Physics 202
Chapter 33
Nov 1, 2007

AC circuits

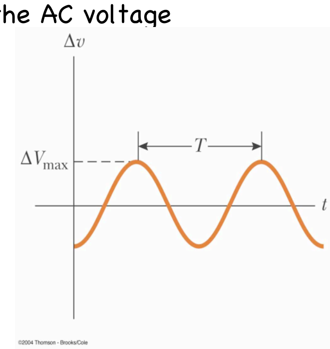
On whiteboard

- AC sources
- All of the material was discussed on the whiteboard.
    - AC sources,
    - RMS
    - L, C and RLC in an AC circuit
    - Resonance
    - Transformer and power
        - Additional discussion on power distribution in the US
    - Filters
        - High pass, low pass, etc.
        - Always remember the frequency dependence of X_L and X_C

AC Voltage sources: $\Delta v = \Delta V_{max} \sin \omega t$

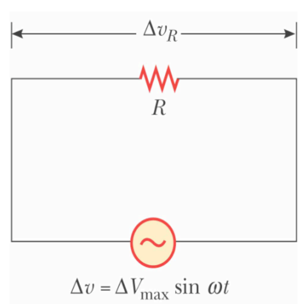
- Output of sinusoidal AC power source:
  - $\Delta v = \Delta V_{max} \sin \omega t$
    - $\Delta v = \Delta v(t)$ is the instantaneous voltage
    - ΔV_{max} is the maximum output voltage of the source
    - ω is the angular frequency of the AC voltage

$$\omega = 2\pi f = \frac{2\pi}{T}$$



- Example: US power grid: f=60Hz

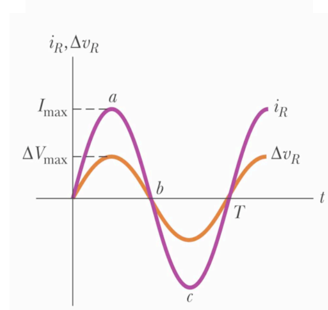
Resistors in an AC Circuit



- $\Delta v = \Delta v_R = \Delta v_{max} \sin \omega t$

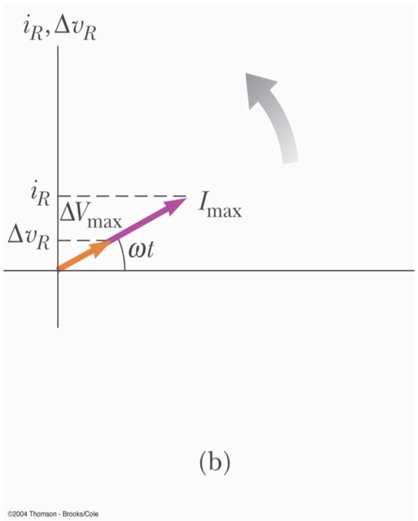
$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

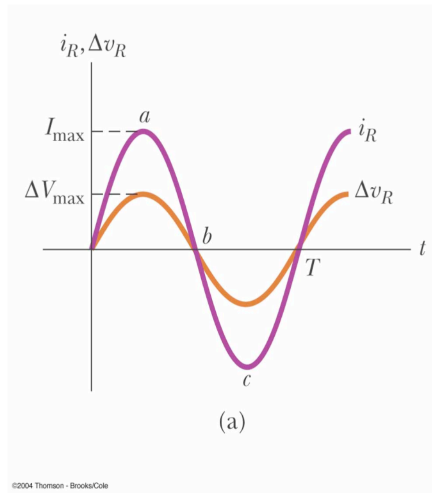
- Current and voltage are "in phase".



