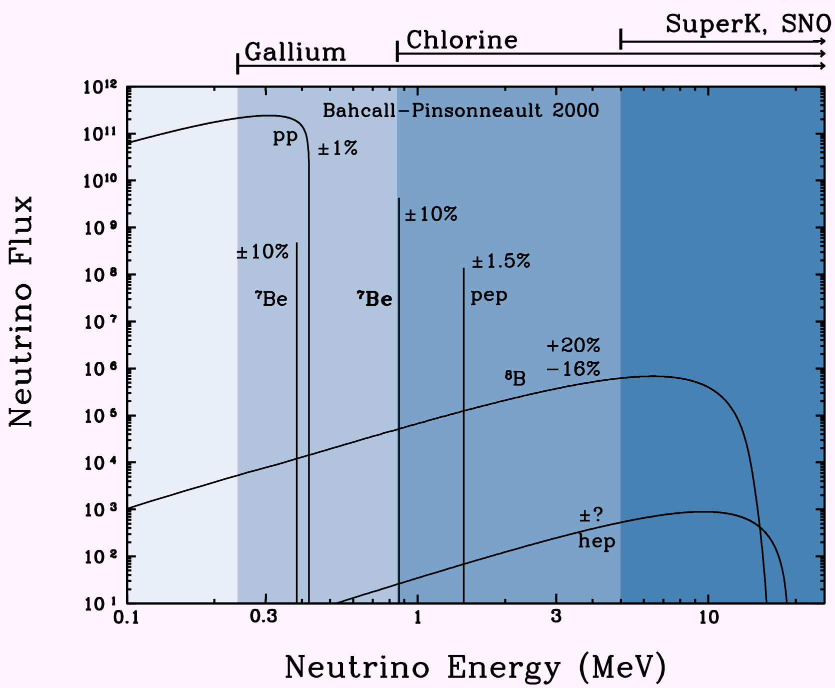
The energy spectrum of solar $\nu_e' s$



$E_\nu \sim 0.1$–10 MeV

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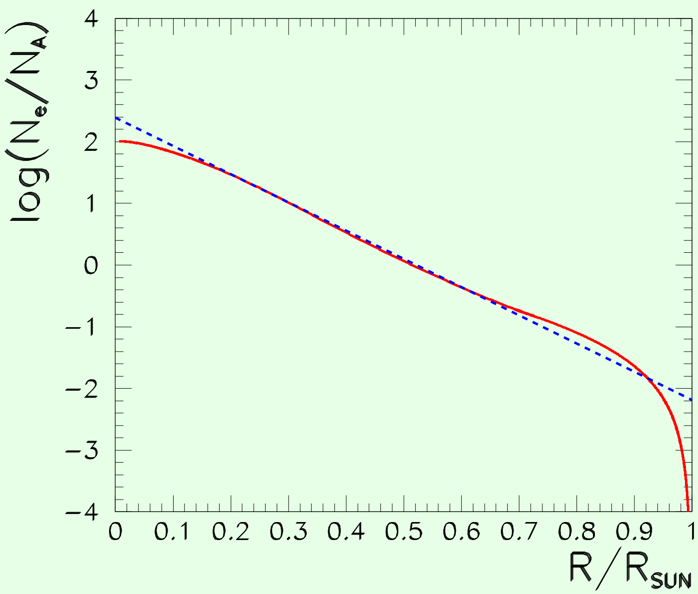

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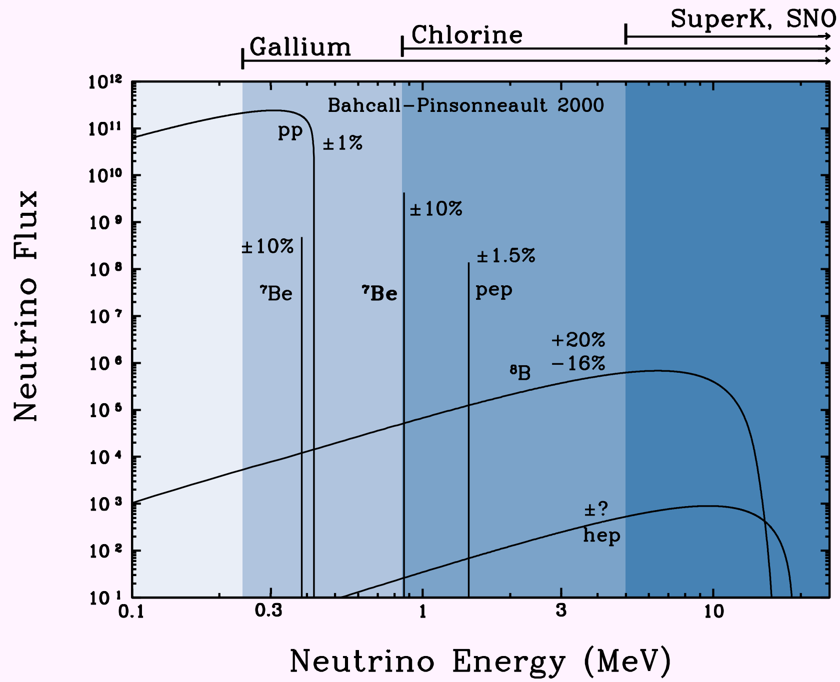

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- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$

- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2 E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

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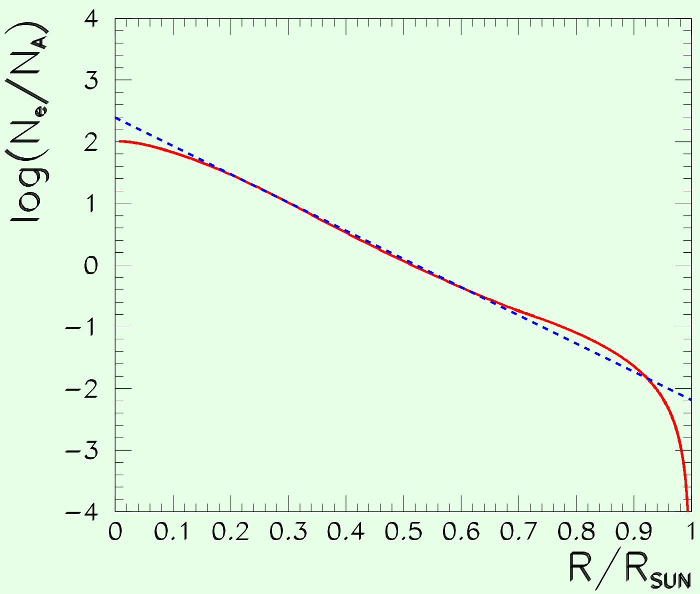

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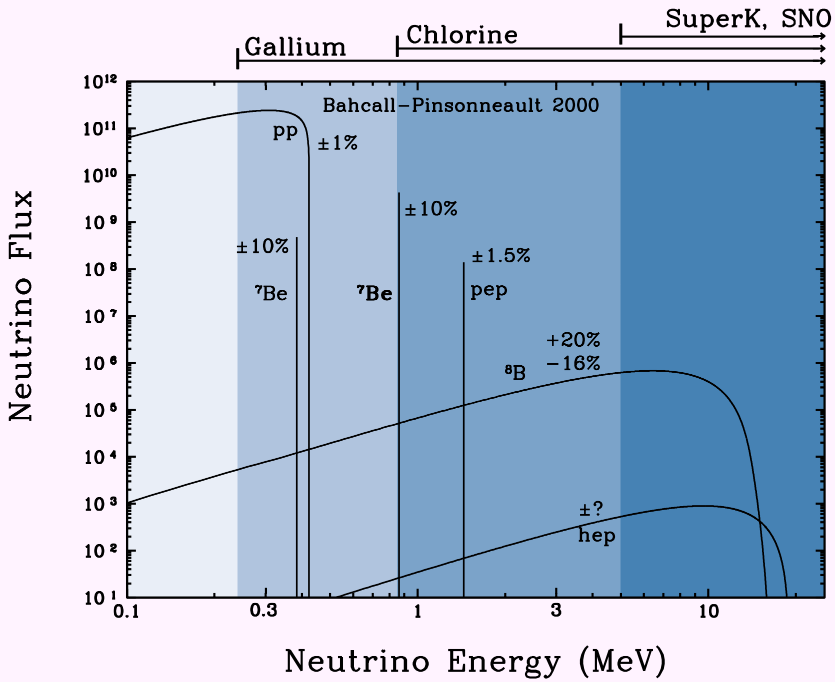

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$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

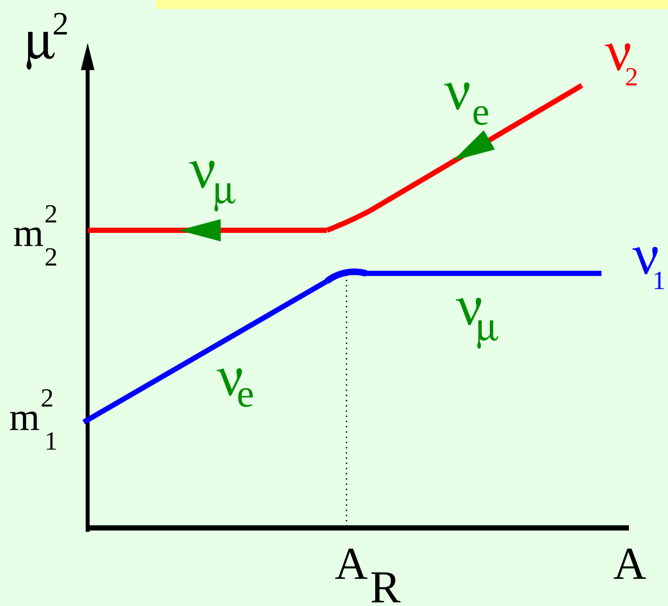
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow$ ν_e component $\downarrow \Rightarrow P_{ee} \downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

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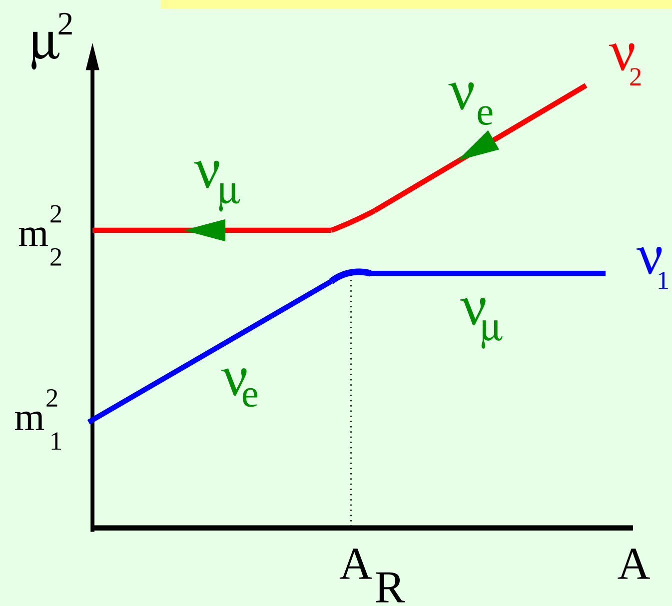


