Gravitational Wave Astronomy

Suggested readings:

Camp and Cornish, Ann Rev Nucl Part Sci 2004

Schutz, gr-qc/0003069

Kip Thorne WEB course
http://elmer.caltech.edu/ph237/week1/week1.html

L. Bergstrom and A. Goobar, Cosmology and Particle Astrophysics (2nd edition), Springer 2004 cap 15

Einstein’s special and general relativity

Special relativity

main motivation: express physical laws in an independent way from the reference frame of the observer. Absolute Newtonian concepts of space and time are inadequate to include gravitation

2 principles:

1) All inertial observers are equivalent
2) The velocity of light c is the same in all inertial frames

Redefinition of space an time in accordance with Lorenz transformations: for 2 inertial frames in relative motion with velocity v along x

$$x' = \gamma\,(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$

$$\gamma = 1/(1 - v^2/c^2)^{1/2}$$

But gravity cannot be accommodated in framework of special relativity

General relativity

The equivalence of gravitational and inertial mass leads to an understanding of gravity as the physical manifestation of the curvature of space-time
Space curvature is associated with the energy-stress tensor of matter fields:

$$G_{\mu v} = 8\pi G_N T_{\mu v}.$$

Where $G_{\mu\nu}$ = Einstein tensor = $R_{\mu\nu} - 1/2g_{\mu\nu}R - \Lambda g_{\mu\nu}$
With $g_{\mu\nu}$=space-time metric
$T_{\mu\nu}$ = stress-energy tensor of matter fields
G_N = Newton gravitation constant
$R_{\mu\nu}$ is the Ricci tensor that can be obtained from the Riemann tensor that tells how much the direction of a vector changes when it is parallel transported around a closed curve (zero for flat space-time) and the metric as $R_{\mu\nu} = g^{\alpha\gamma}R_{\alpha\mu\gamma\nu}$
And the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$
The Einstein tensor is a complicated non linear function of the metric and its derivatives (10 non linear partial differential equations for the metric)

Minkowski metric

Einstein equations are both field equations (matter tells space-time how to curve) and motion equations (tell how matter moves)

At lowest order they imply that matter follows geodesics in space-time (the shortest possible line on a curved surface)

Lorentz transformations leave invariant $ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$ (and also the velocity of light)

The 4-dimensional metric is

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

And 4-vectors

$$x_\mu = \eta_{\mu\nu} x^\nu$$

$$ds^2 = dx^\mu dx_\mu = dx_\mu dx^\mu$$

Weak gravitational waves can be seen as small perturbations of the Minkowski space-time. The proper distance between events x^μ and $x^\mu + dx^\mu$ is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \approx (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu dx^\nu.$$

$h_{\mu\nu}$ = linearized gravitational field

Analogy to em waves

In elecromagnetism a 4-potential is introduced $A^{\mu} = (\Phi, \mathbf{A})$ and a tensor
(*) $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ so that Maxwell equations can be summarized by
$\partial_{\mu}F^{\mu\nu} = j^{\nu}$ with $j^{\nu} = (\rho,\mathbf{j})$ current 4-vector
Since (*) is unchanged under a gauge transformation
$A^{\mu} \rightarrow A^{\mu} + \partial f(\mathbf{r},t)$
We can use this freedom to impose a gauge condition on A^{μ} eg. $A^0 = 0$ and
$\partial_{\nu}A^{\nu} = \nabla\cdot \mathbf{A} = 0$ that lead to a simple wave equation
$\square A^{\mu} = 0$ (Dalambertian or wave operator $\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2$) that has solutions of the form

$$A^{\mu}(\mathbf{r},t) = \varepsilon^{\mu} e^{\pm i(\omega t - \mathbf{k}\cdot\mathbf{r})} = \varepsilon^{\mu} e^{\pm i k^{\mu} x_{\mu}}$$

Where the gauge conditions translate into $\varepsilon^0 = 0$ and $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$ showing that the 2 degrees or freedom are traverse to the direction of propagation

Gravitational waves

For gravitational wave, inserting the first order expansion of the metric tensor field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

And inserting it in the Einstein equations for vacuum ($T_{\mu\nu}$ = 0) we find that the Ricci tensor $R_{\mu\nu}$ = 0. Writing it in terms of the perturbations $h_{\mu\nu}$ leads to a simple wave equation $\Box h_{\mu\nu} = 0$ with similar solutions to em waves:

$$h_{\mu\nu} = E_{\mu\nu} \cos(\omega t - \mathbf{k} \cdot \mathbf{r})$$

As em waves do, also gw propagate at the velocity of light (if the graviton exists it is massles like the photon)

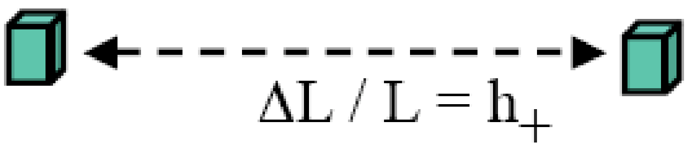
If we choose the z direction as the direction of propagation we find 2 possible polarization states and a plane gw can be written as

