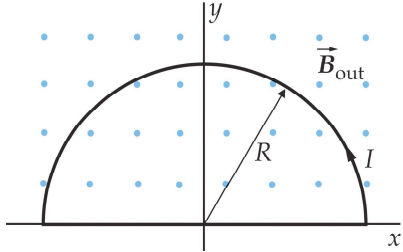


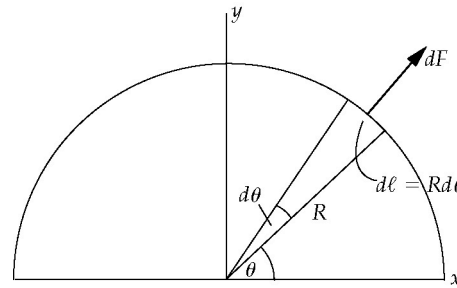
Problems on Force Exerted by a Magnetic Fields from Ch 26 T&M

Problem 26.27

A current-carrying wire is bent into a semicircular loop of radius R that lies in the xy plane. There is a uniform magnetic field $\mathbf{B} = B\mathbf{k}$ perpendicular to the plane of the loop. Verify that the force acting on the loop is zero.



Picture the Problem With the current in the direction indicated and the magnetic field in the z direction, pointing out of the plane of the page, the force is in the radial direction and we can integrate the element of force dF acting on an element of length $d\ell$ between $\theta = 0$ and π to find the force acting on the semicircular portion of the loop and use the expression for the force on a current-carrying wire in a uniform magnetic field to find the force on the straight segment of the loop.



Express the net force acting on the semicircular loop of wire:

$$F = F_{\text{semicircular loop}} + F_{\text{straight segment}} \quad (1)$$

Express the force acting on the straight segment of the loop:

$$\vec{F}_{\text{straight segment}} = I\vec{\ell} \times \vec{B} = -2RIB$$

Express the force dF acting on the element of the wire of length $d\ell$:

$$dF = Id\ell B = IRBd\theta$$

Express the x and y components of dF :

$$dF_x = dF \cos \theta$$

and

$$dF_y = dF \sin \theta$$

Because, by symmetry, the x component of the force is zero, we can integrate the y component to

$$dF_y = IRB \sin \theta d\theta$$

and

find the force on the wire:

$$F_y = RIB \int_0^\pi \sin \theta \, d\theta = 2RIB$$

Substitute in equation (1) to obtain:

$$F = 2RIB - 2RIB = \boxed{0}$$

26.57 Torque on a loop with current

A rigid circular loop of radius R and mass m carries a current I and lies in the xy plane on a rough, flat table. There is a horizontal magnetic field of magnitude B . What is the minimum value of B so that one edge of the loop will lift off the table?

Picture the Problem The loop will start to lift off the table when the magnetic torque equals the gravitational torque.

Express the magnetic torque acting on the loop:

$$\tau_{\text{mag}} = \mu B = I\pi R^2 B$$

Express the gravitational torque acting on the loop:

$$\tau_{\text{grav}} = mgR$$

Because the loop is in equilibrium under the influence of the two torques:

$$I\pi R^2 B = mgR$$

Solve for B to obtain:

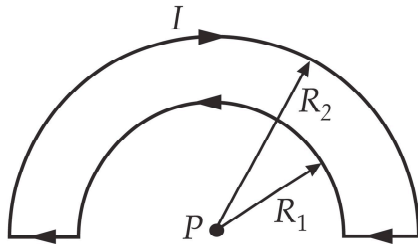
$$B = \boxed{\frac{mg}{I\pi R}}$$

Problem 26.61+27.101 Magnetic moment of a loop and Magnetic field calculation

A wire loop consists of two semicircles connected by straight segments. The inner and outer radii are $R_1 = 0.3$ and $R_2 = 0.5$ m, respectively. A current I of 1.5 A flows in this loop with the current in the outer semicircle in the clockwise direction.

A) What is the magnetic moment of the current loop?

B) Find the magnetic field in P, which is at the common center of the 2 semicircular arcs.



Picture the Problem We can use the definition of the magnetic moment to find the magnetic moment of the given current loop and a right-hand rule to find its direction.

