Image formation

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Physics 202
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Chapter 36

- Mirrors
- Images
- Ray diagrams
- Lenses

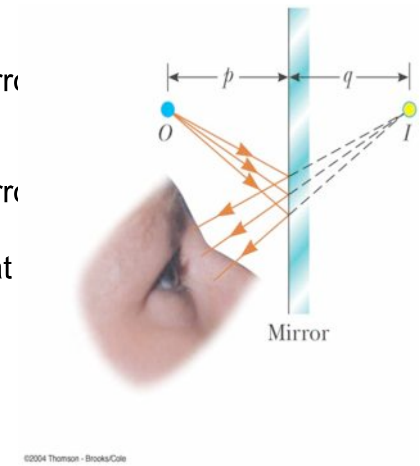
As usual, these notes are only a complement to the notes on the whiteboard.

Types of Images

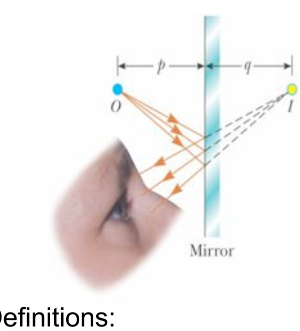
- A *real image* is formed when light rays pass through and diverge from the image point
  - Real images can be displayed on screens
- A *virtual image* is formed when light rays do not pass through the image point but only appear to diverge from that point
  - Virtual images cannot be displayed on screens

Images Formed by Flat Mirrors

- Simplest possible mirr
- Light rays leave the source and are reflected from the mirr
- Point I is called the **image** of the object at point O
- The image is virtual

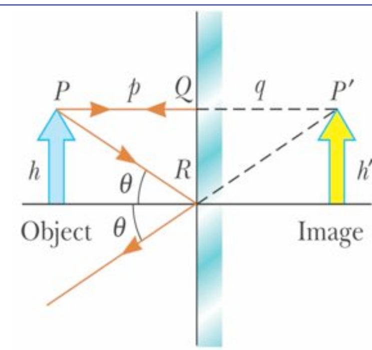


Images Formed by Flat Mirrors



Definitions:

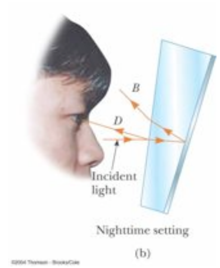
- The **object distance**
  - Denoted by p
- The **image distance**
  - Denoted by q
- The **lateral magnification**
  - Denoted by $M=h'/h$



Flat mirror:

- $-|p|=|q|$
- $M = 1$
- The image is virtual
- The image is upright

Application – Day and Night Settings on Auto Mirrors

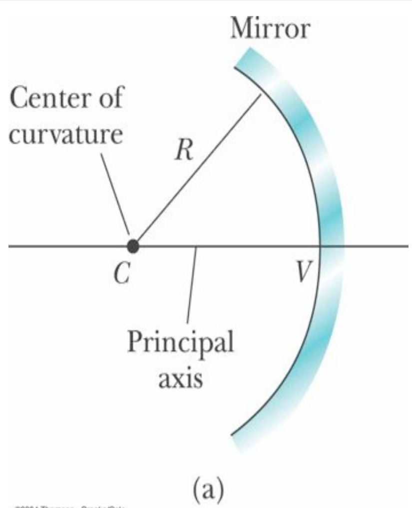


- With the daytime setting, the bright beam of reflected light is directed into the driver's eyes
- With the nighttime setting, the dim beam of reflected light is directed into the driver's eyes, while the bright beam goes elsewhere

Spherical mirrors: Concave Mirror

Notation

- Radius *of curvature*: *R*
- *Center of curvature*: *C*
- A line drawn from *C* to *V*: *principal axis* of the mirror



Spherical Aberration

- Rays that are far from the principal axis converge to other points on the principal axis
- This produces a blurred image
- The effect is called **spherical aberration**

