Physics 202, Lecture 12

Today's Topics

- Magnetic Forces (Ch. 29)
  - Review: magnetic force, magnetic dipoles
  - Motion of charge in uniform B field:
    Applications: cyclotron, velocity selector, Hall effect
- Sources of the Magnetic Field (Ch. 30, part 1)
  - Calculating the B field due to currents (Biot-Savart)

Homework #5: due 10/15 ,10 PM.
Optional reading quiz: due 10/12, 7 PM

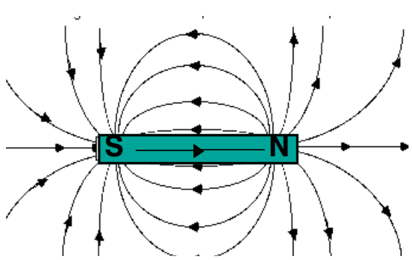
Magnetic Fields and Forces: Recap

- Magnetic Force: experienced by moving charges

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = \int I\, d\vec{l} \times \vec{B}$$

(point charges) (currents)

- Magnetic Field B: sourced by moving charges
  direction: as indicated by north pole of compass



Units: 1 Tesla (T) = 1 N/(A m)

Field lines: closed loops!
Outside magnet: N to S
Inside magnet: S to N

Torque on Current Loop in Uniform B field

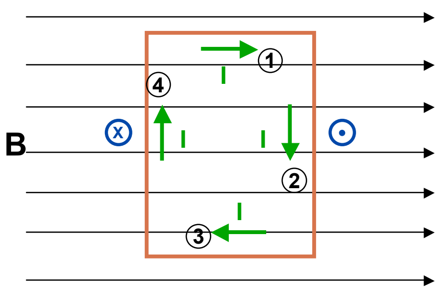
- Force and torque on current loop:

$$\vec{F}_{net} = 0, \vec{\tau}_{net} = \vec{\mu} \times \vec{B}$$

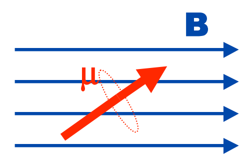
magnetic dipole moment: $\vec{\mu} = NI\vec{A}$

Loop rotates to minimize $U = -\vec{\mu} \bullet \vec{B}$

I.e., until $\vec{\mu} \parallel \vec{B}$

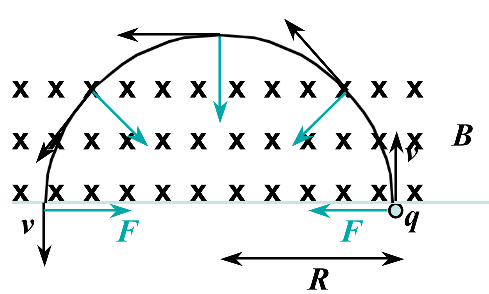


(*N*=# of turns of loop, A=area)



Charged Particle in Uniform B Field

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$$



Force perpendicular to velocity: uniform circular motion

Magnetic force does no work on charge:
kinetic energy constant

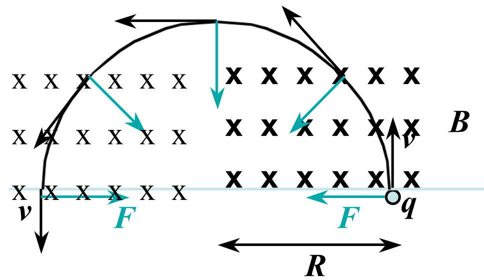
Trajectory in Uniform B Field (2)

- **Force:**

  $F = qvB$

- **centripetal acc:**

  $a = \frac{v^2}{R}$



- **Newton's 2nd Law:**

$$F = ma \quad \Rightarrow \quad qvB = m\frac{v^2}{R}$$

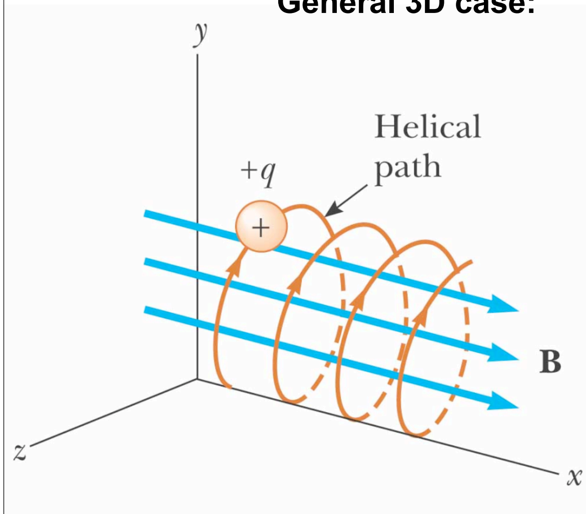
$$\Rightarrow \quad R = \frac{mv}{qB} = \frac{p}{qB}$$

(an important result, with useful experimental consequences!)