Using its definition, express the magnetic moment of the current loop:

$$\mu = IA$$

Express the area bounded by the loop:

$$A = \frac{1}{2} (\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2) = \frac{\pi}{2} (R_{\text{outer}}^2 - R_{\text{inner}}^2)$$

Substitute to obtain:

$$\mu = \frac{\pi I}{2} (R_{\text{outer}}^2 - R_{\text{inner}}^2)$$

Substitute numerical values and evaluate μ :

$$\begin{aligned} \mu &= \frac{\pi(1.5 \text{ A})}{2} [(0.5 \text{ m})^2 - (0.3 \text{ m})^2] \\ &= \boxed{0.377 \text{ A} \cdot \text{m}^2} \end{aligned}$$

Apply the right-hand rule for determining the direction of the unit normal vector (the direction of μ) to conclude that \vec{i} points into the page.

Problem 27.101 •

Picture the Problem Let out of the page be the positive x direction. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P . Hence, the resultant magnetic field at P will be the sum of the magnetic fields due to the current in the two semicircles, and we can use the expression for the magnetic field at the center of a current loop to find \vec{B}_P .

Express the resultant magnetic field at P :

$$\vec{B}_p = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2} \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Express \vec{B}_1 and \vec{B}_2 :

$$\vec{B}_1 = \frac{\mu_0 I}{4R_1} \hat{i}$$

and

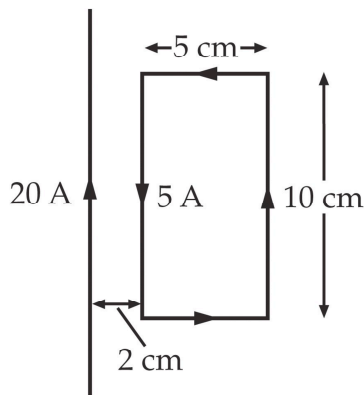
$$\vec{B}_2 = -\frac{\mu_0 I}{4R_2} \hat{i}$$

Substitute to obtain:

$$\vec{B}_p = \frac{\mu_0 I}{4R_1} \hat{i} - \frac{\mu_0 I}{4R_2} \hat{i} = \boxed{\frac{\mu_0 I}{4} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \hat{i}}$$

Problem 27.105 Force between current wires

A long straight wire carries a current of 20 A, as shown in the figure. A rectangular coil with 2 sides parallel to the straight wire has sides 5 cm and 10 cm with the near side at a distance 2 cm from the wire. The coil carries a current of 5 A. (a) Find the force on each segment of the rectangular coil due to the current in the long straight wire. (b) What is the net force on the coil?



Picture the Problem Let I_1 and I_2 represent the currents of 20 A and 5 A, \vec{F}_1 , \vec{F}_2 , \vec{F}_3 , and \vec{F}_4 the forces that act on the horizontal wire at the top of the loop, and the other wires following the current in a counterclockwise direction, and \vec{B}_1 , \vec{B}_2 , \vec{B}_3 , and \vec{B}_4 the magnetic fields at these wires due to I_1 . Let the positive x direction be to the right and the positive y direction be upward. Note that only the components into or out of the paper

of \vec{B}_1 , \vec{B}_2 , \vec{B}_3 , and \vec{B}_4 contribute to the forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 , and \vec{F}_4 , respectively.

(a) Express the forces \vec{F}_2 and \vec{F}_4 in terms of I_2 and \vec{B}_2 and \vec{B}_4 :

$$\vec{F}_2 = I_2 \vec{\ell}_2 \times \vec{B}_2$$

and

$$\vec{F}_4 = I_2 \vec{\ell}_4 \times \vec{B}_4$$

Express \vec{B}_2 and \vec{B}_4 :

$$\vec{B}_2 = -\frac{\mu_0}{4\pi} \frac{2I_1}{R_1} \hat{k}$$

and

$$\vec{B}_4 = -\frac{\mu_0}{4\pi} \frac{2I_1}{R_4} \hat{k}$$

Substitute to obtain:

$$\begin{aligned} \vec{F}_2 &= -I_2 \ell_2 \hat{j} \times \left(-\frac{\mu_0}{4\pi} \frac{2I_1}{R_1} \hat{k} \right) \\ &= \frac{\mu_0 \ell_2 I_1 I_2}{2\pi R_2} \hat{i} \end{aligned}$$

and

$$\begin{aligned} \vec{F}_4 &= I_2 \ell_4 \hat{j} \times \left(-\frac{\mu_0}{4\pi} \frac{2I_1}{R_4} \hat{k} \right) \\ &= -\frac{\mu_0 \ell_4 I_1 I_2}{2\pi R_4} \hat{i} \end{aligned}$$