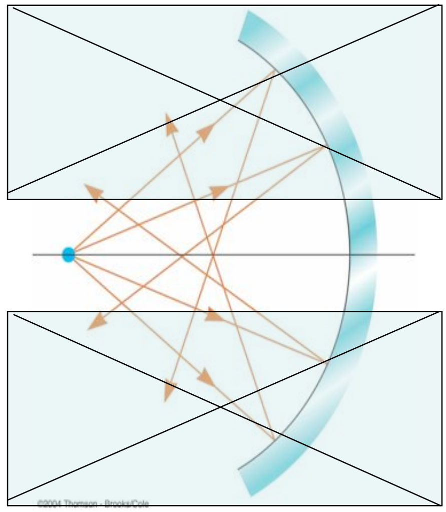


Image Formed by a Concave Mirror

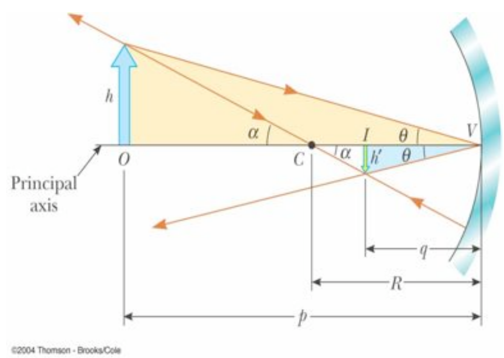
Geometrical analysis shows:

1. *magnification:*

$$M = \frac{h'}{h} = -\frac{q}{p}$$

2. *mirror equation:*

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

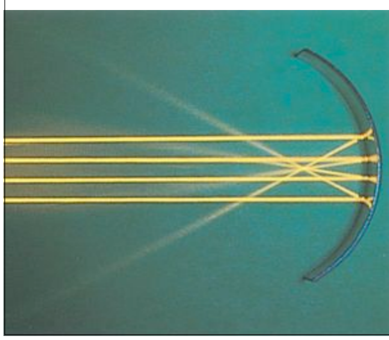


Special case: $p \to \infty$; $q \to R/2$

Focal Length

- Object very far away: $p \to \infty$ incoming rays are essentially parallel
  - image point = **focal point = F**
  - distance from mirror to focal point = **focal length = f**
  - f = R/2

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



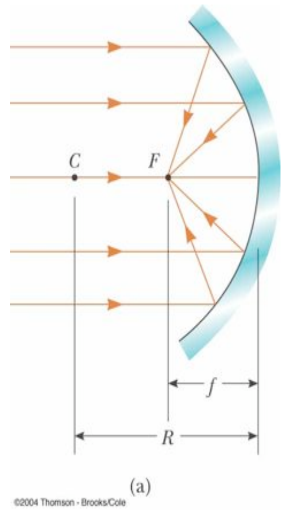
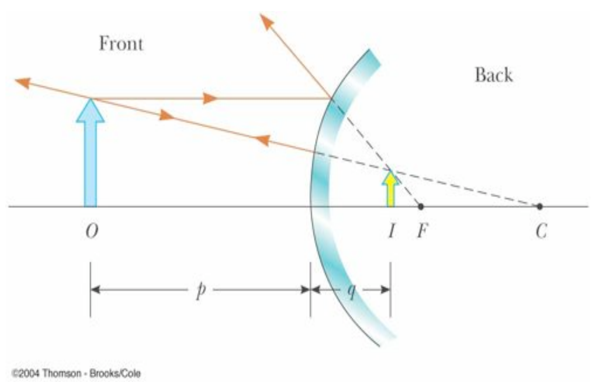


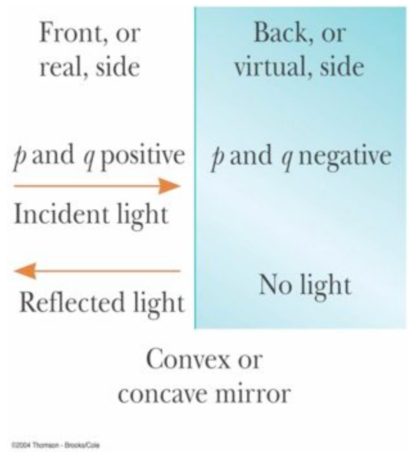
Image Formed by a Convex Mirror



- In general, the image formed by a convex mirror is upright, virtual, and smaller than the object

Sign Conventions

- These sign conventions apply to both concave and convex mirrors
- The equations used for the concave mirror also apply to the convex mirror