$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

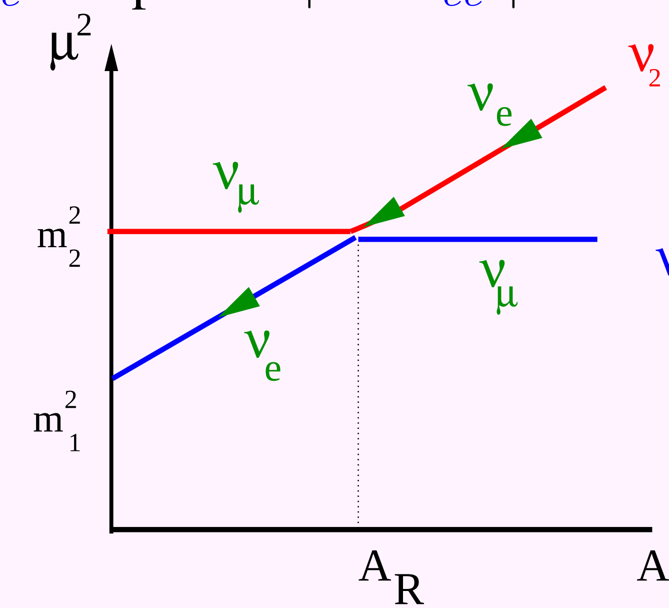
If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \lesssim 3\times 10^{-9}$

$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

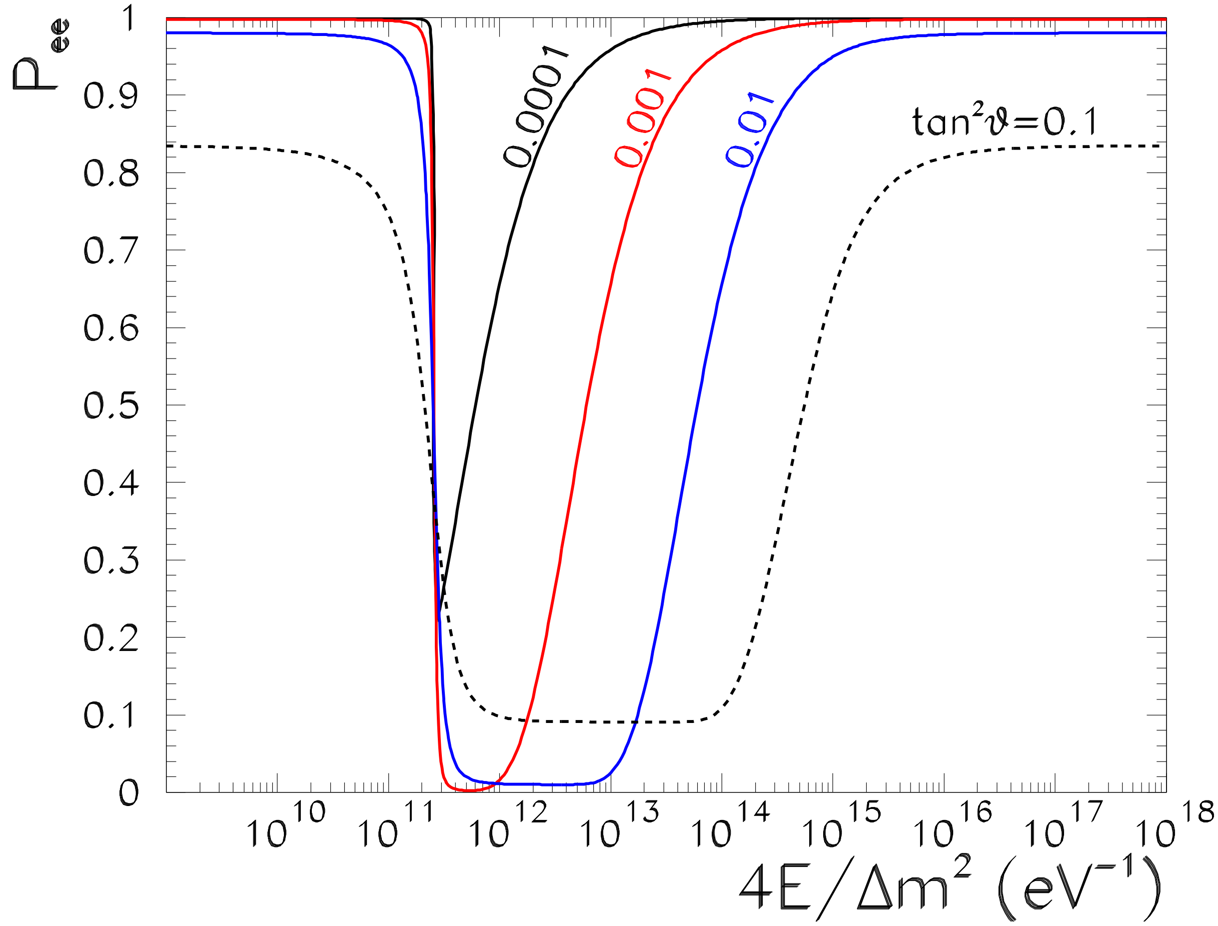
$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow$ ν_e component $\uparrow \Rightarrow P_{ee} \uparrow$



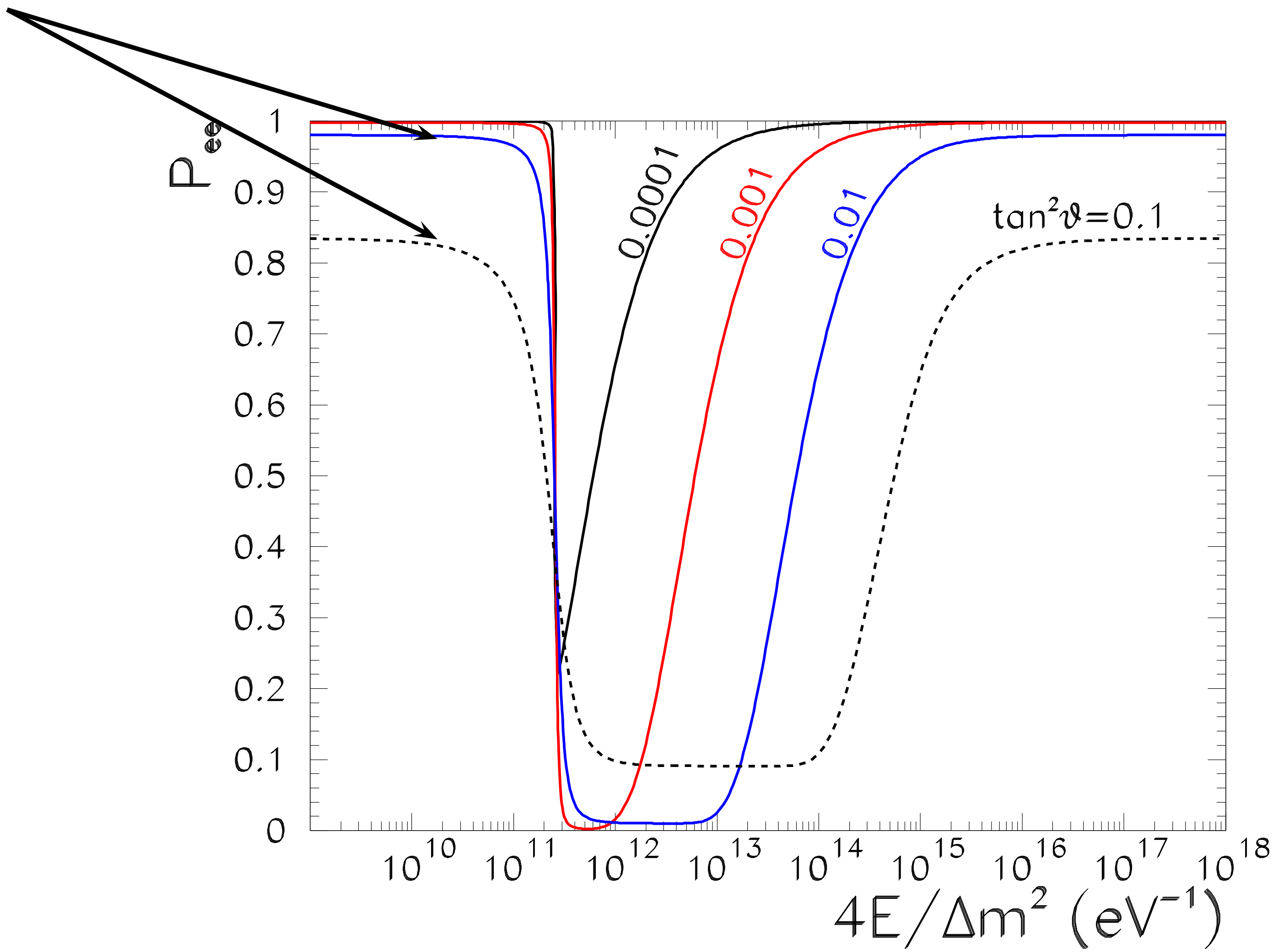
$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect



Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



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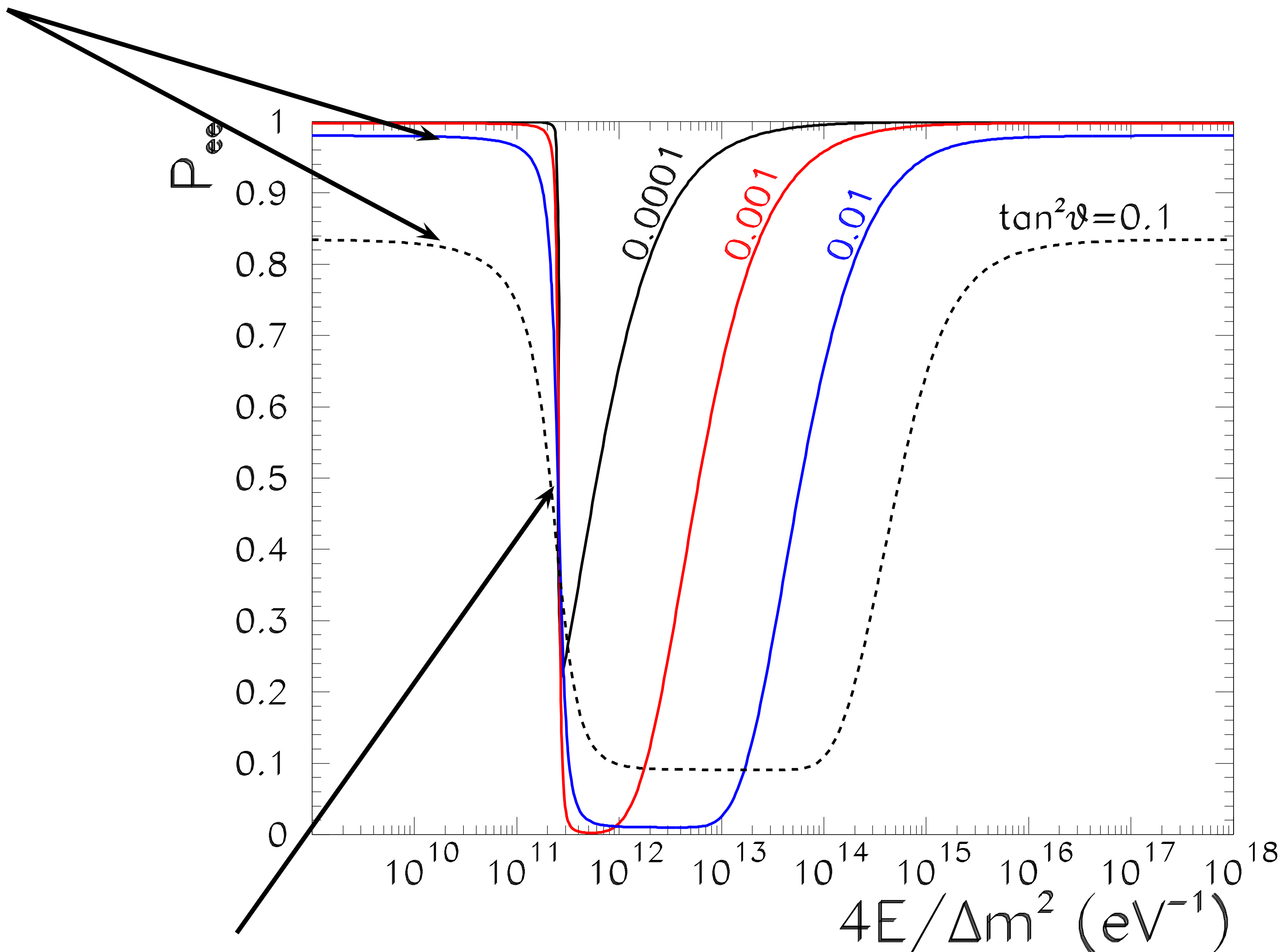