Substitute numerical values and evaluate \vec{F}_2 and \vec{F}_4 :

$$\vec{F}_2 = \frac{(4\pi \times 10^{-7} \text{ N/A}^2)(0.1 \text{ m})(20 \text{ A})(5 \text{ A})}{2\pi(0.02 \text{ m})} \hat{i} = \boxed{(1.00 \times 10^{-4} \text{ N}) \hat{i}}$$

and

$$\vec{F}_4 = -\frac{(4\pi \times 10^{-7} \text{ N/A}^2)(0.1 \text{ m})(20 \text{ A})(5 \text{ A})}{2\pi(0.07 \text{ m})} \hat{i} = \boxed{(-0.286 \times 10^{-4} \text{ N}) \hat{i}}$$

(b) Express the net force acting on the coil:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \quad (1)$$

Because the lengths of segments 1 and 3 are the same and the currents in these segments are in opposite directions:

$$\vec{F}_1 + \vec{F}_3 = 0$$

and

$$\vec{F}_{\text{net}} = \vec{F}_2 + \vec{F}_4$$

Substitute for \vec{F}_2 and \vec{F}_4 in equation (1) and simplify to obtain:

$$\begin{aligned}\vec{F}_{\text{net}} &= (-0.250 \times 10^{-4} \text{ N})\hat{j} + (1.00 \times 10^{-4} \text{ N})\hat{i} + (0.250 \times 10^{-4} \text{ N})\hat{j} \\ &\quad + (-0.286 \times 10^{-4} \text{ N})\hat{i} \\ &= \boxed{(0.714 \times 10^{-4} \text{ N})\hat{i}}\end{aligned}$$

Problem 27.59 Magnetic field in a solenoid

A solenoid with length 30 cm, radius 1.2 cm, and 300 turns carries a current of 2.6 A. Find B on the axis of the solenoid (a) at the center, (b) inside the solenoid at a point 10 cm from one end, and (c) at one end.

Picture the Problem We can use $B_x = \frac{1}{2} \mu_0 n I \left(\frac{b}{\sqrt{b^2 + R^2}} + \frac{a}{\sqrt{a^2 + R^2}} \right)$ to find B at any point on the axis of the solenoid. Note that the number of turns per unit length for this solenoid is 300 turns/0.3 m = 1000 turns/m.

Express the magnetic field at any point on the axis of the solenoid:

$$B_x = \frac{1}{2} \mu_0 n I \left(\frac{b}{\sqrt{b^2 + R^2}} + \frac{a}{\sqrt{a^2 + R^2}} \right)$$

Substitute numerical values to obtain:

$$\begin{aligned}B_x &= \frac{1}{2} (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (1000) (2.6 \text{ A}) \left(\frac{b}{\sqrt{b^2 + (0.012 \text{ m})^2}} + \frac{a}{\sqrt{a^2 + (0.012 \text{ m})^2}} \right) \\ &= (1.63 \text{ mT}) \left(\frac{b}{\sqrt{b^2 + (0.012 \text{ m})^2}} + \frac{a}{\sqrt{a^2 + (0.012 \text{ m})^2}} \right)\end{aligned}$$

(a) Evaluate B_x for $a = b = 0.15$ m:

$$B_x = (1.63 \text{ mT}) \left(\frac{0.15 \text{ m}}{\sqrt{(0.15 \text{ m})^2 + (0.012 \text{ m})^2}} + \frac{0.15 \text{ m}}{\sqrt{(0.15 \text{ m})^2 + (0.012 \text{ m})^2}} \right) = \boxed{3.25 \text{ mT}}$$

(b) Evaluate B_x for $a = 0.1$ m and $b = 0.2$ m:

$$B_x(0.2 \text{ m}) = (1.63 \text{ mT}) \left(\frac{0.2 \text{ m}}{\sqrt{(0.2 \text{ m})^2 + (0.012 \text{ m})^2}} + \frac{0.1 \text{ m}}{\sqrt{(0.1 \text{ m})^2 + (0.012 \text{ m})^2}} \right)$$

$$= \boxed{3.25 \text{ mT}}$$

(c) Evaluate $B_x (= B_{\text{end}})$ for $a = 0$ and $b = 0.3 \text{ m}$:

$$B_x = (1.63 \text{ mT}) \left(\frac{0.3 \text{ m}}{\sqrt{(0.3 \text{ m})^2 + (0.012 \text{ m})^2}} \right) = \boxed{1.63 \text{ mT}}$$

Note that $B_{\text{end}} = \frac{1}{2} B_{\text{center}}$.