(a)

Phasor diagrams



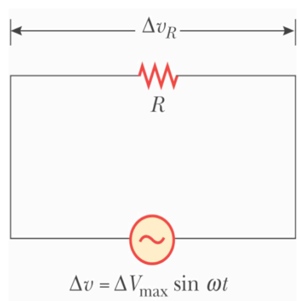


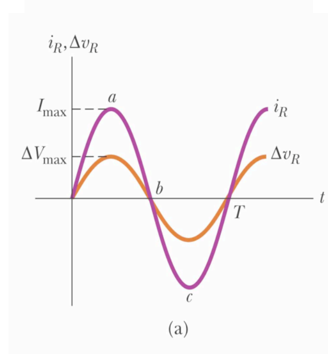
Resistors in an AC Circuit

- $\Delta v = \Delta v_R = \Delta v_{max} \sin \omega t$
- Δv_R is the instantaneous voltage across the resistor

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- Current and voltage are "in phase".





rms Current and Voltage

- The average current in one cycle is zero: <I> = 0
- The *rms current* is the average of importance in an AC circuit
  - rms stands for *root mean square*

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

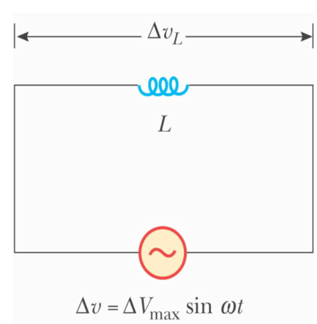
- Alternating voltages can also be discussed in terms of rms values

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

- Instantaneous power:
  - $P = i^2 R$

$$P_{av} = I_{rms}^2 R$$

Inductors in an AC Circuit



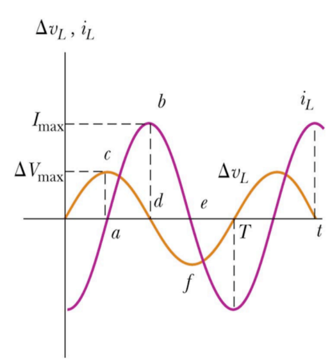
- Kirchhoff's loop rule

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = V_{\max} \sin\omega t$$

$$i = \frac{1}{L}\int \Delta v dt = -\frac{\Delta V_{\max}}{\omega L}\cos\omega t$$

- The instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are "*out* of phase" by 90°
- i_L lags Δv_L



Inductive Reactance

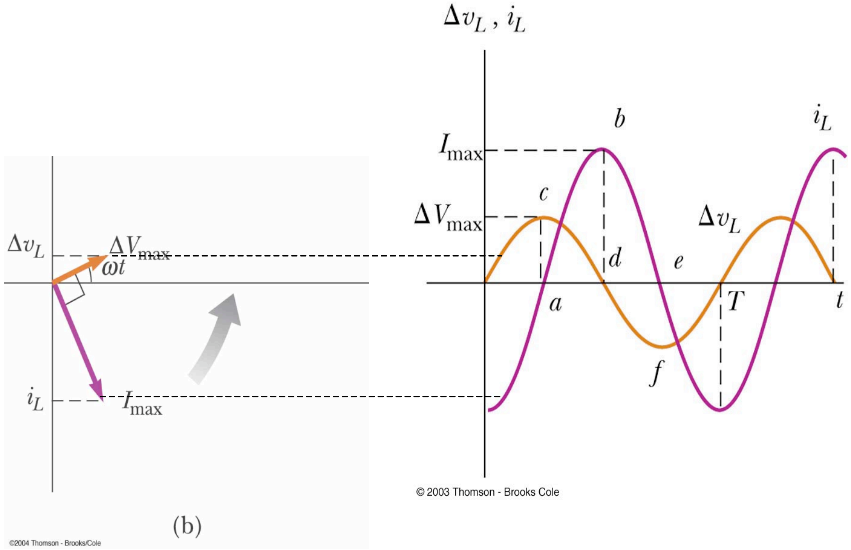
$$\Delta v = V_{\max} \sin \omega t \qquad i = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

- The factor ωL has the same units as resistance
- It is called the **inductive reactance**:
  - $X_L = \omega L$
- *Current* $$I_{\max} = \frac{\Delta V_{\max}}{X_L}$$
- As the frequency increases, the inductive reactance increases
  - This is consistent with Faraday's Law:
    - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the inductance and a decrease in the current

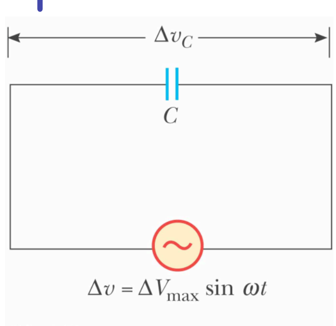
AC circuits continued

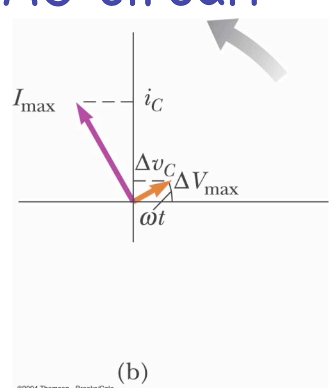
- Nov 1:

Phasor diagram for inductor in AC circuit



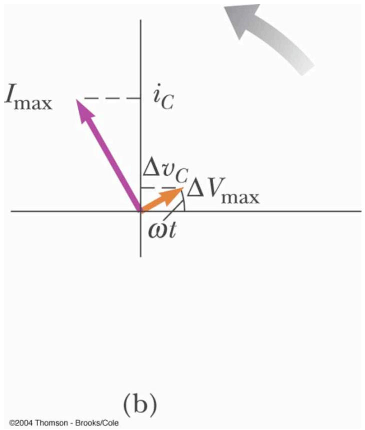
Capacitors in an AC circuit

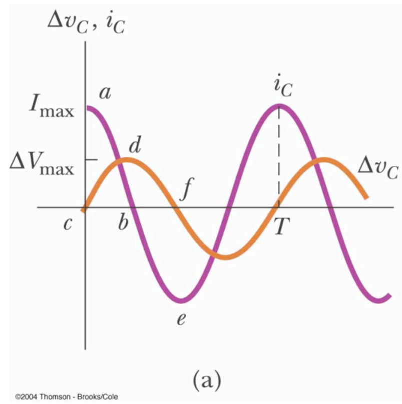




- Analyze using Kirchhoff's loop rule
- Current, voltage
- Phase relationship
- Phasor diagram

Capacitors: i leads v





Capacitors in an AC Circuit

- The charge is $q = q(t) = C\,\Delta V_{max} \sin \omega t$
- The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

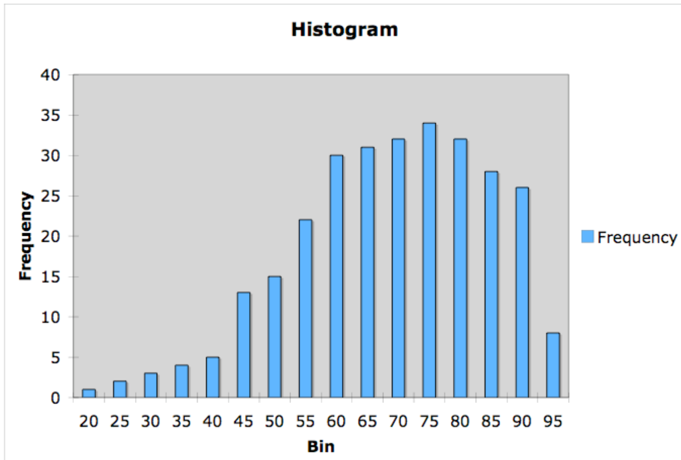
$$\text{or } i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

- The current is $\pi/2$ rad = 90° out of phase with the voltage
- We introduce the capacitive reactance:

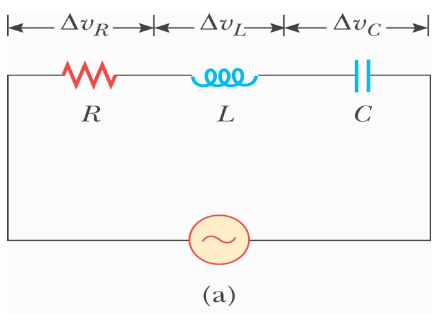
$$X_C \equiv \frac{1}{\omega C} \quad \text{and} \quad I_{max} = \frac{\Delta V_{max}}{X_C}$$

AC

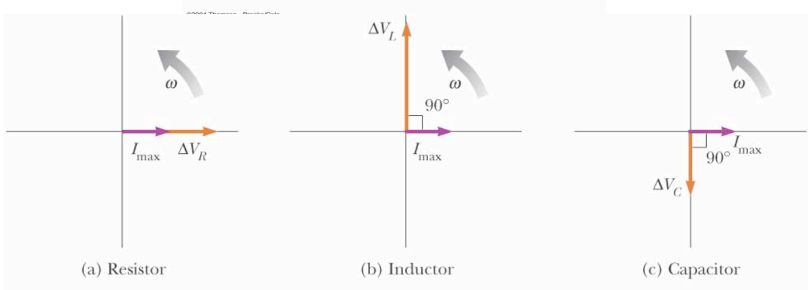
- No



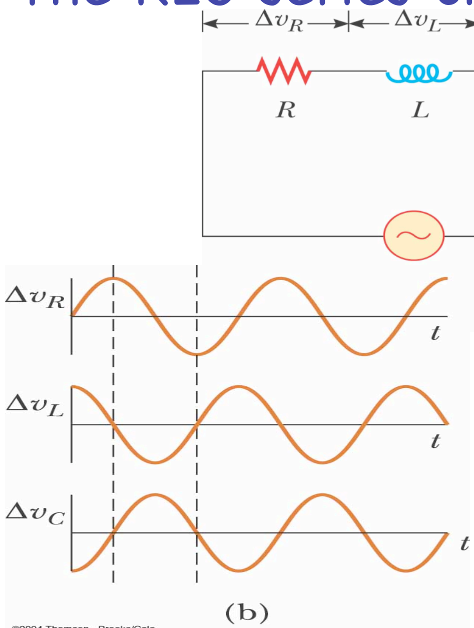
The RLC series circuit



Demonstration



The RLC series circuit



- The instantaneous voltage across the resistor is in phase with the current
- The instantaneous voltage across the inductor leads the current by 90°
- The instantaneous voltage across the capacitor lags the current by 90°

i and *v* Phase Relationships – Equations

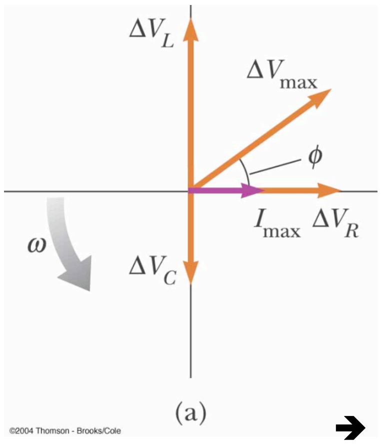
- The instantaneous voltage across each of the three circuit elements can be expressed as

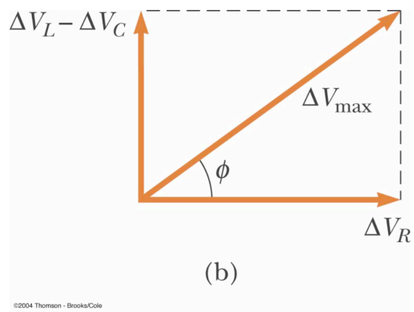
$$\Delta v_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

Vector addition of the phasor diagram

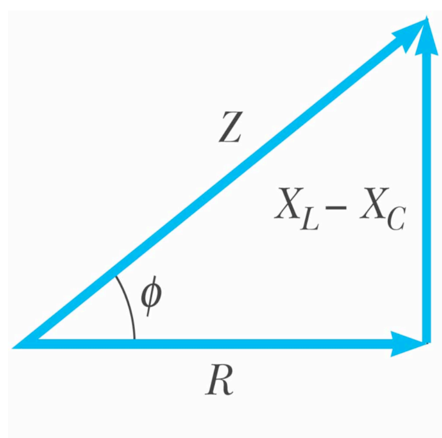




➔ **Whiteboard**

Impedance Triangle

- Since I_{max} is the same for each element, it can be removed from each term in the phasor diagram
- The result is an impedance triangle



Summary of Circuit Elements, Impedance and Phase Angles

Table 33.1

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
R	R	0°
C	X_C	− 90°
L	X_L	+ 90°
R C	$\sqrt{R^2 + X_C^2}$	Negative, between − 90° and 0°
R L	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

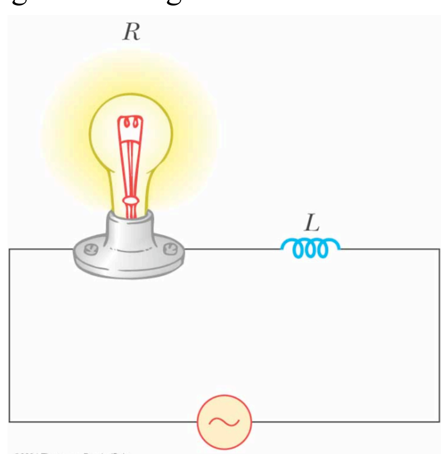
Quick Quiz 33.4

Consider the AC circuit in the figure below. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at

(a) high frequencies

(b) low frequencies

(c) The brightness will be the same at all frequencies.



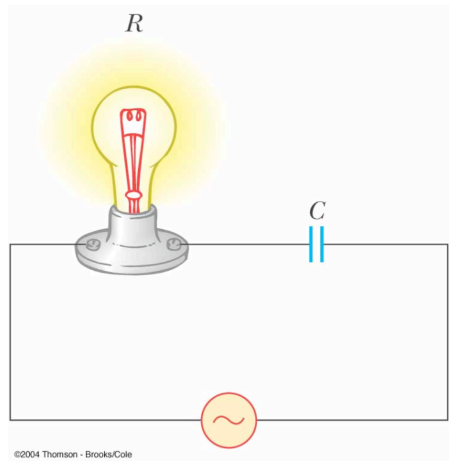
Quick Quiz 33.5

Consider the AC circuit in the figure below. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at

(a) high frequencies

(b) low frequencies

(c) The brightness will be same at all frequencies.



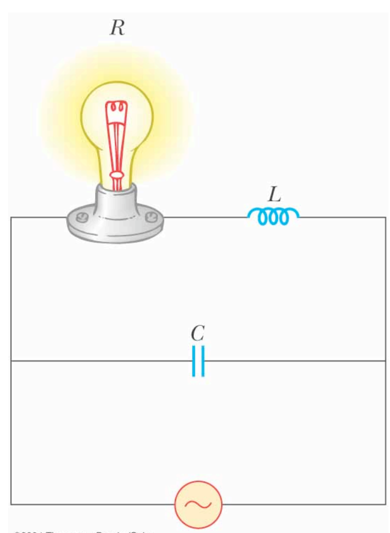
Quick Quiz 33.6

Consider the AC circuit in this figure. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at

(a) high frequencies

(b) low frequencies

(c) The brightness will be same at all frequencies.



Resonance in an AC RLC series Circuit

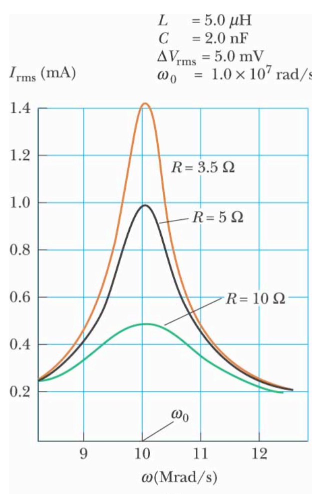
- *Resonance* occurs at the frequency ω_o where the current has its maximum value
  - To achieve maximum current, the impedance must have a minimum value
  - This occurs when $X_L = X_C$
  - Solving for the frequency gi

$$\omega_o = \frac{1}{\sqrt{LC}}$$

- The **resonance frequency** also corresponds to the natural frequency of oscillation of an *LC* circuit

Resonance, cont.