ν crosses resonance

MSW effect

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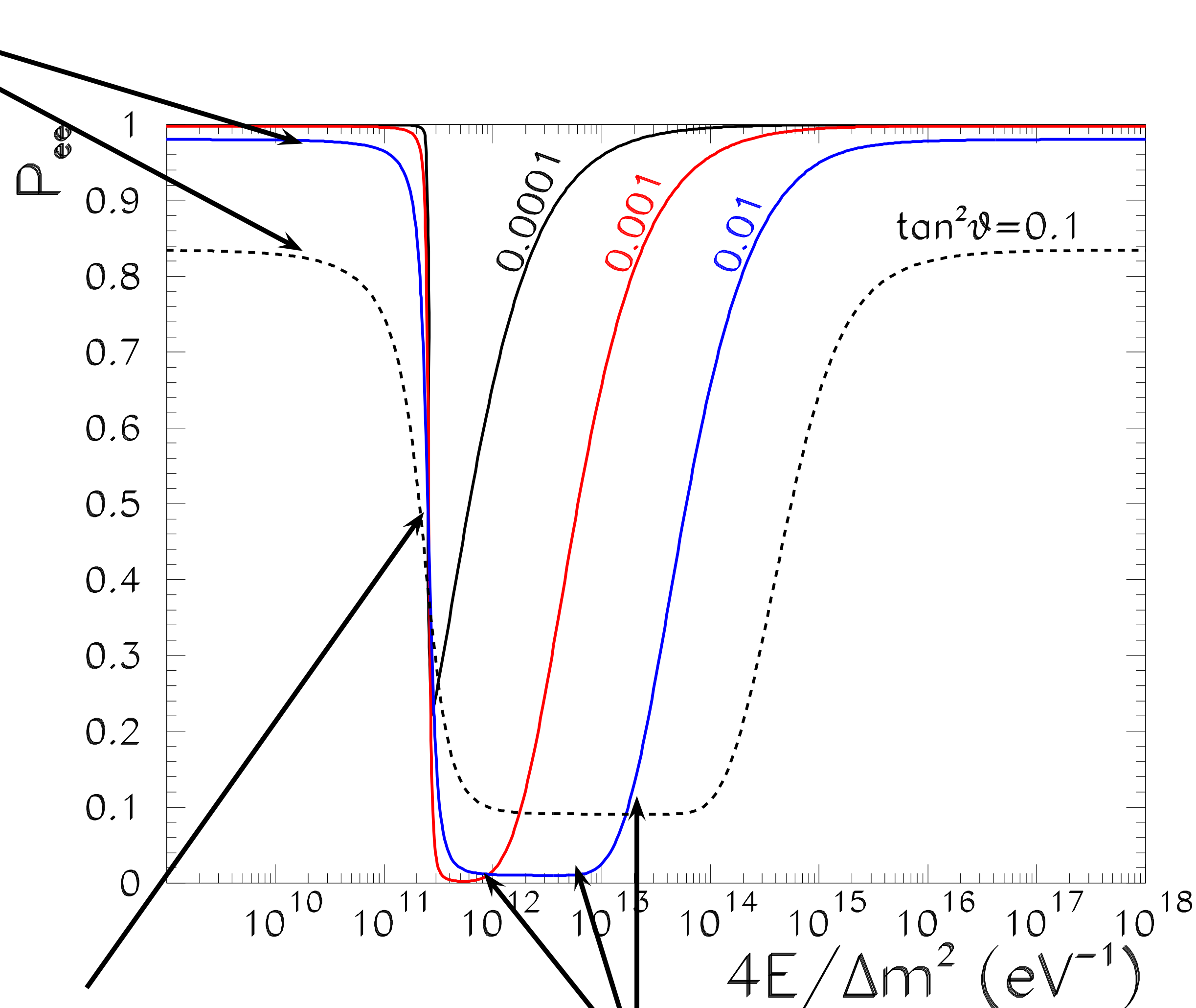

ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

Neutrinos in The Sun : MSW Effect

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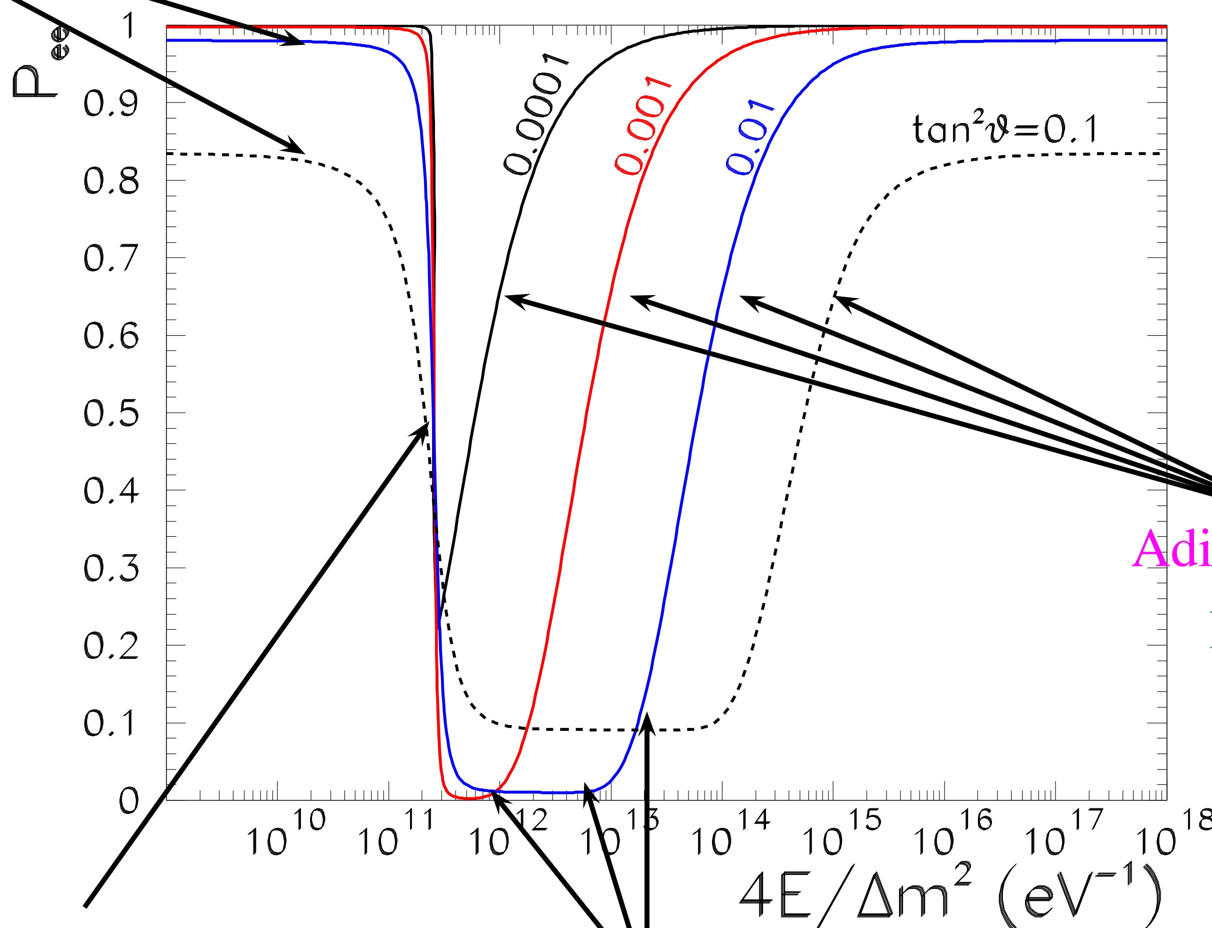


Adiabacity breaking

Effect of P_{LZ}

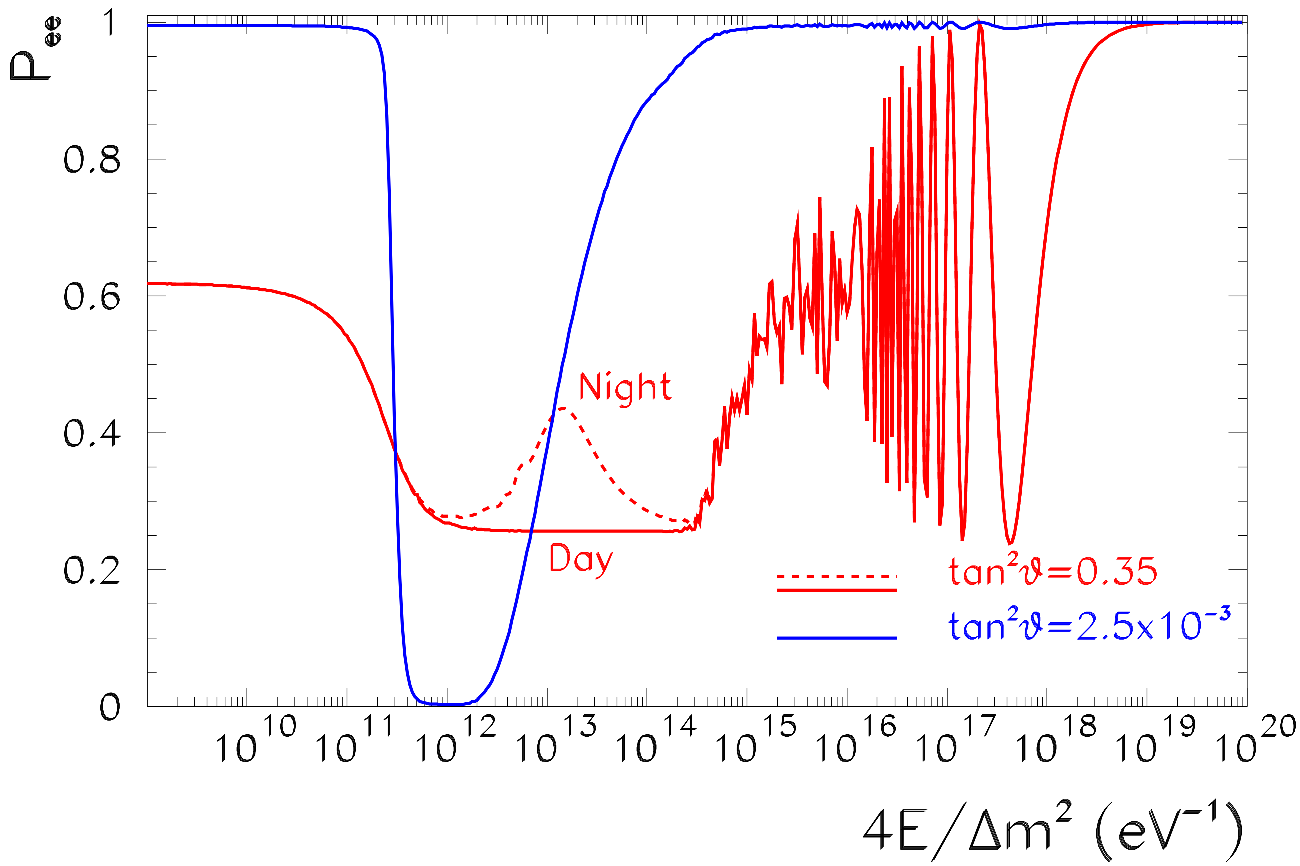
ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