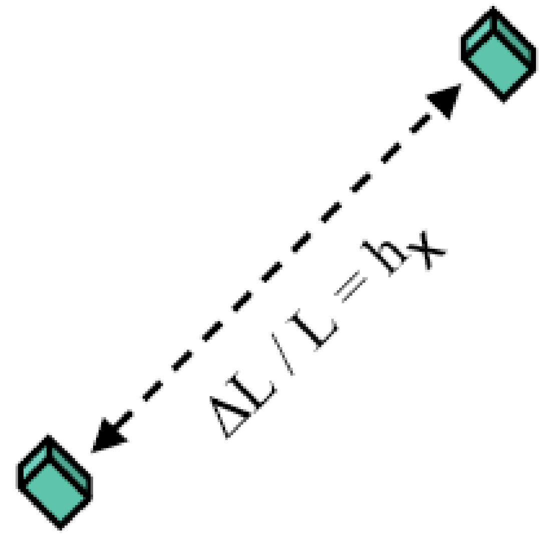
$$h_{\mu v} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

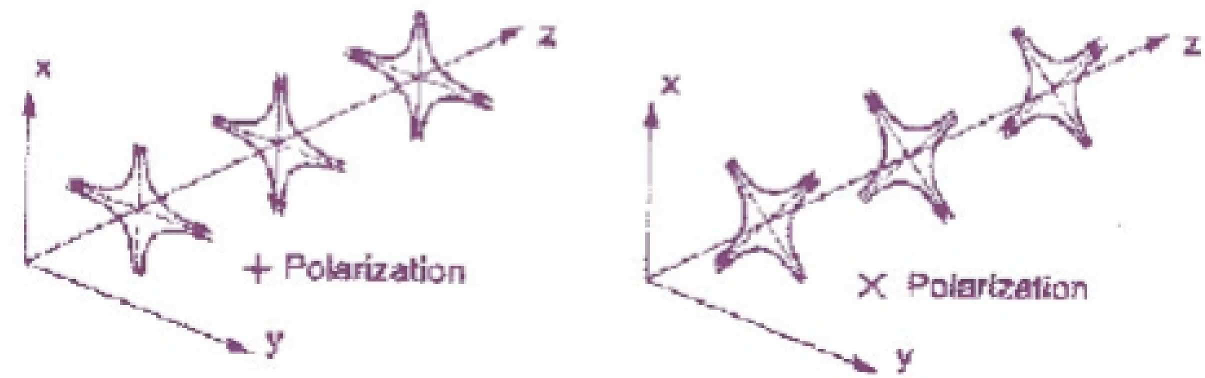
$h_{xx} = h_+$ and $h_{xy} = h_\times$ describe the 2 polarization states of the gw

The 2 polarizations

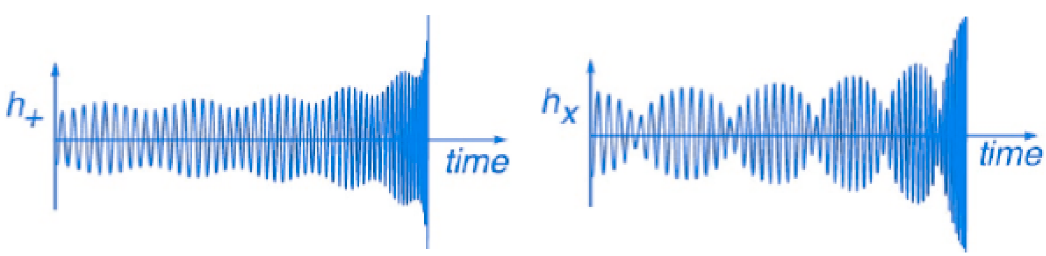
- Each polarization has its own gravitational-wave field





