Physics 202, Lecture 6

Today's Topics

- Review: Potential, Conductors, Equipotentials
- Capacitance (Ch. 26.1-26.3)
  Calculating capacitance, Combinations of capacitors
- Reminder: Homework #2 due 9/24, 10 PM
  Homework #3 due 9/28, 10 PM
- Next lecture:
  - Energy stored in charged capacitors, dielectrics
  - About Exam 1.

Review (I)

Electric potential: $V(r) = -\int_{ref}^{r} \vec{E} \bullet d\vec{s}$ Unit: 1 J/C= 1 V

Finite region of charge: take ref point (zero of potential) at infinity

Calculating the potential:

Point charges: $V(\vec{r}) = k_e \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i'|}$

Distributions: $V(\vec{r}) = k_e \int \frac{dq}{|\vec{r} - \vec{r}'|}$

$|\vec{r} - \vec{r}'|$: distance from source to observation point

Obtaining E from V:

$\vec{E} = -\nabla V$

$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$

$E_r = -\frac{\partial V}{\partial r}, \ldots$

Review (II)

Conductors:

-- Electric field is zero inside conductor

-- All net charge on surface (Gauss's Law)

-- Electric field zero inside any empty cavity within the conductor (Gauss's Law)

-- Electric field just outside a conductor is perpendicular to surface, magnitude $E = \frac{\sigma}{\varepsilon_0}$

Surface of conductor is an equipotential

Equipotentials

Defined as: The locus of points with the same potential.

- Example: for a point charge, the equipotentials are spheres centered on the charge.

The electric field is always perpendicular to an equipotential surface!

$$V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{s}$$

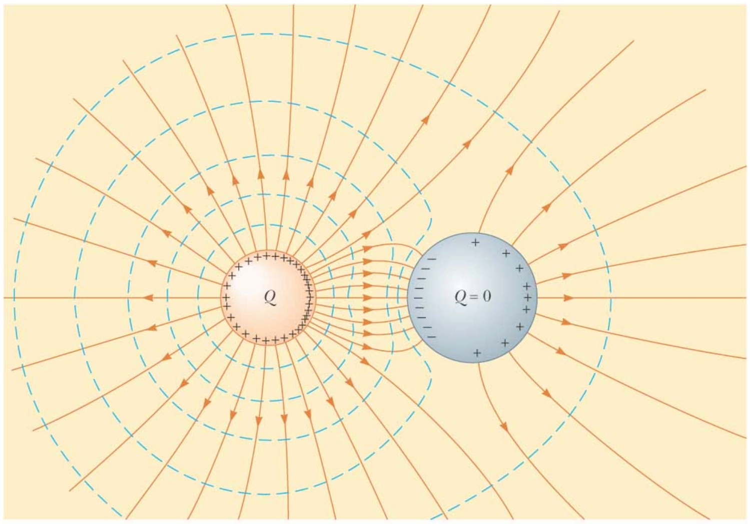
Along equipotential surface, no change in V

$$-\int_A^B \vec{E} \bullet d\vec{s} = \Delta V = 0$$

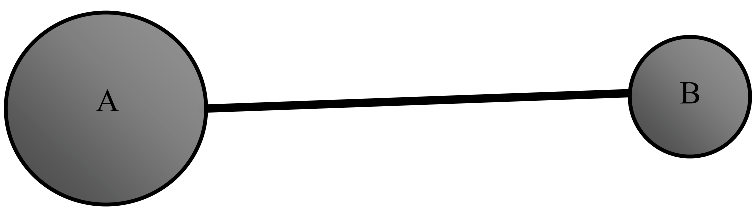
Therefore, $\vec{E} \bullet d\vec{s} = 0 \Longrightarrow \vec{E} \perp d\vec{s}$

Electric field perpendicular to surfaces of constant V

Charge Distribution on Conductors: Field lines and Equipotential Surfaces



Question 1.