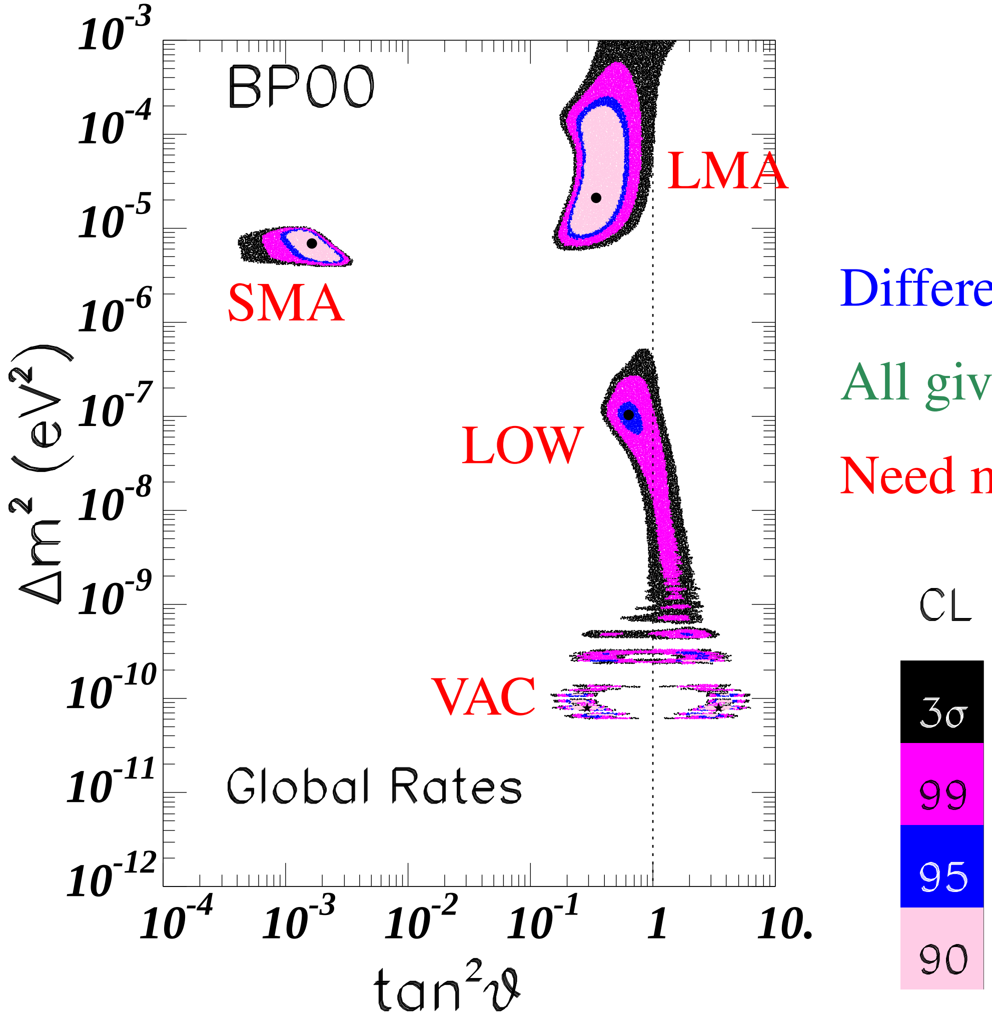
Neutrinos from The Sun : The Full Story

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e)$$
$$+A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$



Solar Neutrinos: Oscillation Solutions

Allowed regions by Fit to Total Rates: Cl, Ga, SK and SNO CC



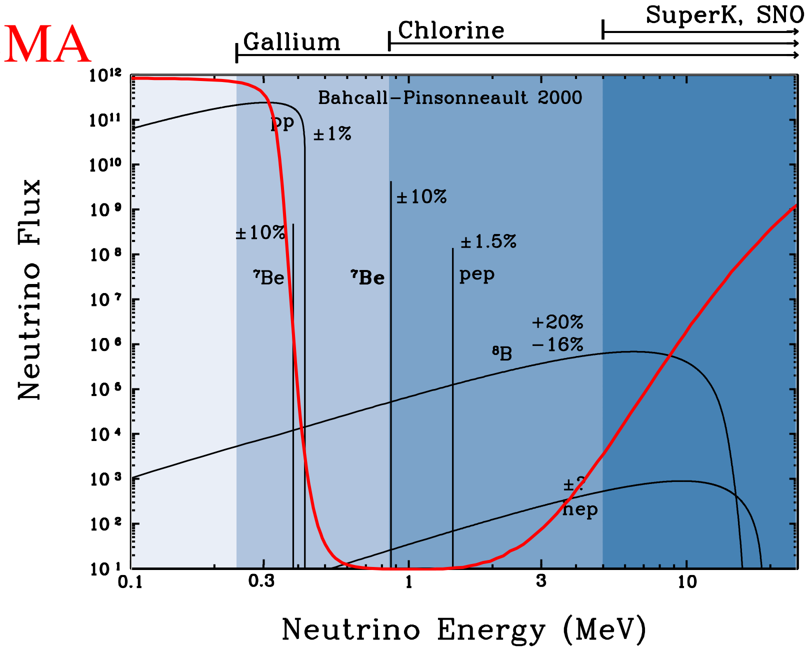
Different regimes can explain the Total Rates

All give similar $\langle P_{ee}\rangle_L$, $\langle P_{ee}\rangle_I$,$\langle P_{ee}\rangle_H$