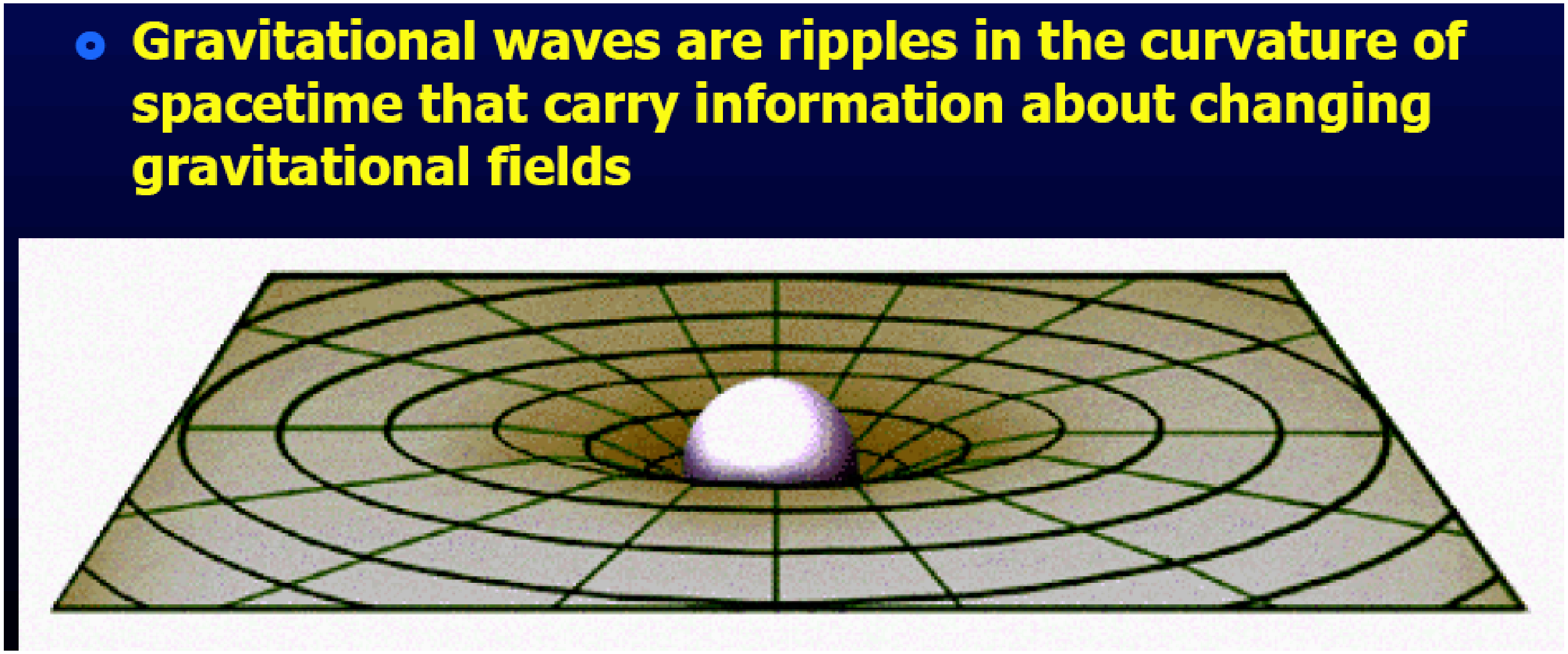


  - These fields' evolutions $h_+(t)$ & $h_x(t)$ are the ***waveforms***



Waveforms carry detailed Information about source

How is space distorted by a gw?



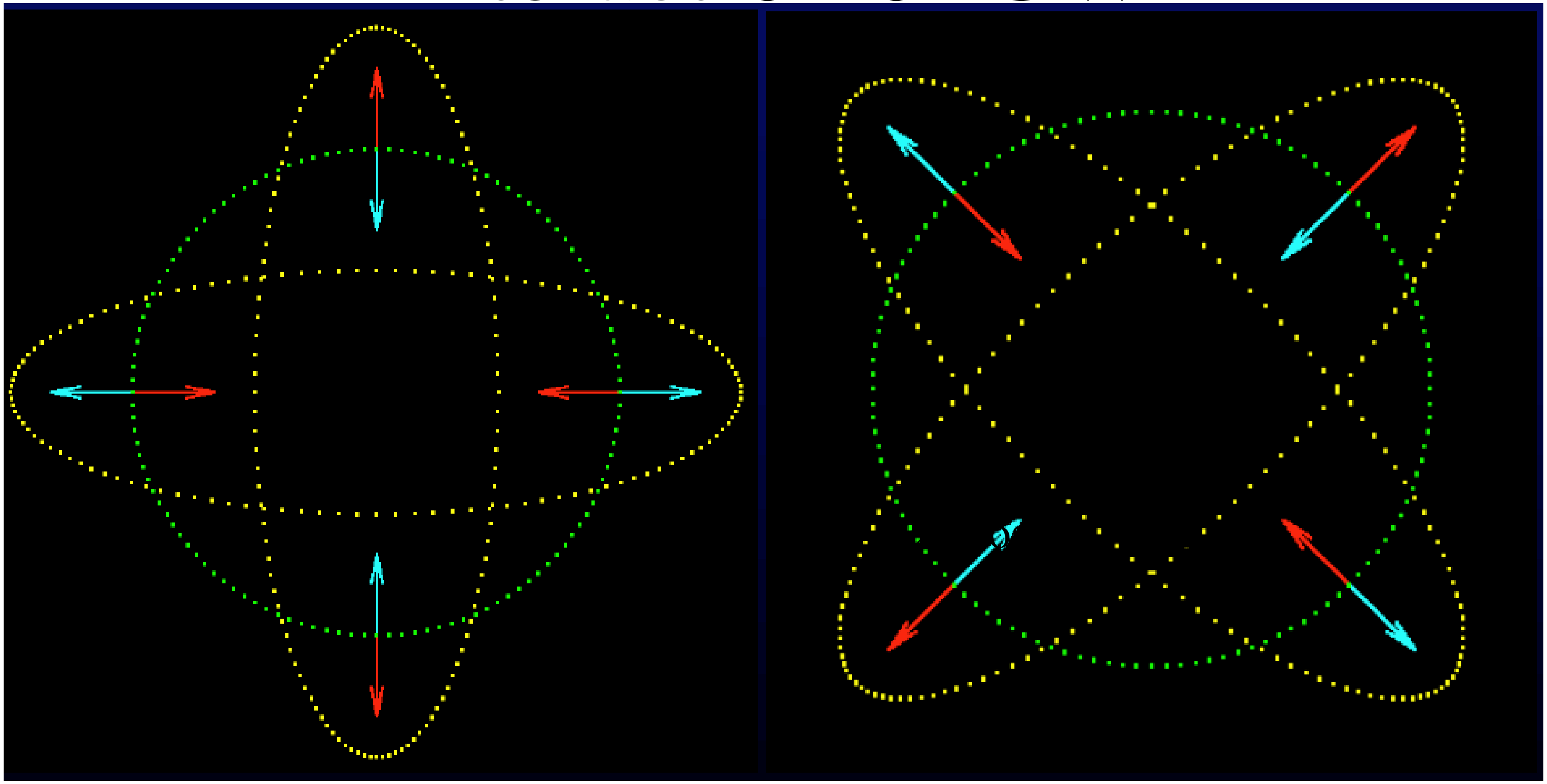
A unit circle in the xy plane is distorted by the h_+ type: the distance between 2 points on the x axis is larger by $h_+/2$ and that of 2 points on y is smaller by the same amount

For the h_x type the same happens rotating the system by $\pi/4$ around the z axis

