Physics 202, Lecture 14

Today's Topics

- Sources of the Magnetic Field (Ch. 30)
  - Review: Biot-Savart Law, Ampere's Law
  - Displacement Current: Ampere-Maxwell Law
  - Magnetism in Matter
  - Maxwell's Equations (prelude)

- Faraday's Law of Induction (Ch. 31)

Homework #6: due 10/22 ,10 PM.
Optional reading quiz: due 10/19, 7 PM

Sources of E and B Fields: An overview

- Sources of electric fields:
  - Static electric charges: Coulomb's Law/Gauss's Law
  - Change of B field (magnetic flux): Faraday's Law (today and next lectures)

- Sources of magnetic fields:
  - Electric current: Biot-Savart Law/Ampere's Law
  - Change of E field (electric flux): Ampere-Maxwell Law (today)

➔ summarized in Maxwell's Equations (later today)

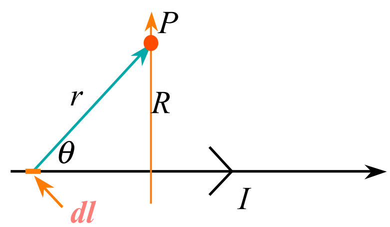
Magnetic Forces and Fields

Magnetic Force: $\vec{F} = \int I d\vec{l} \times \vec{B}$

Magnetic Field:

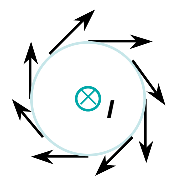
Biot-Savart Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$



Ampere's Law:

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I_{enclosed}$$



$\mu_0 = 4\pi x 10^{-7}$ T·m/A: permeability of free space

Direction of integration along path: use right-hand rule

Magnetic Fields: Examples (Biot-Savart)

- Current loop, distance x on loop axis (radius R):

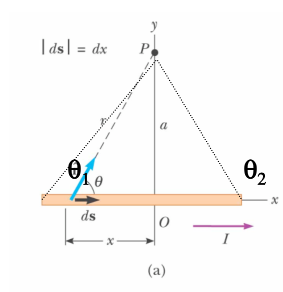
$$B_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \qquad B_{center} = \frac{\mu_0 I}{2R}$$

- Center of arc (radius R, angle θ): $B_{center} = \frac{\mu_0 I \theta}{4\pi R}$

- Straight wire: finite length

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

infinite wire: $B = \frac{\mu_0 I}{2\pi a}$



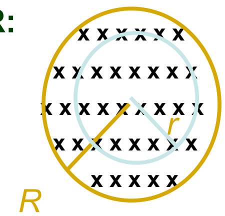
Magnetic Fields: Examples (Ampere's Law)

- **Infinite wire:**

$$B = \frac{\mu_0 I}{2\pi r}$$

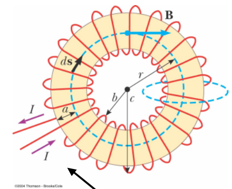
- **Inside wire, radius R:**

$$B = \frac{\mu_0 I r}{2\pi R^2}$$



- **Axial field inside toroid (N turns)**

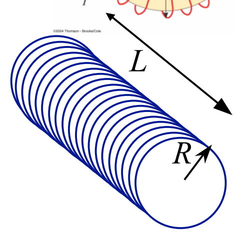
$$B_\varphi = \frac{\mu_0 N I}{2\pi r}$$



- **B field inside long solenoid (L>>R) (n turns/length)**

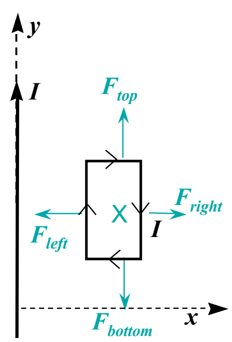
$$B = \mu_0 n I$$

Uniform field!



Question

- A current I flows in the $+y$ direction in an infinite wire; a current I also flows in loop. What is F_x, the net force on the loop in the x-direction?



(a) $F_x < 0$ (b) $F_x = 0$ (c) $F_x > 0$

- Recall: net force on a current loop in a uniform B-field is zero -- but the B-field of an infinite wire is not uniform!
- Forces cancel on the top and bottom of the loop.
- Forces do not cancel on the left and right sides of the loop.

