Homework # 5 Solutions

**1.** (a) The magnetic force on the proton is

$$F = qvB\sin\theta$$

.

(b) Its acceleration is

$$a = \frac{F}{m_p} = \frac{qvB\sin\theta}{m_p}.$$

**2.** (a) The electron accelerates through a potential difference $\Delta V$, giving it kinetic energy

$$|q_e|\Delta V = \frac{1}{2}m_e v^2,$$

such that its speed $v$ is

$$v = \sqrt{\frac{2|q_e|\Delta V}{m_e}}.$$

The maximum value of the force is when the velocity is perpendicular to the B field, in which case

$$|F| = |q_e vB|.$$

(b) The minimum value of the force occurs when the velocity is parallel to the B field, in which case $F = 0$.

**3.** Given the velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$ and the magnetic field $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$, the magnetic force on the proton is

$$\vec{F} = q_p\vec{v}\times\vec{B} = (v_yB_z - v_zB_y)\hat{\imath} + (v_zB_x - v_xB_z)\hat{\jmath} + (v_xB_y - v_yB_x)\hat{k}.$$

The magnitude of the force is $|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$.

**4.** We are given that the velocity of the electron is $\vec{v} = v_e\hat{\imath}$, its acceleration is $\vec{a} = a_e\hat{k}$, and the electric field is $\vec{E} = E\hat{k}$. The components of the magnetic field can be determined using the Lorentz force law:

$$\vec{F} = m_e\vec{a} = q_e\vec{E} + q_e\vec{v}\times\vec{B},$$

which in this case takes the form

$$m_e a_e\hat{k} = q_eE\hat{k} + q_ev_e\hat{\imath}\times\vec{B}.$$

Since there is no acceleration in the $y$ direction, we can conclude that $B_z = 0$. Furthermore, $B_x$ is undetermined since it has a vanishing contribution to the magnetic force (since $B_x$ is parallel to the velocity). The $y$ component of the magnetic field will provide a magnetic force in the $z$ direction. The Lorentz force law becomes

$$m_e a_e = q_e E + q_e v_e B_y,$$

such that the $y$ component of the magnetic field is

$$B_y = \frac{1}{v_e}\left(-\frac{m_e a_e}{|q_e|} - E\right).$$

**5.** The rod experiences a magnetic force $F = IBd$ directed toward the right. The work done as it rolls a distance $L$ is equal to the change in its kinetic energy. Since the rod is rolling without slipping, the kinetic energy equals the translational kinetic energy $\frac{1}{2}mv^2$ plus the rotational kinetic energy $\frac{1}{2}I_{\text{rod}}\omega^2$, where $\omega = v/R$ is the angular velocity, and the moment of inertia $I_{\text{rod}} = \frac{1}{2}mR^2$. Therefore, the work-energy relation takes the form

$$IBdL = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{rod}}\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2,$$

such that

$$v = \sqrt{\frac{4IBdL}{3m}}.$$

**6.** (a) The magnetic moment of the current loop of circumference $2\pi r$ is

$$\mu = IA = I\pi r^2.$$

(b) The torque exerted by the magnetic field on the loop is

$$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = |\mu B|.$$

**7.** For the singly charged ion, the velocity is obtained from

$$q\Delta V = \frac{1}{2}mv^2,$$

such that

$$v = \sqrt{\frac{2q\Delta V}{m}}.$$

The radius of its semicircular orbit is

$$R = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2\Delta V m}{q}}.$$

If the radius of the orbit of a multiply charged ion with $q' = nq$ and mass $m'$ is $R' = NR$, we have the relation

$$\frac{R'}{R} = N = \sqrt{\frac{m'}{m}\frac{q}{q'}} = \sqrt{\frac{1}{n}\frac{m'}{m}},$$

such that

$$\frac{m'}{m} = N^2 n.$$

**8.** The velocity selected in the velocity selector is

$$v = \frac{E}{B}.$$

The radius of the circular orbit of a singly charged ion (*i.e.*, $q = q_p = 1.6 \times 10^{-19}$ C) in the mass spectrometer is

$$r = \frac{mv}{qB} = \frac{mE}{qB^2}.$$

**9.** (a) The Hall voltage is related to the magnetic field as follows:

$$\Delta V_H = \frac{IB}{nqt}. \tag{1}$$

For the Hall probe with current $I$, the initial data for the Hall voltage and the magnetic field allows us to solve for $(nqt)^{-1}$:

$$\frac{1}{nqt} = \frac{\Delta V_H}{IB}.$$

We can then use Eq. (1) to solve for the new magnetic field $B'$ given a new Hall voltage $\Delta V_H'$:

$$B' = nqt\frac{\Delta V_H'}{I} = B\frac{\Delta V_H'}{\Delta V_H}.$$

(b) To determine $n$, use Eq. (1):

$$n = \frac{1}{qt}\frac{IB}{\Delta V_H}.$$

**10.** The magnetic field a distance $r$ away from a long current-carrying wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$