Physics 202, Lecture 3

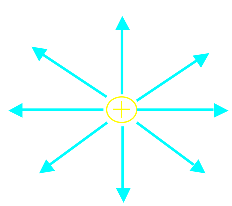
Today's Topics

- Electric Field
  - Quick Review
  - Motion of Charged Particles in an Electric Field
- Gauss's Law (Ch. 24, Serway)
- Conductors in Electrostatic Equilibrium (Ch. 24)
- Homework #2: on WebAssign tonight. Due 9/24, 10 PM

The Electric Field

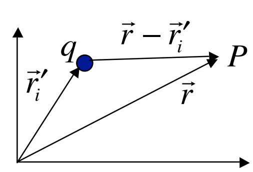
1. Charges are a source of electric fields.

Single point charge at origin: $\vec{E}(\vec{r}) = k_e \frac{q}{r^2}\hat{r}$



Visualization: field lines

Multiple charges: $\vec{E}(\vec{r}) = k_e \sum_i \frac{q_i(\vec{r}-\vec{r}_i')}{|\vec{r}-\vec{r}_i'|^3}$

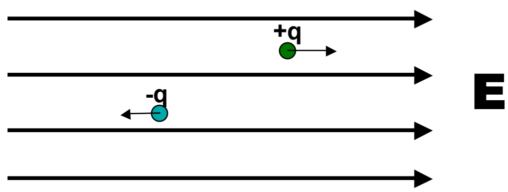


Distributions: $\vec{E}(\vec{r}) = k_e \int \frac{dq(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

The Electric Field (II)

2. Charges respond to electric fields.

$$\vec{F} = q\vec{E} \qquad \vec{a} = \frac{F}{m} = \frac{q\vec{E}}{m}$$

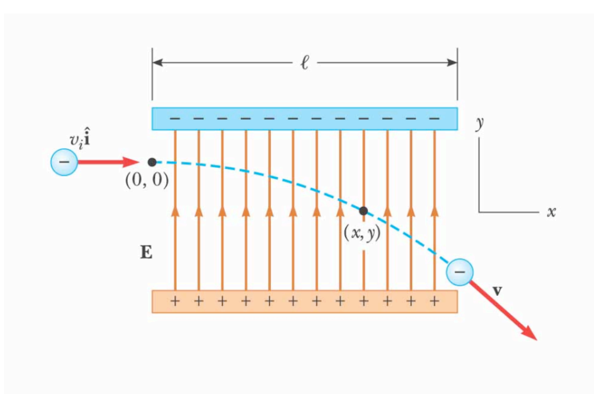


Example: uniform E field, uniformly accel. motion!

$$v = v_0 + at \qquad s = s_0 + v_0 t + \tfrac{1}{2}at^2 \qquad v^2 = v_0^2 + 2as$$

Exercise: Electron in Uniform E Field

- **What is the vertical displacement after an electron passes through a region with uniform electric field E?**
- **Solution: See board. Answer: $dy = -\frac{1}{2}(|e|/m)E\,(l/v_i)^2$**



Calculating E Fields

- We have seen how to calculate electric fields given a charge distribution (discrete or continuous)

  often very complicated to carry out analytically!
  one example: uniform sphere (from last lecture)

- A different but equivalent statement of Coulomb's law:

Karl Friedrich Gauss

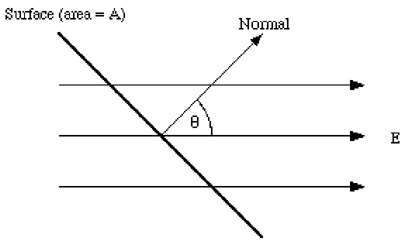
Gauss's Law

one of four fundamental equations of electromagnetism (Maxwell's equations)

Electric Flux

- The electric flux $\Delta\Phi_E$ through an area element $\Delta\mathbf{A}$: dot product of the electric field and the area vector:

  $\Delta\Phi_E = \mathbf{E} \bullet \Delta\mathbf{A} = E\Delta A\cos\theta$



(area vector: normal to surface)

$$\Phi_E = \int \mathrm{E} \bullet d\mathrm{A}$$

- Net electric flux through a closed surface:

$$\Phi_E = \oint \mathrm{E} \bullet d\mathrm{A}$$

Visualization:

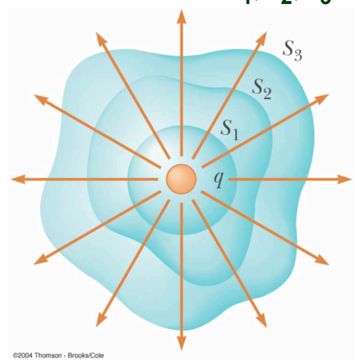
field lines through surface

Flux through Closed Surfaces

- **Compare fluxes through closed surfaces s_1, s_2, s_3:**

$\Phi_{s1}=\Phi_{s2}=\Phi_{s3}$

(# field lines same through all 3 surfaces)



- **Note: if no charge inside surface, $\Phi_{s1}=\Phi_{s2}=\Phi_{s3}=0$!**

(# field lines going in = # field lines going out)

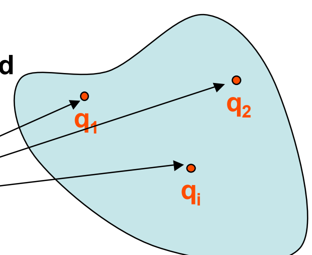
Gauss's Law (1)

- **Gauss's Law: net electric flux through any closed surface ("Gaussian surface") equals the total charge enclosed inside the closed surface divided by the permittivity of free space.**

flux

q_{in} **: all charges enclosed regardless of positions**

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{\sum q_{in}}{\varepsilon_0}$$



Gaussian surface (any shape)

ε_0: permittivity constant $(4\pi\varepsilon_0)^{-1} = k_e$

Gauss's Law (2)

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Gauss's Law:

True in all situations, but not always easy to use…

However, Gauss's Law is a powerful calculational tool in specific cases where the charge distribution exhibits a high degree of symmetry.

Using Gauss' Law

To solve for the electric field using Gauss's Law, it is necessary to choose a closed (Gaussian) surface such that the surface integral is trivial.

How to choose a Gaussian surface: use symmetry arguments.

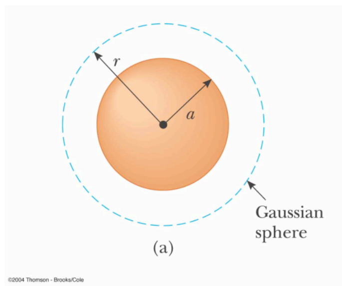
1. **Direction. Choose a surface such that E is known to be either parallel or perpendicular to each piece of surface**
2. **Magnitude. Choose a surface such that E is known to have the same value at all points on the surface**

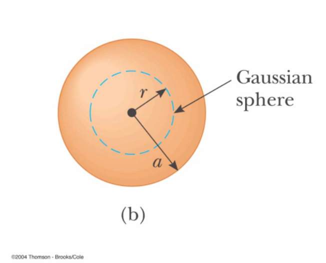
Then: $$\oint \vec{E} \bullet d\vec{A} = \oint E dA = E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

Given q_{in}, can solve for E (at surface), and vice versa

Last Lecture's Example Again: Uniformly Charged Sphere

- A uniformly charged sphere radius a and total charge Q, find electric field outside and inside the sphere.
- Solution: (see board, note the arguments on symmetry)

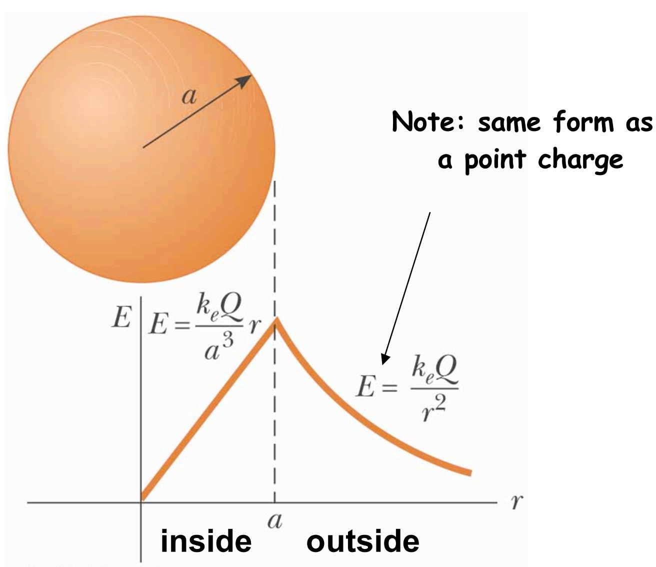




Apply Gauss's Law Outside the Sphere

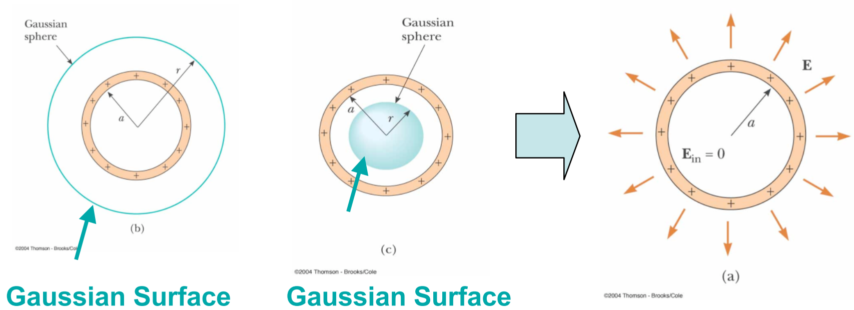
Apply Gauss's Law inside the Sphere

Uniform Charge Sphere: Final Solution



Another Example: Thin Spherical Shell

- ❑ Find E field inside/outside a uniformly charged thin spherical shell. Solution: see board.



Gaussian Surface for E(outside)

Gaussian Surface for E(inside)

Result

Other examples:
Be familiar with them!

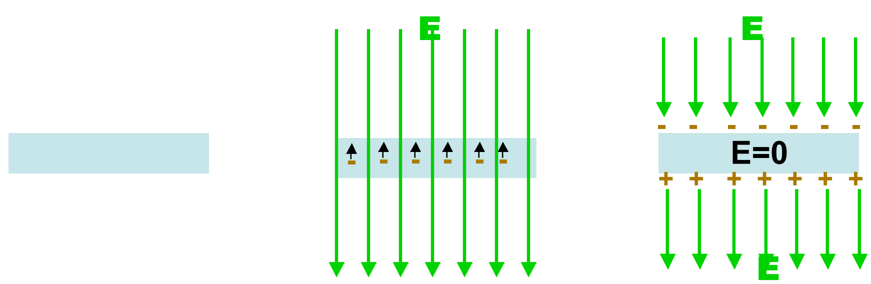
Infinite line of charge

Infinite uniform charged sheet

Infinite charged cylinder

Conductors And Electrostatic Equilibrium

- ❑ Conductors: charges (electrons) able to move freely
  → Charges redistribute when subject to E field.
- ❑ Charge redistribution → electrostatic equilibrium.



Initial → transient, $<10^{-16}$s (right after E applied) → equilibrium

Properties of Electrostatic Equilibrium

- E field is always zero inside the conductor.
- E field on the surface of conductor: normal to the surface, and magnitude $E = \frac{\sigma}{\varepsilon_0}$
  (show using Gauss's law)
- All charges reside on the surface of conductor.

E field is also zero inside any cavity within the conductor. (why?)

The above properties are valid regardless of the shape of and the total charge on the conductor!

How not to apply Gauss's Law

- Two charges +2Q and –Q are placed at locations shown. Find the electric field at point P.
- 1. Draw a Gaussian surface passing P
  2. Apply Gauss's law:

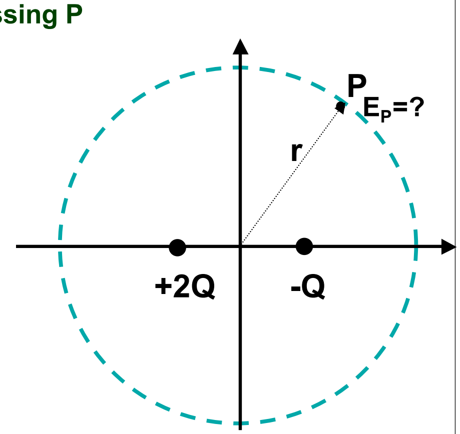
$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

q_{in}= +2Q +(-Q)=Q

3. Surface integral:

$$\oint \vec{E} \bullet d\vec{A} = 4\pi r^2 E$$

➔ E=1/4$\pi\varepsilon_0$(Q/r²)



Is this correct? No! Which step is wrong? Last step

Example

- What is the electric flux through closed surface S?

  $\Phi = 0$

  $\Phi = (q_1+q_2+q_3+q_4+q_5)/\varepsilon_0$

  ➔ $\Phi = (q_1+q_2+q_3)/\varepsilon_0$