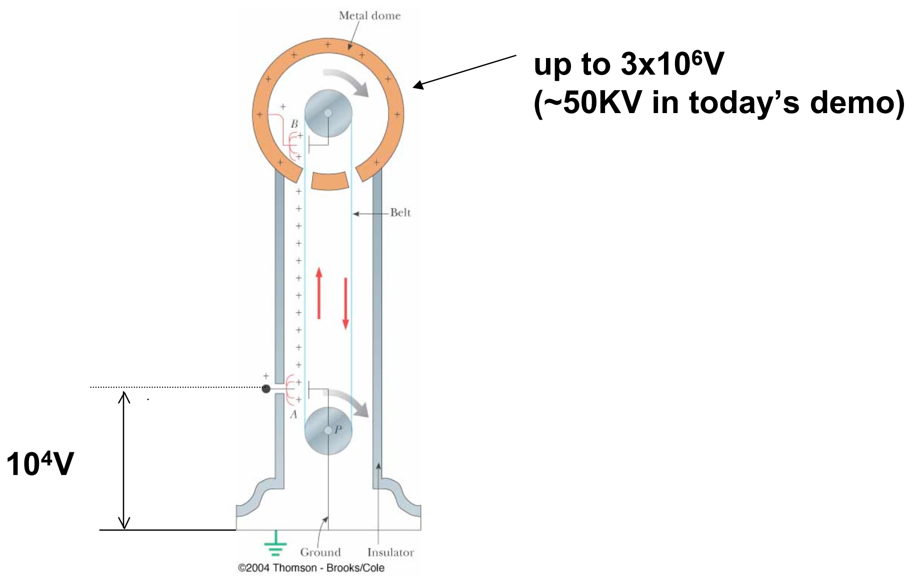
Two conductors are connected by a wire. How do the potentials at the conductor surfaces compare?

a) $V_A > V_B$ b) $V_A = V_B$ c) $V_A < V_B$

What happens to the charge on conductor A after it is connected to conductor B ?

a) Q_A increases b) Q_A decreases c) Q_A doesn't change

High Voltage Electrostatic Generator: Van de Graaff Generator



Capacitance

Charged conductor: characterized by a constant potential.

Two such (spatially separated) conductors, charged to +Q and -Q, will have a potential difference ΔV

This is a capacitor:

a device that stores electrical energy (by storing charge) which can be released in a controlled manner during a short period of time.

Capacitance: the ratio of the charge on one conductor to the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

Units: Farad (F)
1 F= 1 C/V

Calculating Capacitance

Capacitance is independent of charge, voltage on capacitor: it depends on geometry of the conductors.

Examples:

- **Parallel conducting plates**
- **Concentric spherical conductors**
- **Concentric cylindrical conductors**

Procedure: determine ΔV as a function of Q

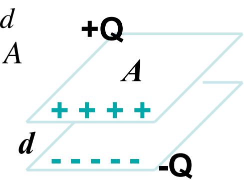
Simple example: capacitance of sphere **(imagine second conductor at infinity)**

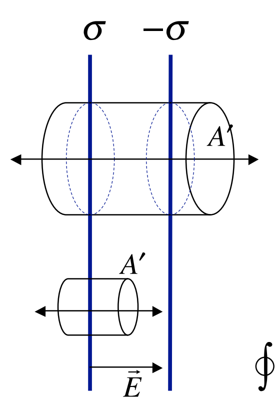
$$\Delta V = V = \frac{k_e Q}{R} \qquad C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\varepsilon_0 R$$

Example: Parallel Plate Capacitor

Consider two metallic parallel plates with area A, separation d:

Step 1. Use Gauss's law to get E: $\sqrt{A} \gg d$, $\sigma = Q/A$





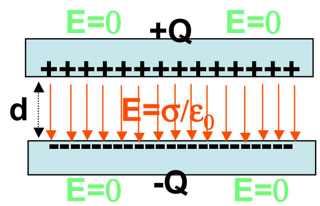
- **Field outside plates is zero**
  (Gaussian surface encloses zero net charge)
- **Field inside plates nonzero**
  (Gaussian surface encloses nonzero net charge)

$$\oint \vec{E} \bullet d\vec{A} = E_{in} A' = \sigma A'/\varepsilon_0 \qquad E_{in} = \sigma/\varepsilon_0$$