- The left segment is in a larger magnetic field than the right:

$$F_{\text{left}} > F_{\text{right}}$$

Magnetic Field Generated by Varying E Eield

- General Form of Ampere's Law
  → Ampere-Maxwell Law:

$$\Phi_E \equiv \int \vec{E} \bullet d\vec{A}$$

$$\oint_{\substack{any \\ closed\ path}} \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 (I + I_d)$$

➔ **Changing electric field also generates a B field!**

displacement current

$$I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt}$$

Example: charging capacitor

Text: problem 30.37

Magnetism in Matter

The B field produced along the axis of a circular loop (radius R) by a current I is:

$$\vec{B} \approx \frac{\mu_0 \mu}{2\pi z^3} \hat{z} \qquad \text{typical dipole behaviour}$$

μ is the magnetic moment $= I \cdot area$

and $z >> R$

Materials are composed of particles that have magnetic moments -- *(negatively charged electrons circling around the positively charged nucleus).*

orbital angular momentum
spin angular momentum

(quantum mechanics)

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \frac{J}{T}$$

Bohr magneton

Magnetization

Text examples: Ch 30, #41,43

Apply external B field B_0. Field is changed within materials by these magnetic moments.

Magnetization: total magnetic moment per unit volume $\vec{M} \equiv \frac{\vec{\mu}_{total}}{V}$

The B field in the material is $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$

Define H (magnetic field strength): $\vec{B} = \mu_0(\vec{H} + \vec{M})$

$(\vec{H} = \vec{B}_0 / \mu_0)$

Magnetic susceptibility: $\vec{M} = \chi \vec{H}$

$$\vec{B} = \mu \vec{H} = \mu_0 (1 + \chi) \vec{H}$$

"permeability" $\mu \ (= \kappa_m \mu_0)$

Magnetic Materials

Materials are classified by magnetic susceptibilities:

- **Paramagnetic** (aluminum, tungsten, oxygen,…)
  Atomic magnetic dipoles line up with the field, increasing it.
  Only small effects due to thermal randomization: $\chi \sim +10^{-5}$
- **Diamagnetic** (gold, copper, water,…as well as superconductors)
  - Applied field induces an opposing field; usually very weak $\chi \sim -10^{-5}$
- **Ferromagnetic** (iron, cobalt, nickel,…)
  - Dipoles prefer to line up with the applied field (similar to paramagnetic), but tend to all line up the same way due to collective effects: very strong enhancements $\chi \sim +10^{+3} - 10^{+5}$

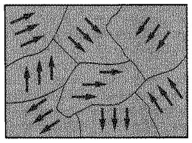
Magnetic susceptibility temperature dependent
(above range of typical values given at T=20 C)

Ferromagnets

- Dipoles tend to strongly align over small patches – "domains" (even w/o external magnetic field). With external field, the domains align to produce a large net magnetization.

"Soft" ferromagnets

Domains re-randomize when magnetic field is removed

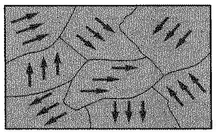


Magnetic Domains

•"Hard" ferromagnets

- Domains persist even when the field is removed
- "Permanent" magnets
- Domains may be aligned in a different direction in a new external field
- Domains may be re-randomized by sudden physical shock
- If temperature is raised above "Curie point" (770 ˚C for iron), domains will also randomize (like a paramagnet)

- Applied field aligns almost all the dipoles – and the "domains". Magnetization is then "saturated": no further increase.



Magnetic Domains

- **"Hard" ferromagnets:**
  - Domains can persist even when the field is removed
  - "Permanent" magnets
    - Domains may be aligned in different directions by changing the applied field…..

      A **"memory"** effect that requires a large reverse field to significantly change the magnetization of the object: **"hysteresis"**.

Maxwell's Equations: Prelude

Gauss's Law: Electric Fields $\oint \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

Gauss's Law: Magnetic Fields $\oint \vec{B} \bullet d\vec{A} = 0$

Ampere-Maxwell: $\oint \vec{B} \bullet d\vec{s} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

What about $\oint \vec{E} \bullet d\vec{s}$?