"Cyclotron" frequency: $\omega = \frac{v}{R} = \frac{qB}{m}$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$

Trajectory in Uniform B Field (3)

General 3D case:



➢ **In the plane perpendicular to B:**

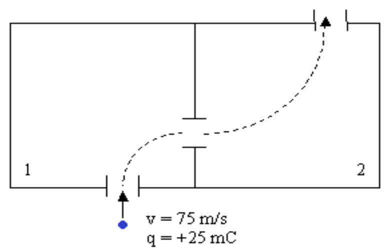
$$R = \frac{mv_\perp}{qB} \qquad T = \frac{2\pi m}{qB}$$

➢ **Parallel to B: spacing b/w turns of helix**

$$d = v_\parallel T = \frac{v_\parallel 2\pi m}{qB}$$

Question 1

The drawing shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle fired into chamber 1 follows the dashed path shown in the figure.

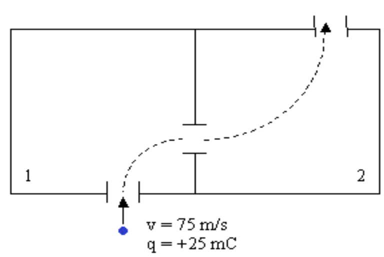


What is the direction of the magnetic field in chamber 1?

a) Up
b) Down
c) Left
d) Right
e) Into page
f) Out of page

Question 2

What is the direction of the magnetic field in chamber 2?



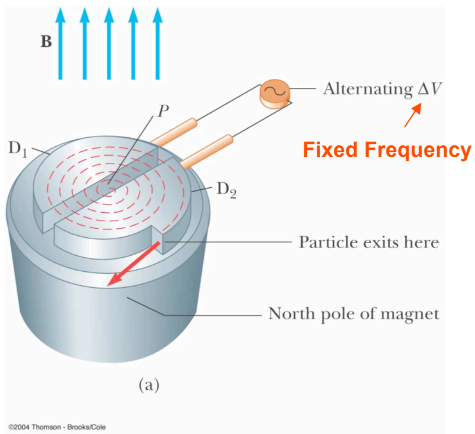
a) Up
b) Down
c) Left
d) Right
e) Into page
f) Out of page

Which field is larger, B_1 or B_2?

a) $\boldsymbol{B}_1 > \boldsymbol{B}_2$ b) $\boldsymbol{B}_1 = \boldsymbol{B}_2$ c) $\boldsymbol{B}_1 < \boldsymbol{B}_2$

Application: Cyclotron

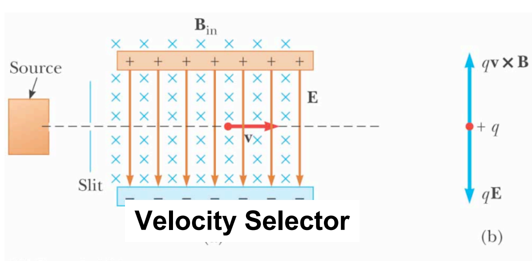
- First Modern Particle Accelerator



First Cyclotron (1934)
Lawrence & Livingston

Application: Velocity, Mass Selectors

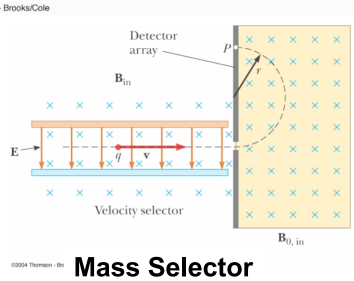
- Velocity and mass selector:



Velocity Selector

speed selected:

$$v = \frac{E}{B}$$



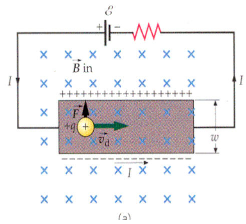
Mass Selector

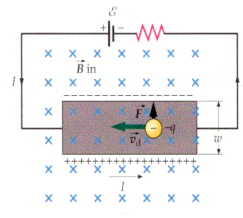
mass selected:

$$\frac{m}{q} = \frac{rB_0}{v} = \frac{rB_0}{(E/B)}$$

The Hall Effect (1)

Potential difference on current-carrying conductor in B field:





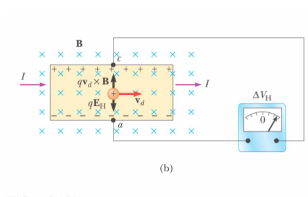
positive charges moving counterclockwise: upward force, upper plate at higher potential

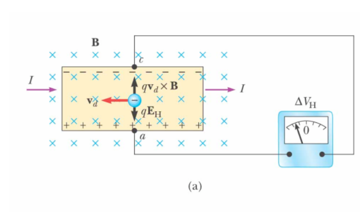
negative charges moving clockwise: upward force Upper plate at lower potential