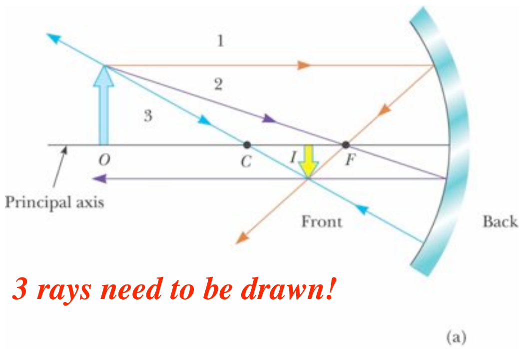
Sign Conventions, Summary Table

Table 36.1

Sign Conventions for Mirrors

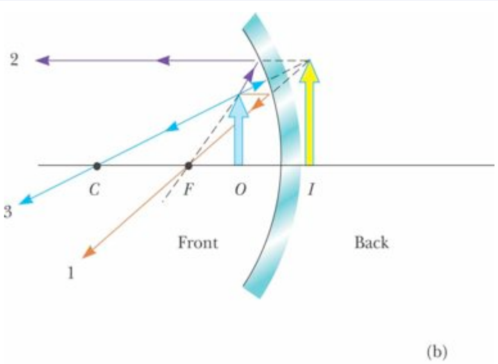
Quantity	Positive When	Negative When
Object location (p)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

Ray Diagram for a Concave Mirror, $p > R$



- The center of curvature is between the object and the concave mirror surface
- The image
  - real
  - inverted
  - smaller than the object (reduced)

Ray Diagram for a Concave Mirror, $p < f$



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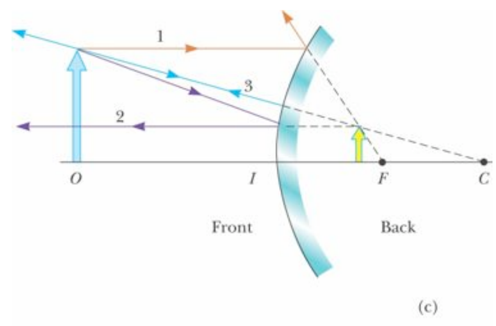
- The object is between the mirror surface and the focal point
- The image is
  - virtual
  - upright
  - is larger than the object (enlarged)

Ray Diagram for a Convex Mirror



- The object is in front of a convex mirror
- The image is
  - virtual
  - upright
  - smaller than the object (reduced)

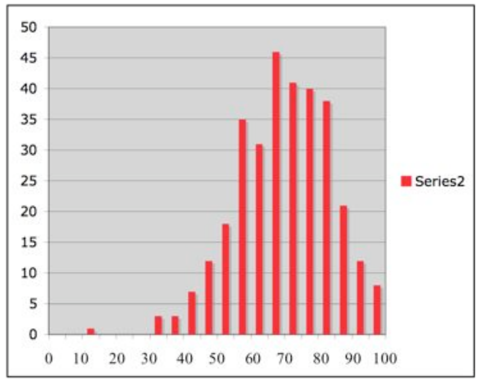
Notes on Images

- With a concave mirror, the image may be either real or virtual
- With a convex mirror, the image is always virtual and upright
- An image is real when the rays are passing through, virtual when this is not the case.

Midterm 3 results

Statistics:

- Minimum 12
- Maximum 99.5
- Points 316
- Mean 69.3
- Median 70.0
- Std Deviation 13.8

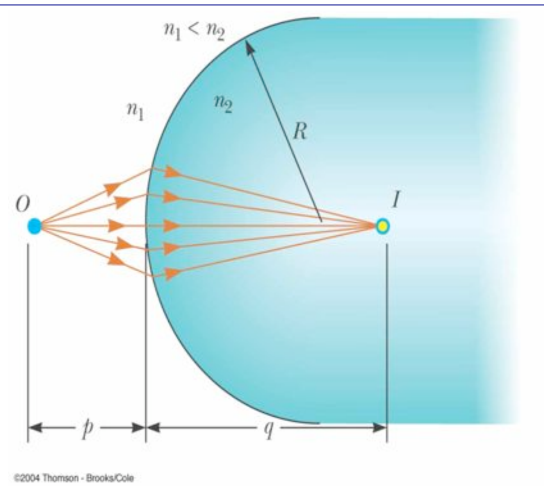


Key solutions to be posted by end of week.