The amplitude of oscillations of h is

$$h \sim 2\epsilon \frac{GM}{rc^2}$$ ε is a number <1

Interaction of GW

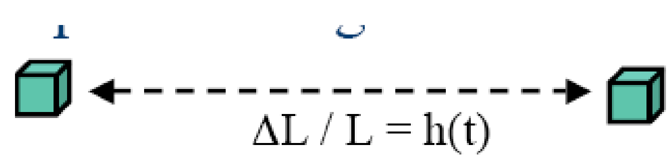


The distorsion of the circle into an ellipse oscillates with time according to

$$h_{\mu\nu} = E_{\mu\nu} \cos(\omega t - \mathbf{k} \cdot \mathbf{r})$$

Waves squeeze and stretch freely

floating objects



Quadrupole approximation

Analogous to em: monopole moment = total charge = const ⇒ no radiation

Dipole: $$d_i = \int \rho x_i d^3x.$$ ρ = charge density x_i = spatial coordinate

If it is time dependent the amplitude of waves $\propto dd_i/dt$ and radiated energy $\propto \sum |d^2d_i/dt^2|^2$.

For general relativity the equivalent of the monopole is the mass-energy (the source of gravitational field constant as long as the radiation is weak) and for the dipole ρ = density of mass-energy

v is the velocity

$$\dot{d}_i = \int \rho v_i d^3x.$$

To lowest order it is constant and there is no energy radiated (it reflects the fact that the source is moving in space).

To find radiation one should consider the quadrupole terms

$$Q_{jk} = \int \rho x_j x_k d^3x.$$

Quadrupole formula

As in em, the amplitude of the radiation is $\propto$ to $\frac{d^2 Q_{jk}}{dt^2}$.

A factor G_N/c^4 is needed for making h adimensional

Einstein equation

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right) \bar{h}_{\mu v} = 16\pi\, G_N T_{\mu v}.$$

has solutions $$h_{jk} = \frac{2G_N}{rc^4}\frac{d^2 Q_{jk}}{dt^2}.$$ quadrupole radiation formula

If the source is highly non-spherical such as binary systems $d^2 Q_{jk}/dt^2$

has magnitude $M v_{N.S.}^2$:

Hence

$$h \sim \frac{2GMv_{N.S.}^2}{rc^4}$$

Amplitude of GWs

For n star of radius R and frequency f and a bump of mass m:

$$v_{N.S.} = 2\pi R f.$$

The radiation amplitude is

$$h_{bump} \sim 2(2\pi R f/c)^2 Gm/rc^2$$

For a binary system (the dynamical f is about 2 f gw) of 2 stars of mass M and R is the semimajor orbit axis

$$f_{\rm dyn} = \frac{1}{2\pi}\,(\pi\, G_N \rho)^{1/2} \sim \left(\frac{G_N M}{16\pi\, R^3}\right)^{1/2}.$$

$$h \sim 2\left(\frac{G_N M}{Rc^2}\right)\left(\frac{G_N M}{rc^2}\right)$$

Internal gravitational potential of the system for a binary system

r=distance observer-source grav. potential of the system at observer location

$$\pi c^5/G_N = 1.1 \times 10^{53}\ \mathrm{W}$$

is an upper limit since the other factor <1

$$L \sim \frac{\pi c^5}{G_N}\left(\frac{G_N M}{Rc^2}\right)^5$$

$$3.6 \times 10^{52}\ \mathrm{W},$$

The luminosity of a source is

Sun luminosity $3.8 \times 10^{26}\ \mathrm{W}.$

Binary star system

2 n stars of 1.4 solar masses separated by 90 km at 15 Mpc distance (approximately distance of centre of Virgo cluster of galaxies). The evolution driven by gw emission is:

- inspiral phase: orbit shrinks as gw carry off energy and angular momentum
- eventually driving the 2 star to merge
- end product of the merger is a highly distorted star or a BH

$$h \approx 10^{-21} \left(\frac{15\,\mathrm{Mpc}}{r}\right) \left(\frac{M}{2.8\,M_\odot}\right)^2 \left(\frac{90\,\mathrm{km}}{R},\right)$$ Amplitude

$$f = \left(\frac{M}{2.8\,M_\odot}\right)^{1/2} \left(\frac{90\,\mathrm{km}}{R}\right)^{3/2} 100\,\mathrm{Hz},$$ Frequency

$$\tau_{\mathrm{gw}} = \left(\frac{R}{90\,\mathrm{km}}\right)^4 \left(\frac{2.8\,M_\odot}{M}\right)^3 0.5\,\mathrm{s}.$$ Duration of the source

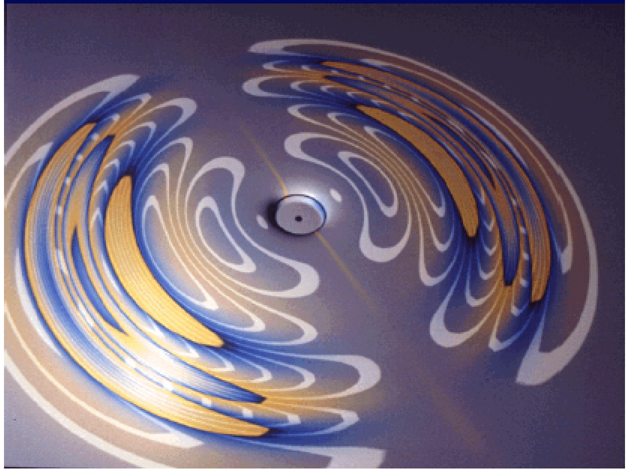
Examples

TABLE 1 Strain amplitude estimates for supernova and binary inspirals

Source	H	f (Hz)	M ($M_\odot$)	R (km)	r (Mpc)
Supernova	10^{-21}	—	1.4	—	10
NS-NS Inspiral	10^{-21}	100	2.8	90	15
MBH-MBH Inspiral	10^{-16}	10^{-4}	10^{7}	10	1000

