Electromagnetic waves

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Physics 202
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Note: These slides are not a complete representation of the lecture. Details are presented on whiteboard.

- Maxwell's equations, review
- Wave equation
- Electromagnetic waves
- Speed of em waves (light)
- Antenna, radio waves
- Electromagnetic spectrum

Demonstrations

- Radio emission from dipole transmitter,
  - Receiver with small light bulb
  - Length of antenna
  - Orientation and distance of receiving antenna
  - Magnetic loop antenna (with LC circuit)
- Microwave transmitter
  - Polarized waves get absorbed or reflected by grid when grid is oriented in direction of dipole.

Maxwell's Equations

$$\oint_S \vec{E} \cdot \vec{dA} = \frac{q}{\varepsilon_0}$$

- Gauss's law (electric)

$$\oint_S \vec{B} \cdot \vec{dA} = 0$$

- Gauss's law in Magnetism

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$$

- Faraday's Law

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

- Ampere-Maxwell Law

Connections between Maxwell's equations:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I + \varepsilon_o \mu_o \frac{d\Phi_E}{dt}$$

Derivation of Speed – Some Details

- From Maxwell's equations applied to empty space, the following partial derivatives can be found:

$$\frac{\partial^2 E}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2} \quad and \quad \frac{\partial^2 B}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B}{\partial t^2}$$

- These are in the form of a general wave equation, with

$$v = c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

- Substituting the values for μ_o and ε_o gives c = 2.99792 x 10^8 m/s

E to B Ratio – Some Details

- The simplest solution to the partial differential equations is a sinusoidal wave:
  - $E = E_{max} \cos (kx - \omega t)$
  - $B = B_{max} \cos (kx - \omega t)$
- The angular wave number is $k = 2\pi/\lambda$
  - λ is the wavelength
- The angular frequency is $\omega = 2\pi f$
  - f is the wave frequency

E to B Ratio – Details, cont.

- The speed of the electromagnetic wave is

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

- Taking partial derivations also gives

$$\frac{E_{max}}{B_{max}} = \frac{\omega}{k} = \frac{E}{B} = c$$

Relation between E and B

- $E = E_{max} \cos(kx - \omega t)$
- $B = B_{max} \cos(kx - \omega t)$
- First derivatives:

$$\frac{\partial E}{\partial x} = -kE_{max} \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{max} \sin(kx - \omega t)$$

From: $\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$

Speed of em waves
(speed of light in vacuum)

$$\boxed{\frac{E_{max}}{B_{max}} = \frac{\omega}{k} = \frac{E}{B} = c}$$

This relation comes from Maxwell's equations!

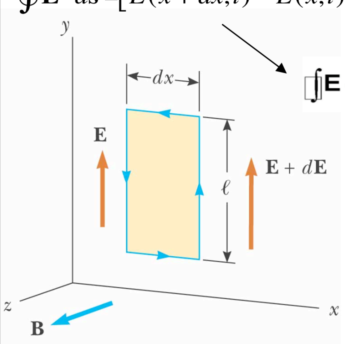
Let's demonstrate: $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

A wave at instant t in x and x+dx:
the E field varies from **E** to **E**+d**E**

– $E = E_{max} \cos(kx - \omega t)$ $\quad\quad B = B_{max} \cos(kx - \omega t)$

$$E(x+dx,t) \approx E(x,t) + \frac{\partial E}{\partial x}dx \Rightarrow$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = [E(x+dx,t) - E(x,t)]\ell = \frac{\partial E}{\partial x}dx\ell$$



$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Magnetic flux through rectangle:

