Physics 202
Chapter 32 continued
Oct 30, 2007

Induction:

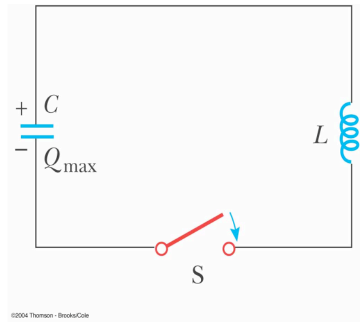
LC circuit, RLC circuit

On whiteboard

- LC circuit
  - Solve Kirchhoff's equation
  - Harmonic oscillator
    - Discussion
    - analogy to mass spring system
    - Energy
- RLC circuit
  - Solution
  - Demonstration with a weakly damped RLC circuit

LC Circuits

- Assume the capacitor is initially charged and then the switch is closed
- Assume no resistance and no energy losses to radiation
- The current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values
- With zero resistance, no energy is dissipated.

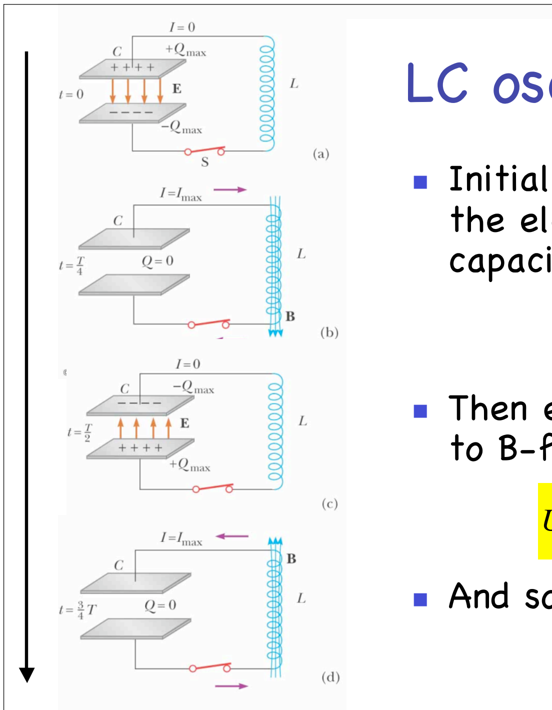


- Kirchoff's loop rule:

$$-\frac{Q}{C} - L\frac{dI}{dt} = 0$$

Discussion on whiteboard….

Derivation of LC and RLC circuit.



LC oscillator

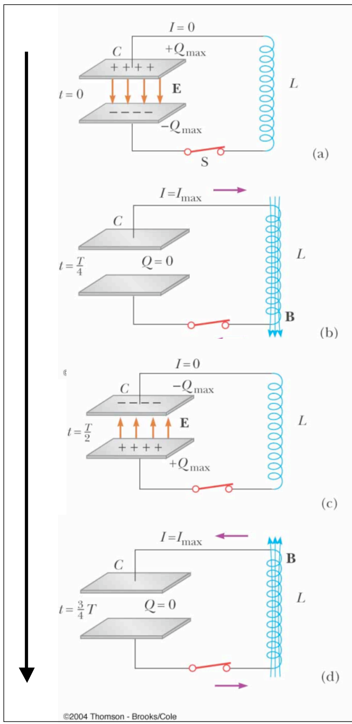
- Initially energy stored in the electric field in the capacitor

$$U = \frac{Q^2}{2C}$$

- Then energy transferred to B-field

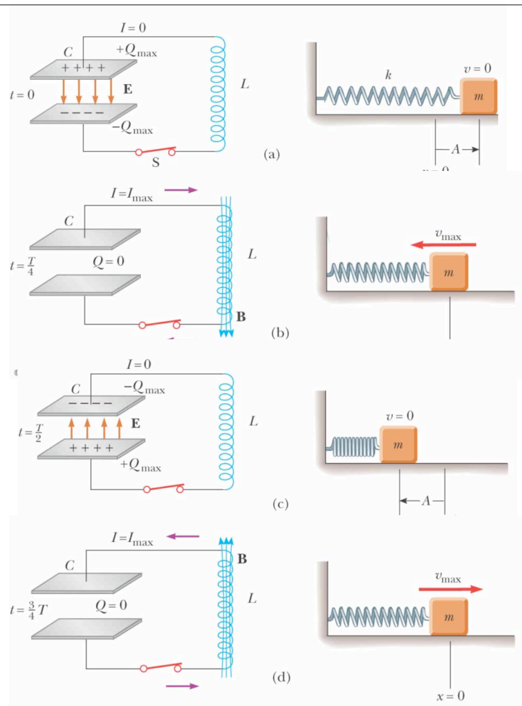
$$U = \frac{1}{2}L \cdot I^2$$

- And so forth ….