Parallel Plate Capacitor (continued)

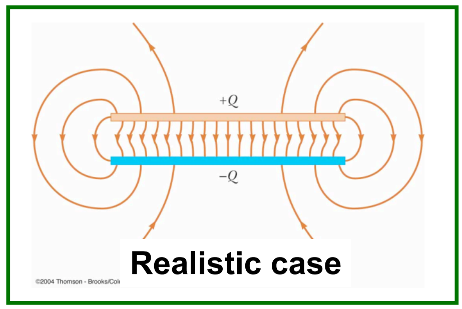
Step 2. Get potential difference:

$$\Delta V = -\int \vec{E}\bullet d\vec{s} = \frac{\sigma}{\varepsilon_0}\int ds = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}$$



Step 3. Get Capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$



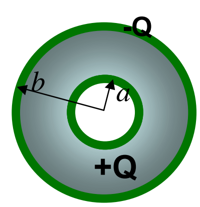
Spherical Capacitor

Consider two concentric conducting spherical shells, radii a, b

$a < b$

Step 1. Get electric field:

Gauss's Law: $\vec{E} = \frac{k_e Q}{r^2}\hat{r} \qquad (a < r < b)$



Step 2. Get potential difference:

$$\Delta V = -\int \vec{E}\bullet d\vec{s} = -\int_b^a \frac{k_e Q}{r^2}dr = k_e Q\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{k_e Q(b-a)}{ab}$$

Step 3: $$C = \frac{ab}{k_e(b-a)}$$

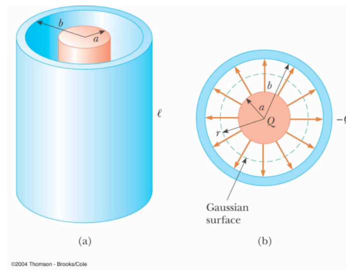
Cylindrical Capacitor

Consider two concentric cylindrical conducting shells, radii a, b

$Q = \lambda \ell \qquad a < b$

Step 1. Get electric field:

Gauss's Law: $\vec{E} = \dfrac{2k_e\lambda}{r}\hat{r} \qquad (a < r < b)$



Step 2. Get potential difference:

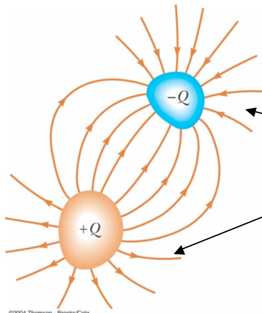
$$\Delta V = -\int \vec{E}\bullet d\vec{s} = -\int_b^a \frac{2k_e\lambda}{r}dr = 2k_e\lambda \ln(b/a)$$

Step 3:

$$C = \frac{\ell}{2k_e \ln(b/a)}$$

Capacitors: Summary

- **Generic capacitor:**

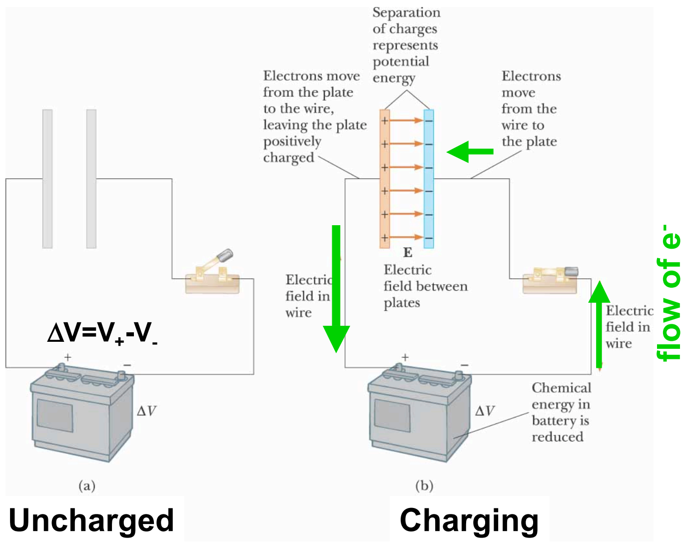


two conductors oppositely charged

Q = CΔV

- **First example of "circuit element"**
- **Capacitors are very useful devices:**
  - **Timing control, noise filters, energy buffer, frequency generator/selector/filter, sensors, memories...**