- Man made GW are very weak to observe
- For typical astronomical sources $h \sim 10^{-21}$

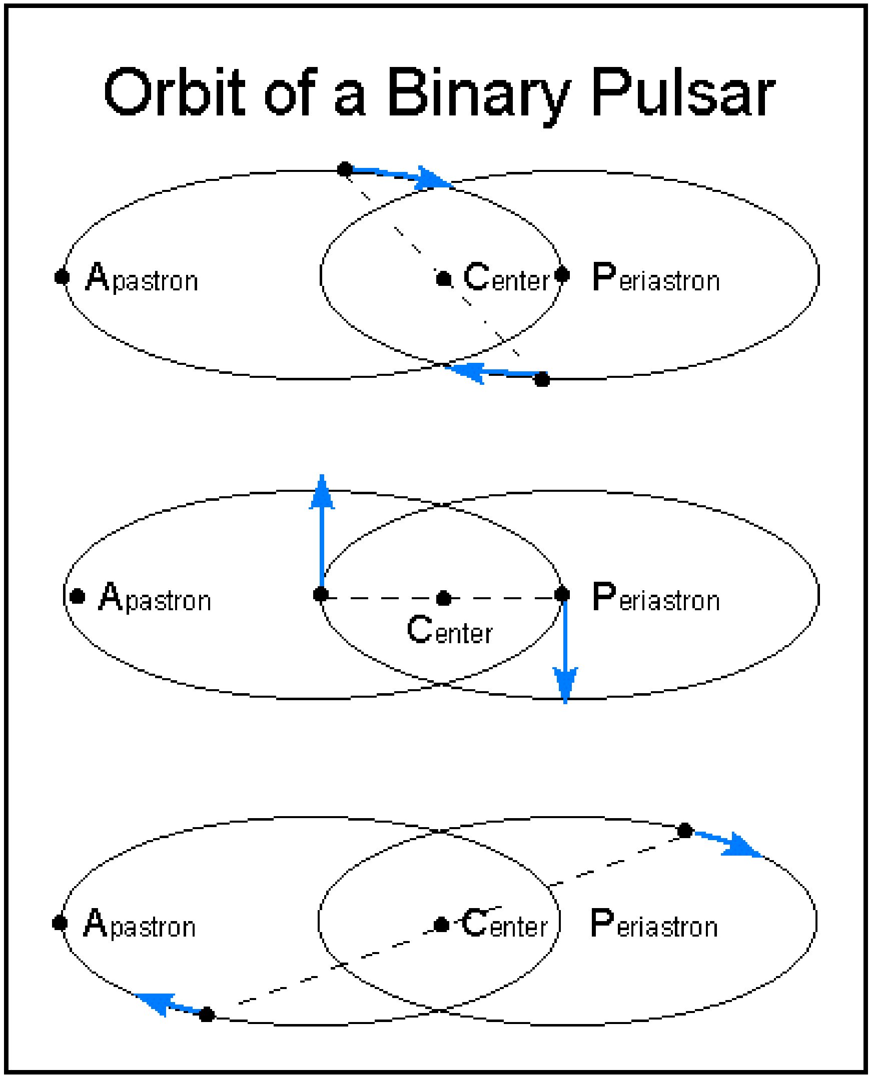
Collision of two black holes at 300 ly can generate detectable amplitudes



Do GW exist?

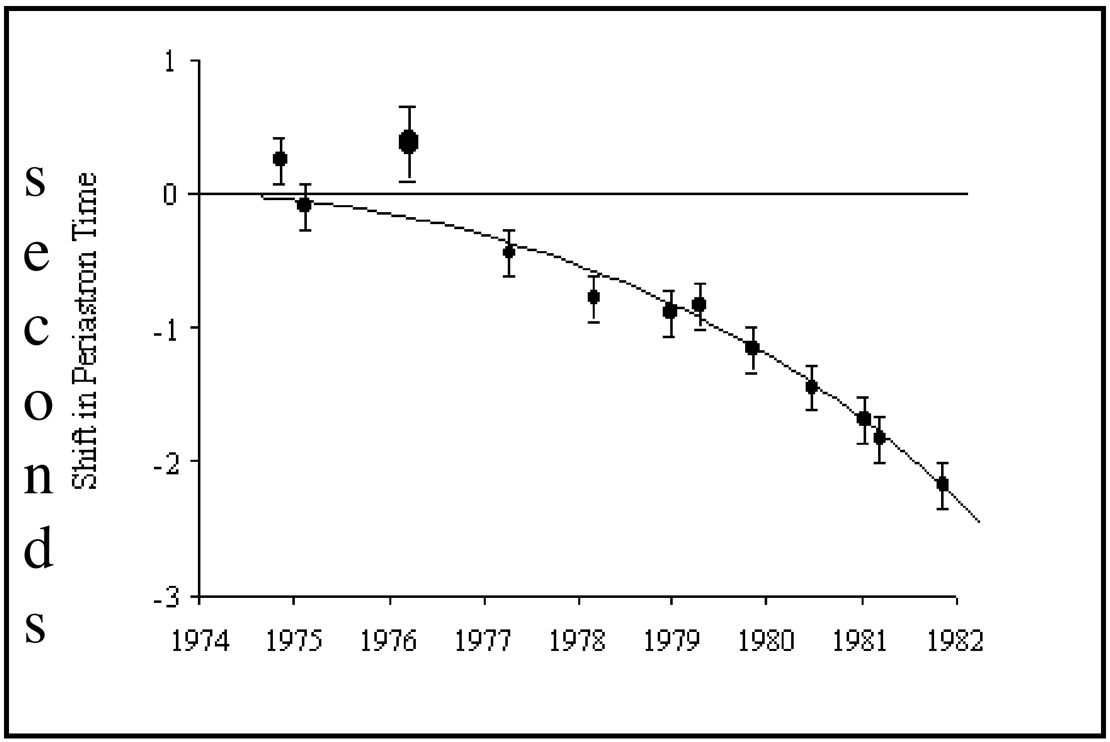
The fact that binary systems lose energy emitting gravitational radiation has turned out to be a useful tool for testing Einstein's theory of relativity.