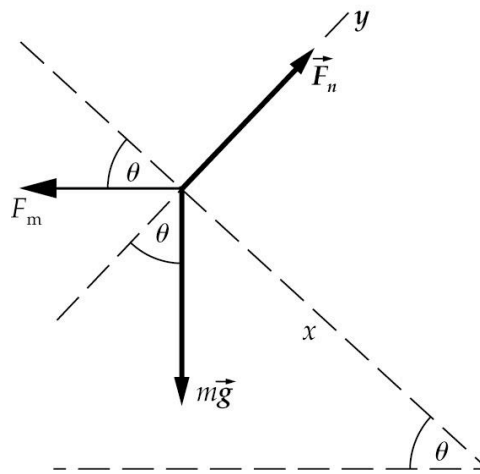
Conceptual Problem 27.67

Show that a uniform magnetic field with no fringing field, such as that shown in the figure, is impossible because it violates Ampère's law. Do this by applying Ampère's law to the rectangular curve shown by the dashed line.

Determine the Concept The contour integral consists of four portions, two horizontal portions for which $\oint_C \vec{B} \cdot d\vec{\ell} = 0$, and two vertical portions. The portion within the magnetic field gives a nonvanishing contribution, whereas the portion outside the field gives no contribution to the contour integral. Hence, the contour integral has a finite value. However, it encloses no current; thus, it appears that Ampère's law is violated. What this demonstrates is that there must be a fringing field so that the contour integral does vanish.

Problem 28.45

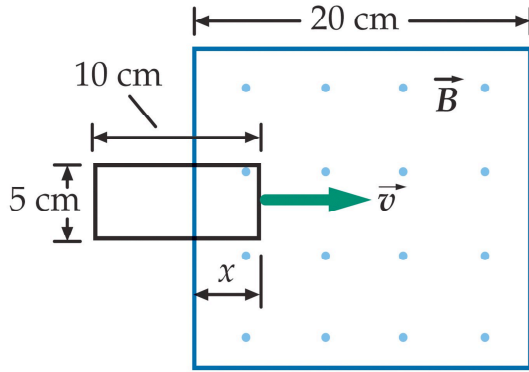
Picture the Problem The free-body diagram shows the forces acting on the rod as it slides down the inclined plane. The retarding force is the component of F_m acting up the incline, i.e., in the $-x$ direction. We can express F_m using the expression for the force acting on a conductor moving in a magnetic field. Recognizing that only the horizontal component of the rod's velocity \vec{v} produces an induced emf, we can apply the expression for a motional emf in conjunction with Ohm's law to find the induced current in the rod. In part (b) we can apply Newton's 2nd law to obtain an expression for dv/dt and set this expression equal to zero to obtain v_t .



(a) Express the retarding force acting on the rod:	$F = F_m \cos \theta \quad (1)$ <p>where</p> $F_m = I\ell B$ <p>and I is the current induced in the rod as a consequence of its motion in the magnetic field.</p>
Express the induced emf due to the motion of the rod in the magnetic field:	$\mathcal{E} = B\ell v \cos \theta$
Using Ohm's law, relate the current I in the circuit to the induced emf:	$I = \frac{\mathcal{E}}{R} = \frac{B\ell v \cos \theta}{R}$
Substitute in equation (1) to obtain:	$F = \left(\frac{B\ell v \cos \theta}{R} \right) \ell B \cos \theta$ $= \boxed{\frac{B^2 \ell^2 v}{R} \cos^2 \theta}$
(b) Apply $\sum F_x = ma_x$ to the rod:	$mg \sin \theta - \frac{B^2 \ell^2 v}{R} \cos^2 \theta = m \frac{dv}{dt}$ <p>and</p> $\frac{dv}{dt} = g \sin \theta - \frac{B^2 \ell^2 v}{mR} \cos^2 \theta$
When the rod reaches its terminal velocity v_t , $dv/dt = 0$ and:	$0 = g \sin \theta - \frac{B^2 \ell^2 v_t}{mR} \cos^2 \theta$
Solve for v_t to obtain:	$v_t = \boxed{\frac{mgR \sin \theta}{B^2 \ell^2 \cos^2 \theta}}$