- Resonance occurs at the same frequency regardless of the value of *R*
- As *R* decreases, the curve becomes narrower and taller
- Theoretically, if *R* = 0 the current would be infinite at resonance
  - Real circuits always have some resistance

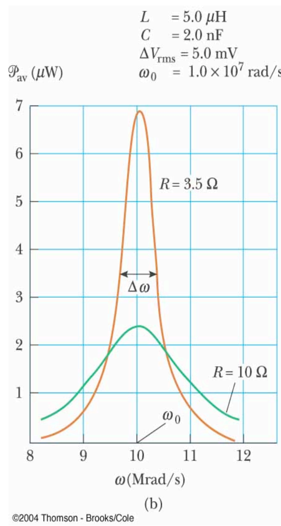


Power as a Function of Frequency

- Power can be expressed as a function of frequency in an *RLC* circuit

$$P_{av} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_o^2)^2}$$

- the average power is a maximum

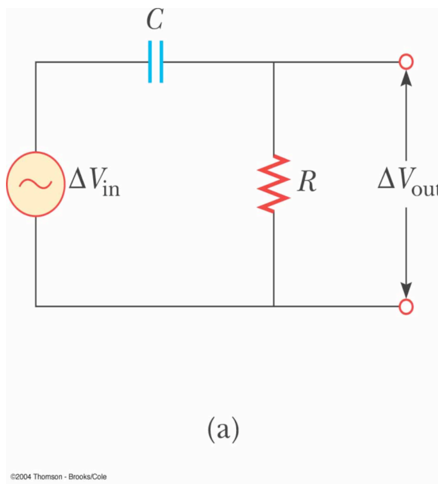


Filter Circuit, Example

- A **filter circuit** is one used to smooth out or eliminate a time-varying signal
- After rectification, a signal may still contain a small AC component
  - This component is often called a *ripple*
- By filtering, the ripple can be reduced
- Filters can also be built to respond differently to different frequencies

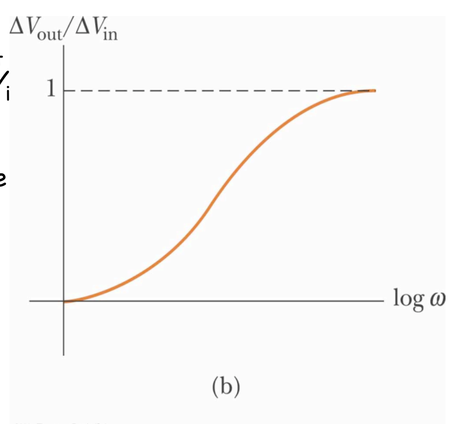
High-Pass Filter

- The circuit shown is one example of a **high-pass filter**
- A high-pass filter is designed to preferentially pass signals of higher frequency and block lower frequency signals

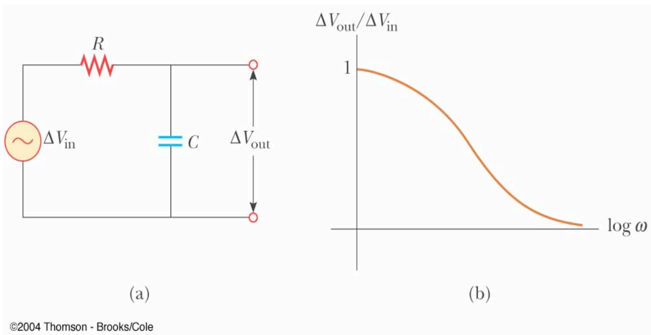


High-Pass Filter, cont

- At low frequencies, ΔV_{out} is much smaller than ΔV_i
  - At low frequencies, the capacitor has high reactance and much of the applied voltage appears across the capacitor
- At high frequencies, the two voltages are equal
  - At high frequencies, the capacitive reactance is small and the voltage appears across the resistor