1974 Hulse & Taylor discovery of the 1st binary pulsar PSR 1913+16 with the 300 m radio telescope at Arecibo, Puerto Rico: 2 n stars with 1.4 Msun and orbital period 0f 7.5 hrs. It was found that the orbit period is declining by about 1/75e6 of a second/yr. They reported a systematic shift in the observed time of periastron relative to that expected if the orbital separation remained constant.



Do GW exist?

In 1982 the pulsar was arriving at its periastron more than 2 s earlier compared to 1974. The orbit shrinks by 3.1 mm/orbit and the 2 stars will merge in 300e6 yrs



Signal frequency

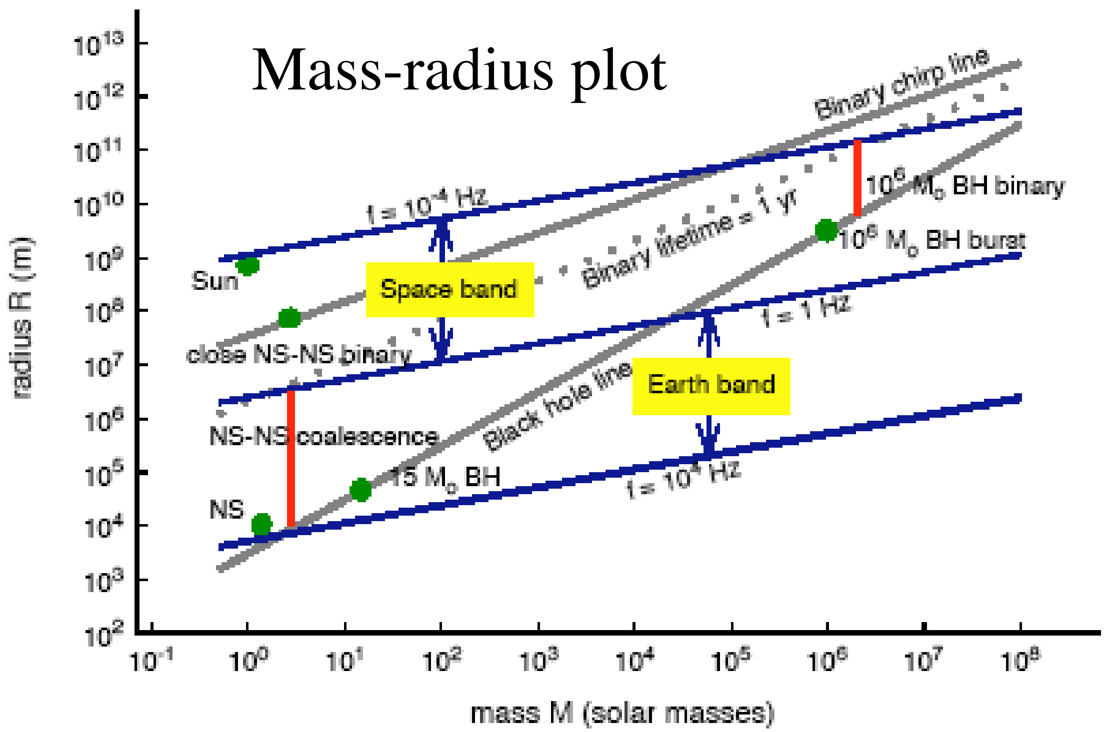
$\bar{\rho}$ is the mean density of mass-energy $f_0 = \sqrt{G\bar{\rho}/4\pi},$ $\boxed{\bar{\rho} = 3M/4\pi R^3.}$

Neutron Star: M = 1.4 Msun and R = 10 km f = 1.9 kHz

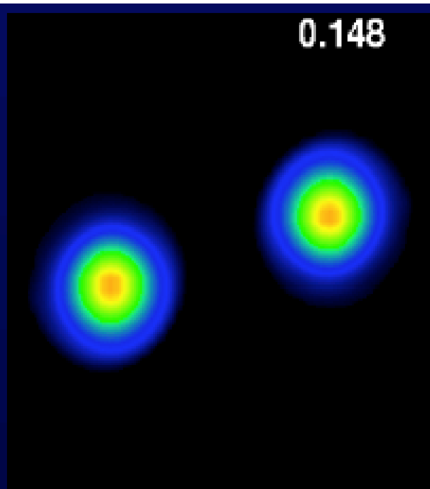
BH: 10 Msun and radius $2GM/c^2$ = 30 km it is f_0 = 1 kHz