Images Formed by Refraction

- Consider two transparent media having indices of refraction n_1 and n_2
- Analysis of geometry yields:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

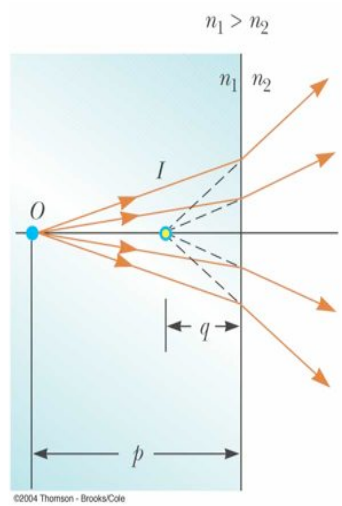


Flat Refracting Surfaces

- If a refracting surface is flat, then R is infinite

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

- Then $q = -(n_2 / n_1)p$
  - The image formed by a flat refracting surface is on the same side of the surface as the object
- A virtual image is formed



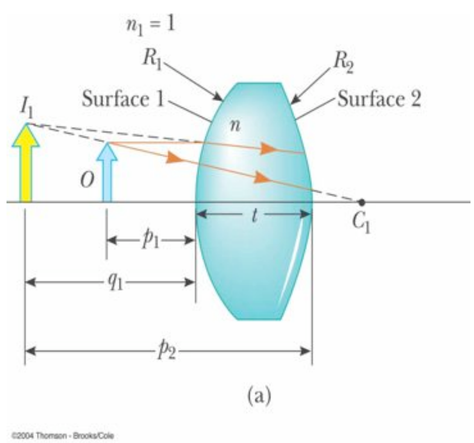
Lenses

- Lenses are commonly used to form images by refraction
  - All is a result of Snell's law and can be derived from that.
- Lenses are used in optical instruments
  - Cameras
  - Telescopes
  - Microscopes

Locating the Image Formed by a Lens

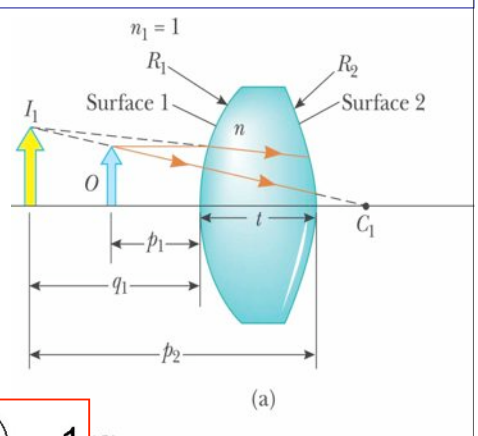
- The lens has an index of refraction n and two spherical surfaces with radii of R_1 and R_2
  - R_1 is the radius of curvature of the lens surface that the light of the object reaches first
  - R_2 is the radius of curvature of the other surface
- The object is placed at point O at a distance of p_1 in front of the first surface
- Now, after
  - Some algebra
  - And the assumption t << R

we get the lensmakers equation….



Lens Makers' Equation

- The focal length of a thin lens is the image distance that corresponds to an infinite object distance
  - This is the same as for a mirror
- The **lens makers' equation** is



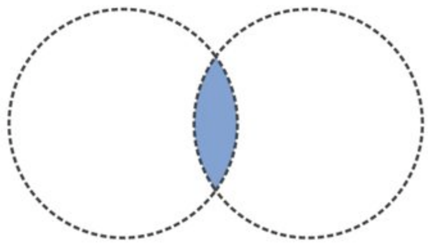
$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

QUICK QUIZ 36.2

(For the end of section 36.4)

A certain biconvex thin lens is designed by drawing two circles of the same radius next to one another and using the overlapping region as the area to be used for the lens. If the radius of each circle is *r*, and the index of refraction of the glass for the lens is 1.5, the focal length of the lens (when used in air) will be

a) *r*/1.5,
b) 0.75*r*,
c) *r*,
d) 2*r*,
e) infinity.



$$\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{r}-\frac{1}{(-r)}\right)=(0.5)\left(\frac{2}{r}\right)=\frac{1}{r}.$$

Thin Lens Equation

- Thin Lens equation (same as for mirror):