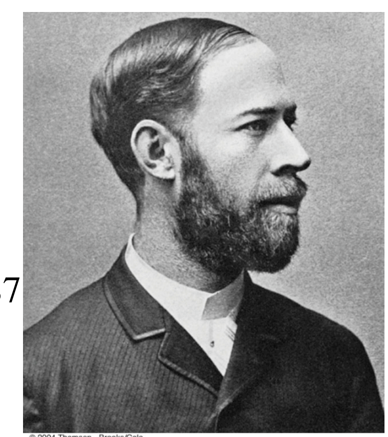
$$d\Phi_B = B\ell dx$$

$$\frac{d\Phi_B}{dt} = \ell dx\frac{\partial B}{\partial t} \Rightarrow \oint \mathbf{E}\cdot d\mathbf{s} = \frac{\partial E}{\partial x}dx\ell = -\frac{d\Phi_B}{dt} = -dx\ell\frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Hertz's Confirmation of Maxwell's Predictions

- Heinrich Hertz was the first to generate and detect electromagnetic waves in a laboratory setting
- The most important discoveries were in 1887
- He also showed other wave aspects of light



Announcements

- Reminder: Exam 3 on Monday, Nov 26
- Review sessions
  - Nov 18 7PM: Hao
  - Nov 19 6PM: Ming
  - Nov 20 7PM: Stephen
  - Location: room 2103
- No labs next week and the week after

From LC circuit to antenna:

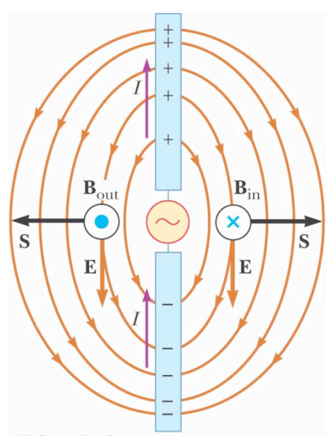
- --> on the whiteboard

Production of em Waves by an Antenna

- Neither stationary charges nor steady currents can produce electromagnetic waves
- The fundamental mechanism responsible for this radiation is the acceleration of a charged particle
- Whenever a charged particle accelerates, it must radiate energy

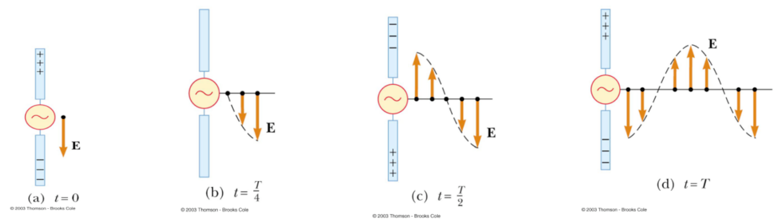
Production of em Waves by an Antenna, 2

- This is a *half-wave* antenna
- Two conducting rods are connected to a source of alternating voltage
- The length of each rod is one-quarter of the wavelength of the radiation to be emitted

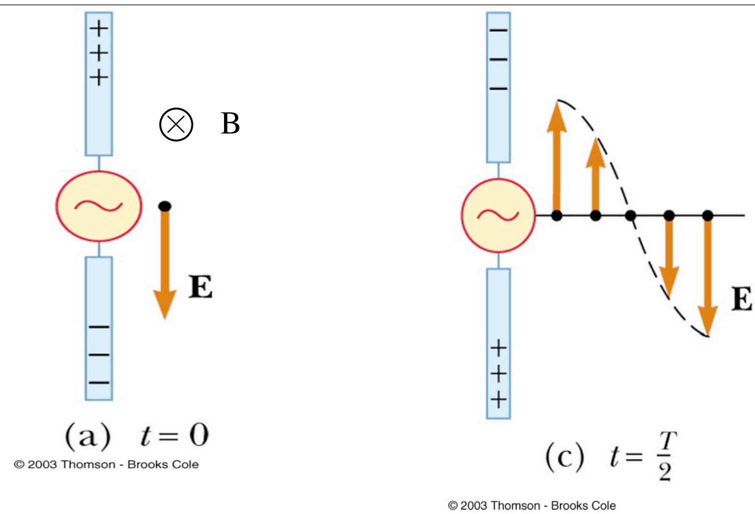


- Accelerated charged particles are sources of EM waves:
- EM waves are radiated by any circuit carrying alternating current

