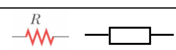
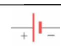
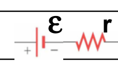
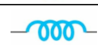
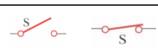
Physics 202, Lecture 10

Today's Topics

- DC Circuits (2)
  - Kirchhoff's Rules
  - RC Circuits
- Announcements:
  - Homework #4 due Monday, 10/8 at 10 PM
  - Reading quiz (optional): open until Friday, 10/5 5 PM

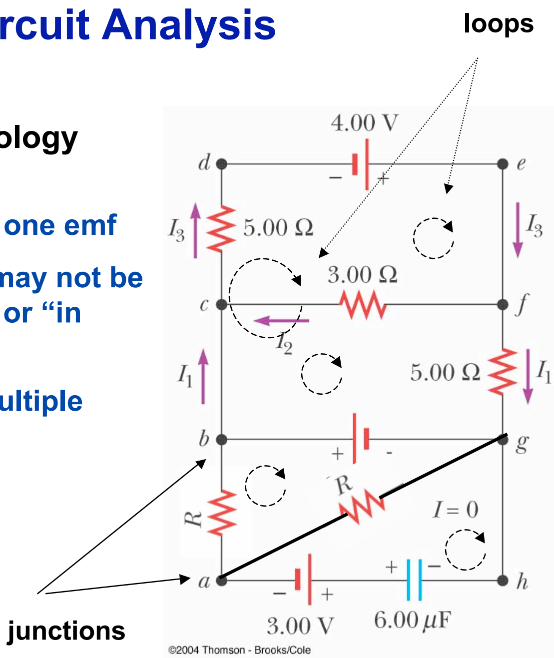
Basic Circuit Components

Component	Symbol	Behavior in circuit
Ideal battery, emf		$\Delta V=V_+-V_- = \varepsilon$
Resistor		$\Delta V= -IR$
Realistic Battery		→ 
(Ideal) wire		$\Delta V=0$ (→$R=0$, $C=0$)
Capacitor		$\Delta V=V_- - V_+ = -q/C$, $dq/dt = I$
Inductor		later in semester
(Ideal) Switch		$C=0$, $R=0$ (on), $R=\infty$ (off)
Transformer	Future Topics	
Diodes, Transistors,…		

Circuit Analysis

General circuits:
more complicated topology

- may contain more than one emf
- resistor combinations may not be as simple as “in series” or “in parallel”
- Circuits may contain multiple loops and junctions.



Circuit Analysis

Kirchoff’s Laws

#1 Conservation of electric charge

All the charge that flows into a junction of conductors per unit time, the same amount must leave in the same time interval.

#2 Conservation of energy

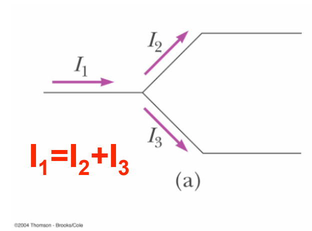
A complete trip around the circuit (the end point is the same as the beginning point) must result in zero net energy change.

Kirchhoff's Rules: Junction Rule

- **Rule #1: Junction Rule**

The net current entering any junction equals the net current leaving that junction.

$\Sigma I_{in} = \Sigma I_{out}$



- **Determined by assigned direction for each current:**
  - "in" : current with assigned direction towards junction
  - "out" : current with assigned direction off junction

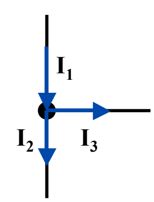
(Very) Quick Quiz: Junction Rule

- **What is the junction rule for the current assignment shown?**

$I_1+I_2=I_3$

$I_1=I_2+I_3$ ←

$I_1-I_2=I_3$



Although equation 2 and 3 are equivalent, equation 3 does not follow template form $I_{in}=I_{out}$

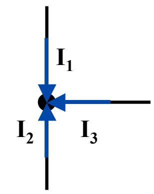
Quick Quiz: Junction Rule

- ❑ What is the junction rule for the current assignment shown?

