Physics 202, Lecture 13

Today's Topics

- Sources of the Magnetic Field (Ch. 30)
  - Calculating the B field due to currents
    - Biot-Savart Law
      - Examples: ring, straight wire
      - Force between parallel wires
    - Ampere's Law: infinite wire, solenoid, toroid
  - Displacement Current: Ampere-Maxwell
  - Magnetism in Matter

On WebAssign Tonight: Homework #6: due 10/22 ,10 PM.
Optional reading quiz: due 10/19, 7 PM

Magnetic Fields of Current Distributions

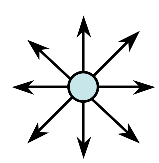
First, review: back to electrostatics:

Two ways to calculate the electric field directly:

– Coulomb's Law

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

"Brute force"



– Gauss' Law

$$\int \vec{E}\bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

"High symmetry"

$\varepsilon_0 = 8.85\text{x}10^{-12}\ C^2/(N\ m^2)$: permittivity of free space

Magnetic Fields of Current Distributions

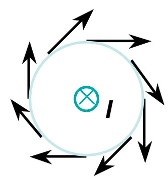
Two Ways to calculate the magnetic field:

– **Biot-Savart Law** **(“Brute force”)**

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

– **Ampere’s Law** **(“high symmetry”)**

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I_{enclosed}$$



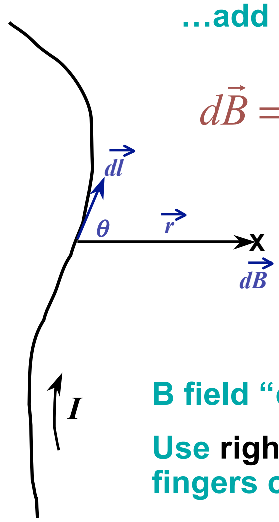
–AMPERIAN LOOP

μ_0 = 4πx10^{-7} T·m/A: permeability of free space

Biot-Savart Law...

…add up the pieces

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$



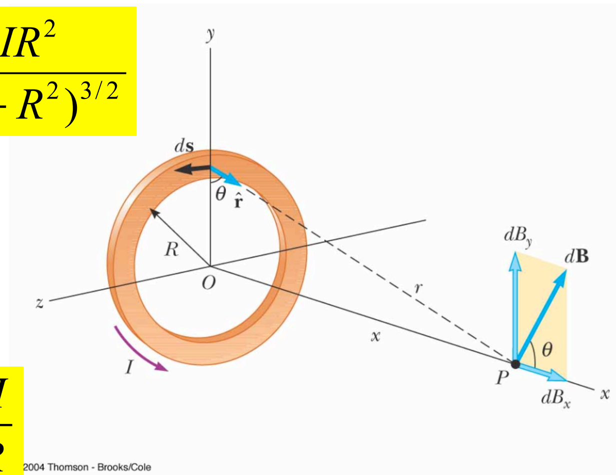
B field “circulates” around the wire

Use right-hand rule: thumb along I, fingers curl in direction of B.

B Field of Circular Current Loop on Axis

❑ **(Text example 30.3) B field on axis of current loop is:**

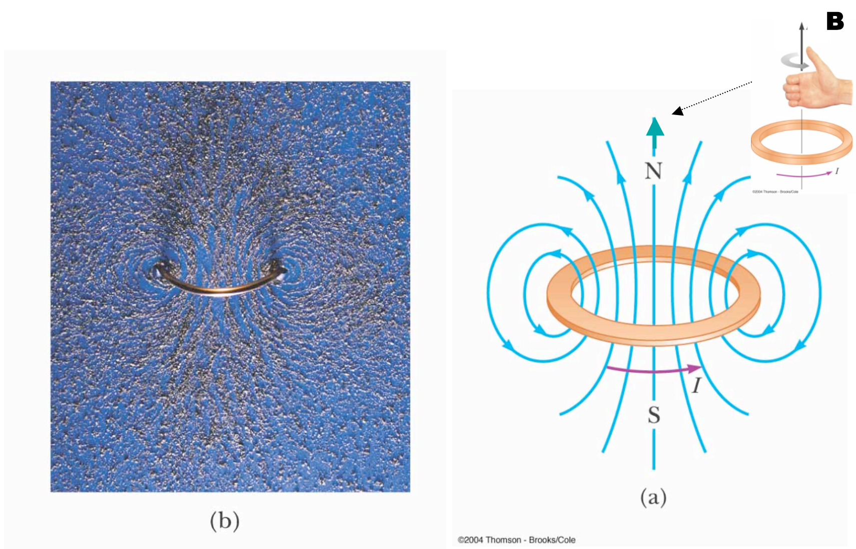
$$B_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$



$$B_{center} = \frac{\mu_0 I}{2R}$$

See also: center of arc (text example 30.2)