EM Waves from an Antenna



- Two rods are connected to an AC source, charges oscillate between the rods (a)
- As oscillations continue, the rods become less charged, the field near the charges decreases and the field produced at t = 0 moves away from the rod (b)
- The charges and field reverse (c)
- The oscillations continue (d)

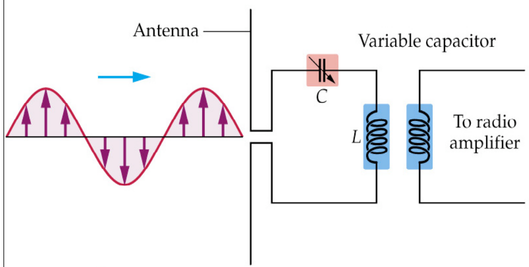


In each case:

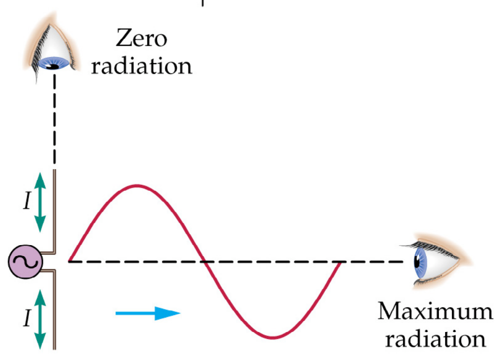
•*What is the direction and amplitude of the current?*

•*What is the direction of the B-filed?*

Receiving radio waves



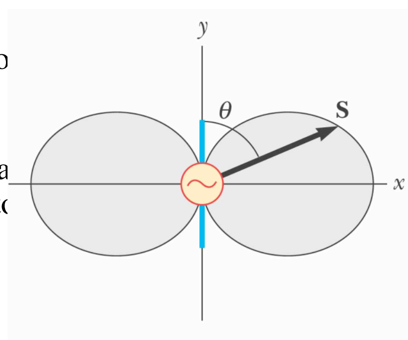
Basic elements of a tuning circuit used to receive radio waves.
- First, an incoming wave sets up an alternating current in the antenna.
- Next, the resonance frequency of the LC circuit is adjusted to match the frequency of the radio wave, resulting in a relatively large current in the circuit.
- This current is then fed into an amplifier to further increase the signal.



- Electromagnetic radiation is greatest when charges accelerate at right angles to the line of sight.
- Zero radiation is observed when the charges accelerate along the line of sight.
- These observations apply to electromagnetic waves of all frequencies.

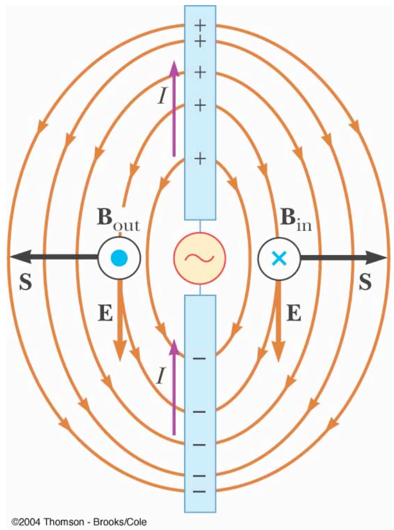
Angular Dependence of Intensity

- This shows the angular dependence of the radiation intensity produced by a dipo antenna
- The intensity and power radiated are a maximum in a plane that is perpendicular tc the antenna and passing through its midpoint
- The intensity varies as $(\sin^2 \theta) / r^2$



Quick Quiz 34.6

If the antenna in the figure represents the source of a distant radio station, rank the following points in terms of the intensity of the radiation, from greatest to least:
(**1**) a distance *d* to the right of the antenna
(**2**) a distance 2*d* to the left of the antenna
(**3**) a distance 2*d* in front of the antenna (out of the page)
(**4**) a distance *d* above the antenna (toward the top of the page).