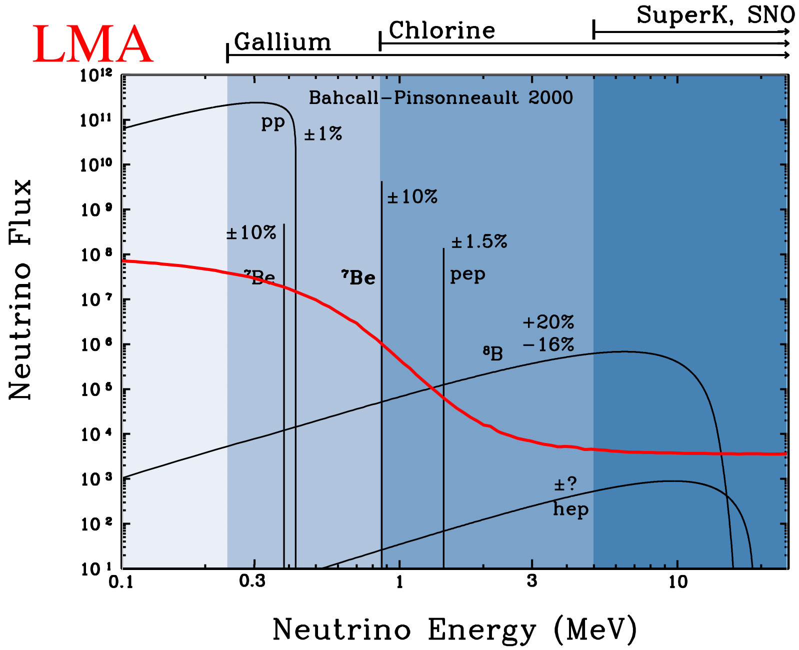
Need more observables to discriminate

Energy Dependence of P_{ee} for Different Solutions

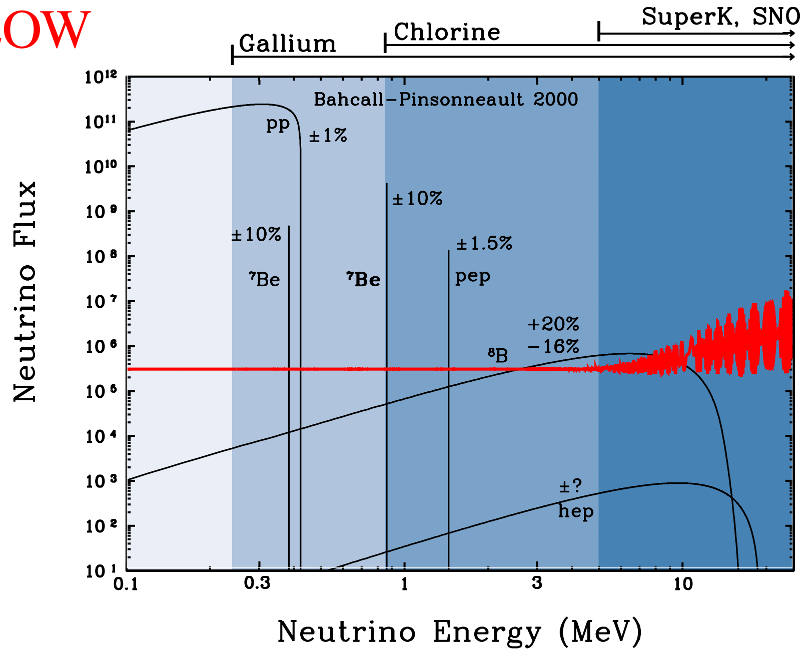
SMA



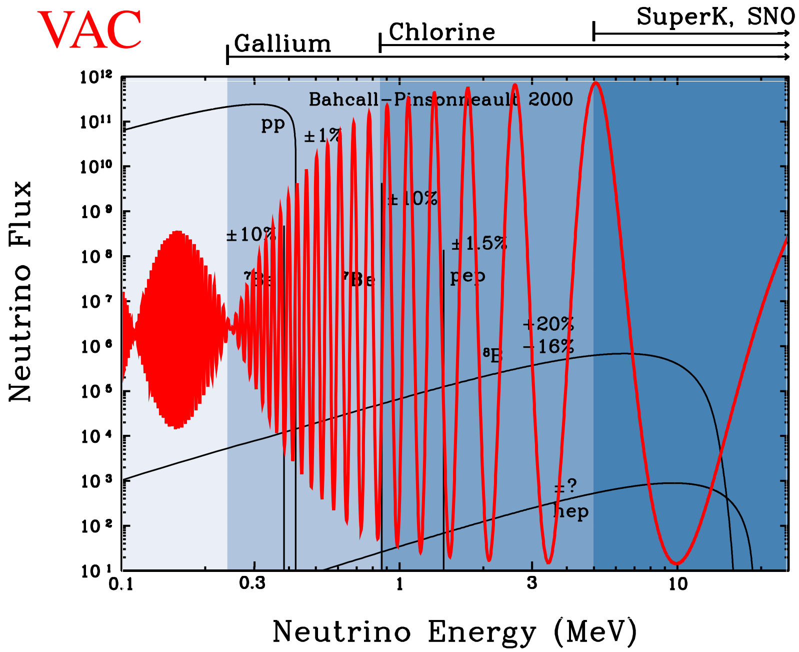
LMA



LOW



VAC



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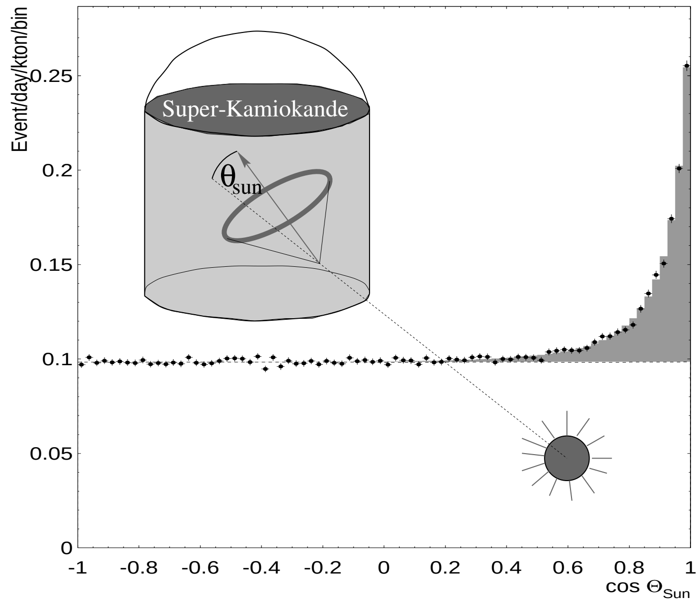

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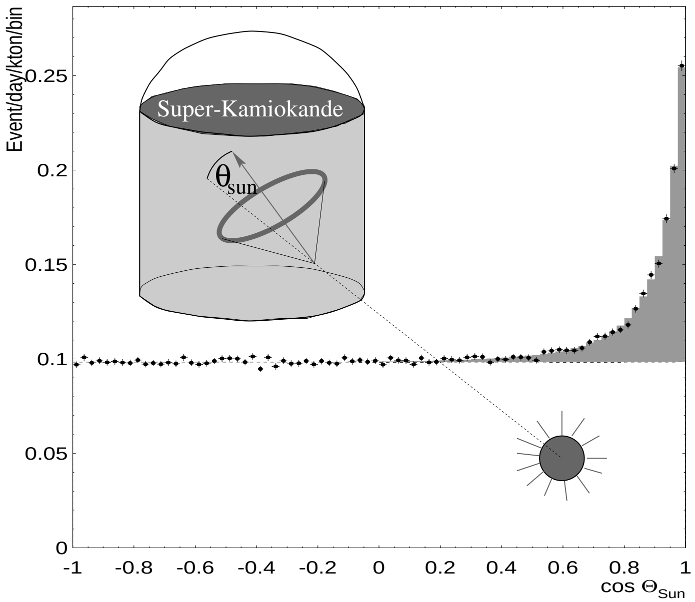
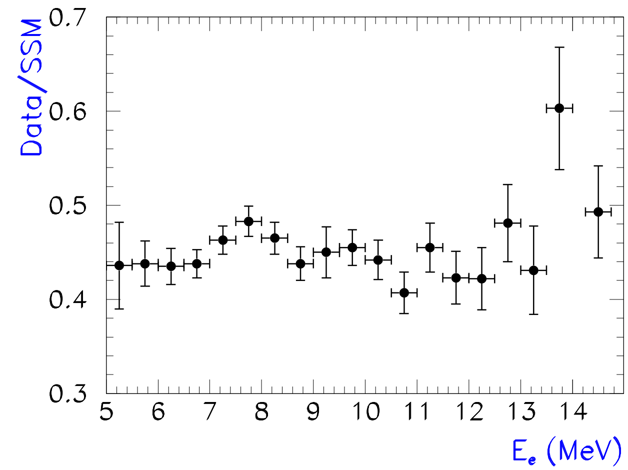


No Energy Distorsion

Deficit indep $E_\nu \gtrsim 5$ MeV



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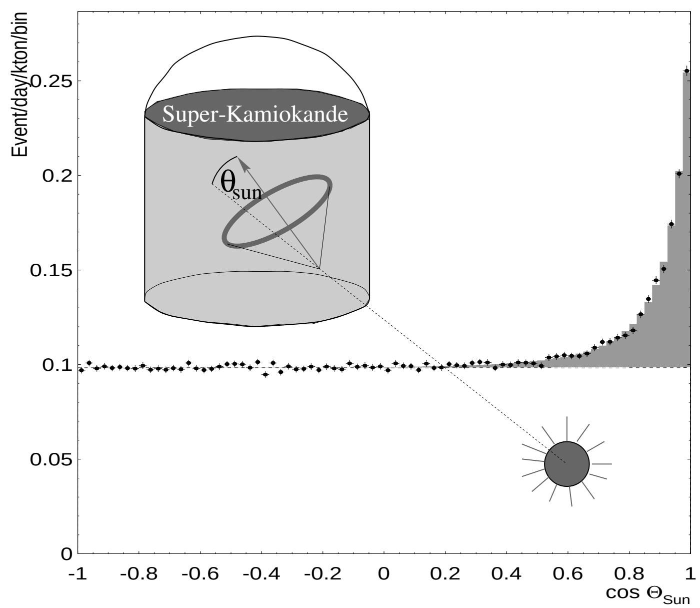

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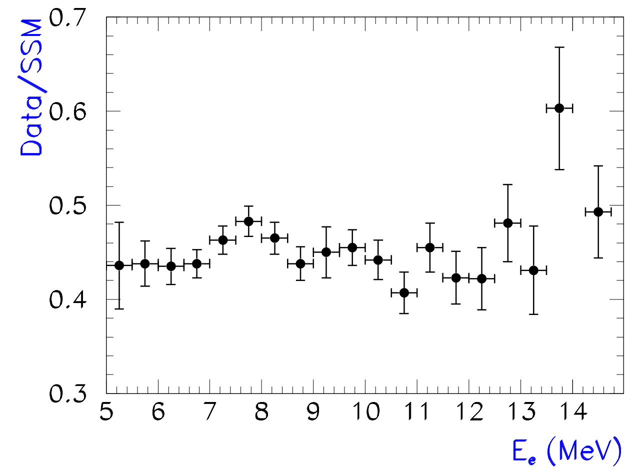

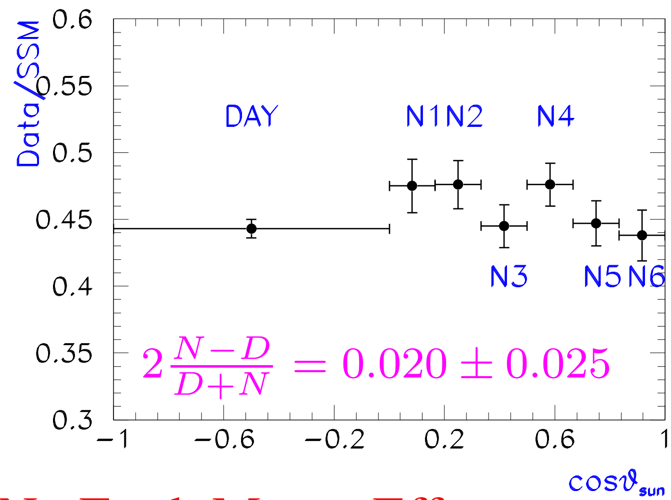

No Earth Matter Effect:

Small Day-Night Asymmetry

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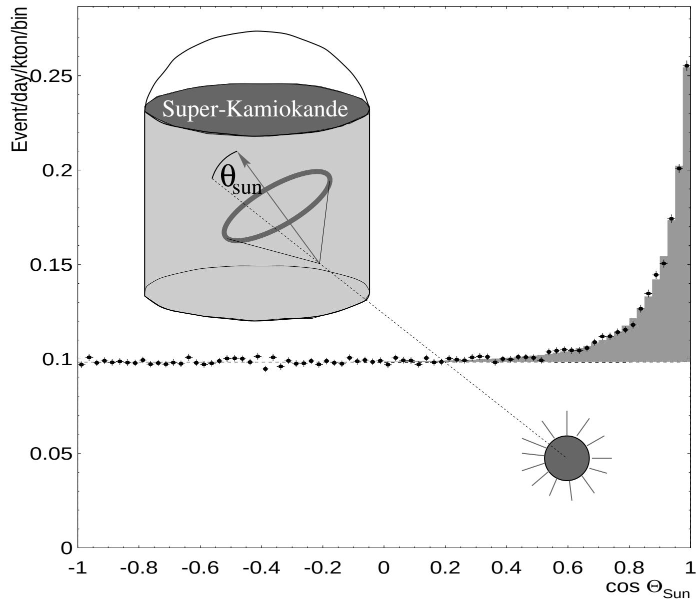

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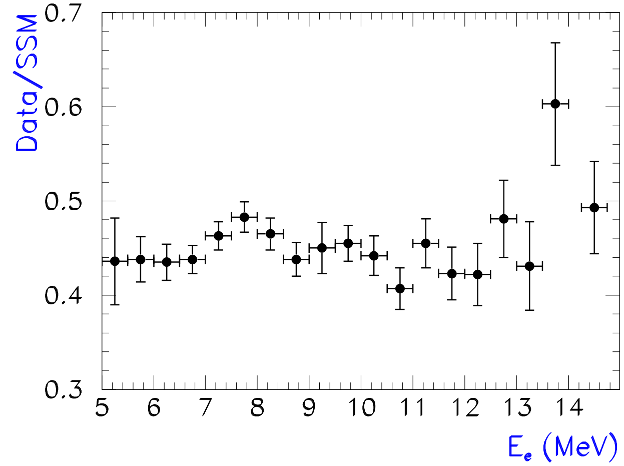

