Physics 202, Lecture 5

Today's Topics

- **Announcements:**
  Homework #3 on WebAssign by tonight
  Due (with Homework #2) on 9/24, 10 PM
- **Review: (Ch. 25-Part I)**
  - **Electric Potential Energy, Electric Potential**
- **Electric Potential (Ch. 25-Part II)**
  - **Electric Potential For Various Charge Distributions**
    - **Point charges, Continuous distributions (uniform ring, sphere, shell)**
  - **More on Conductors in Electric Fields, Equipotentials**

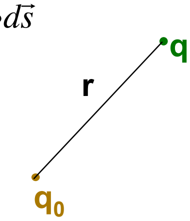
Electric Potential Energy, Electric Potential

Work required to move q_0 from A to B in E field: potential energy

$$\Delta U = \Delta W = \int_A^B \vec{F}_{us} \bullet d\vec{s} = -\int_A^B \vec{F}_e \bullet d\vec{s} = -q_0 \int_A^B \vec{E} \bullet d\vec{s}$$

Point charges:

$$U(r) = \frac{k_e q_0 q}{r} \qquad U = \sum_{i<j} \sum_j \frac{k_e q_i q_j}{r_{ij}}$$



(work required to assemble charge configuration)

Electric potential difference: **(independent of test charge q_0)**

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \bullet d\vec{s} = V_B - V_A$$

Example: uniform E field parallel to path $\Delta V = -Ed$

Example: Uniform Electric Field

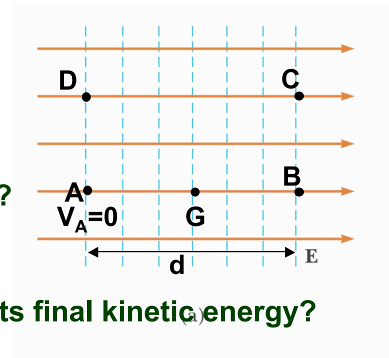
In the uniform electric field shown:

1. Find potential at B,C,D,G

2. If a charge +q is placed at B, what is the potential energy U_B?

3. If now a –q is at B, what is U_B?

4. If a -q is initially at rest at G, will it move to A or B? What is its final kinetic energy?



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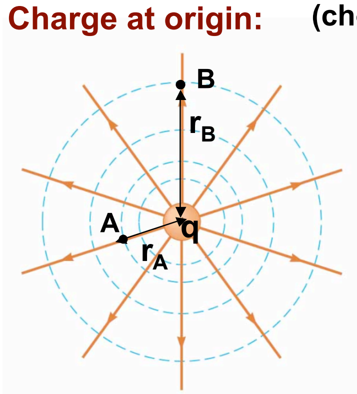
Recall: Particles will move to minimize their final potential energy!

Electric Potential: Point Charges

Charge at origin: (choose zero of potential at infinity)



$$V(r) = -\int \vec{E} \bullet d\vec{s} = -\int_{\infty}^{r} \frac{k_e q}{r^2} dr = \frac{k_e q}{r}$$

Potential difference:

$$V(r_B) - V(r_A) = \frac{k_e q}{r_B} - \frac{k_e q}{r_A}$$

General: (b)

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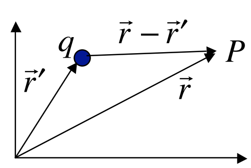
$$V(\vec{r}) = k_e \frac{q}{|\vec{r} - \vec{r}'|}$$

Single charge

$$V(\vec{r}) = k_e \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i'|}$$

Multiple charges



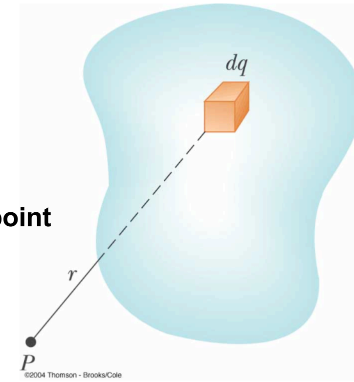
Electric Potential: Continuous Charge Distributions

Finite charge distributions: usually set V=0 at infinity.

If charge distribution is known:

$$V = k_e \int \frac{dq}{r}$$

Here: $r =$ **distance b/w source and obs. point**



Note: scalar integral!

