Physics 202 - Fall 2007
Homework Set 12

Problem 1:

In

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$p^{-1}+q^{-1}=\text{constant},$

we differentiate with respect to time

$$-1\left(p^{-2}\right)\frac{dp}{dt}-1\left(q^{-2}\right)\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}.$$

We must find the momentary image location q:

$$\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$$

$q=0.305\text{ m}.$

Now $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.001\,16\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}.$

Problem 2:

Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{2.00\text{ m}}-\frac{1}{2.50\text{ cm}}\right]=0.050\,0\text{ cm}^{-1}$$

so $f=\boxed{20.0\text{ cm}}.$

Problem 3:

In the first arrangement the lens is used as a magnifying glass, producing an upright, virtual enlarged image:

$$M=\frac{h'}{h}=\frac{120\text{ cm}}{3.6\text{ cm}}=33.3=-\frac{q}{p}$$

$q=-33.3p=-33.3(20\text{ cm})=-667\text{ cm}$

For the lens,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{20\text{ cm}}+\frac{1}{-667\text{ cm}}=\frac{1}{f}$$

$f=20.62\text{ cm}$

In the second arrangement the lens us used as a projection lens to produce a real inverted enlarged image:

$$-\frac{120\text{ cm}}{3.6\text{ cm}}=-33.3=-\frac{q_2}{p_2}\qquad q_2=33.3p_2$$

$$\frac{1}{p_2}+\frac{1}{33.3p_2}=\frac{1}{20.62\text{ cm}}\qquad \frac{34.3}{33.3p_2}=\frac{1}{20.62\text{ cm}}\qquad p_2=21.24\text{ cm}$$

The lens was moved $21.24\text{ cm} - 20.0\text{ cm} = \boxed{1.24\text{ cm}}$.

Problem 4:

(a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^2$$

$$350\text{ W} = \left(1\,000\text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334\text{ m or larger}}$.

(b) In $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{2}{R}$

we have $p \to \infty$

so $q = \dfrac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q\left(\dfrac{h}{p}\right) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where $\dfrac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \dfrac{\mathcal{P}}{\pi h'^2/4} = \dfrac{4 I_0 \pi R_a^2}{\pi h'^2} = \dfrac{4 I_0 R_a^2}{(R/2)^2\left(9.30\times 10^{-3}\text{ rad}\right)^2}$

$$120\times 10^3\text{ W/m}^2 = \frac{16\left(1\,000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times 10^{-3}\text{ rad}\right)^2}$$

so $\boxed{\dfrac{R_a}{R} = 0.025\,5\text{ or larger}}$.

Problem 5:

Underwater, the wavelength of the light would decrease, $\lambda_{\text{water}} = \dfrac{\lambda_{\text{air}}}{n_{\text{water}}}$. Since the positions of light and dark bands are proportional to λ, (according to Equations 37.2 and 37.3), the underwater fringe separations will decrease.

Problem 6:

The threads that are woven together to make the cloth have small meshes between them. These bits of space act as pinholes through which the light diffracts. Since the cloth is a grid of such pinholes, an interference pattern is formed, as when you look through a diffraction grating.

Problem 7:

P37.3 Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

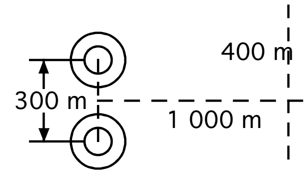

FIG. P37.3

(a) At the $m = 2$ maximum, $\tan\theta = \frac{400\text{ m}}{1\,000\text{ m}} = 0.400$

$$\theta = 21.8°$$

so

$$\lambda = \frac{d\sin\theta}{m} = \frac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$$

.

(b) The next minimum encountered is the $m = 2$ minimum;

and at that point, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

which becomes $d\sin\theta = \frac{5}{2}\lambda$

or $\sin\theta = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{55.7\text{ m}}{300\text{ m}}\right) = 0.464$

and $\theta = 27.7°$

so $y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}$.

Therefore, the car must travel an additional $\boxed{124\text{ m}}$.

If we considered Fresnel interference, we would more precisely find (a) $\lambda = \frac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2\text{ m}$ and (b) 123 m.

Problem 8:

Location of A = central maximum,

Location of B = first minimum.

So, $\Delta y = [y_{\min} - y_{\max}] = \frac{\lambda L}{d}\left(0 + \frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}$.

Thus, $d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$.