For a large BH of 2.5e6 Msun (like the one in the centre of the Galaxy) 4 mHz

Gravitational Dynamics

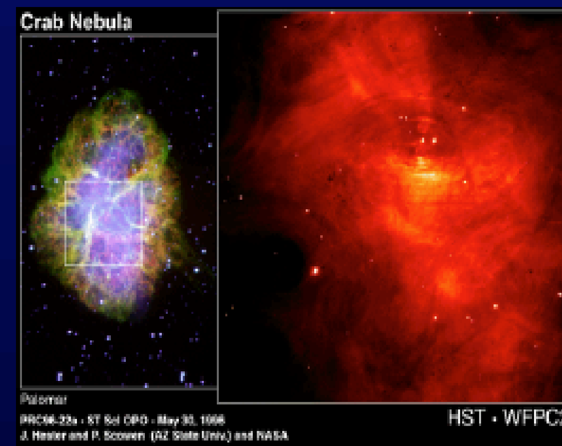


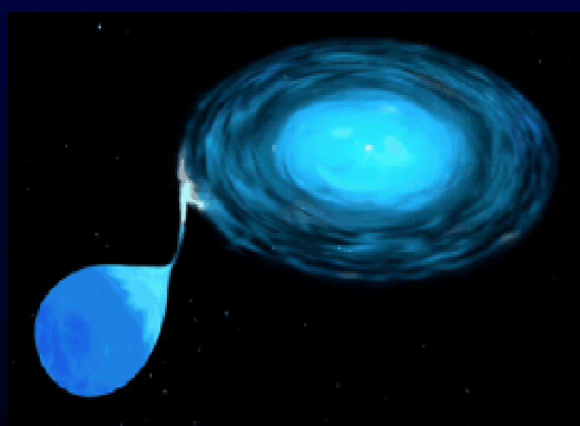
The signals for ground based detectors



Binaries of black holes and neutron stars

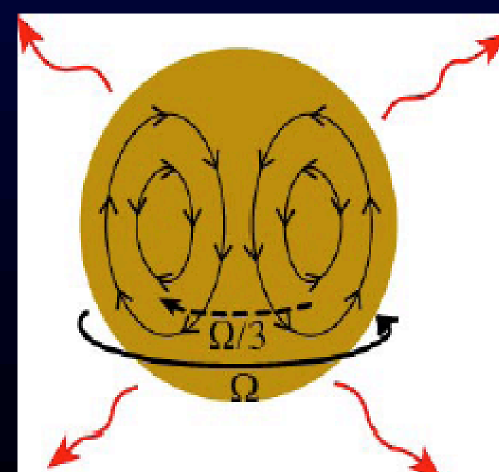
Supernovae and birth of NS and BH





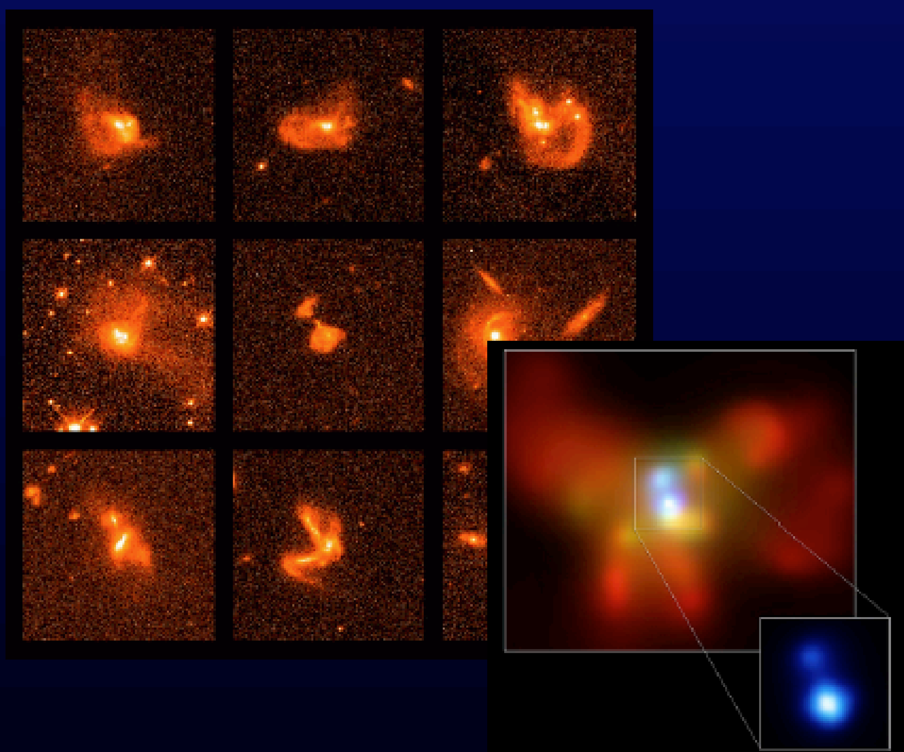
Spinning neutron stars in X-ray binaries

Relativistic Instabilities in young NS



And for space interferometers (LISA)