More precisely (see board):

$$V(\vec{r}) = k_e \int \frac{dq}{|\vec{r} - \vec{r}'|}$$

$\vec{r}$ **: field (observation) point** $\vec{r}'$ **: source point**

Calculating Electric Field, Electric Potential

Three ways to calculate E field:

Coulomb's Law: (lecture 2) $\vec{E}(\vec{r}) = k_e \int \frac{dq(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$

Gauss's Law: (lecture 3,4) $\Phi_e = \oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

Potential: (lecture 4,today) $V(\vec{r}) = k_e \int \frac{dq}{|\vec{r} - \vec{r}'|}$

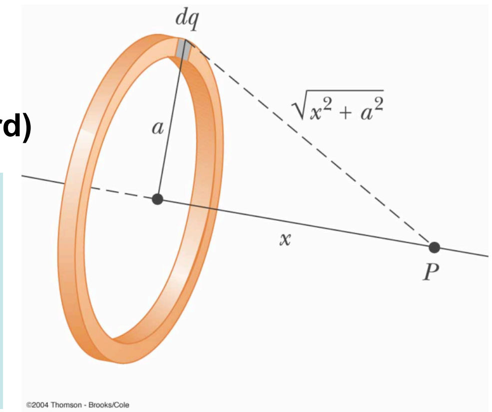
$$V = -\int \vec{E} \bullet d\vec{s} \Longrightarrow dV = -\vec{E} \bullet d\vec{s} \Longrightarrow$$

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z} \Longrightarrow \vec{E} = -\nabla V$$

Example: Uniformly Charged Ring

- For a uniformly charged ring, show that the potential along the central axis is

$$V = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$



Solution: (also see board)

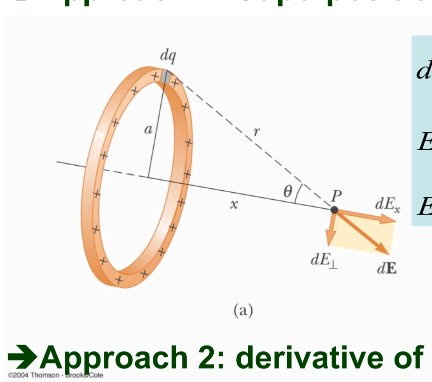
$$V = \int \frac{k_e dq}{r} = \int \frac{k_e dq}{\sqrt{x^2 + a^2}} = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq$$

↑ =Q

Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.

➔Approach 1: Superposition. (Eg. 23.18)



$$dE_x = dE\cos\theta = \frac{k_e dq}{r^2}\frac{x}{r}$$

$$E_x = \int dE_x = \frac{k_e x Q}{r^3} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}}$$

$E_\perp = 0$ **due to symmetry**

➔Approach 2: derivative of potential

$$V(x) = \frac{k_e Q}{(x^2 + a^2)^{1/2}}$$

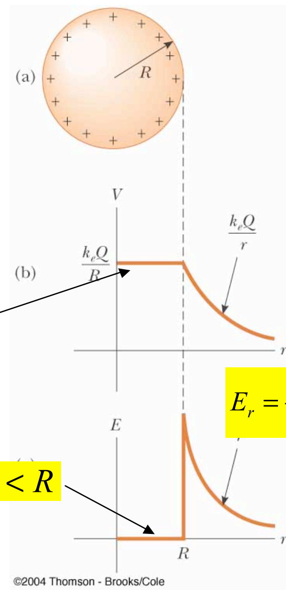
$$E_x = -\frac{\partial V}{\partial x} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}}$$

Example: Uniformly Charged Spherical Shell

- For uniformly charged spherical shell.

