Homework 8:

Problem 1:

P32.52 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times 10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$

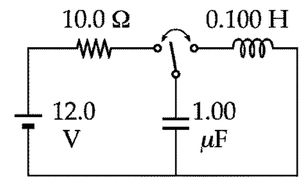

FIG. P32.52

(b) $Q = C\varepsilon = (1.00\times 10^{-6}\text{ F})(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_{\max}^2$

$I_{\max} = \varepsilon\sqrt{\frac{C}{L}} = 12\text{ V}\sqrt{\frac{1.00\times 10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$

(d) At all times $U = \frac{1}{2}C\varepsilon^2 = \frac{1}{2}(1.00\times 10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$.

Problem 2:

P57: The period of damped oscillation is $T = \frac{2\pi}{\omega_d}$. After one oscillation the charge returning to the capacitor is $Q = Q_{\max}e^{-RT/2L} = Q_{\max}e^{-2\pi R/2L\omega_d}$. The energy is proportional to the charge squared, so after one oscillation it is $U = U_0 e^{-2\pi R/L\omega_d} = 0.99U_0$. Then

$$e^{2\pi R/L\omega_d} = \frac{1}{0.99}$$

$$\frac{2\pi 2\ \Omega}{L\omega_d} = \ln(1.010\,1) = 0.001\,005$$

$$L\omega_d = \frac{2\pi 2\ \Omega}{0.001\,005} = 1\,250\ \Omega = L\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{1/2}$$

$$1.563\times 10^6\ \Omega^2 = \frac{L}{C} - \frac{(2\ \Omega)^2}{4}$$

$$\frac{L}{C} = 1.563\times 10^6\ \Omega^2$$

We are also given

$$\omega = 2\pi\times 10^3/\text{s} = \frac{1}{\sqrt{LC}}$$

$$LC = \frac{1}{(2\pi\times 10^3/\text{s})^2} = 2.533\times 10^{-8}\text{ s}^2$$

Solving simultaneously,

$$C = 2.533\times 10^{-8}\ \text{s}^2/L$$

$$\frac{L^2}{2.533\times 10^{-8}\text{ s}^2} = 1.563\times 10^6\ \Omega^2 \quad \boxed{L = 0.199\text{ H}}$$

$$C = \frac{2.533\times 10^{-8}\text{ s}^2}{0.199\text{ H}} = \boxed{127\text{ nF} = C}$$

PROBLEM 3:

$$i_L(t) = \frac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) = \frac{(80.0\text{ V})\sin[(65.0\pi)(0.015\,5) - \pi/2]}{(65.0\pi\text{ rad/s})(70.0\times 10^{-3}\text{ H})}$$

$$i_L(t) = (5.60\text{ A})\sin(1.59\text{ rad}) = \boxed{5.60\text{ A}}$$

PROBLEM 4:

P33.33 (a) $P = I_{rms}(\Delta V_{rms})\cos\phi = (9.00)180\cos(-37.0°) = 1.29\times 10^3\text{ W}$

$P = I_{rms}^2 R$ so $1.29\times 10^3 = (9.00)^2 R$ and $R = \boxed{16.0\ \Omega}$.

(b) $\tan\phi = \frac{X_L - X_C}{R}$ becomes $\tan(-37.0°) = \frac{X_L - X_C}{16}$: so $X_L - X_C = \boxed{-12.0\ \Omega}$.

PROBLEM 5:

(a) $$f = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(10^{10}/\text{s})^2\, 400\times 10^{-12}\text{ Vs}}\text{A}\left(\frac{\text{C}}{\text{As}}\right) = \boxed{6.33\times 10^{-13}\text{ F}}$$

(b) $$C = \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0 \ell^2}{d}$$

$$\ell = \left(\frac{Cd}{\kappa \in_0}\right)^{1/2} = \left(\frac{6.33\times 10^{-13}\text{ F}\times 10^{-3}\text{ mm}}{1\times 8.85\times 10^{-12}\text{ F}}\right)^{1/2} = \boxed{8.46\times 10^{-3}\text{ m}}$$

(c) $$X_L = 2\pi f L = 2\pi\times 10^{10}/\text{s}\times 400\times 10^{-12}\text{ Vs/A} = \boxed{25.1\ \Omega}$$

PROBLEM 6:

P33.46 (a) $(\Delta V_{2,\text{ rms}}) = \frac{N_2}{N_1}(\Delta V_{1,\text{ rms}})$ $\qquad N_2 = \frac{(2\,200)(80)}{110} = \boxed{1\,600\text{ windings}}$

(b) $I_{1,\text{ rms}}(\Delta V_{1,\text{ rms}}) = I_{2,\text{ rms}}(\Delta V_{2,\text{ rms}})$ $\qquad I_{1,\text{ rms}} = \frac{(1.50)(2\,200)}{110} = \boxed{30.0\text{ A}}$

(c) $0.950 I_{1,\text{ rms}}(\Delta V_{1,\text{ rms}}) = I_{2,\text{ rms}}(\Delta V_{2,\text{ rms}})$ $\qquad I_{1,\text{ rms}} = \frac{(1.20)(2\,200)}{110(0.950)} = \boxed{25.3\text{ A}}$

PROBLEM 7:

(a) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{68.0^2 + (16.0 - 101)^2} = \boxed{109\ \Omega}$

$X_L = \omega L = (100)(0.160) = 16.0\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(100)(99.0 \times 10^{-6})} = 101\ \Omega$

(b) $I_{max} = \dfrac{\Delta V_{max}}{Z} = \dfrac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{16.0 - 101}{68.0} = -1.25$:

$\phi = -0.896\ \text{rad} = -51.3°$

$\boxed{I_{max} = 0.367\ \text{A}}$ $\boxed{\omega = 100\ \text{rad/s}}$ $\boxed{\phi = -0.896\ \text{rad} = -51.3°}$

PROBLEM 8:

P33.53 For this RC high-pass filter, $\dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$.

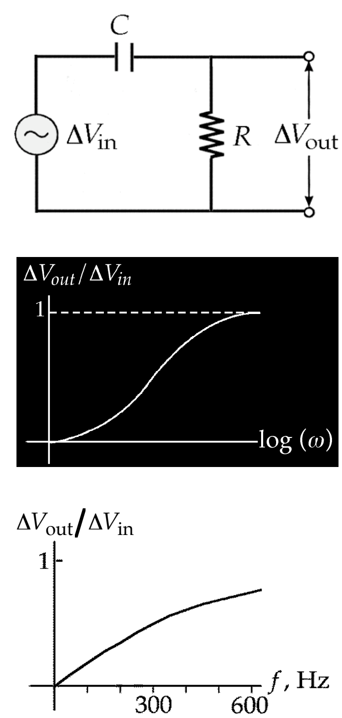

FIG. P33.53

(a) When $\dfrac{\Delta V_{out}}{\Delta V_{in}} = 0.500$,

then $\dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + X_C^2}} = 0.500$ or $X_C = 0.866\ \Omega$.

If this occurs at $f = 300$ Hz, the capacitance is

$C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(300\ \text{Hz})(0.866\ \Omega)} = 6.13 \times 10^{-4}\ \text{F} = \boxed{613\ \mu\text{F}}$.

(b) With this capacitance and a frequency of 600 Hz,

$X_C = \dfrac{1}{2\pi(600\ \text{Hz})(6.13 \times 10^{-4}\ \text{F})} = 0.433\ \Omega$

$\dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{R}{\sqrt{R^2 + X_C^2}} = \dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$.

PROBLEM 9:

P33.35 Consider a two-wire transmission line:

$$I_{\text{rms}} = \frac{\mathcal{P}}{\Delta V_{\text{rms}}} \text{ and power loss } = I_{\text{rms}}^2 R_{\text{line}} = \frac{\mathcal{P}}{100}.$$

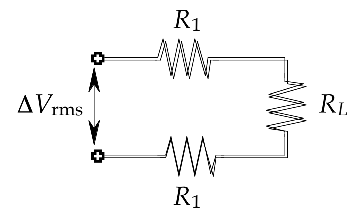

FIG. P33.35

Thus, $\left(\frac{\mathcal{P}}{\Delta V_{\text{rms}}}\right)^2 (2R_1) = \frac{\mathcal{P}}{100}$ or

$$R_1 = \frac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$$

$$R_1 = \frac{\rho d}{A} = \frac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}} \quad \text{or}$$

$$A = \frac{\pi(2r)^2}{4} = \frac{200\rho\mathcal{P}\, d}{(\Delta V_{\text{rms}})^2}$$

and the diameter is

$$2r = \boxed{\sqrt{\frac{800\rho\mathcal{P}\, d}{\pi(\Delta V)^2}}}.$$