(a) **1**, **2**, **3**

(b) **2**, **4**, **1**

(c) **3**, **4**, **1**

(d) **4**, **3**, **1**

Poynting Vector

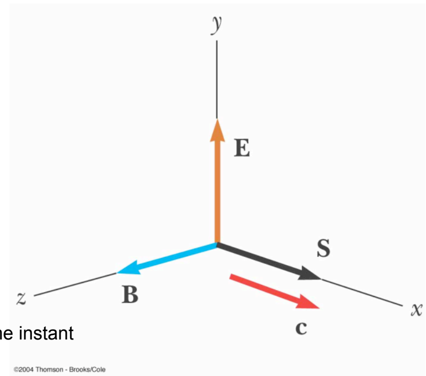
- Electromagnetic waves carry energy
- As they propagate through space, they can transfer that energy to objects in their path
- The rate of flow of energy in an em wave is described by a vector, **S**, called the **Poynting vector**

Poynting Vector, cont.

- The Poynting vector is defined as

$$\boxed{\mathbf{S} \equiv \frac{1}{\mu_o}\mathbf{E} \times \mathbf{B}}$$

- Its direction is the direction of propagation
- This is time dependent
  - Its magnitude varies in time
  - Its magnitude reaches a maximum at the same instant as **E** and **B**



- The magnitude **S** represents the rate at which energy flows through a unit surface area perpendicular to the direction of the wave propagation
  - **This is the *power per unit area***
- The SI units of the Poynting vector are J/s·m^2 = W/m^2

Intensity

- The *wave intensity, I,* is the time average of S (the Poynting vector) over one or more cycles
- When the average is taken, the time average of $\cos^2(kx - \omega t) = 1/2$ is involved

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_o} = \frac{E_{max}^2}{2\mu_o c} = \frac{c\,B_{max}^2}{2\mu_o}$$

Energy Density

- The energy density, u, is the energy per unit volume
- For the electric field, $u_E = 1/2\ \varepsilon_o E^2$
- For the magnetic field, $u_B = 1/2\ \mu_o B^2$
- Since $B = E/c$ and $c = 1/\sqrt{\mu_o \varepsilon_o}$

$$u_B = u_E = \frac{1}{2}\varepsilon_o E^2 = \frac{B^2}{2\mu_o}$$

Energy Density, cont.

- The instantaneous energy density associated with the magnetic field of an em wave equals the instantaneous energy density associated with the electric field
  - In a given volume, the energy is shared equally by the two fields

Energy Density, final

- The **total** instantaneous energy density is the sum of the energy densities associated with each field
  - $u = u_E + u_B = \varepsilon_o E^2 = B^2 / \mu_o$
- When this is averaged over one or more cycles, the total average becomes
  - $u_{av} = \varepsilon_o (E^2)_{av} = 1/2\ \varepsilon_o E^2_{max} = B^2_{max} / 2\mu_o$
- In terms of I, $I = S_{av} = cu_{av}$
  - The intensity of an em wave equals the average energy density multiplied by the speed of light

Momentum

- Electromagnetic waves transport momentum as well as energy
- As this momentum is absorbed by some surface, pressure is exerted on the surface
- Assuming the wave transports a total energy U to the surface in a time interval Δt, the total momentum is $p = U / c$ for complete absorption

Pressure and Momentum

- Pressure, P, is defined as the force per unit area

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{c}\frac{dU/dt}{A}$$

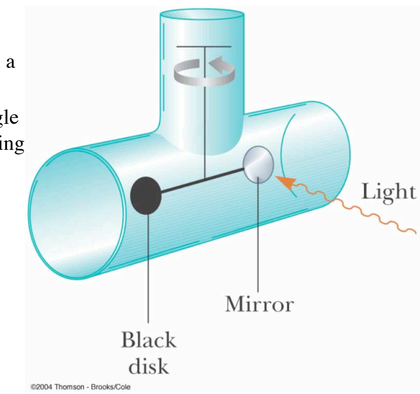
- But the magnitude of the Poynting vector is $(dU/dt)/A$ and so $P = S / c$
  - For a perfectly absorbing surface

Pressure and Momentum, cont.