Again, use:

$$\Delta V = -\int_A^B \vec{E} \bullet d\vec{s} = V_B - V_A$$



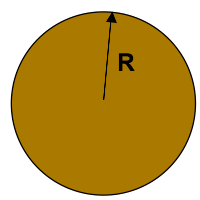
Tip:
V is the same inside E=0 region

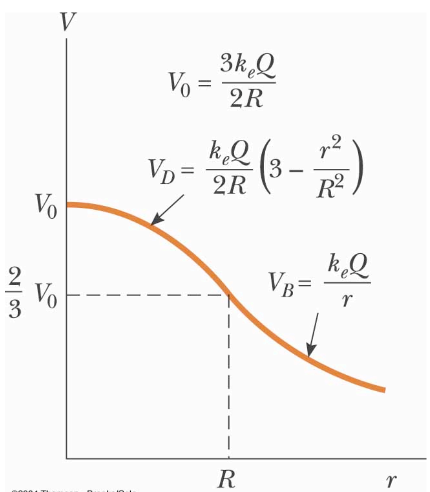
$$E_r = \frac{k_eQ}{r^2} \qquad r > R$$

$$E = 0 \qquad r < R$$

Example: Uniformly Charged Sphere

- Show that the potential of a uniformly charged sphere is:





Hint: more convenient to use $\Delta V = -\int_A^B \mathrm{E} \bullet ds = V_B - V_A$

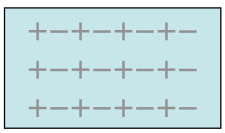
since from Gauss's law:

$$E_r = \frac{k_eQ}{r^2} \qquad r > R$$

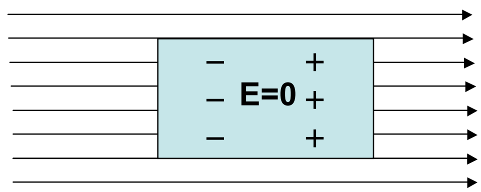
$$E_r = \frac{k_eQr}{R^3} \qquad r < R$$

Conductors (Lecture 3 Review)

- For conductors, electrons inside able to move freely
- Key points:
  - Free electrons inside the conductor move under the influence of any applied electric field
  - Electron redistribution: additional electric fields. Eventually (~10^{-16}s), reach electrostatic equilibrium (E=0 inside conductor).



No Applied Field

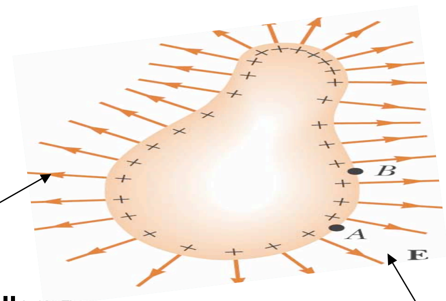


In Applied Field

Conductors in Electrostatic Equilibrium

- Regardless of shape :
  - Electric field inside conductor is zero
  - All net charges reside on the surface
  - Electric field on surface of conductor always normal to the surface, magnitude σ/ε_0
  - Electric field also zero inside any empty cavity within the conductor
  - sharper edge → larger field.
- Since E=0 inside conductor,
  - Potential is the same throughout the conductor: Equipotential

Charge Distribution On Conductor



|E| higher at smaller radius of curvature (more charge density)

|E| lower (less charge density)

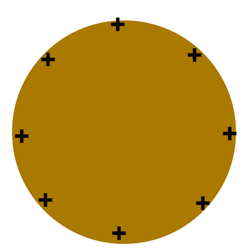
Example: Charge Distribution On Conductors (I)

❑ The total charge on this conducting sphere is 5q. How is the charge distributed?

Evenly distributed throughout the body

➔ $Q_{surface}$=5q, Q_{body}=0

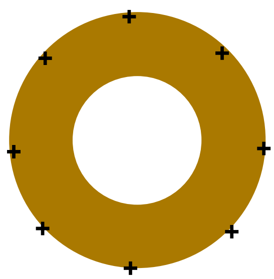
None of above

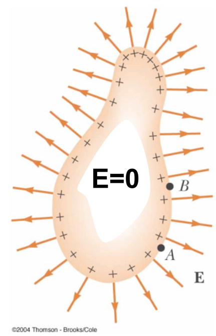


Note: Regardless of shape, charge resides only on the surface of a conductor

Example: Charge Distribution On Conductors (II)

- ❑ The total charge on this conducting shell is 5q, How is the charge distributed? ($R_{outer}=2R_{inner}$)
  - $Q_{Inner\_surface}$ = 2.5q, $Q_{Outer\_surface}$=2.5q, Q_{body}=0
  - $Q_{Inner\_surface}$ = q, $Q_{Outer\_surface}$=4q, Q_{body}=0
  - ➔ $Q_{Inner\_surface}$ = 0, $Q_{Outer\_surface}$=5q, Q_{body}=0