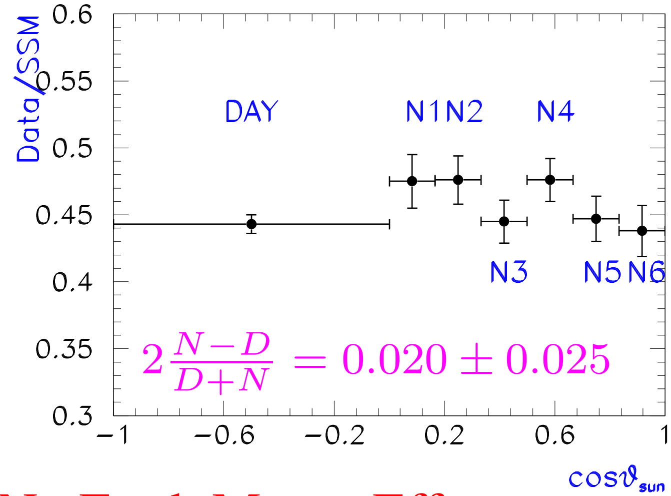

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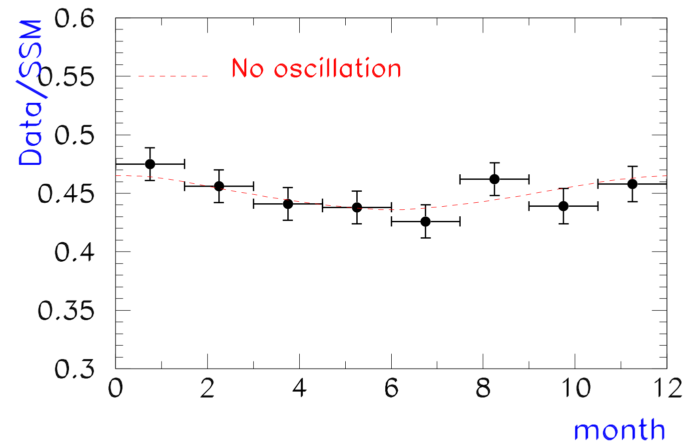
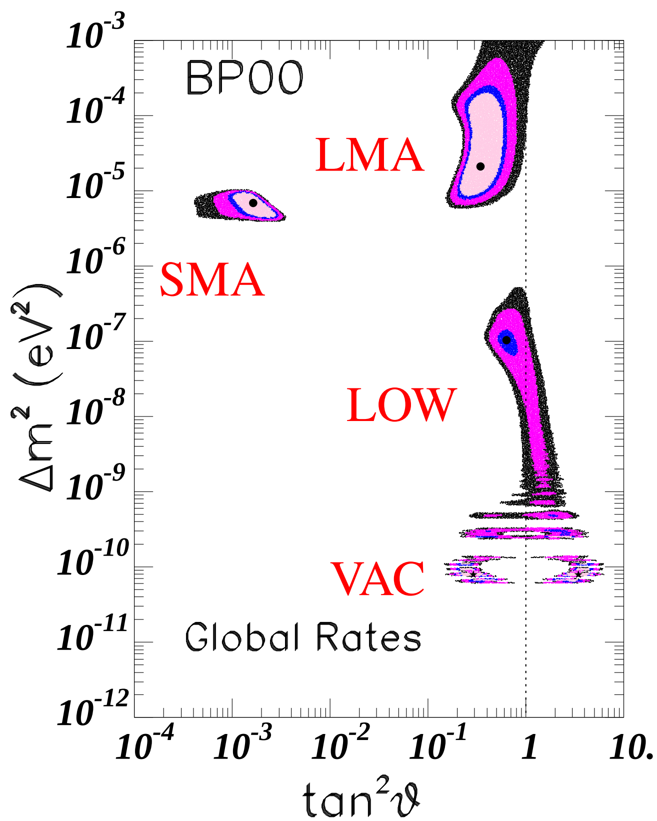


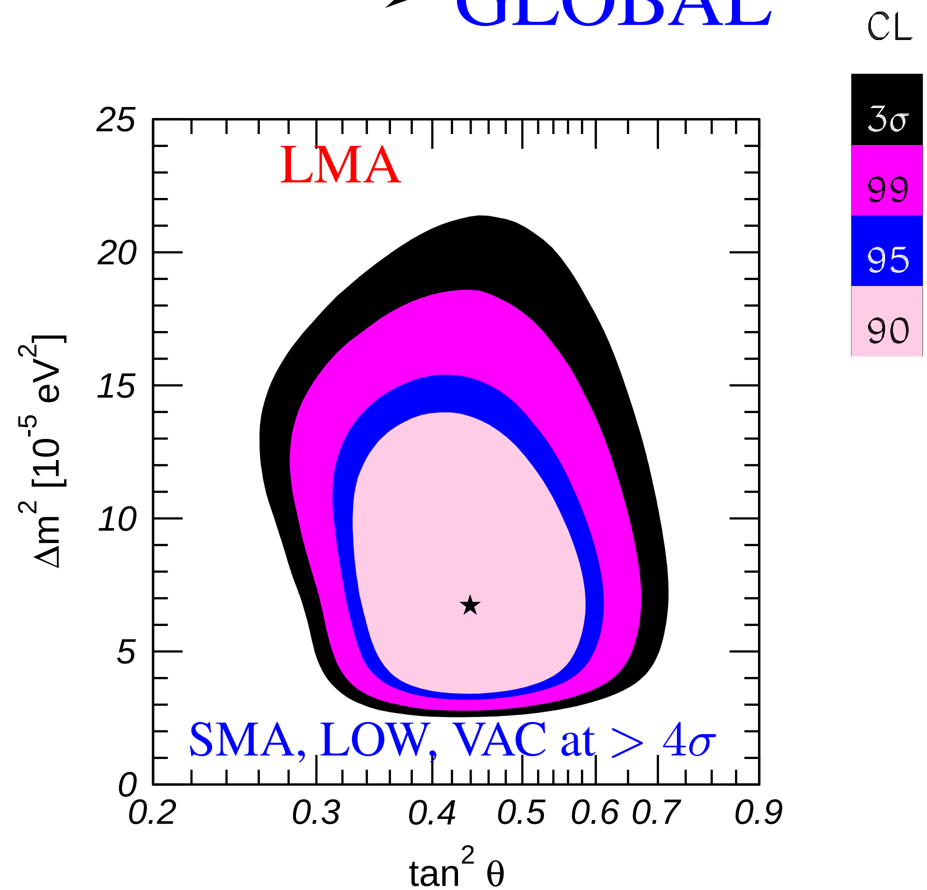
Seasonal Variation

Nothing beyond $1/R^2$

Solar Neutrinos: Oscillation Solutions

RATES ONLY → SK and SNO E and t dependence → GLOBAL





Best fit

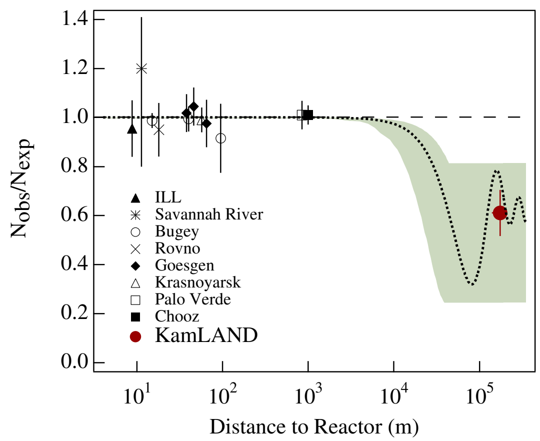
$$\Delta m^2 = 6.3 \times 10^{-5}\ \mathrm{eV}^2$$

$$\tan^2\theta = 0.44$$

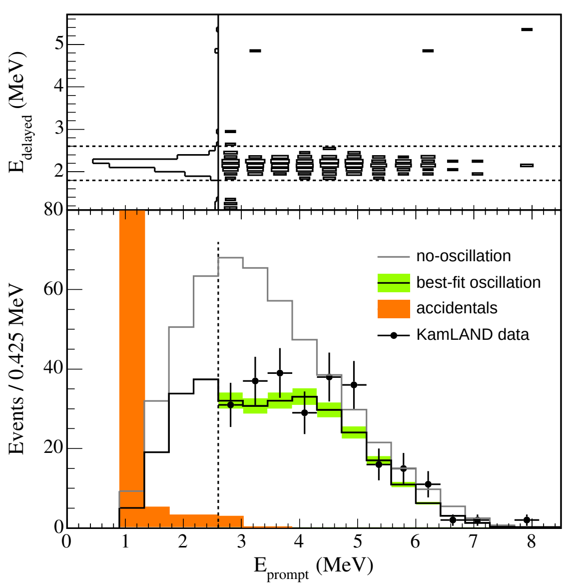
Terrestrial Test of LMA: KamLAND

- Search on $\overline{\nu_e}$ at L$\sim$ 180 km reactors, $E_{\overline{\nu}} \sim$ few MeV: $\bar{\nu}_e + p \rightarrow n + e^+$

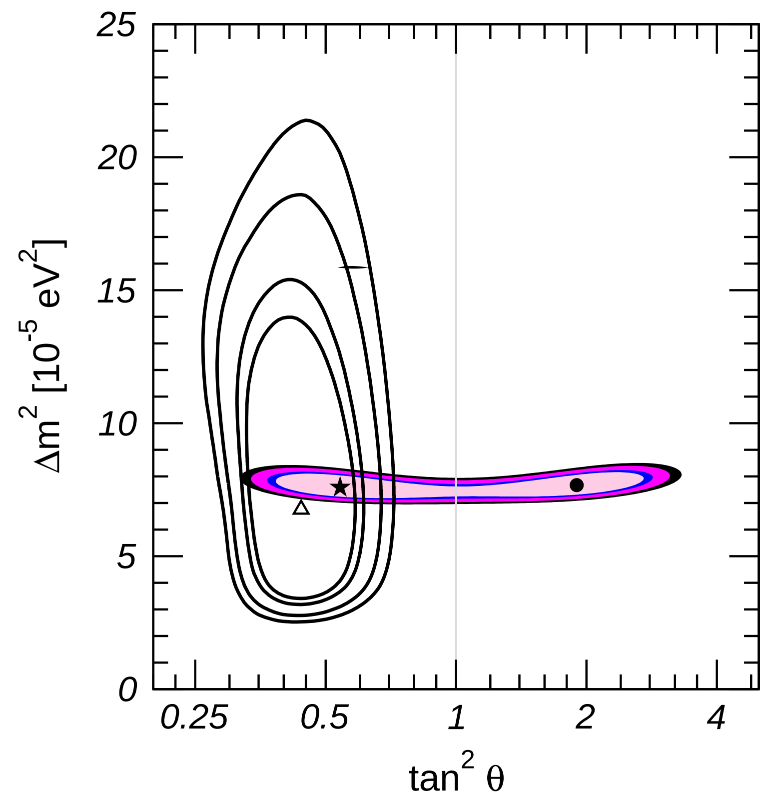
2002: Deficit $R_{\rm KLAND} = 0.611 \pm 0.094$



2004: Significant Energy Distortion



Oscillation Analysis



Learning How the Sun Shines

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- The Sun shines converting protons into α, e^+ and $\nu' s$

