Homework 9:

Problem 1:

From $v = \sqrt{\frac{T}{\mu}}$, we must increase the tension by a factor of 4.

Problem 2:

From $y = (12.0\text{ cm})\sin\left((1.57\text{ rad/m})x - (31.4\text{ rad/s})t\right)$

(a) The transverse velocity is $\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12\text{ cm}(31.4\text{ rad/s}) = \boxed{3.77\text{ m/s}}$

(b) $a_y = \frac{\partial v_y}{\partial t} = \frac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12\text{ m})\left(31.4\text{ s}^{-1}\right)^2 = \boxed{118\text{ m/s}^2}$

Problem 3:

(a) $f = \frac{v}{\lambda} = \frac{(1.00\text{ m/s})}{2.00\text{ m}} = \boxed{0.500\text{ Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14\text{ rad/s}}$

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.00\text{ m}} = \boxed{3.14\text{ rad/m}}$

(c) $y = A\sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100\text{ m})\sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$\boxed{y = (0.100\text{ m})\sin(-3.14t/\text{s})}$

(e) $\boxed{y = (0.100\text{ m})\sin(4.71\text{ rad} - 3.14\,t/\text{s})}$

(f) $v_y = \frac{\partial y}{\partial t} = 0.100\text{ m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and –1, so

$v_y \le (0.314\text{ m/s})$

Problem 4:

Comparing $y = 0.35\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$ we have

$$k = \frac{3\pi}{m},\ \omega = 10\pi/\text{s},\ A = 0.35\ \text{m}. \text{ Then } v = f\lambda = 2\pi f\frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33\ \text{m/s}.$$

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\,3.33\ \text{m/s} = \boxed{15.1\ \text{W}}.$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P}\,T = \frac{1}{2}\mu\omega^2A^2\lambda = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\,\frac{2\pi\ \text{m}}{3\pi} = \boxed{3.02\ \text{J}}.$$

Problem 5:

$v = \sqrt{\frac{T}{\mu}}$ where $T = \mu x g$, the weight of a length x, of rope.

Therefore, $v = \sqrt{gx}$

But $v = \frac{dx}{dt}$, so that $dt = \frac{dx}{\sqrt{gx}}$

and $$t = \int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}}\left.\frac{\sqrt{x}}{\frac{1}{2}}\right|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

Problem 6:

(a) $\Delta x = \sqrt{9.00 + 4.00} - 3.00 = \sqrt{13} - 3.00 = 0.606\ \text{m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{300\ \text{Hz}} = 1.14\ \text{m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$ of a wave,

or $\Delta\phi = 2\pi(0.530) = \boxed{3.33\ \text{rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $f = \frac{v}{2\Delta x} = \frac{343}{2(0.606)} = \boxed{283\ \text{Hz}}$

Problem 7:

$y = (1.50\ \text{m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos\omega t$

Therefore, $k = \frac{2\pi}{\lambda} = 0.400\ \text{rad/m}$

$$\lambda = \frac{2\pi}{0.400\ \text{rad/m}} = \boxed{15.7\ \text{m}}$$

and $\omega = 2\pi f$ so

$$f = \frac{\omega}{2\pi} = \frac{200\ \text{rad/s}}{2\pi\ \text{rad}} = \boxed{31.8\ \text{Hz}}$$

The speed of waves in the medium is

$$v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{\omega}{k} = \frac{200\ \text{rad/s}}{0.400\ \text{rad/m}} = \boxed{500\ \text{m/s}}$$

Problem 8:

(a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$.

Thus, $$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

and also $$f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$$

Thus, $$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\ \text{kg})g}{(16.0\ \text{kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $$f = \frac{4}{2(2.00\ \text{m})}\sqrt{\frac{(25.0\ \text{kg})(9.80\ \text{m/s}^2)}{0.002\ 00\ \text{kg/m}}} = \boxed{350\ \text{Hz}}$$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ so $$350\ \text{Hz} = \frac{1}{2(2.00\ \text{m})}\sqrt{\frac{m(9.80\ \text{m/s}^2)}{0.002\ 00\ \text{kg/m}}}$$

yielding $m = \boxed{400\ \text{kg}}$

Problem 9:

(a) The string could be tuned to either $\boxed{521\ \text{Hz or } 525\ \text{Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\ \text{Hz}}$.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \text{ and } T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523\text{ Hz}}{526\text{ Hz}}\right)^2 T_1 = 0.989T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change } = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.011\,4 = 1.14\% \text{ lower.}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.