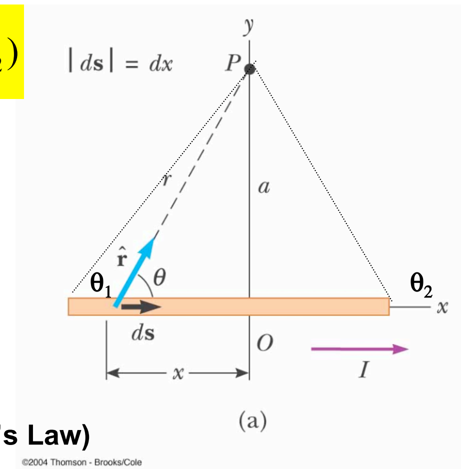
B of Circular Current Loop: Field Lines



B Field of Straight Wire, length L

- (Text example 30.1) B field at point P is:

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$



→When the length of the wire is infinity:

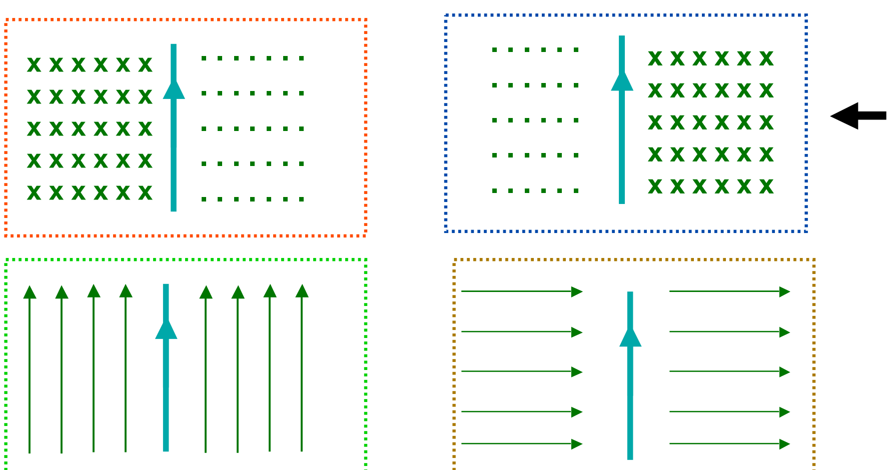
$$B = \frac{\mu_0 I}{2\pi a}$$

(return to this later with Ampere's Law)

Superposition: example Ch. 30, #3

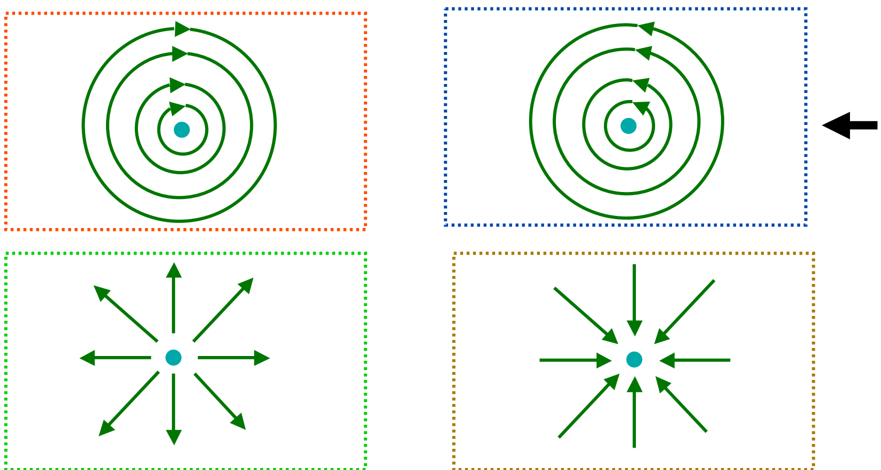
Quick Question 1: Magnetic Field and Current

- Which figure represents the **B** field generated by the current **I**.



Quick Question 2: Magnetic Field and Current

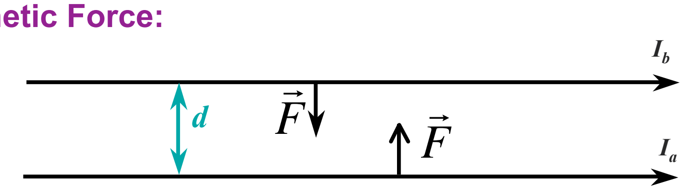
- Which figure represents the **B** field generated by the current **I**.