Low-Pass Filter

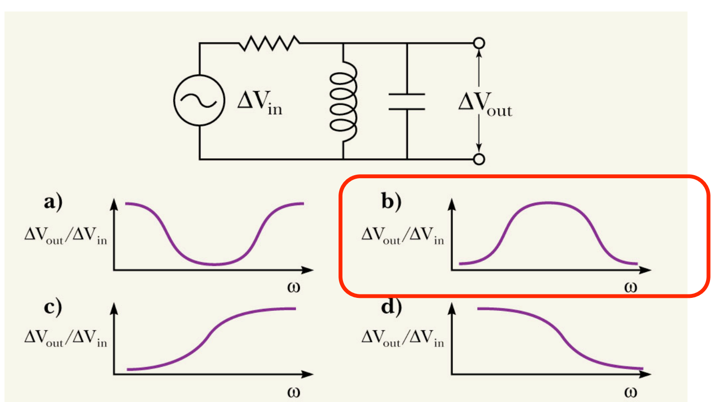


- At low frequencies, the reactance and voltage across the capacitor are high
- As the frequency increases, the reactance and voltage decrease
- This is an example of a low-pass filter

QUICK QUIZ 33.5

(For the end of section 33.9)

For the circuit below, which graph best represents the ratio of output voltage to input voltage as a function of frequency?



Quick Quiz 33.9

The impedance of a series *RLC* circuit at resonance is

(a) larger than R

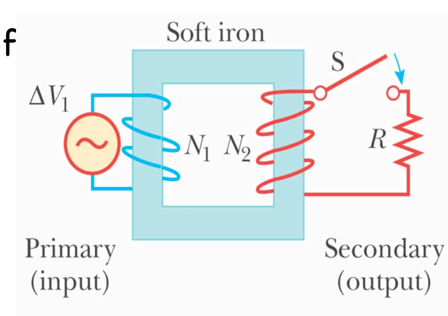
(b) less than R

(c) equal to R

(d) impossible to determine

Transformers

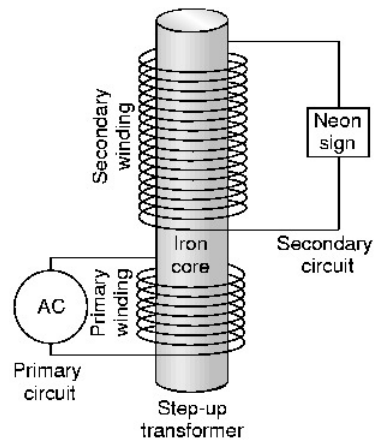
- An **AC transformer** consists of two coils of wire wound around a core of soft iron
- The side connected to the input AC voltage source is called the *primary* and has N_1 turns



Step-up transformer

Example: Neon sign:

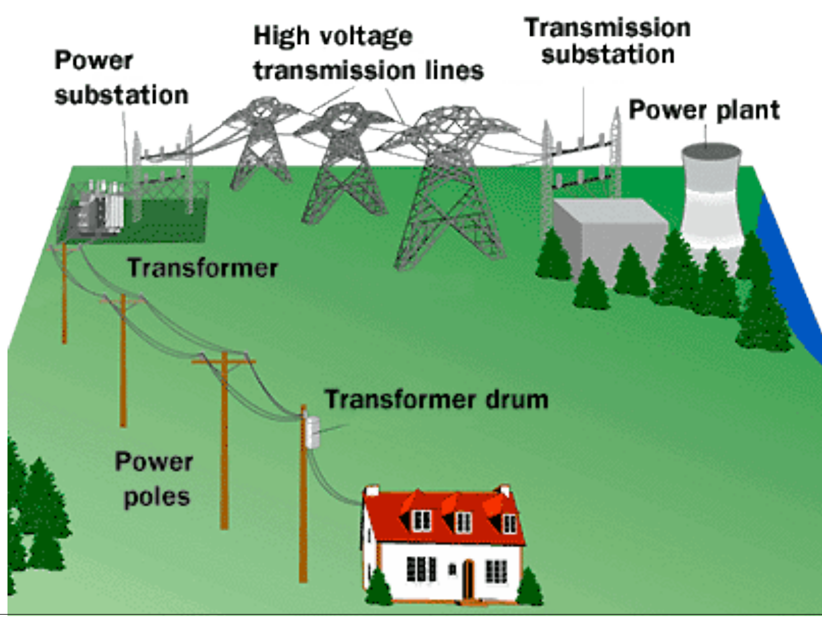
- Needs 10000V AC
- Supply: 240V
- What is the ratio of N_2/N_1?