Equilibrium between electrostatic & magnetic forces:

$$F_{\text{up}} = qv_d B \qquad F_{\text{down}} = qE_{\text{ind}} = q\frac{V_{\text{H}}}{\text{w}} \qquad V_{\text{H}} = v_{\text{d}} Bw = \text{"Hall Voltage"}$$

The Hall Effect (2)





$$I = nqv_d A = nqv_d wt \qquad V_H = v_d Bw = \frac{IB}{nqt}$$

Hall coefficient: $$R_H \equiv \frac{V_H}{IB} = \frac{1}{nqt}$$

Hall effect: determine sign,density of charge carriers

(first evidence that electrons are charge carriers in most metals)

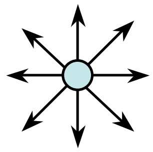
Magnetic Fields of charges, currents

Review: back to electrostatics:

Two Ways to calculate the electric field:

- **Coulomb's Law**

$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$



"Brute force"

- **Gauss' Law**

$$\int \vec{E}\bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

"High symmetry"

Are there analogous equations for the *Magnetic* Field?

Calculation of Magnetic Fields (Currents)

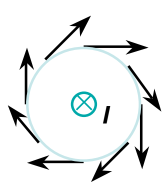
Two Ways to calculate the magnetic field:

- **Biot-Savart Law** **("Brute force")**

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

- **Ampere's Law** **("High symmetry")**

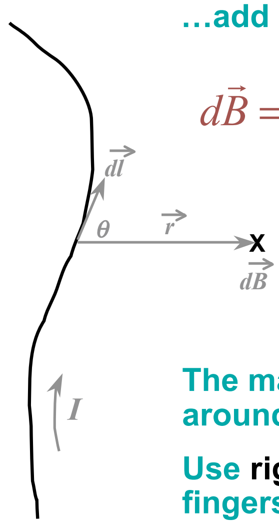
$$\oint \vec{B}\bullet d\vec{l} = \mu_0 I$$



–AMPERIAN LOOP

(Tuesday's lecture)

Biot-Savart Law...



…add up the pieces

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$$

The magnetic field "circulates" around the wire

Use **right-hand rule**: thumb along **I**, fingers curl in direction of **B**.

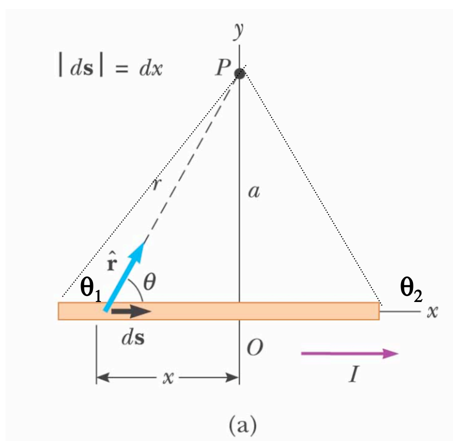
B Field of Straight Wire, length L

- (Text example 30.1) Show that B at point P is:

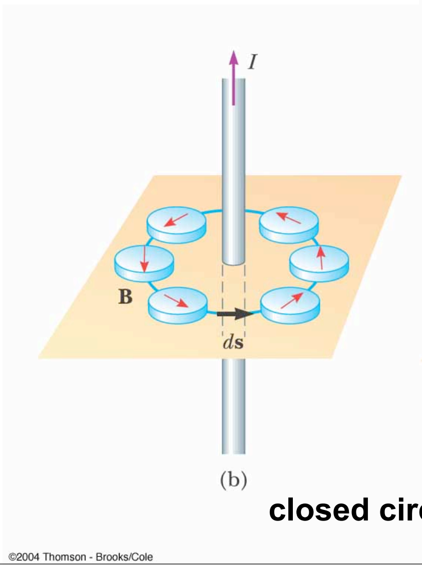
$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

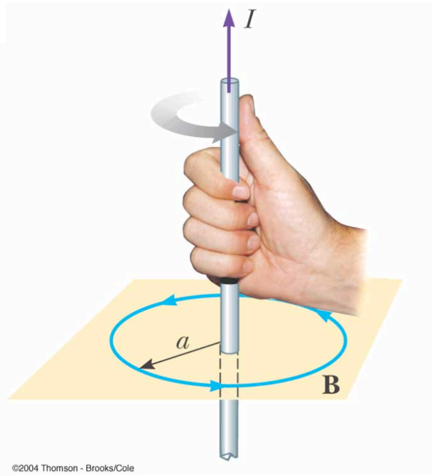
→When the length of the wire is infinity:

$$B = \frac{\mu_0 I}{2\pi a}$$



Direction: another right-hand rule





closed circular loops centered on current