Problem 28.39

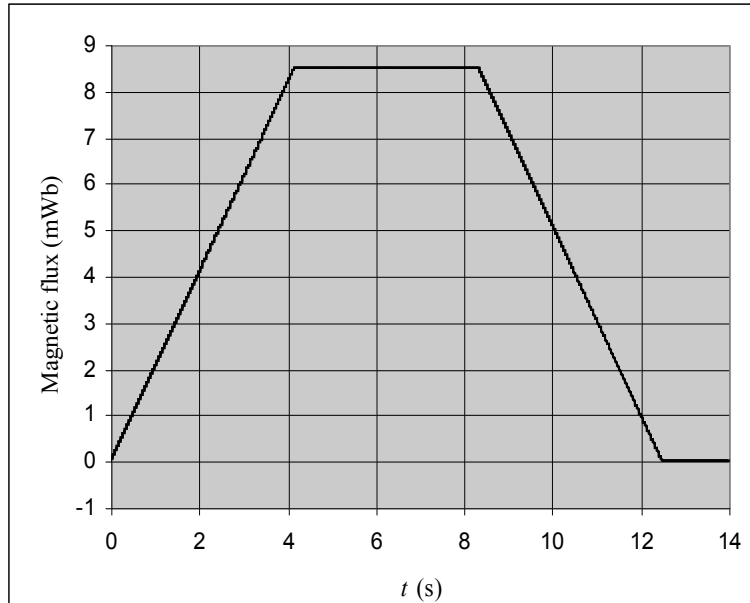
Picture the Problem We'll need to determine how long it takes for the loop to completely enter the region in which there is a magnetic field, how long it is in the region, and how long it takes to leave the region. Once we know these times, we can use its definition to express the magnetic flux as a function of time. We can use Faraday's law to find the induced emf as a function of time.



(a) Find the time required for the loop to enter the region where there is a uniform magnetic field:	$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10 \text{ cm}}{2.4 \text{ cm/s}} = 4.17 \text{ s}$
Letting w represent the width of the loop, express and evaluate ϕ_m for $0 < t < 4.17 \text{ s}$:	$\begin{aligned} \phi_m &= NBA = NBwvt \\ &= (1.7 \text{ T})(0.05 \text{ m})(0.024 \text{ m/s})t \\ &= (2.04 \text{ mWb/s})t \end{aligned}$
Find the time during which the loop is fully in the region where there is a uniform magnetic field:	$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10 \text{ cm}}{2.4 \text{ cm/s}} = 4.17 \text{ s}$ i.e., the loop will begin to exit the region when $t = 8.33 \text{ s}$.
Express ϕ_m for $4.17 \text{ s} < t < 8.33 \text{ s}$:	$\begin{aligned} \phi_m &= NBA = NB\ell w \\ &= (1.7 \text{ T})(0.1 \text{ m})(0.05 \text{ m}) \\ &= 8.50 \text{ mWb} \end{aligned}$
The left-end of the loop will exit the field when $t = 12.5 \text{ s}$. Express ϕ_m for $8.33 \text{ s} < t < 12.5 \text{ s}$:	$\phi_m = mt + b$ where m is the slope of the line and b is the ϕ_m -intercept.
For $t = 8.33 \text{ s}$ and $\phi_m = 8.50 \text{ mWb}$:	$8.50 \text{ mWb} = m(8.33 \text{ s}) + b \quad (1)$
For $t = 12.5 \text{ s}$ and $\phi_m = 0$:	$0 = m(12.5 \text{ s}) + b \quad (2)$
Solve equations (1) and (2) simultaneously to obtain:	$\phi_m = -(2.04 \text{ mWb/s})t + 25.5 \text{ mWb}$
The loop will be completely out of the magnetic field when $t > 12.5 \text{ s}$	$\phi_m = 0$

and:

The following graph of $\phi_m(t)$ was plotted using a spreadsheet program.