- For a perfectly reflecting surface, $p = 2U/c$ and $P = 2S/\mathrm{c}$
- For a surface with a reflectivity somewhere between a perfect reflector and a perfect absorber, the momentum delivered to the surface will be somewhere in between U/c and $2U/c$
- For direct sunlight, the radiation pressure is about 5×10^{-6} N/m^2

Determining Radiation Pressure

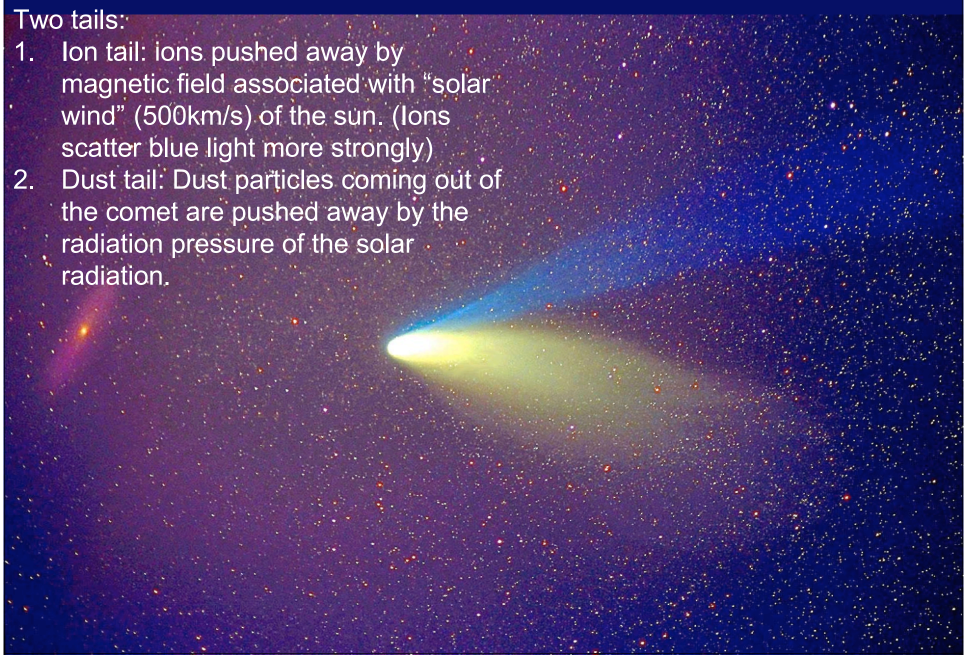
- This is an apparatus for measuring radiation pressure
- In practice, the system is contained in a high vacuum
- The pressure is determined by the angle through which the horizontal connecting rod rotates



Comet Hale-Bopp, 1997

Two tails:

1. Ion tail: ions pushed away by magnetic field associated with "solar wind" (500km/s) of the sun. (Ions scatter blue light more strongly)
2. Dust tail: Dust particles coming out of the comet are pushed away by the radiation pressure of the solar radiation.



QUICK QUIZ 34.4

(For the end of section 34.5)

An interplanetary dust particle of radius ~ 0.2 μm is at a certain distance away from the sun so that the sum of the attractive gravitational force from the sun and the repulsive radiation force is zero. If this particle is moved to a distance that is twice as far away from the sun as its original distance, the net force on the particle will be

a) attractive,
b) repulsive,
c) zero,
d) impossible to determine without knowing the mass of the particle.

QUICK QUIZ 34.3

(For the end of section 34.4)

An interplanetary dust particle of radius 1 mm is influenced by the force of gravity from the sun and the force due to radiation pressure from the sun's light. The net force on such a particle is determined to be F_1. The net force on a similar particle of the same density but of radius 2 mm will be approximately

a) $F_1/8$,
b) $F_1/4$,
c) F_1
d) $2F_1$,
e) $8F_1$.

