Reweighting and Livetime Normalization of dCORSIKA Monte Carlo

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1 Introduction

In order to reduce simulation time, the 2005 AMANDA dCORSIKA Monte Carlo is generated with a harder spectrum than is present in nature. Using the DSLOPE steering file option, the primary spectrum is altered roughly from $E^{-2.7}$ to $E^{-1.7}$. More accurately, the DSLOPE applies a slope difference to each component of the slope parametrization used, which in our case is Hörandel. The generated events must then be reweighted to the original spectrum and an appropriate normalization factor applied.

2 Event Weighting (Single Power Law)

In this section, we derive a simpler reweighting result for a single-component power-law spectrum $E^{-\gamma}$ generated with minimum energy E_L and maximum energy E_H. We first derive the normalization on the flux generating N events:

$$N = \int_{E_L}^{E_H} A\ E^{-\gamma} \mathrm{d}E \tag{1}$$

and in our case, since $E_H \gg E_L$, we approximate $E_H \approx \infty$, so we have for the normalization factor A:

$$A \approx \frac{N\ (\gamma - 1)}{E_L^{-\gamma+1}}\ . \tag{2}$$

So, suppose we are generating a sample of N events with a modified slope of $\tilde{\gamma} = \gamma + \Delta$, where $\Delta =$ DSLOPE (note the sign convention here). This sample corresponds to a flux of

$$\frac{N\ (\tilde{\gamma} - 1)}{E_L^{-\tilde{\gamma}+1}}\ E^{-\tilde{\gamma}}\ . \tag{3}$$

We will apply two weighting factors to weight the sample as if it were N events of the original slope: $w_E(E)$ to correct the slope, and w_N to correct the normalization. That is,

$$w_N \; w_E(E) \; \frac{N \; (\tilde{\gamma}-1)}{E_L^{-\tilde{\gamma}+1}} \; E^{-\tilde{\gamma}} \; = \frac{N \; (\gamma-1)}{E_L^{-\gamma+1}} \; E^{-\gamma} \; . \tag{4}$$

By inspection, we have:

$$w_N = \frac{\gamma-1}{\tilde{\gamma}-1} \; \frac{E_L^{-\tilde{\gamma}+1}}{E_L^{-\gamma+1}} \tag{5}$$

and

$$w_E(E) = \frac{E^{-\gamma}}{E^{-\tilde{\gamma}}} \; . \tag{6}$$

Therefore, the event weight for the ith event w_i is:

$$\begin{aligned} w_i &= w_N \; w_E(E_i) \\ &= \frac{\gamma-1}{\tilde{\gamma}-1} \; \frac{E_L^{-\tilde{\gamma}+1}}{E_L^{-\gamma+1}} \frac{E_i^{-\gamma}}{E_i^{-\tilde{\gamma}}} \\ &= \frac{\gamma-1}{\gamma-1+\Delta} \; E_L^{-\Delta} \; E_i^{\Delta} \; . \end{aligned} \tag{7}$$

In the specific case of the 2005 dCORSIKA simulation, $\Delta = -1$, $\gamma = 2.7$, and $E_L = 800$ GeV, so the approximate weight w_i would be

$$w_i = \frac{1.7}{0.7} \; \frac{800 \text{ GeV}}{E_i \text{ (GeV)}} \; . \tag{8}$$

However, see section 4 for a more accurate result.

3 Livetime

The advantage of reweighting the normalization back to N events of the unmodified spectrum is that it makes the livetime reweighting simple, since one can now use the livetime of an unmodified dCORSIKA run. We simply use an additional factor w_L, which is as follows:

$$w_L = \frac{T}{N_{file} \; t_{file}} \tag{9}$$

where T is the data livetime (including any prescaling factors), N_{file} is the number of MC files, and t_{file} is the livetime of one file with an *unmodified* spectrum (with the same number of events, of course). This technique avoids any confusion about dCORSIKA / ucr calculation of livetimes on runs with modified DSLOPE.

For the 2005 CORSIKA MC, $t_{file} = 0.0787$ s (obtained with one run with an unmodified spectrum), and the preliminary livetime of the filtered data set is 199.25 d. This results in a final livetime-adjusted weight of:

$$w_L \; w_i = \frac{1.7}{0.7} \; \frac{800 \text{ GeV}}{E_i \text{ (GeV)}} \; \frac{199.25 \cdot 86400 \text{ s}}{N_{file} \cdot 0.0787 \text{ s}} \; . \tag{10}$$

4 Event Weighting (Hörandel)

One finds in practice that the expression in eq. 10 is only accurate to about 30% when applied to dCORSIKA generated with Hörandel. This is for two reasons: first, there are multiple components in the flux, each with different spectral slope γ_k; second, with the `SPRIC` steering card enabled, the minimum energy for a primary with mass A_k amu is $A_k \cdot E_L$, not E_L.

In theory one could construct a composite expression using the above equations for each component, knowing the parameters of the Hörandel flux; however, dCORSIKA makes our life easier by providing a composite integral FLUXSUM in the log file. The only trick is that the energy integral is calculated internally in units of TeV; so when using it in our reweighting expression, we need to correct for this.

The analogue to expression 7 using the FLUXSUM approach is:

$$w_i = \frac{\tilde{F}_\gamma}{F_\gamma} \; 1000^{-\Delta} \; E_i^{\Delta} \tag{11}$$

where, as before, $\tilde{\gamma} = \gamma + \Delta$, and $\tilde{F}_\gamma$ is the FLUXSUM for one file with slope $\tilde{\gamma}$, and F_γ is the FLUXSUM for one file with slope γ.

The FLUXSUM ratio allows us to use the original unweighted livetime in the full weight, so adding the same livetime weight w_L, we have

$$w = w_i \; w_L = \frac{\tilde{F}_\gamma}{F_\gamma} \; 1000^{-\Delta} \; E_i^{\Delta} \; \frac{T}{N_{file} \; t_{file}} \tag{12}$$

where T is the data livetime (including any prescaling factors), N_{file} is the number of MC files, and t_{file} is the livetime of one file with an unmodified spectrum (with the same number of events). The $1000^{-\Delta}$ term corrects the units to GeV.

However, knowing the unit conversion of FLUXSUM allows us to use the modified livetime as calculated by ucr – in which case we can calculate w without using ratios between two different dCORSIKA runs. Equivalently,

$$w = 1000^{-\Delta}\ E_i^{\Delta}\ \frac{T}{N_{file}\ \tilde{t}_{file}} \tag{13}$$

where $\tilde{t}_{file}$ is the livetime reported by ucr for one file with *modified* spectrum.

For the original 2005 dCORSIKA generation, with Hörandel spectrum, $E_L = 800$ GeV, DSLOPE=-1, and 10K events/file, this results in a final reweighting expression of

$$w(\Delta = -1) = \frac{1000}{E_i(\mathrm{GeV})}\ \frac{199.25 \cdot 86400\ \mathrm{s}}{N_{file}\ 0.0306\ \mathrm{s}}\ . \tag{14}$$

More recently, we have generated another sample of 2005 MC with DSLOPE=-0.4 and 1M events/file. For this sample, $\tilde{t}_{file} = 6.20$ s, so the weight is

$$w(\Delta = -0.4, 1\mathrm{M\ events}) = \frac{15.8}{\mathrm{E_i^{0.4}}}\ \frac{199.25 \cdot 86400\ \mathrm{s}}{\mathrm{N_{file}}\ 6.20\ \mathrm{s}}\ . \tag{15}$$