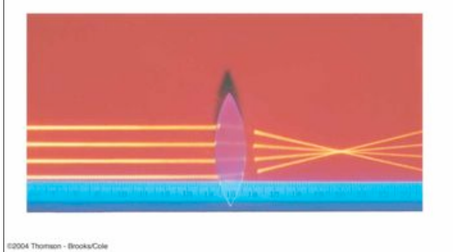
$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

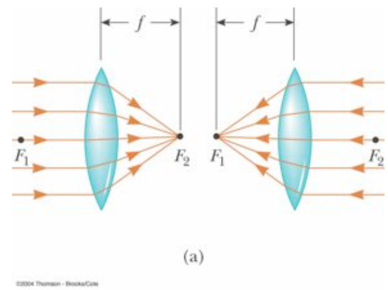
- Magnification:

$$M=\frac{h'}{h}=-\frac{q}{p}$$

  - When *M* is positive, the image is upright and on the same side of the lens as the object
  - When *M* is negative, the image is inverted and on the side of the lens opposite the object

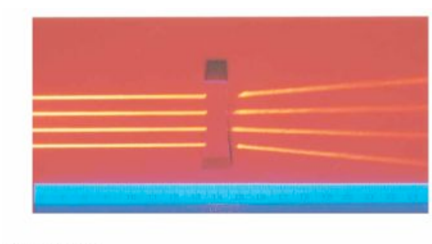
Focal Length of a Converging Lens

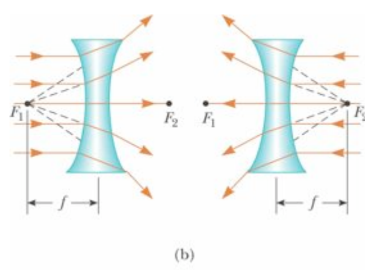




- The parallel rays pass through the lens and converge at the focal point
- The parallel rays can come from the left or right of the lens

Focal Length of a Diverging Lens





- The parallel rays diverge after passing through the diverging lens
- The focal point is the point where the rays appear to have originated

Sign Conventions for Thin Lenses

Table 36.3

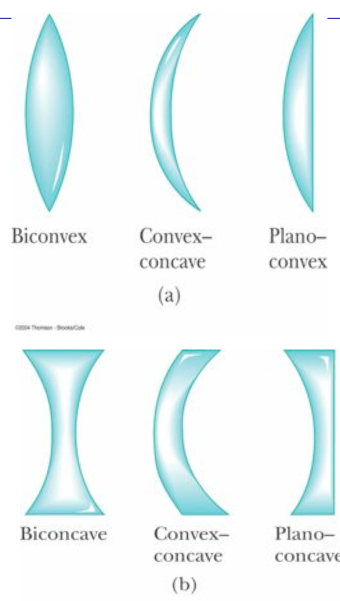
Sign Conventions for Thin Lenses

Quantity	Positive When	Negative When
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