Recall definition of flux: $\Phi_E = \int \vec{E} \bullet d\vec{A}$ $\quad \Phi_M = \int \vec{B} \bullet d\vec{A}$

The Path Integral of the E field

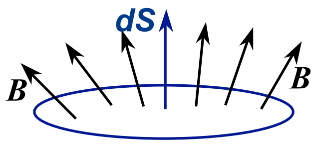
Recall in electrostatics: Coulomb force conservative

$$\oint \vec{E} \bullet d\vec{s} = 0$$

define electrostatic potential: $V_b - V_a = -\int_a^b \vec{E} \bullet d\vec{s}$

But in the presence of moving charges (B fields):

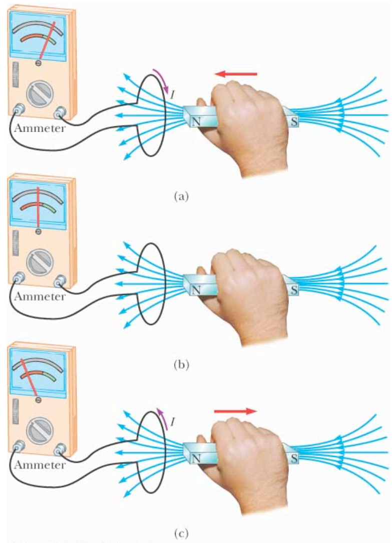
new contribution to E field, with



$$\varepsilon = \oint \vec{E}_{NC} \bullet d\vec{s} \neq 0$$

Induced emf! (Faraday)

Emf and Change of Magnetic Flux



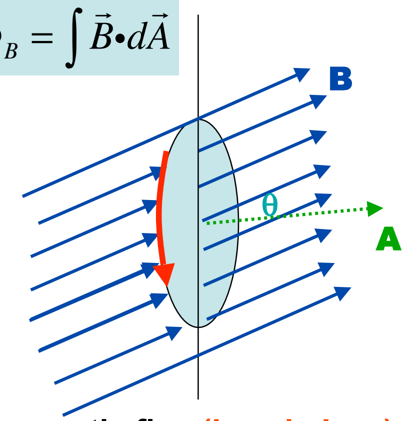
Also: battery-less flashlight

Faraday's Law of Induction

- **Emf induced in a "circuit" is proportional to the time rate of change of magnetic flux through the "circuit".**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

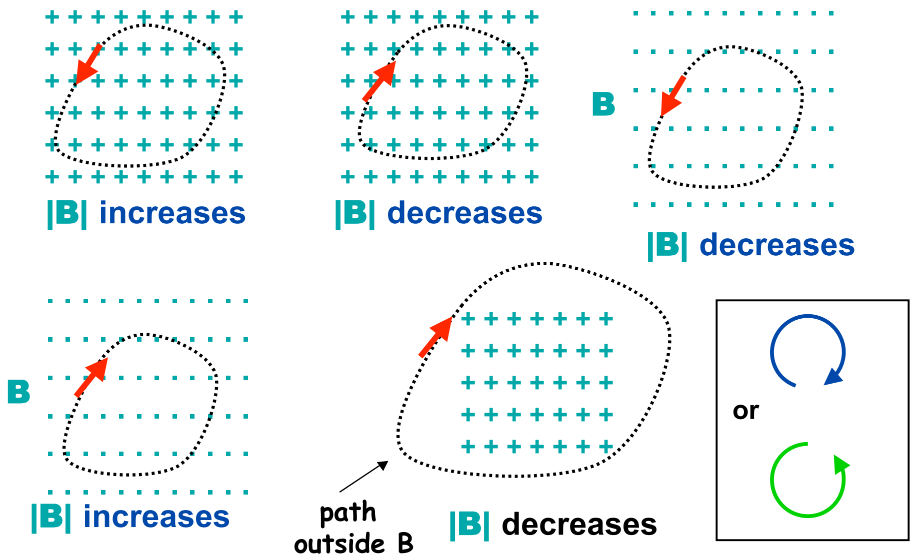


"Circuit": any closed path (does not have to be a real conducting circuit)

Direction: opposes change in magnetic flux (Lenz's Law)

Exercises: Determine Direction of Induced Emf

- Indicate the direction of emf in the following cases:



Methods to Change Electric Flux

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \underset{uniform\ \mathbf{B}}{=} -\frac{d(BA\cos\theta)}{dt}$$

Change of Φ_B → emf

- To change Φ_B:
  - Change B → emf produced by an induced E field
  - Change A → motional emf
  - Change θ → motional emf
  - Combination of above

Example: Ch. 31, #1