$I_1+I_2=I_3$

$I_1+I_2+I_3=0$ ←

Neither



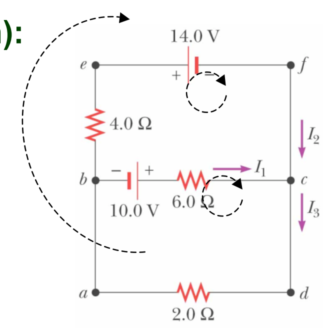
While the actual currents can not all goes into a junction, the assigned currents can.

Kirchhoff's Rules: Loop Rule

- ❑ Loop Rule (Energy Conservation):

The sum of potential drops across components along any closed circuit loop must be zero.

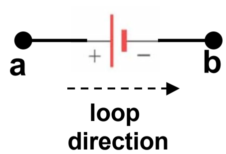
$$\sum \Delta V = 0$$



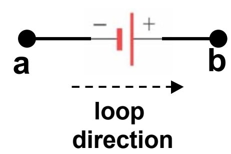
- ➢ The exact expression of the potential drop is determined by the type of component and the assigned current direction (see next slides)

Determine Potential Difference(1)

Choose a direction for the current, and move around the loop in that direction.



$\Delta V = V_b - V_a = -\varepsilon$



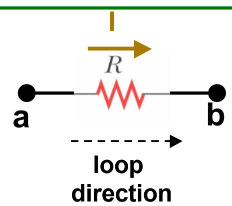
$\Delta V = V_b - V_a = +\varepsilon$

When a battery is traversed from the positive terminal to the negative terminal, the voltage drops (- sign).

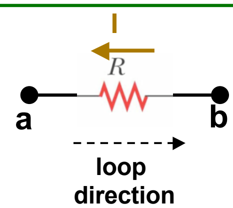
When a battery is traversed from the negative terminal to the positive terminal, the voltage increases (+ sign).

Determine Potential Differences (2)

Choose a direction for the current, and move around the loop in that direction.



$\Delta V = V_b - V_a = -IR$



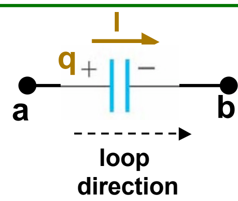
$\Delta V = V_b - V_a = +IR$

When moving across a resistor in the direction of the assigned current, the voltage drops (- sign).

When moving across a resistor In a direction opposite to the assigned current, the voltage increases (+ sign).

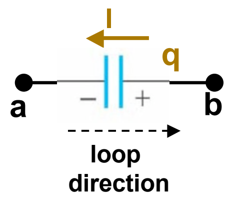
Determine Potential Differences (3)

Choose a direction for the current, and move around the loop in that direction.



$\Delta V = V_b - V_a = -\,q/C$

$I = dq/dt$



$\Delta V = V_b - V_a = +\,q/C$

$I = dq/dt$

When moving across a capacitor from the positive plate to the negative plate, the voltage drops (- sign).

When moving across a capacitor from the negative plate to the positive plate, the voltage increases (+ sign).

Steps to Apply Kirchhoff's Rules

1. **Assign directional currents for each branch of the circuit.**

2. **Set up junction rules at certain (any) junctions. Normally, # of junctions = # of currents -1.**

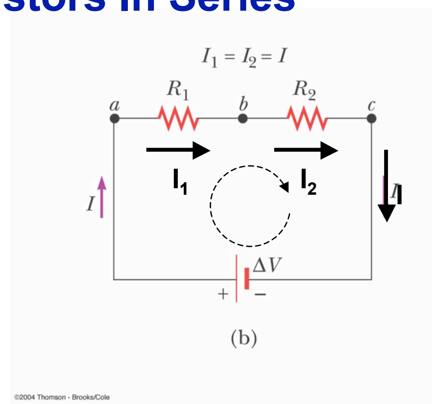
3. **Select a number of closed loops to apply loop rule.**
   For each, follow a clockwise (or counterclockwise) direction, find ΔV across each component, apply loop rule.
   # of loops determined by # of unknowns.

4. **Solve for unknowns. (A negative current indicates its direction is opposite to the assigned direction.)**

Example 1: Resistors In Series

- Show $R_{eq}=R_1+R_2$.

Applying Kirchhoff Rules:
Junction a: $I_1=I$
Junction c: $I_2=I$
Loop: $\Delta V - I_1R_1 - I_2R_2 = 0$

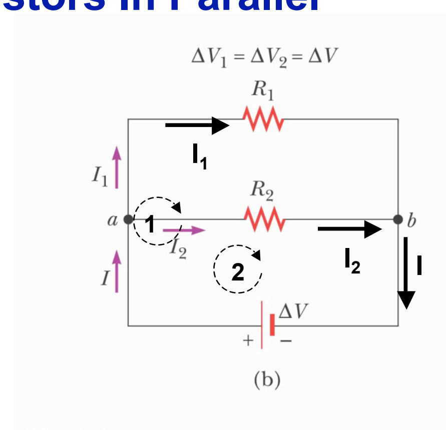


- Questions:
  - What are the voltage drops across R_1 and R_2?
  - What is the power delivered to R_1 and R_2 ?

Example 2: Resistors In Parallel

- Show $1/R_{eq}=1/R_1+1/R_2$.

Applying Kirchhoff Rules:
Junction: $I_1+I_2=I$
Loop 1: $\Delta V - I_1R_1 = 0$
Loop 2: $\Delta V - I_2R_2 = 0$



- Questions:
  - What are the voltage drops across R_1 and R_2?
  - What is the power delivered to R_1 and R_2 ?

Multi-loop examples

- On board: two loop circuit example
  Ch. 28 # 21 (28 if time)
- On board: three loop circuit example
  Ch. 28, #26 (25 if time)
- On slides: textbook example 28.9

Example 3: Multi-Loop

- (Text example 28.9)

Find out I_1, I_2, I_3

- Kirchhoff's Rules:

Junction c:

$I_1+I_2=I_3$

Loop 1:

$\varepsilon_1 - I_1R_1 - I_3R_3 = 0$

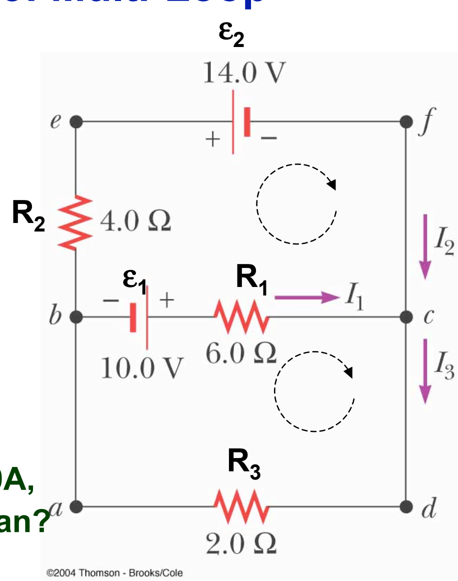
Loop 2:

$-\varepsilon_2 + I_1R_1 - \varepsilon_1 - I_2R_2 = 0$

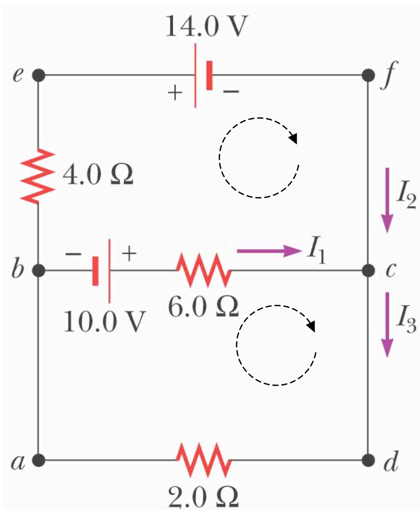
Solving three equations:

$I_1= 2.0A$, $I_2= -3.0A$, $I_3= -1.0A$,

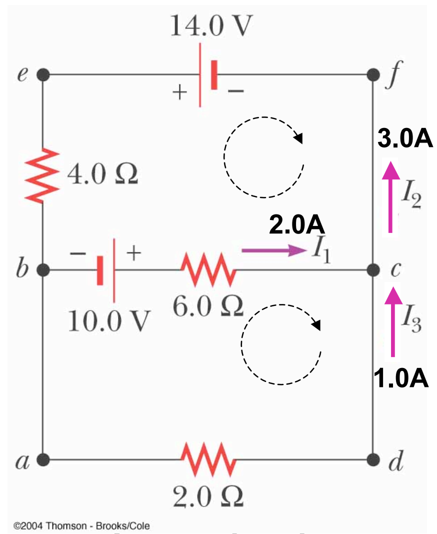
What does the – sign mean?



Example 3: Interpretation of Results



$I_1 = 2.0A$, $I_2 = -3.0A$, $I_3 = -1.0A$,



Actual situation

Example 3 Again: Different Initial Directions

Different initial direction for I_1, I_2

❑ Apply Kirchhoff's Rules:

Junction c:

$0 = I_3 + I_1 + I_2$

Loop 1:

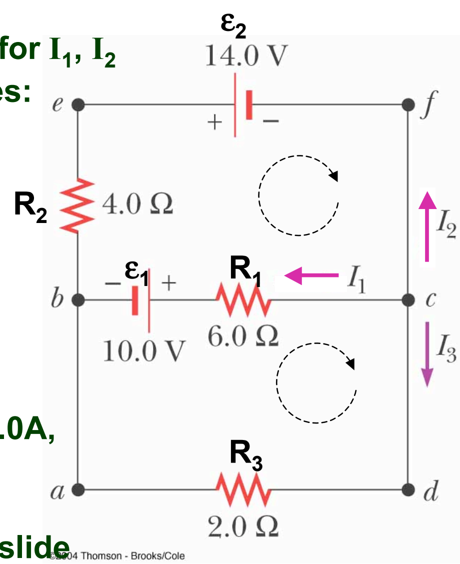
$\varepsilon_1 + I_1 R_1 - I_3 R_3 = 0$

Loop 2:

$-\varepsilon_2 - I_1 R_1 - \varepsilon_1 + I_2 R_2 = 0$

Solving three equations:

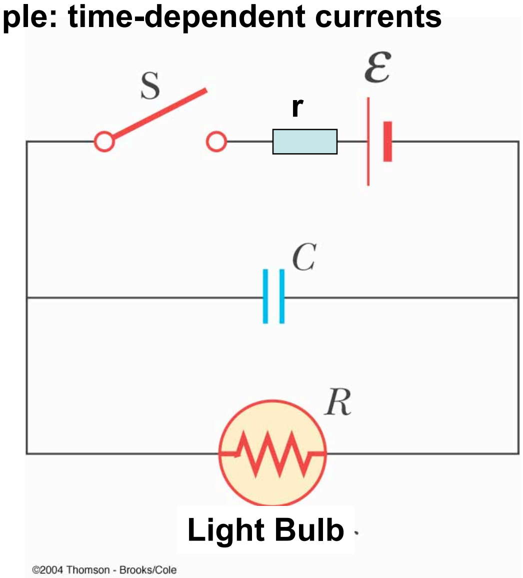
$I_1 = -2.0A$, $I_2 = +3.0A$, $I_3 = -1.0A$,



Same result as previous slide

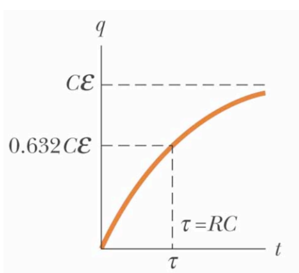
RC Circuits

First example: time-dependent currents

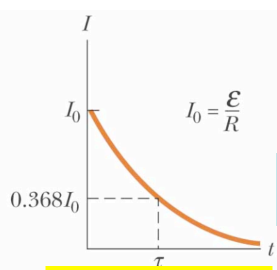


Charging A Capacitor in RC Circuit

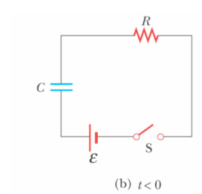
- ❑ **Find I and q when a capacitor is being charged in a RC circuit (see board).**



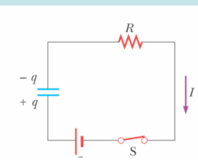
$$q(t) = \mathcal{E}C(1 - e^{-t/RC})$$



$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$



$$\mathcal{E} - q(t)/C - R\frac{dq(t)}{dt} = 0$$



Charging

Note: τ≡RC is the "time constant"

Discharging A Capacitor in RC Circuit

- Find I, q when a capacitor is being discharged in a RC circuit (See board).

$$q(t) = Qe^{-t/RC}$$

$$I(t) = -\frac{Q}{RC}e^{-t/RC}$$

Note the time constant τ=RC