Homework # 2 Solutions

**1.** Gauss's law dictates that the net flux through any closed surface is given by the charge enclosed divided by $\epsilon_0$ ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N\,m}^2$). The charge enclosed by the sphere of radius $R < a$ is just the point charge $q$, so the total electric flux is $q/\epsilon_0$.

**2.** Using Gauss's law, the electric flux $\Phi_{S_i}$ through the surfaces are:

$$\Phi_{S_1} = (-2Q + Q)/\epsilon_0 = -Q/\epsilon_0, \quad \Phi_{S_3} = (-2Q + Q - Q)/\epsilon_0 = -2Q/\epsilon_0, \quad \Phi_{S_2} = \Phi_{S_4} = 0.$$

**3.** The electric field of a uniform solid sphere of radius $R$ with charge $Q$ distributed throughout its volume is

$$\begin{aligned} \vec{E}(r) &= \frac{k_e Q r}{R^3}\hat{r}, \quad (r < R) \\ \vec{E}(r) &= \frac{k_e Q}{r^2}\hat{r}. \quad (r > R). \end{aligned}$$

Here $R = 0.4$ m. The field values are as follows:

$$\begin{aligned} &(a) \quad E(r=0) = 0 \\ &(b) \quad E(r = 0.10\,\mathrm{m}) = \frac{k_e Q (0.10\ \mathrm{m})}{(0.4\ \mathrm{m})^3} \\ &(c) \quad E(r = R) = \frac{k_e Q}{R^2} = \frac{k_e Q}{(0.4\ \mathrm{m})^2} \\ &(d) \quad E(r = 0.6\,\mathrm{m}) = \frac{k_e Q}{(0.6\,\mathrm{m})^2}. \end{aligned}$$

**4.** (a) Begin by defining a linear surface charge density $\lambda = Q/L$, where $L$ is the length of the cylinder and $Q$ is the net charge on the shell. Since $L$ is much larger than the field point $r$ at which we know the electric field, the the length of the cylinder can be approximated as infinite. Gauss's law can be used to obtain the electric field; the electric flux through a Gaussian surface (cylinder) of arbitrary length $l$ enclosing the cylindrical shell is:

$$\int \vec{E} \cdot d\vec{A} = E(r) 2\pi r l = \frac{q_{enclosed}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}. \tag{1}$$

This gives

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}, \tag{2}$$

and thus

$$Q = \lambda L = 2\pi\epsilon_0 r E(r) L. \tag{3}$$

(b) To determine the electric field at a value $r < R$, draw a Gaussian surface at $r$. There is no charge enclosed by this surface, so the electric field is zero in this region.

**5.** The electric field just outside the spherical shell (inner radius $r_i$, outer radius $r_0$) can be determined by Gauss's law:

$$\int \vec{E} \cdot d\vec{A} = E(r) 4\pi r^2 = \frac{q_{enclosed}}{\epsilon_0} = \frac{Q_{sph} + q}{\epsilon_0}, \tag{4}$$

such that

$$\vec{E}(r) = k_e \frac{Q_{sph} + q}{r^2} \hat{r}. \tag{5}$$

In the above,

$$Q_{sph} = \rho V_{sph} = \rho \frac{4}{3} \pi (r_o^3 - r_i^3),$$

$\rho$ is the volume charge density, and $q$ is the charge at the center of the shell. Since both $q$ and $Q_{sph}$ are negative, the force on the orbiting proton provides a centripetal acceleration:

$$F = q_p E(r) = q_p k_e \frac{(|Q_{sph}| + |q|)}{r^2} = \frac{m_p v^2}{r}. \tag{6}$$

The speed of the proton's orbit is

$$v = \sqrt{\frac{q_p k_e (|Q_{sph}| + |q|)}{m_p r_0}}. \tag{7}$$

**6.** The middle of a large uniformly charged sheet can be approximated as an infinite uniform sheet of charge. Upon drawing a pillbox with cross-sectional area A, Gauss's law gives

$$\int \vec{E} \cdot d\vec{A} = EA = \frac{q_{enclosed}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}, \tag{8}$$

such that

$$E = \frac{\sigma}{2\epsilon_0}. \tag{9}$$

It points upward (*i.e.*, away from the charged sheet) since the surface charge density $\sigma > 0$.

**7.** (a) To obtain the charge per unit length on the inner surface of the cylinder, first recall that inside a conductor, the electric field must be zero. By Gauss's law,

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}. \tag{10}$$

The charge enclosed by a Gaussian cylinder of length $l$ with a radius just inside the conductor must then be zero:

$$\int \vec{E} \cdot d\vec{A} = 0 = \frac{(\lambda l + \lambda_{inner} l)}{\epsilon_0}, \tag{11}$$

such that $\lambda_{inner} = -\lambda$.

(b) Since the total charge per unit length on the cylinder is $2\lambda$, the charge per unit length on the outer surface of the cylinder is given by $\lambda_{outer} = 2\lambda - \lambda_{inner} = 2\lambda - (-\lambda) = 3\lambda$.

(c) To obtain the E field outside the cylinder, draw a Gaussian cylinder of length $l$ at radius r. Using Gauss's Law,

$$\int \vec{E} \cdot d\vec{A} = E(r) 2\pi r l = \frac{q_{enclosed}}{\epsilon_0} = \frac{3\lambda l}{\epsilon_0}. \tag{12}$$

The electric field is then

$$E(r) = \frac{3\lambda}{2\pi\epsilon_0 r}. \tag{13}$$

**8.** The potential difference needed to stop an electron with initial speed $v_i$ is

$$\frac{1}{2}mv^2 = q_e \Delta V \tag{14}$$

$$\Delta V = -\frac{(9.11 \times 10^{-31}\,\mathrm{kg})v^2}{2(1.6 \times 10^{-19}\,\mathrm{C})}. \tag{15}$$

**9.** The electric field is given by

$$E = \frac{\Delta V}{d}. \tag{16}$$

**10.** To evaluate the potential difference, we need to determine the electric field and the height of the ball's trajectory. Given the initial velocity of the ball $v_0$ and the time for a round trip trajectory $t_r$, we can solve for the acceleration using the fact that at the top of the trajectory, $v = 0$:

$$v = 0 = v_0 + a\frac{t_r}{2}, \tag{17}$$

such that

$$a = -\frac{2v_0}{t_r}, \tag{18}$$

(the minus sign reflects that it is directed downward). The magnitude of the acceleration is related to the electric field as follows:

$$g + \frac{|q|E}{m} = |a| = \frac{2v_0}{t_r}, \tag{19}$$

where $g = 9.8m/s^2$. Solving this equation for $E$ yields

$$E = \frac{m}{|q|}\left(\frac{2v_0}{t_r} - g\right). \tag{20}$$

The distance to the top of the trajectory can be obtained by

$$\begin{aligned} \Delta y &= v_0\frac{t_r}{2} + \frac{1}{2}a\left(\frac{t_r}{2}\right)^2 & (21) \\ &= v_0\frac{t_r}{2} - \frac{1}{2}\frac{2v_0}{t_r}\left(\frac{t_r}{2}\right)^2 = \frac{1}{4}v_0 t_r. & (22) \end{aligned}$$

The potential difference is given by

$$\Delta V = -\int_{y_0}^{y} \vec{E} \cdot d\vec{s} = E\Delta y, \tag{23}$$

such that

$$\Delta V = \frac{m}{|q|}\left(\frac{2v_0}{t_r} - g\right)\left(\frac{1}{4}v_0 t_r\right). \tag{24}$$