Physics 202
Chapter 32
Oct 25, 2007

- Induction:
  - Inductance,
  - Self-Inductance,
  - RL circuit
  - Energy of B-field

On whiteboard

- RL circuit
  - Solve Kirchhoff's equation for the case of
  - Switching on, in detail
  - Switching off
- Power
- Energy of B-field, dissipation in R

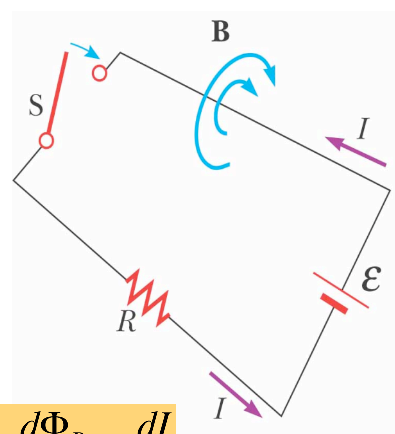
Demonstrations

- Eddy currents
  - Conducting plate entering localized strong B-field
    - Discussion on whiteboard (remember the difference to a plate moving inside a homogenous field!)
  - Spherical magnet rolling slowly inside copper tube
  - Magnet bouncing of a thick cold conducting copper plate because of Eddy currents

- RL circuit: voltage and current when switching on and off

Self-Inductance

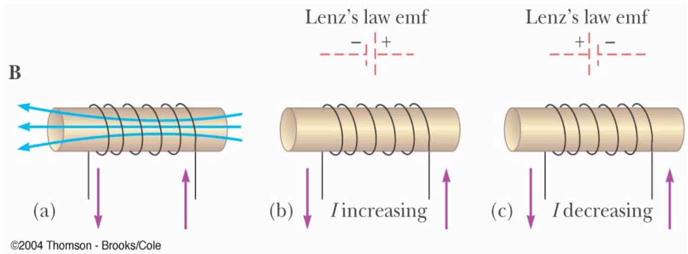
- When the switch is closed, the current does not immediately reach its maximum value
- Faraday's law can be used to describe the effect:
  - Close switch: I increases,
  - $dI/dt \neq 0$
  - $\Rightarrow$ Φ_B increases
  - $\Rightarrow$ Self-induced emf, according to Faraday's law
  - $\Rightarrow$ Induced current in opposite direction
  - $\Rightarrow$ Net current is reduced.



$$\varepsilon = -\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

Self-Inductance, Coil Example



- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)

Self-Inductance

- A induced emf is always proportional to the time rate of change of the current
- L := inductance
  - a measure of the opposition to a change in current
  - L depends on the specifics of the coil
- Symbol for an inductor:

- Inductance of a solenoid on the whiteboard

$$\varepsilon_L = -L\frac{dI}{dt}$$

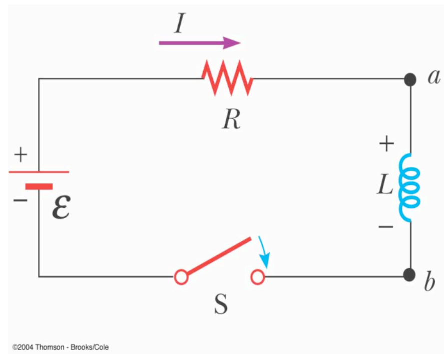
$$L = -\frac{\varepsilon_L}{dI/dt}$$

$$1\text{H} = 1\frac{\text{V}\cdot\text{s}}{\text{A}}$$

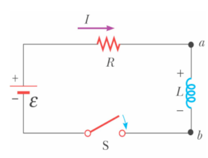
Unit: Henry (named for Joseph Henry)

RL Circuit, Analysis

- An *RL* circuit contains an inductor and a resistor
- When the switch is closed (at time $t = 0$), the current begins to increase
- At the same time, a back emf is induced in the inductor that opposes the change, - the increasing current.



RL Circuit, Analysis



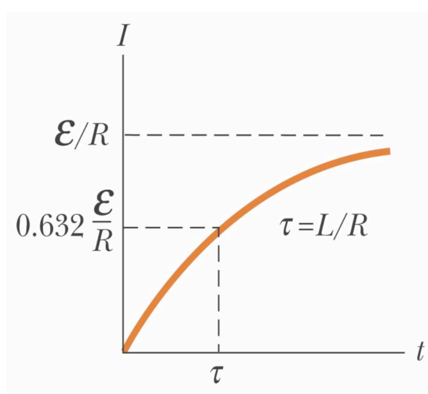
- Applying Kirchhoff's loop rule to the previous circuit gives

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

- Looking at the current and doing some math (whiteboard), we find

$$I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

- where $\tau = L / R$



RL Circuit

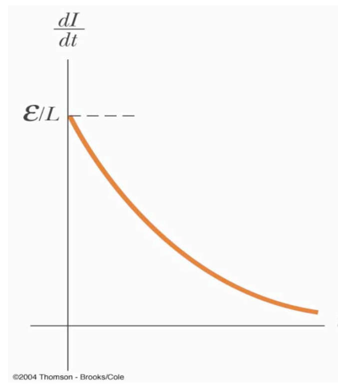
- Applying Kirchhoff's loop rule:

$$-IR - L\frac{dI}{dt} = 0$$

- This one is very easy to solve. Separation of variables and integration (whiteboard) yields:

$$I = \frac{\varepsilon}{R}e^{-Rt/L} = \frac{\varepsilon}{R}e^{-t/\tau}$$

- where $\tau = L / R$



Energy in a Magnetic Field

- In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor
- Part of the energy supplied by the battery appears as internal energy in the resistor
- The remaining energy is stored in the magnetic field of the inductor

$$U = \frac{1}{2}L \cdot I^2$$

- Compare to the energy stored in the electric field in a capacitor

$$U = \frac{1}{2}C \cdot (\Delta V)^2$$

Energy density of the Magnetic Field

- Energy stored in the magnetic field:
  - Just as energy can be stored in the electric field in a capacitor

$$u_B = \frac{B^2}{2\mu_0}$$

- The energy density of the electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$