Problem 9:

P37.7 (a) For the bright fringe,

$$y_{\text{bright}} = \frac{m\lambda L}{d} \quad \text{where } m = 1$$

$$y = \frac{\left(546.1\times 10^{-9}\text{ m}\right)\left(1.20\text{ m}\right)}{0.250\times 10^{-3}\text{ m}} = 2.62\times 10^{-3}\text{ m} = \boxed{2.62\text{ mm}}.$$

(b) For the dark bands, $y_{\text{dark}} = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right);\ m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1)$$

$$= \frac{\left(546.1\times 10^{-9}\text{ m}\right)\left(1.20\text{ m}\right)}{0.250\times 10^{-3}\text{ m}}$$

$$\Delta y = \boxed{2.62\text{ mm}}.$$

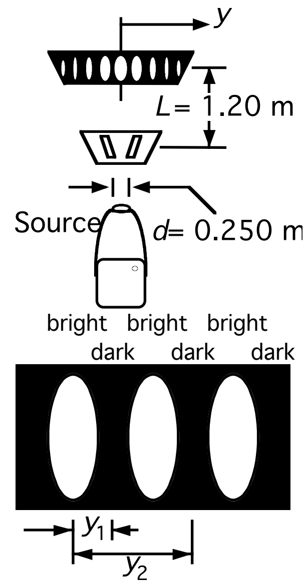

FIG. P37.7

Problem 10:

(a) From Equation 37.8,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2+D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times 10^{-3}\text{ m}\right)\left(2.50\times 10^{-3}\text{ m}\right)}{\left(600\times 10^{-9}\text{ m}\right)\left(2.80\text{ m}\right)} = \boxed{7.95\text{ rad}}$$

(b)
$$\frac{I}{I_{\max}} = \frac{\cos^2\left[(\pi d/\lambda)\sin\theta\right]}{\cos^2\left[(\pi d/\lambda)\sin\theta_{\max}\right]} = \frac{\cos^2(\phi/2)}{\cos^2 m\pi}$$

$$\frac{I}{I_{\max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

Problem 11:

If the path length difference $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}.$$

Problem 12:

P37.38 The condition for bright fringes is

$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1,\ 2,\ 3,\ \ldots.$$

From the sketch, observe that

$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}.$$

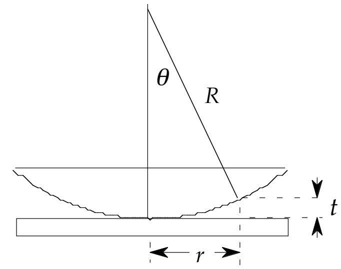

FIG. P37.38

The condition for a bright fringe becomes $\frac{r^2}{R} = \left(m - \frac{1}{2}\right)\frac{\lambda}{n}$.

Thus, for fixed m and λ, $nr^2 = \text{constant}$.

Therefore, $n_{\text{liquid}}r_f^2 = n_{\text{air}}r_i^2$ and $n_{\text{liquid}} = (1.00)\frac{(1.50\text{ cm})^2}{(1.31\text{ cm})^2} = \boxed{1.31}$.

Problem 13:

P37.61 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is

$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n - 1).$$

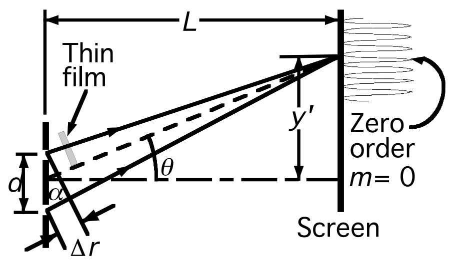

FIG. P37.61

The corresponding difference in **path length** Δr is

$$\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n - 1)\left(\frac{\lambda_a}{2\pi}\right) = t(n - 1).$$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$.

Eliminating Δr by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$.

Problem 14:

The positions of the first-order minima are $\frac{y}{L} \approx \sin\theta = \pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\frac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10 \times 10^{-3}\text{ m}}{2}\right)\left(\frac{0.550 \times 10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}.$$

Problem 15:

$$1.22\frac{\lambda}{D} = \frac{d}{L} \qquad\qquad \lambda = \frac{c}{f} = 0.020\,0\text{ m}$$

$$D = 2.10\text{ m} \qquad\qquad L = 9\,000\text{ m}$$

$$d = 1.22\frac{(0.020\,0\text{ m})(9\,000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$$