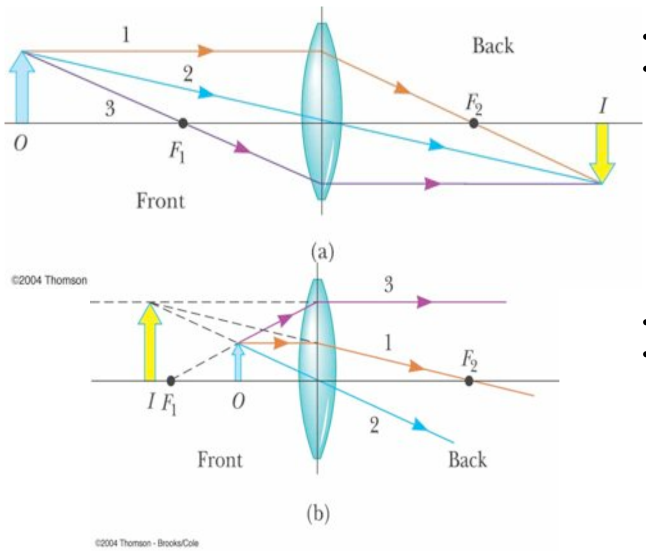
Thin Lens Shapes

- *Converging* lenses
- They have positive focal lengths

- *Diverging* lenses
- They have negative focal lengths



Ray Diagram for Converging Lens

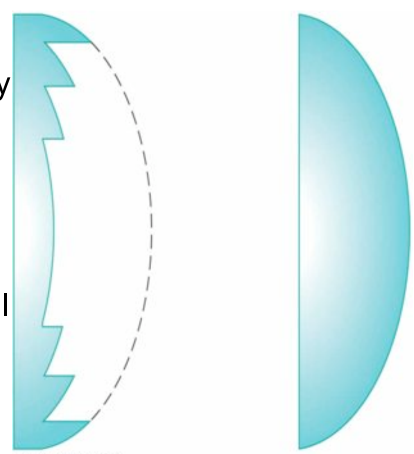


- *p > f* :
- The image is
  - real
  - inverted
  - on the back side of the lens

- p < f:
- The image is
  - virtual
  - upright
  - larger than the object
  - on the front side of the lens

Fresnel Lens

- Refraction occurs only at the surfaces of the lens
- A *Fresnal lens* is designed to take advantage of this fact
- It produces a powerful lens without great thickness

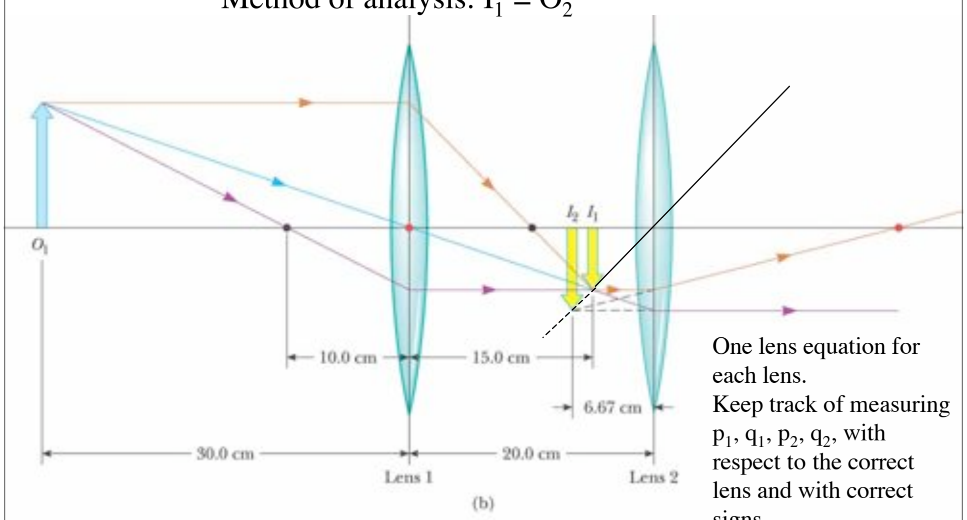


Fresnel Lens, cont.

- Only the surface curvature is important in the refracting qualities of the lens
- The material in the middle of the Fresnal lens is removed
- Because the edges of the curved segments cause some distortion, Fresnal lenses are usually used only in situations where image quality is less important than reduction of weight

Combination of Thin Lenses

Method of analysis: $I_1 = O_2$



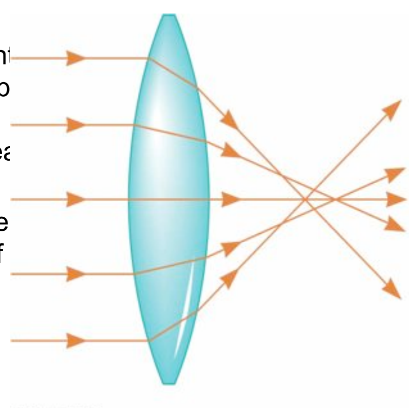
One lens equation for each lens.
Keep track of measuring p_1, q_1, p_2, q_2, with respect to the correct lens and with correct signs.

Lens Aberrations

- Assumptions have been:
  - Rays make small angles with the principal axis
  - The lenses are thin
- The rays from a point object do not focus at a single point
  - The result is a blurred image
- The departures of actual images from the ideal predicted by our model are called **aberrations**

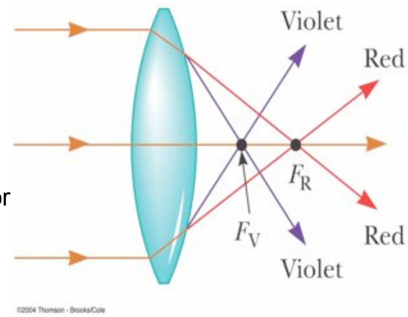
Spherical Aberration

- This results from the focal poin of light rays far from the princip axis being different from the focal points of rays passing nea the axis
- For a camera, a small aperture allows a greater percentage of the rays to be paraxial
- For a mirror, parabolic shapes can be used to correct for spherical aberration



Chromatic Aberration

- Different wavelengths of light refracted by a lens focus at different points
  - Violet rays are refracted more than red rays
  - The focal length for red light is greater than the focal length for violet light
- Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses made of different materials

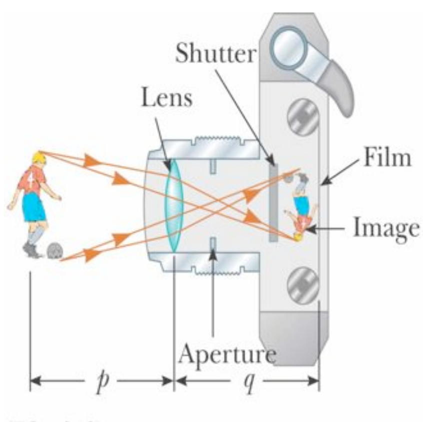


The Camera

- The photographic camera is a simple optical instrument
- Components
  - Light-tight chamber
  - Converging lens
    - Produces a real image
  - Film behind the lens
    - Receives the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

- The lens, therefore f is constant:
- Adjust q to change p.

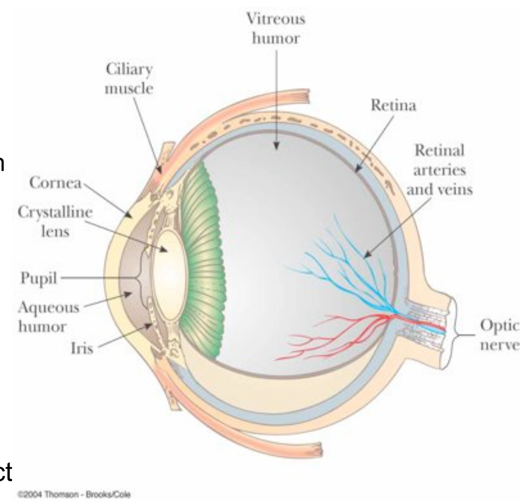


The Eye

- The normal eye focuses light and produces a sharp image
- Essential parts of the eye:
  - Cornea – light passes through this transparent structure
  - Aqueous Humor – clear liquid behind the cornea

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

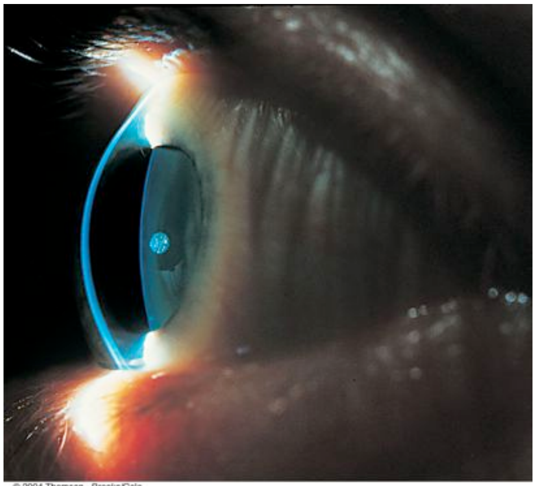
- q constant
- Adjust the focal length f of the lens to focus on different object distances p



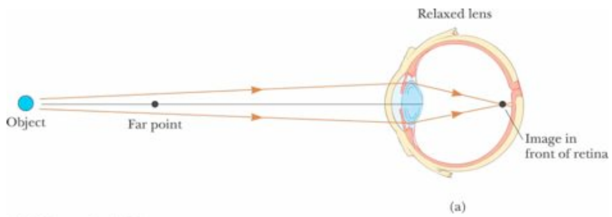
The Eye – Parts, cont.

- The pupil
  - A variable aperture
  - An opening in the iris
- The crystalline lens
- Most of the refraction takes place at the outer surface of the eye
  - Where the cornea is covered with a film of tears

The Eye – Close-up of the Cornea

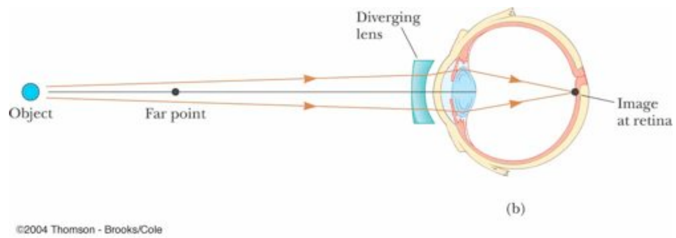


Nearsightedness



- Also called *myopia*
- The far point of the nearsighted person is not infinity and may be less than one meter
- The nearsighted person can focus on nearby objects but not those far away

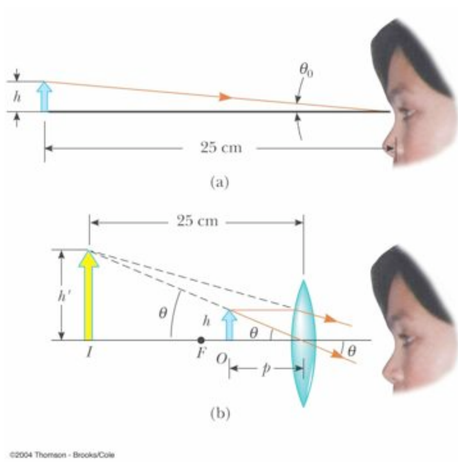
Correcting Nearsightedness



- A diverging lens can be used to correct the condition
- The lens refracts the rays away from the principal axis before they enter the eye
  - This allows the rays to focus on the retina

The Size of a Magnified Image

- When an object is placed at the near point, the angle subtended is a maximum
  - The near point is about 25 cm
- When the object is placed near the focal point of a converging lens, the lens forms a virtual, upright, and enlarged image



Angular Magnification

- **Angular magnification** is defined as

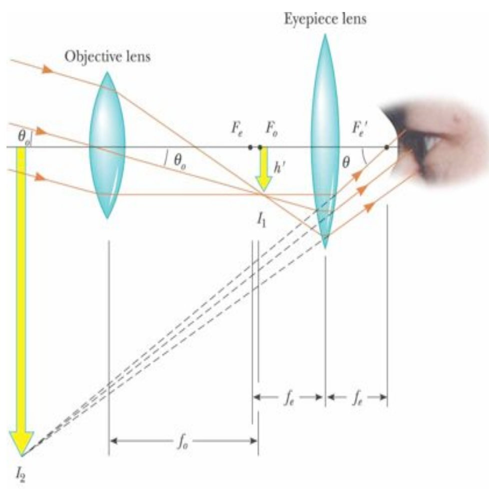
$$m \equiv \frac{\theta}{\theta_o} = \frac{\text{angle with lens}}{\text{angle without lens}}$$

- The angular magnification is at a maximum when the image formed by the lens is at the near point of the eye
  - $q = -25$ cm
  - Calculated by

$$m_{\max} = 1 + \frac{25 \text{ cm}}{q}$$

Refracting Telescope

- The two lenses are arranged so that the objective forms a real, inverted image of a distant object
- The image is near the focal point of the eyepiece
- The two lenses are separated by the distance $f_o + f_e$ which corresponds to the length of the tube
- The eyepiece forms an enlarged, inverted image of the first image



Angular Magnification of a Telescope

- The angular magnification depends on the focal lengths of the objective and eyepiece

$$m = \frac{\theta}{\theta_o} = -\frac{f_o}{f_e}$$

  - The negative sign indicates the image is inverted
- Angular magnification is particularly important for observing nearby objects
  - Nearby objects would include the sun or the moon
  - Very distant objects still appear as a small point of light

Reflecting Telescope, Newtonian Focus

- The incoming rays are reflected from the mirror and converge toward point *A*
  - At *A*, an image would be formed
- A small flat mirror, M, reflects the light toward an opening in the side and it passes into an eyepiece
  - This occurs before the image is formed at *A*