  $4\,p \;\rightarrow\; ^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$

  $4m_p - m_{^4He} - 2m_e \simeq 26$ MeV Thermal energy mostly in γ

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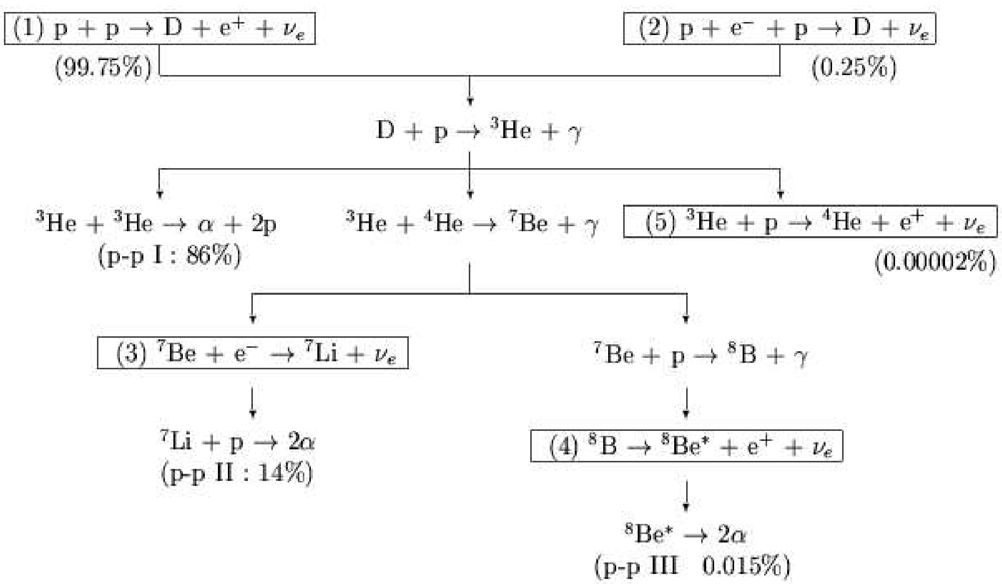

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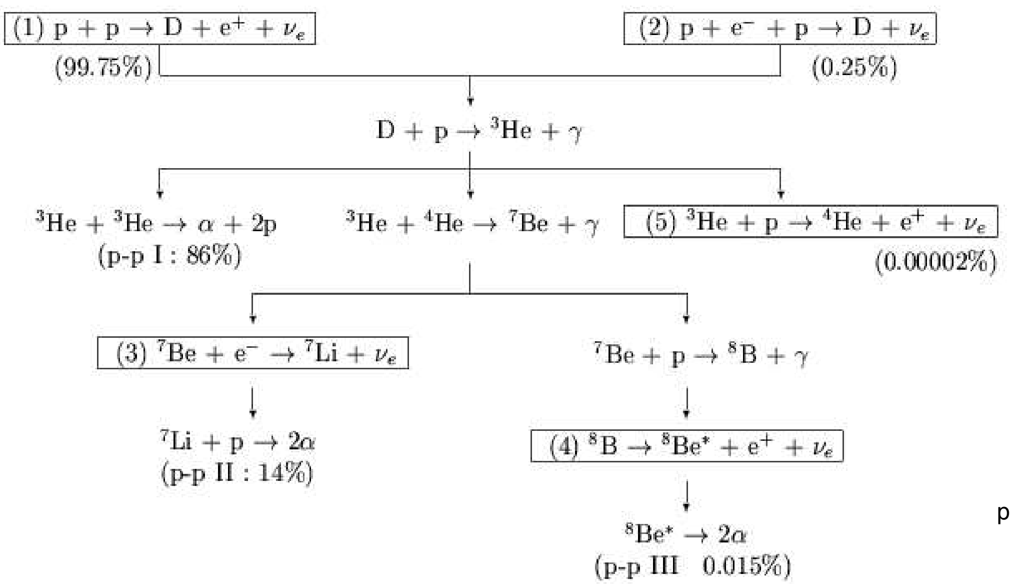
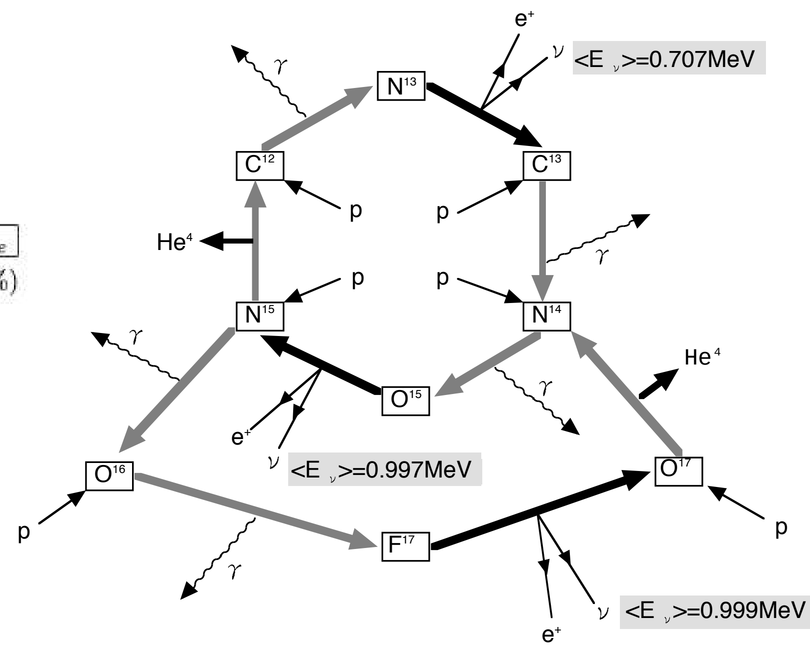


CNO cycle:



- The ratio pp/CNO very sensitive to T_{core}

- First proposal by Bethe (1939) was that CNO dominated

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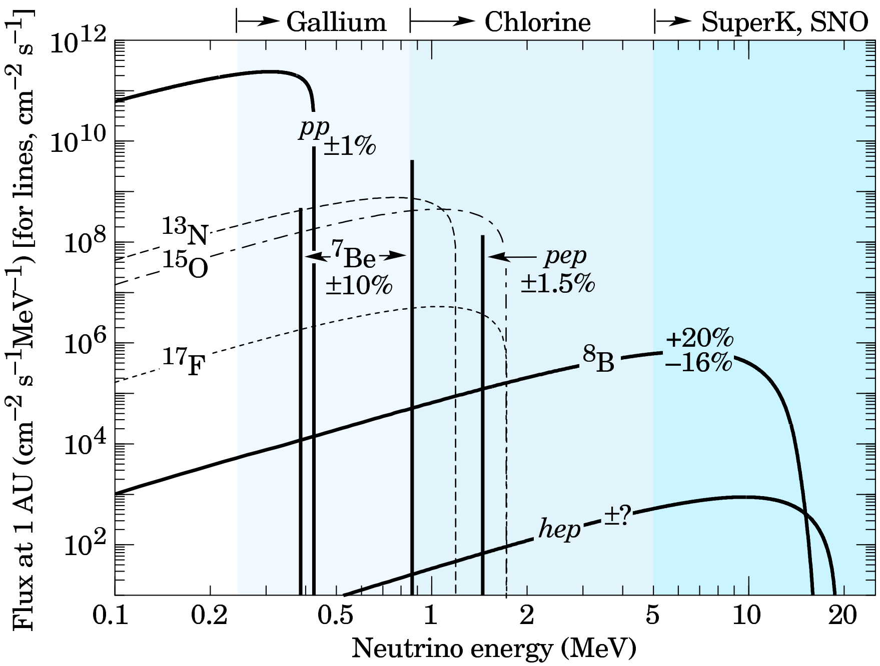

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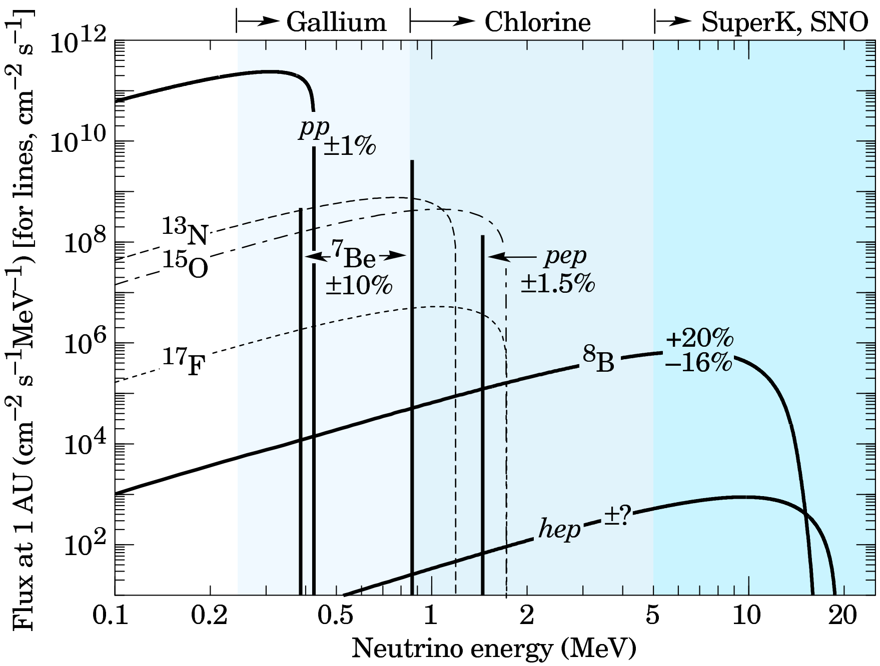

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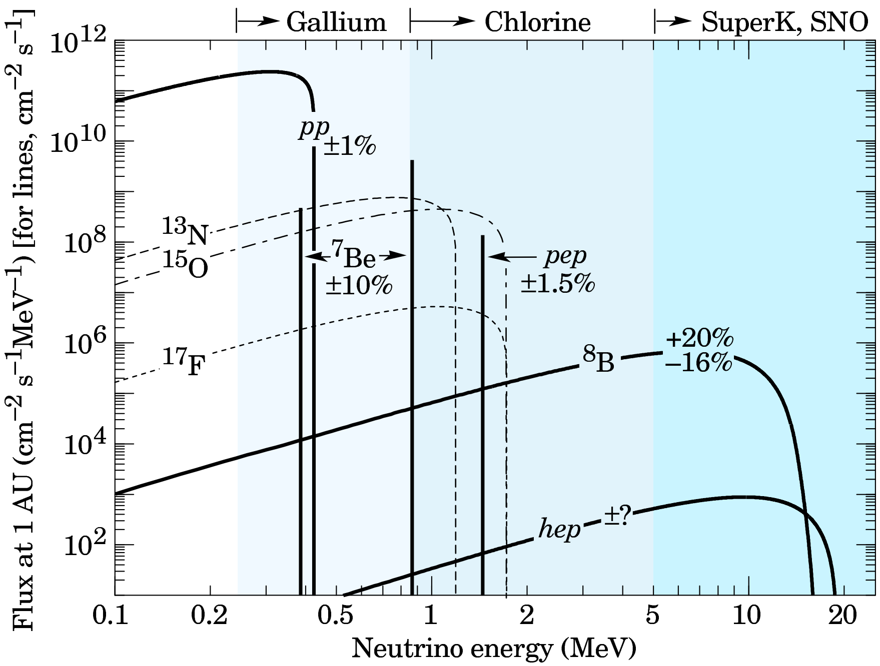

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- Can this be tested experimentally? Difficult