LC oscillator

Time [T]	Q [Q_{Max}]	I [I_{Max}]
0	1	0
1/4	0	1
1/2	-1	0
3/4	0	-1
1	1	0



Analogy to mechanical harmonic oscillator

Oscillations in an *LC* Circuit

- The energy continues to oscillate between the inductor and the capacitor
- The total energy stored in the LC circuit remains constant in time and equals

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2} L I^2$$

Time Functions of an *LC* Circuit

- In an LC circuit, charge can be expressed as a function of time
  - $Q = Q_{max} \cos(\omega t + \varphi)$
  - This is for an ideal *LC* circuit
- The angular frequency, ω, of the circuit depends on the inductance and the capacitance
  - It is the *natural frequency* of oscillation of the circuit

$$\omega = 1 \big/ \sqrt{LC}$$

Time Functions of an *LC* Circuit, 2

- The charge can be expressed as a function of time

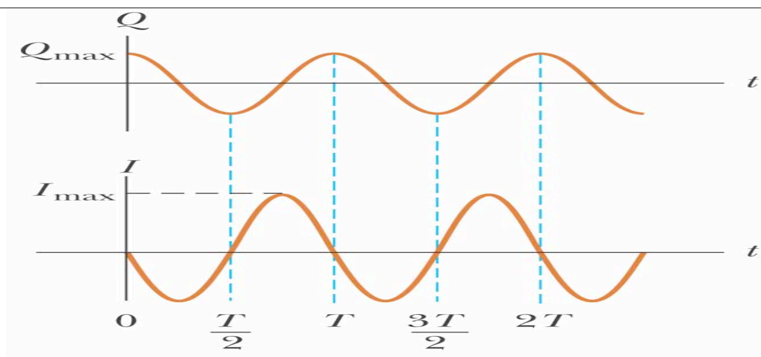
$$Q = Q_{max} \cos(\omega t + \varphi)$$

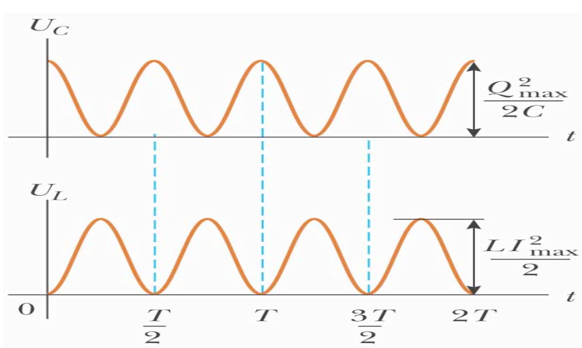
- The current:

$$I = \frac{dQ}{dt} = -\omega Q_{max} \sin(\omega t + \varphi)$$

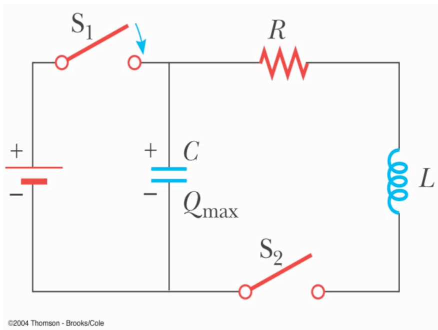
- The total energy

$$U = U_C + U_L = \frac{Q_{max}^2}{2c} \cos^2 \omega t + \frac{1}{2} L I_{max}^2 \sin^2 \omega t$$





The *RLC* Circuit



Kirchoff's rule gives:

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

RLC Circuit – a damped harmonic oscillator

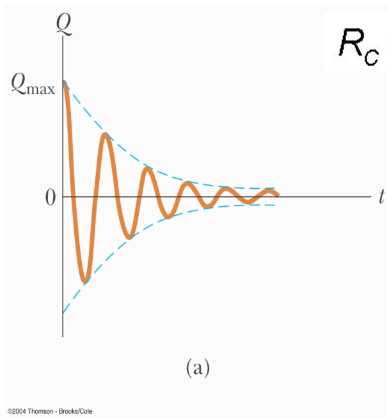
- When *R* is small:
  - The *RLC* circuit is analogous to light damping in a mechanical oscillator
  - $Q = Q_{max}\, e^{-Rt/2L} \cos \omega_d t$
  - ω_d is the angular frequency of oscillation for the circuit and

$$\omega_d=\left[\frac{1}{LC}-\left(\frac{R}{2L}\right)^2\right]^{1/2}$$

RLC Circuit – Damped Oscillator

- When *R* is very large, the oscillations damp out very rapidly
- There is a critical value of *R* above which no oscillations occur
- If $R = R_C$, the circuit is said to be *critically damped*
- When $R > R_C$, the circuit is said to be *overdamped*

$$R_C = \sqrt{4L/C}$$



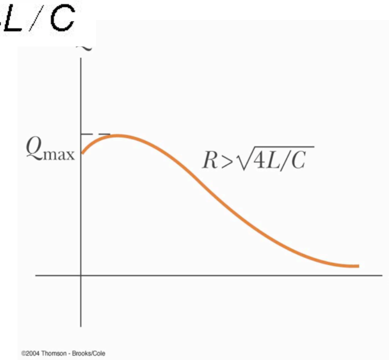


Table 32.1

Analogies Between Electrical and Mechanical Systems

Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring