Calculation of Sensitivity to a Gaussian Galactic Plane Source

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Our method in estimating the sensitivity to a new distribution using the sensitivity to a galactic plane line source is the approximation that by equalizing the number of events in the on-source region, we can calculate the effective sensitivity to the new distribution. This approximation should be valid as long as the zenith angle does not vary too much over the on-source region.

In previous work, we have calculated a sensitivity Φ_{gal} to a line source from the galactic equator. The response to this delta-function signal results in the line spread function of the detector and reconstruction, approximated by a Gaussian of width σ_{lsf}. We also approximate that this width is independent of galactic longitude.

The convolution of a Gaussian signal of width σ_{sig} with the line spread function results in a wider Gaussian of the form:

$$\Phi_{gaussian}(b,l) = \Phi_{gauss}\ e^{-b^2/2(\sigma_{lsf}^2+\sigma_{sig}^2)}\ . \tag{1}$$

and Φ_{gauss} is the flux normalization factor (per steradian, galactic coordinates). Note that technically we should be working with $\sin(b)$ instead of b, but the difference is negligible in the region within a few degrees of $b = 0$.

We assume the line source has a normalization of Φ_{gal} per radian. To keep our units consistent in the following discussion, we note that this can also be written as a flux per steradian, using a delta function:

$$\Phi_{initial}(b,l) = \Phi_{gal}\ \delta(b)\ . \tag{2}$$

When this is convolved with the line spread function, it too becomes a Gaussian:

$$\begin{aligned}\Phi_{line}(b,l) &= \Phi_{gal}\ e^{-b^2/2\sigma_{lsf}^2}\ \Big/ \int_{-\infty}^{\infty} e^{-b^2/2\sigma_{lsf}^2}\ db \\ &= \frac{1}{\sqrt{2\pi}\ \sigma_{lsf}}\ \Phi_{gal}\ e^{-b^2/2\sigma_{lsf}^2}\end{aligned} \tag{3}$$

where the integral in the denominator normalizes the line spread function. Equalizing the integrals of $\Phi_{line}(b,l)$ and $\Phi_{gaussian}(b,l)$ over the signal region will give us the normalization of the new distribution that we seek:

$$\begin{aligned}
\int_{l_{min}}^{l_{max}} \int_{-B}^{B} \Phi_{gaussian}(b,l)\ db\ dl &= \int_{l_{min}}^{l_{max}} \int_{-B}^{B} \Phi_{line}(b,l)\ db\ dl \\
\int_{l_{min}}^{l_{max}} \int_{-B}^{B} \Phi_{gauss}\ e^{-b^2/2(\sigma_{lsf}^2+\sigma_{sig}^2)}\ db\ dl &= \int_{l_{min}}^{l_{max}} \int_{-B}^{B} \Phi_{gal}\ e^{-b^2/2(\sigma_{lsf}^2)}\ db\ dl \ / \ \int_{-\infty}^{\infty} e^{-b^2/2\sigma_{lsf}^2}\ db \\
&= \eta \int_{l_{min}}^{l_{max}} \Phi_g\ dl \qquad (4)
\end{aligned}$$

where the efficiency factor η can be written:

$$\begin{aligned}
\eta &= \int_{-B}^{B} e^{-b^2/2\sigma_{lsf}^2}\ db \ / \ \int_{-\infty}^{\infty} e^{-b^2/2\sigma_{lsf}^2}\ db \\
&= \mathrm{erf}(B/\sqrt{2}\sigma_{lsf}) \qquad (5)
\end{aligned}$$

and $\mathrm{erf}(x)$ is defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\ dt\ . \qquad (6)$$

Solving for Φ_{gauss} gives us:

$$\Phi_{gauss} = \frac{\Phi_{gal}}{\sqrt{2\pi(\sigma_{lsf}^2+\sigma_{sig}^2)}}\ \mathrm{erf}(B/\sqrt{2}\sigma_{lsf})\ /\ \mathrm{erf}(B/\sqrt{2(\sigma_{lsf}^2+\sigma_{sig}^2)})\ . \qquad (7)$$

Applying this to a Strong *et al.* distribution, which is approximately a Gaussian with $\sigma_{sig} = 2.1^o = 0.037$, using an on-source region of $B = 2^o = 0.035$, and using the average line spread function $\sigma_{lsf} = 1.5^o = 0.026$, we find:

$$\Phi_{gauss} = 12.9\ \Phi_{gal}\ = 8.6 \cdot 10^{-4}\ \mathrm{GeV}^{-1}\ \mathrm{s}^{-1}\ \mathrm{cm}^{-2}\ \mathrm{sr}^{-1}. \qquad (8)$$

Finally, we note that this on-source region B was originally chosen to minimize Φ_{gal}, but this does not necessarily minimize Φ_{gauss}. Instead, we consider $\Phi_{gal} = \Phi_{gal}(B)$ as the line-flux sensitivity as function of the on-source region (a known function). Then we can reoptimize for a Gaussian signal by minimizing $\Phi_{gauss}(B)$. Performing this reoptimization numerically, we find a new optimal on-source window to the aforementioned Gaussian of $\pm 4.4^o$, and a sensitivity of:

$$\Phi_{gauss} = 7.2 \cdot 10^{-4}\ \mathrm{GeV}^{-1}\ \mathrm{s}^{-1}\ \mathrm{cm}^{-2}\ \mathrm{sr}^{-1}. \qquad (9)$$

This is an improvement of 16% above the non-optimal on-source window.