Careful with this one!
Consider the size of the particle.
Check example 34.3

The Spectrum of EM Waves

- Various types of electromagnetic waves make up the em spectrum
- There is no sharp division between one kind of em wave and the next
- All forms of the various types of radiation are produced by the same phenomenon – accelerating charges

The EM Spectrum

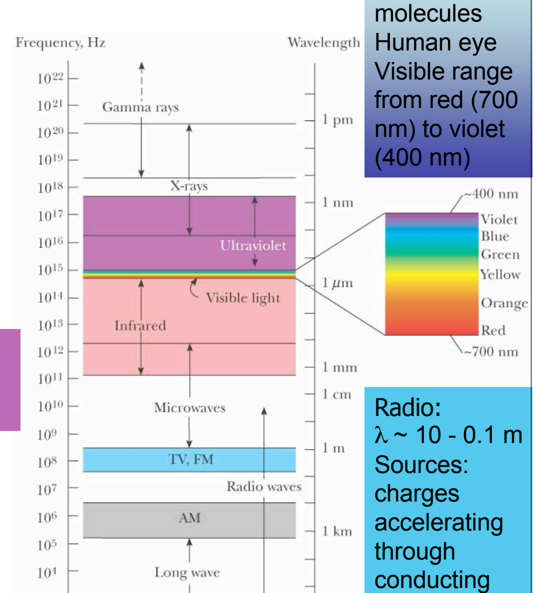
Gamma rays: $\lambda \sim 10^{-14} - 10^{-10}$ m
Source: radioactive nuclei
cause serious damage to living tissues

X-rays: $\sim 10^{-12} - 10^{-8}$ m
source: deceleration of high-energy electrons striking a metal target
Diagnostic tool in medicine

UV $\lambda \sim 6 \times 10^{-10} - 4 \times 10^{-7}$ m
Most UV light from the sun is absorbed in the stratosphere by ozone

Infrared: $\lambda \sim 7 \times 10^{-7} - 10^{-3}$ m
Sources: hot objects and molecules

Microwaves: $\lambda \sim 10^{-4}$ - 0.3 m
sources: electronic devices
radar systems, MW ovens

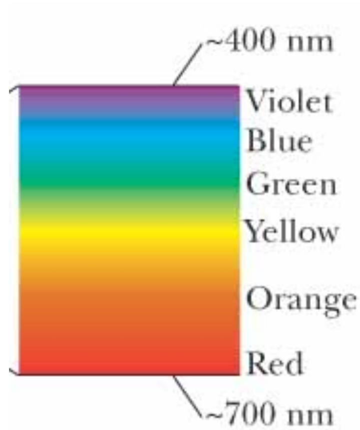


Source: atoms and molecules
Human eye
Visible range from red (700 nm) to violet (400 nm)

Radio:
$\lambda \sim 10 - 0.1$ m
Sources: charges accelerating through conducting wires
Radio and TV

More About Visible Light

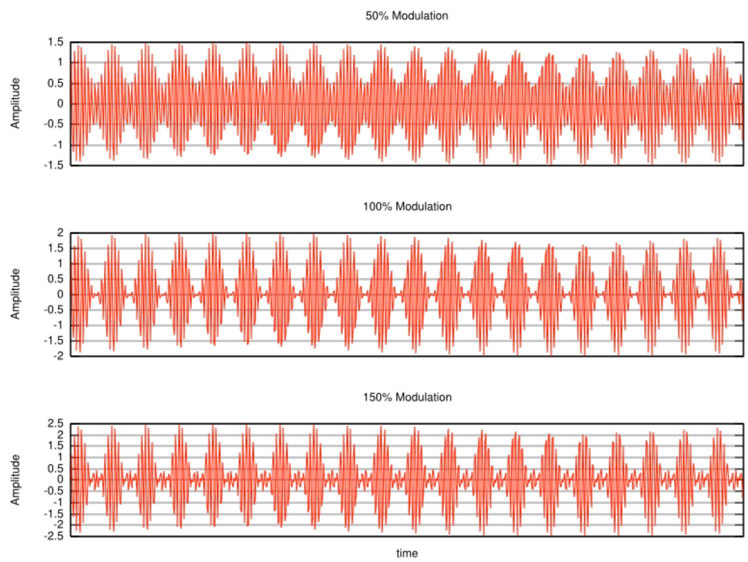
- Different wavelengths correspond to different colors
- The range is from red ($\lambda \sim 7 \times 10^{-7}$ m) to violet ($\lambda \sim 4 \times 10^{-7}$ m)