– Radiochemical experiments sensitive to CNO fluxes

But do not measure $E \Rightarrow$ only integrated flux above E_{th}

– Oscillations modify the E dependence of detected fluxes

$\Rightarrow$ Possible suppression of CNO fluxes $\Rightarrow$ no experimental limit

How the Sun Shines? Older Answer

- Before SK and SNO large CNO solutions allowed

Bahcall, Fukugita, Krastev PLB (1996)

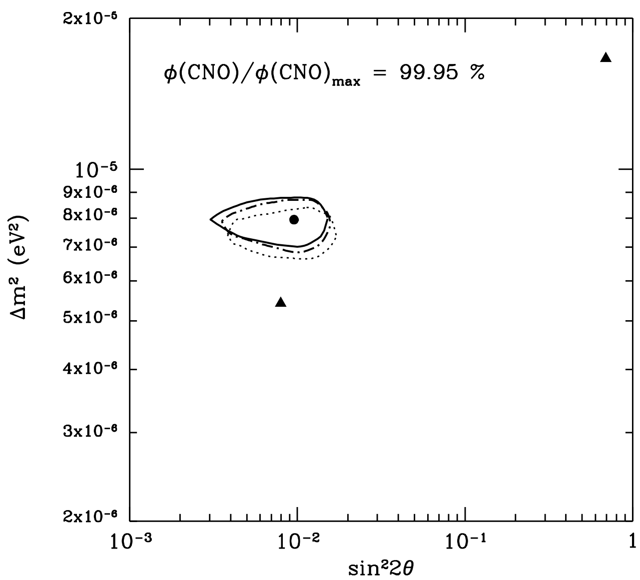
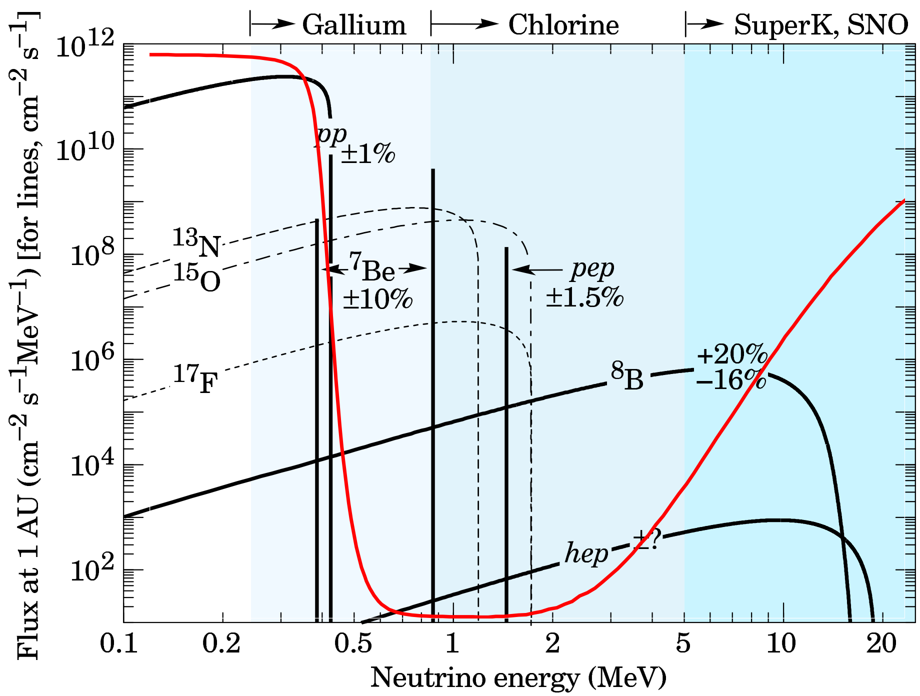


Fig.2



How the Sun Shines? Present Answer

- Fit solar (and KamLAND) data for:
  - 2ν oscillations Δm^2, $\tan^2\theta$
  - 8 free solar ν fluxes under conditions:

    * Luminosity constraint

$$\frac{L_\odot}{4\pi(A.U.)^2} = \sum_{i=1}^{8} \alpha_i \Phi_i \;\Rightarrow\; 1 = \sum_{i=1}^{8} \left(\frac{\alpha_i}{10\text{ MeV}}\right) a_i f_i$$

$$f_i \equiv \frac{\Phi_i}{\Phi_i(\text{BP2000})}, \qquad a_i \equiv \frac{\Phi_i(\text{BP2000})}{(8.5272\times 10^{10}\text{cm}^{-2}\text{s}^{-1})}$$

* Nuclear Physics inequalities:

$$\Phi_{^7\text{Be}} + \Phi_{^8\text{B}} \leq \Phi_{\text{pp}} + \Phi_{\text{pep}} \qquad \Phi_{^{15}\text{O}} \leq \Phi_{^{13}\text{N}}$$

* ^{15}O and ^{17}F fluxes: $\frac{\Phi_{^{15}\text{O}}(\text{BP2000})}{\Phi_{^{13}\text{N}}(\text{BP2000})} < \frac{\Phi_{^{15}\text{O}}}{\Phi_{^{13}\text{N}}} < 1$ and $\frac{\Phi_{^{17}\text{F}}(\text{BP2000})}{\Phi_{^{13}\text{N}}(\text{BP2000})} < \frac{\Phi_{^{17}\text{F}}}{\Phi_{^{13}\text{N}}} \leq 1$

* *pep* flux: $\frac{\Phi_{pep}}{\Phi_{p-p}} = \frac{\Phi_{pep}(\text{BP2000})}{\Phi_{p-p}(\text{BP2000})} \pm 10\%$

* hep flux: within present limits $1 \leq f_{hep} \leq 8$

How the Sun Shines? Present Answer

Study the quality of fit as a function of: $$\frac{L_{CNO}}{L_\odot} = \sum_{i=N,O,F} (\frac{\alpha_i}{10\ \mathrm{MeV}}) a_i\, f_i$$

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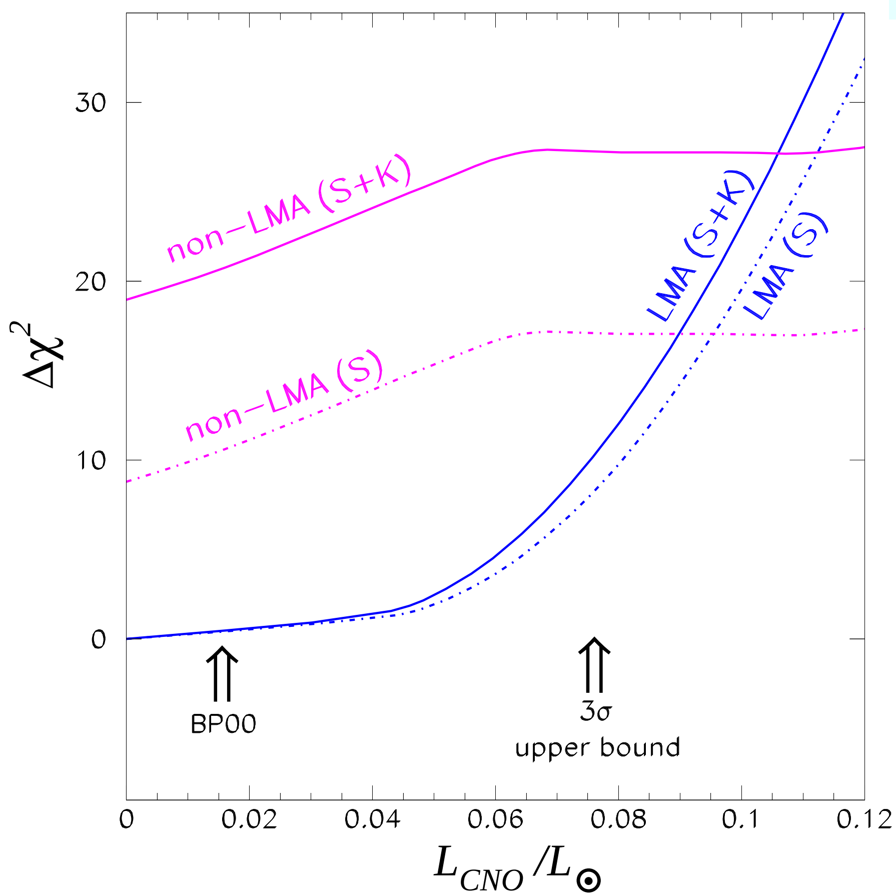

Resulting Limit:

$$\frac{L_{\rm CNO}}{L_\odot} < 7.3\%\ (7.8\%)$$
at 3σ

Old large CNO solutions were SMA-like

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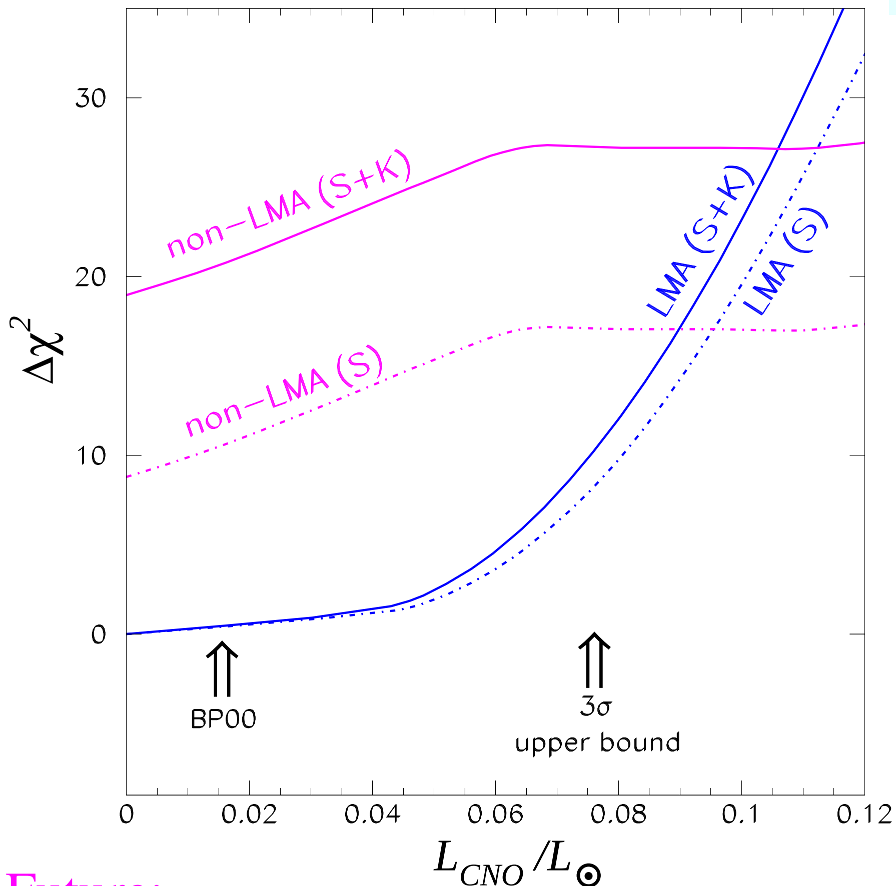

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Future:

– Borexino: $\Rightarrow \frac{L_{\rm CNO}}{L_\odot} < 5.6\%\ [4.9\%]$

– To test BP00 prediction 1.5%: lowE experiment with excellent E resolution

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  Tomorrow