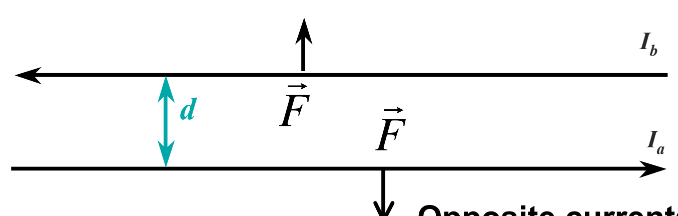
Forces between Current-Carrying Wires

A current-carrying wire experiences a force in an external B-field, but also produces a B-field of its own.

Magnetic Force:



Parallel currents: attract

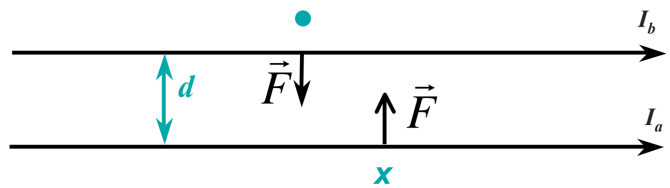


Opposite currents: repel

Example: Two Parallel Currents

- Force on length L of wire b due to field of wire a:

$$B_a = \frac{\mu_0 I_a}{2\pi d} \qquad F_b = \int I_b d\vec{l} \times \vec{B}_a = \frac{\mu_0 I_b I_a L}{2\pi d}$$



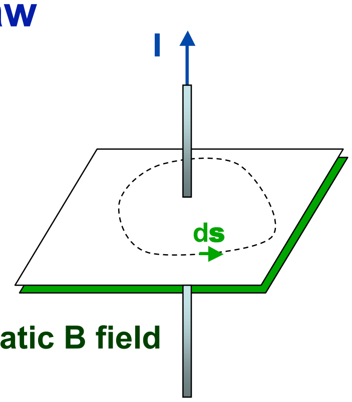
- Force on length L of wire a due to field of wire b:

$$B_b = \frac{\mu_0 I_b}{2\pi d} \qquad F_a = \int I_a d\vec{l} \times \vec{B}_b = \frac{\mu_0 I_a I_b L}{2\pi d}$$

Ampere's Law

❑ **Ampere's Law:**

$$\oint_{\text{any closed path}} \vec{B} \bullet d\vec{s} = \mu_0 I_{enclosed}$$

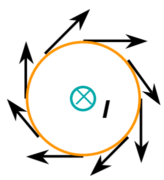


- applies to any closed path, any static B field
- useful for practical purposes
  only for situations with high symmetry

- Ampere's Law can be derived from Biot-Savart Law
- Generalized form: Ampere-Maxwell (later in lecture)

Ampere's Law: B-field of ∞ Straight Wire

- Use symmetry (and Ampere's Law) $\oint \vec{B} \bullet d\vec{s} = \mu_0 I$



Choose loop to be circle of radius R centered on the wire in a plane ⊥ to wire.

Why?

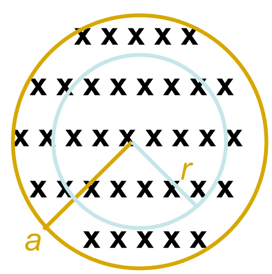
Magnitude of B is constant (function of R only)
Direction of B is parallel to the path.

$$\oint \vec{B} \bullet d\vec{s} = \oint B r\, d\theta = 2\pi r B = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$$

Quicker and easier method for B field of infinite wire (due to symmetry -- compare Gauss' Law)

B Field Inside a Long Wire

Total current *I* flows through wire of radius *a* into the screen as shown.



What is the B field inside the wire?

By symmetry -- take the path to be a circle of radius r:

$$\oint \vec{B} \bullet d\vec{s} = B 2\pi r$$

•Current passing through circle: $I_{enclosed} = \frac{r^2}{a^2} I$

Therefore: Ampere's Law gives

$$\oint \vec{B} \bullet d\vec{s} = B 2\pi r = \mu_o I_{enclosed} \Rightarrow \qquad B = \frac{\mu_0 I r}{2\pi a^2}$$