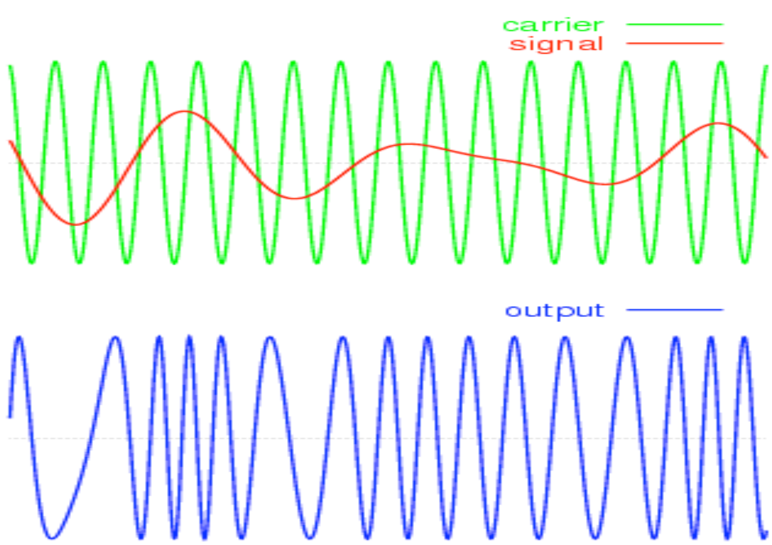
More on Radio

- AM: amplitude modulation
  - Simple technology, susceptible to noise, longer range of transmission, DX in shortwave (reflection in ionosphere)
  - Frequency bands:
    - Long wave is 153–279 kHz;
    - Medium wave is 520–1,710 kHz
    - Short wave is 2,300–26,100 kHz
- FM: frequency modulation
  - Higher performance (why?)
  - Higher bandwidth
  - 87.5 to 108.0 MHz
- Bandwidth:
  - The sinusoidal carrier wave is broadened by the modulation according to the Fourier theory. As a rule of thumb, modulation of 20 kHz will broaden the frequency spectrum by similar amount.

AM

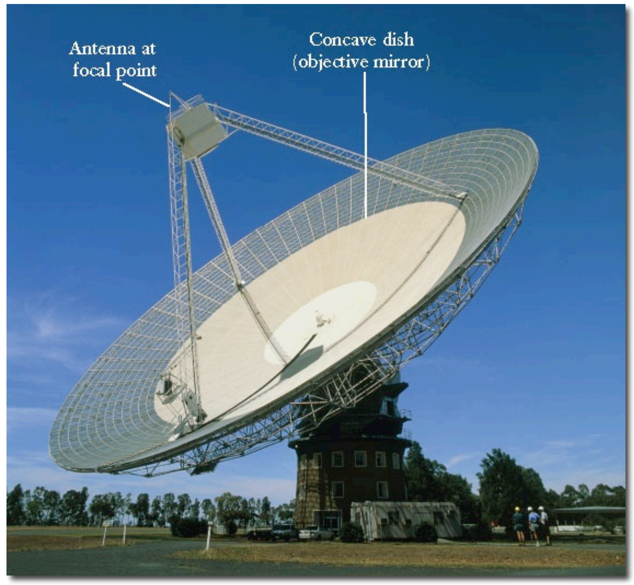

50% Modulation
Amplitude
1.5
1
0.5
0
-0.5
-1
-1.5
100% Modulation
Amplitude
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
150% Modulation
Amplitude
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
time

FM


carrier
signal
output

Radio Telescopes

Parkes 64-m radio telescope in New South Wales, Australia



Radio Telescopes

Arecibo: largest single-dish radio telescope (305 m)


Radio Telescopes
The Very Large Array (VLA),
27 telescopes of 25-m each-
New Mexico -> angular
resolution of a single dish of
km-size radius (~ 1 arcsec)