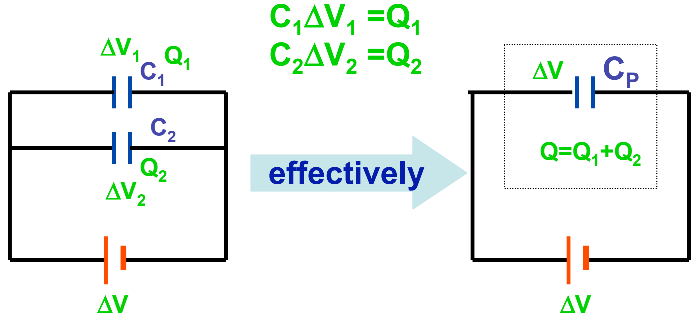
Charging A Pair of Parallel Conductors



Combinations of Capacitors In Parallel

$C_1\Delta V_1 = Q_1$

$C_2\Delta V_2 = Q_2$



$\Delta V_1 = \Delta V_2 = \Delta V$
(why?)

Equivalent Capacitance

$C_P = Q/\Delta V \rightarrow C = C_1 + C_2$

$C_P = C_1 + C_2 + C_3 + \ldots$

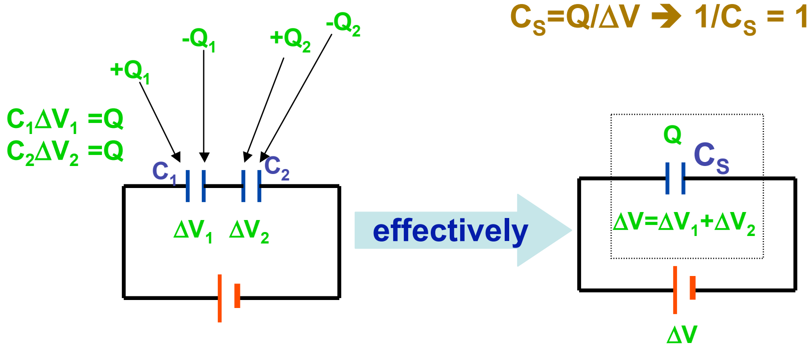
Note: C_P always $> C_i$

Combinations of Capacitors In Series

Charge conservation: $Q_1=Q_2(=Q)$

Equivalent Capacitance

$C_S=Q/\Delta V \Rightarrow 1/C_S = 1/C_1+1/C_2$

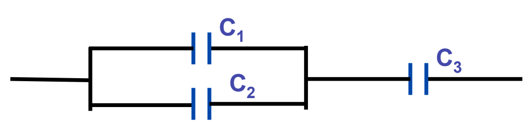


$$1/C_S= 1/C_1+1/C_2+1/C_3+\ldots$$

Note: C_S always < C_i

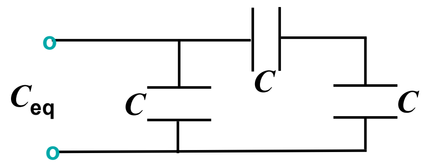
Question 2. Combination of Capacitors

- What is the effective capacitance for this combination? ($C_1=1\mu F$, $C_2=2\mu F$, $C_3=3\mu F$)
  - $C=6\mu F$
  - $C=3\mu F$
  - ➔ $C=1.5\mu F$
  - None of above



Question 3.

- What is the equivalent capacitance, C_{eq}, of the combination of capacitors shown below?



(a) $C_{eq} = (3/2)C$ (b) $C_{eq} = (2/3)C$ (c) $C_{eq} = 3C$

Question 4: Connection of Charged Capacitors

- Two capacitors, C_1=1μF and C_2=2μF are initially charged to ΔV_1=1V and ΔV_2=2V, respectively.
  - What are the charges in each capacitor?
  - If the capacitors are connected in parallel as shown by the dashed lines, what is the charge in each capacitor after the connection?