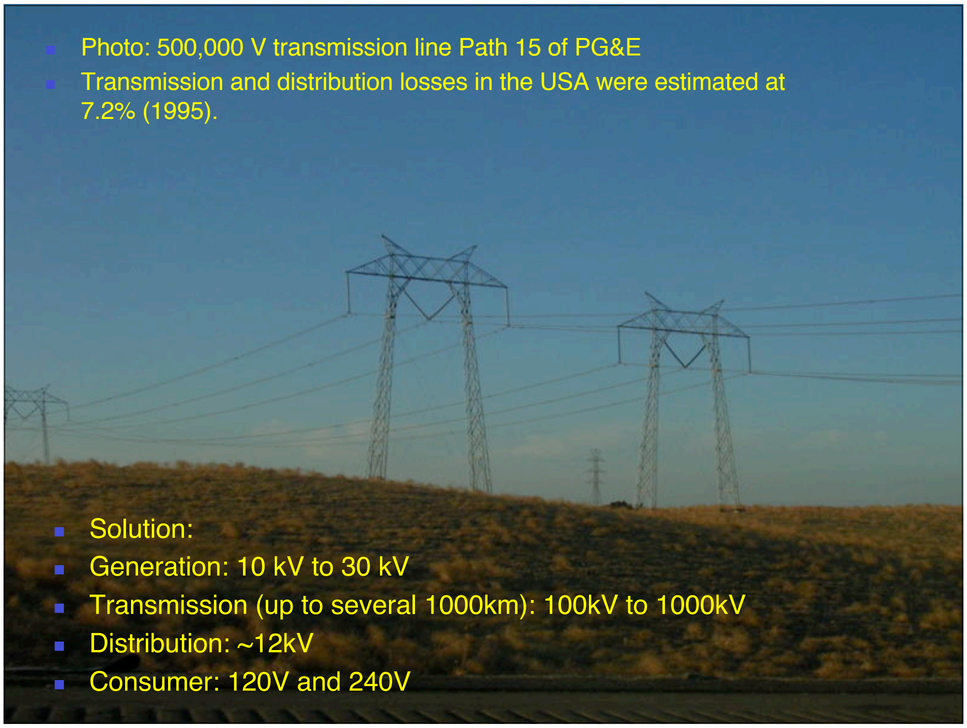


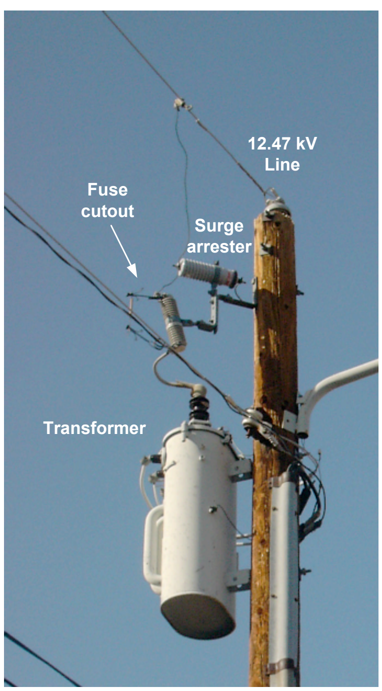
Ratio of turns:

$N_2/N_1 = 10000/240 = 41.6$

Power distribution







Consumer Service Drop