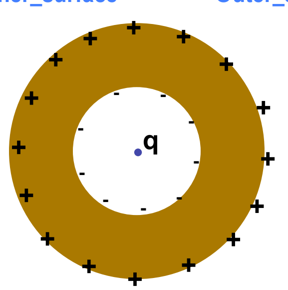




Note: Regardless of shape, charge resides only on outer surface of a conductor if no charge inside cavity (E_{inside}=0).

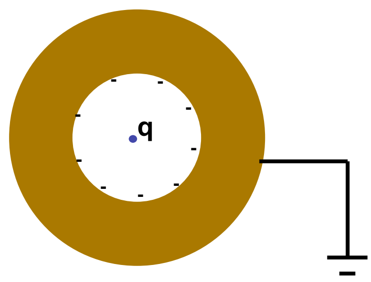
Example: Charge Distribution On Conductors (III)

- ❑ The total charge on this conducting shell ($R_{outer}=2R_{inner}$)
- ❑ is +5q. A point charge of +q is placed at the center. How is the charge distributed?
  - ➔ $Q_{Inner\_surface}$ = - q, $Q_{Outer\_surface}$=6q, Q_{body}=0
  - $Q_{Inner\_surface}$ = q, $Q_{Outer\_surface}$=4q, Q_{body}=0
  - $Q_{Inner\_surface}$ = 0, $Q_{Outer\_surface}$=5q, Q_{body}=0

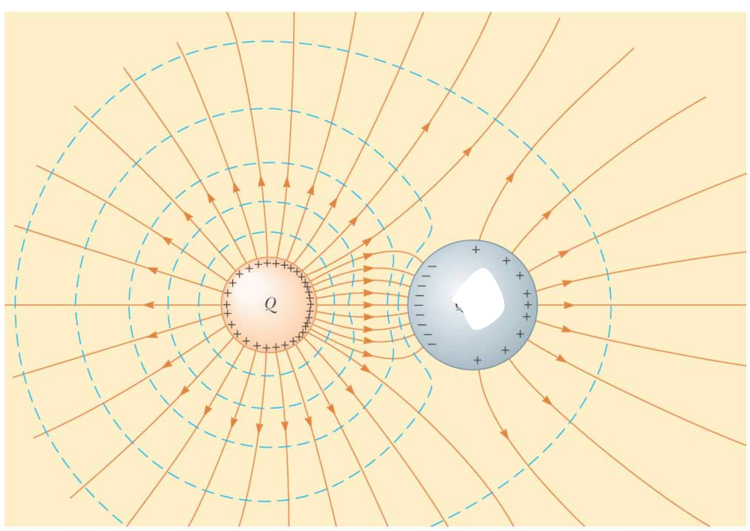


Example: Charge Distribution On Conductors (IV)

- Initially, the total charge on this shell is +5q. A point charge of +q is placed at the center, and the shell is then grounded. How is the charge distributed? ($R_{outer}=2R_{inner}$)
  - $Q_{Inner\_surface} = -q$, $Q_{Outer\_surface}=6q$, $Q_{body}=0$
  - $Q_{Inner\_surface} = q$, $Q_{Outer\_surface}=4q$, $Q_{body}=0$
  - ➔ $Q_{Inner\_surface} = -q$, $Q_{Outer\_surface}=0$, $Q_{body}=0$
  - Q=0 everywhere.



Charge Distribution on Conductors: Field lines and Equipotential Surfaces



Note: equipotentials are normal to field lines

Equipotentials

Defined as: **The locus of points with the same potential.**

- **Example: for a point charge, the equipotentials are spheres centered on the charge.**

The electric field is always perpendicular to an equipotential surface!

$$V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{s}$$

Along equipotential surface, no change in V

$$-\int_A^B \vec{E} \bullet d\vec{s} = \Delta V = 0$$

Therefore, $\vec{E} \bullet d\vec{s} = 0 \Longrightarrow \vec{E} \perp d\vec{s}$

Electric field perpendicular to surfaces of constant V