(b) Using Faraday's law, relate the induced emf to the magnetic flux:

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

During the interval $0 < t < 4.17 \text{ s}$:

$$\mathcal{E} = -\frac{d}{dt}[(2.04 \text{ mWb/s})t] = -2.04 \text{ mV}$$

During the interval $4.17 \text{ s} < t < 8.33 \text{ s}$:

$$\mathcal{E} = -\frac{d}{dt}[8.50 \text{ mWb}] = 0$$

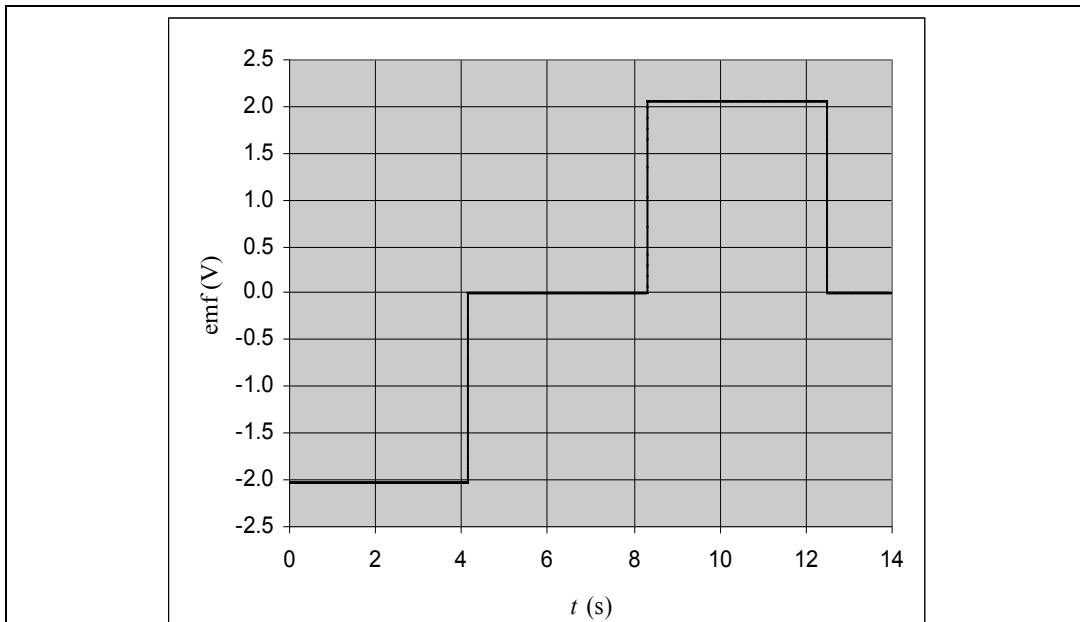
During the interval $8.33 \text{ s} < t < 12.5 \text{ s}$:

$$\begin{aligned} \mathcal{E} &= -\frac{d}{dt}[(-2.04 \text{ mWb/s})t + 25.5 \text{ mWb}] \\ &= 2.04 \text{ mV} \end{aligned}$$

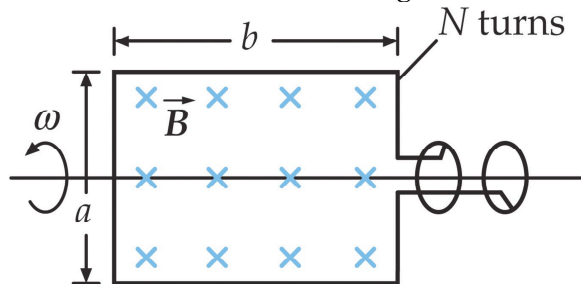
For $t > 12.5 \text{ s}$:

$$\mathcal{E} = 0$$

The following graph of $\mathcal{E}(t)$ was plotted using a spreadsheet program.



Problem 28.85 **The AC generator**



Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the coil (potential difference between the slip rings). In part (b) we can solve this equation for ω under the given conditions.

(a) Use Faraday's law to express the induced emf:	$\mathcal{E} = -\frac{d\phi_m}{dt}$
Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:	$\phi_m(t) = NBA \cos \omega t$
Substitute to obtain:	$\begin{aligned} \mathcal{E} &= -\frac{d}{dt} [NBA \cos \omega t] \\ &= -NBab\omega(-\sin \omega t) \\ &= \boxed{NBab\omega \sin \omega t} \end{aligned}$

(b) Express the condition under which $\varepsilon = \varepsilon_{\max}$:	$\sin \omega t = 1$
Solve for and evaluate ω under this condition:	$\omega = \frac{\varepsilon_{\max}}{NBab}$ $= \frac{110 \text{ V}}{(1000)(2 \text{ T})(0.01 \text{ m})(0.02 \text{ m})}$ $= \boxed{275 \text{ rad/s}}$