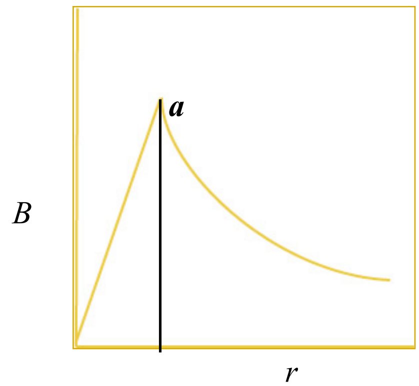
B Field of a Long Wire

Inside the wire: ***(r < a)***

$$B = \frac{\mu_0 I}{2\pi}\frac{r}{a^2}$$

- **Outside the wire:** ***(r > a)***

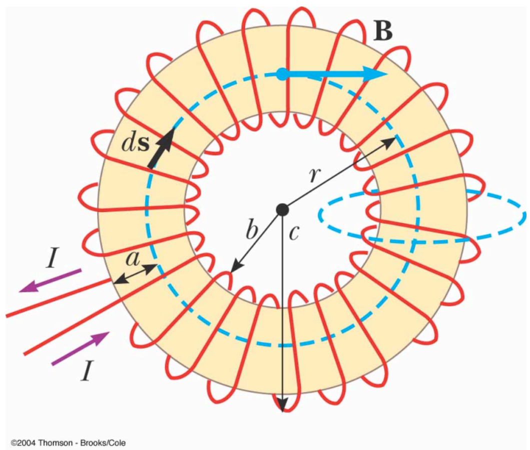
$$B = \frac{\mu_0 I}{2\pi r}$$



Ampere's Law: Toroid

- (Text example 30.5) Using Ampere's Law, B field inside a toroid is found to be:

$$B = \frac{\mu_0 NI}{2\pi r}$$

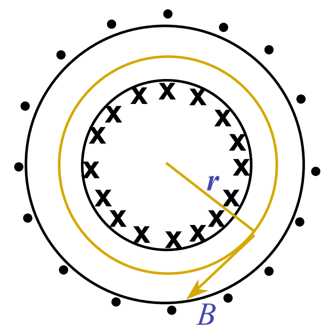


Ampere's Law: Toroid

Toroid: N turns with current I.

$B_\phi = 0$ outside toroid!

(Consider integrating B on circle outside toroid: net current zero)



B_ϕ inside: consider circle of radius *r*, centered at the center of the toroid.

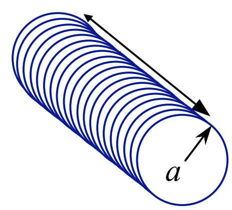
$$\oint \vec{B} \bullet d\vec{s} = B 2\pi r = \mu_o I_{enclosed}$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

B Field of a Solenoid

Solenoid: source of uniform B field (inside of it)

- Solenoid: current I flows through a wire wrapped n *turns per unit length* on a cylinder of radius a and length L.



To calculate the B-field, use Biot-Savart and add up the field from the different loops.

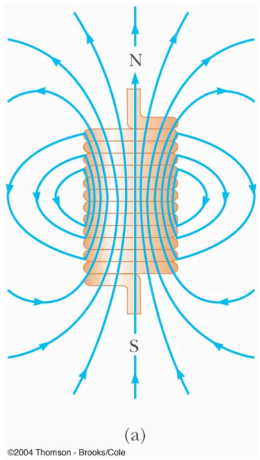
If $a << L$, the B field is to first order contained within the solenoid, in the axial direction, and of constant magnitude.

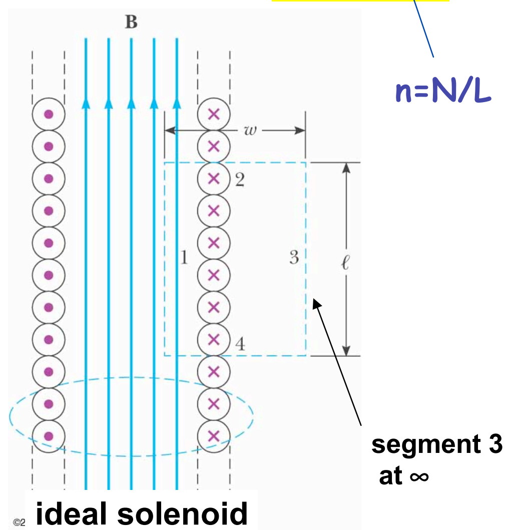
In this limit, can calculate the field using Ampere's Law!

Ampere's Law: Solenoid

- The B field inside an ideal solenoid is: $B = \mu_0 n I$
  (see board)

n=N/L

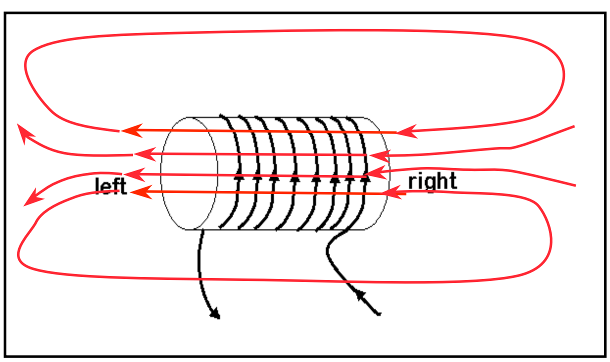




Solenoid: Field Lines

Right-hand rule: direction of field lines.

In this example, since the field lines leave the left end, the left end is the north pole.



Like a bar magnet! (except it can be turned on and off)