Exercise in class 1

Solve at least 2 of these problems in class. The others are due by Feb 23.

1. Work out the threshold energy of a photon for pair production making the relevant considerations on when the process can occur.

2. Calculate the range of a muon of energy 100 GeV in rock (density = 2.65 g/cm^3) under the assumption that in the energy loss expression for a muon $-dE/dx = a+bE$, $a = 2$ MeV/g/cm^2 and $b = 4.4\ 10^{-6}$ cm^2/g and they are energy independent.

3. Which is the minimum kinetic energy of a cosmic ray muon to survive to sea level from a production altitude of 20 km? Consider that the lifetime of a muon in its reference frame is $\tau_\mu = 2.19703$ μs and that its mass is $m_\mu = 105.65837$ MeV. For this problem assume that all muons have the given lifetime in their rest frame though lifetime should be considered more correctly on an average sense.

4. Show that the mass of a charged particle can be inferred from the cosine of the Cherenkov angle $\cos\theta_C = 1/(n\beta)$ and from its momentum.

Suggested Solutions

1. Pair production can occur in the presence of a nucleus of mass M to conserve 4-momentum

$$E_{CM}^2 = s = (2m_e + M)^2 c^4 = E_1^2 - \mathbf{p}_1^2 c^2 + E_2^2 - \mathbf{p}_2^2 c^2 + 2E_1 E_2 (1 - \beta_1 \beta_2 \cos\vartheta)$$

In the rest frame of the nucleus $E_2 = Mc^2$, and $\beta_2 = 0$:

$$(4m_e^2 + M^2 + 4m_e M)c^4 = M^2 c^4 + 2Mc^2 E_\gamma \Rightarrow E_\gamma = \frac{(4m_e^2 + 4m_e M)c^2}{2M} = \frac{2m_e c^2 (m_e + M)}{M} = 2m_e c^2 \left(1 + \frac{m_e}{M}\right)$$

Since $M \gg m_e \Rightarrow E_\gamma \approx 2m_e c^2 = 1.02$ MeV

2.

$$R = \int_E^0 \frac{dE}{dE/dx} = \int_E^0 \frac{dE}{a + bE} = \frac{1}{b} \ln\left(1 + \frac{b}{a} E\right) = 45193.4\, g/cm^2 = 170.5m$$

3.

The lifetime should be considered in an average sense only. It does not mean that a particle with lifetime τ will decay exactly after the time τ after it was produced. Its actual lifetime t is a random number distributed with a probability density function:

$$f(t,\tau)dt = \frac{1}{\tau} e^{-t/\tau} dt$$

giving a probability that the lifetime t lies between t and t+dt. The mean value is τ:

$$\int t f(t,\tau)dt = \int_0^\infty t \frac{1}{\tau} e^{-t/\tau} dt = \left[-te^{-t/\tau} dt\right]_0^\infty + \int_0^\infty e^{-t/\tau} dt = \left[-\tau e^{-t/\tau}\right]_0^\infty = \tau$$

With this in mind we solve the problem. For an unstable particle the mean range before it decays is given by its velocity times the lifetime in its rest frame:

$$L = \beta c \gamma \tau_\mu \Rightarrow \beta\gamma = L / c\tau_\mu = \sqrt{\gamma^2 - 1} \Rightarrow \gamma^2 = \left(\frac{L}{c\tau_\mu}\right)^2 + 1$$

Since $L/c\tau_\mu \gg 1 \Rightarrow \gamma \approx L/c\tau_\mu \Rightarrow E_\mu = \gamma m_\mu c^2 \approx m_\mu c^2 L/c\tau_\mu \Rightarrow$

$E_K = \gamma m_\mu c^2 - m_\mu c^2$=105.65837*20e3/(2.19703e-6*3e8)-105.65837=3.1 GeV

5. $\cos\theta_C = 1/n\beta$ and $p = \gamma m \beta c$

$$\beta = \frac{p}{\gamma mc} \Rightarrow \cos\theta_C = \frac{\gamma mc}{np} \Rightarrow \frac{np\cos\theta_C}{mc} = \frac{E}{mc^2} = \frac{c\sqrt{p^2 + m^2c^2}}{mc^2} \Rightarrow (np\cos\theta_C)^2 = p^2 + m^2c^2 \Rightarrow m = \frac{p\sqrt{n^2\cos^2\theta_C - 1}}{c}$$