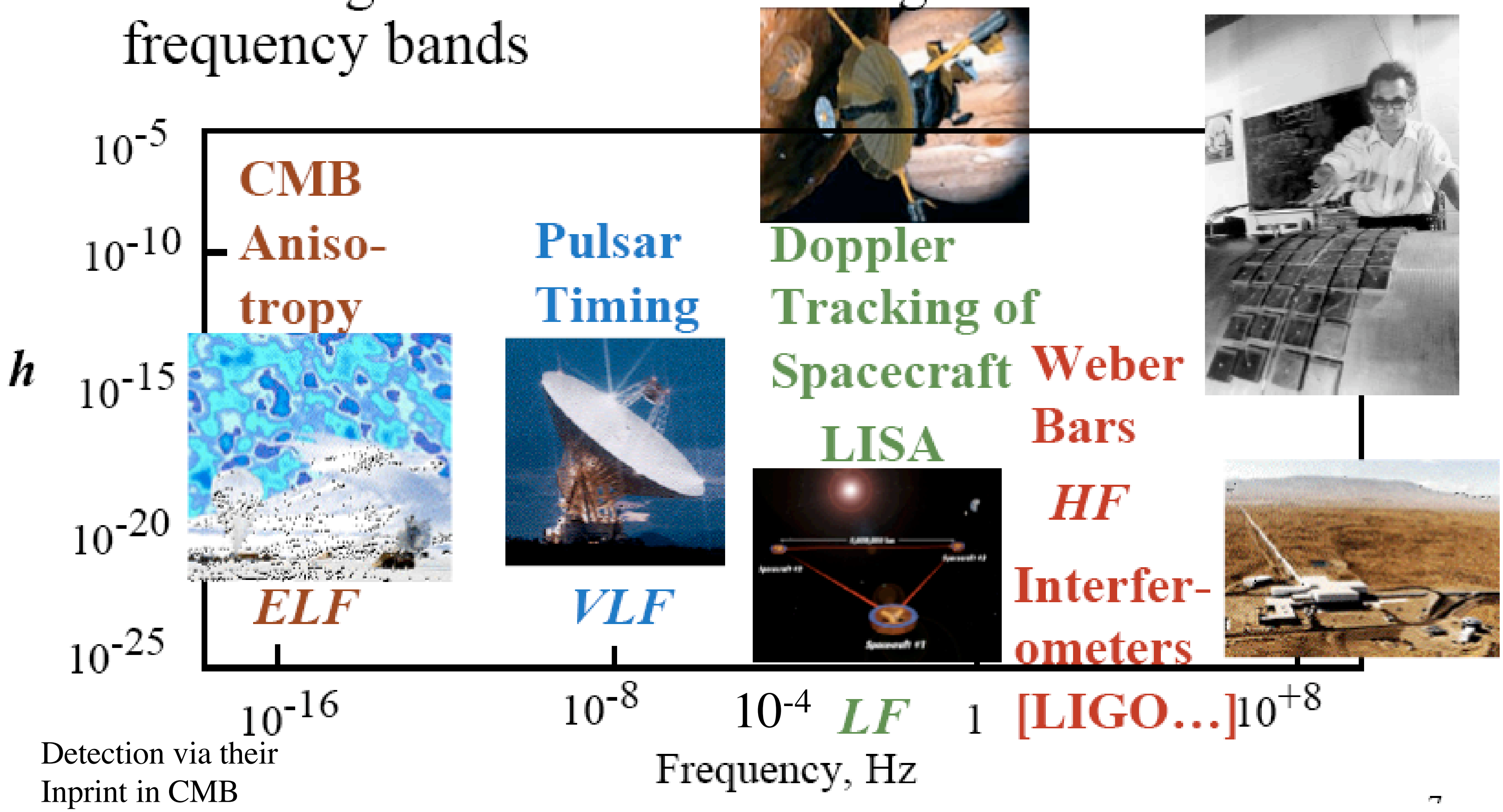
- Merging supermassive black holes (SMBH) in galactic centers
- Signals from gravitational capture of small BHs by SMBHs



X-ray image of a merging SMBH

Sources and experiments

- Spectrum of known and expected sources extends over 22 decades of frequency
- Promising sensitivities are being achieved in four frequency bands



The noise and sensitivity

2 noise, which competes with the actual gw signal by causing motion of experimental setup. A form of displacement noise is thermal noise

- sensing noise associated with the conversion of a small displacement into a readout signal

The output of gw detectors is a time series s(t) that includes instrument noise n(t) and the response to the gw signal h(t)

$$s(t) = F^{+}(t)h_{+}(t) + F^{\times}(t)h_{\times}(t) + n(t).$$

Strain amplitude spectral density

antenna patterns $F^{+}, F^{\times}$

$$h(f) = \sqrt{S_s(f)}. \quad S_s(f) = \tilde{s}^*(f)\tilde{s}(f)$$

Fourier transform of the time series:

$$\tilde{s}(f) = \int_{-\infty}^{\infty} e^{-2\pi i f t} s(t)\, dt.$$

Characteristic strain = rms signal in frequency interval of width Δf = f centered Around f

$$h_c(f) = \sqrt{f\, S_s(f)}.$$

Operation of bar detectors

When a GW passes through an object it exerts a tidal force causing its deformation. If the object is vibrating at a characteristic resonant frequency the deformation will appear as a deviation from its resonant ringing.
It is typically a massive cylinder of several tons. By measuring the acoustic signal at the ends of the bar the gw can be detected at around 700-900 Hz with band widths typically of 50 Hz.
Their sensitivity is sufficient to detect gravitational wave bursts that occur in the Galaxy.
Main noise: thermal motion of atoms the cause bar vibrations and ability to convert acoustic signals into electric ones. To suppress thermal noise bars are cooled at very low temperatures from several K to 0.1 °K.
At room temperature the rms amplitude of oscillation due to thermal noise is

$$x_{\rm rms} = \sqrt{\frac{kT}{4\pi^2 m f_0^2}}.$$

~10^{-16} m

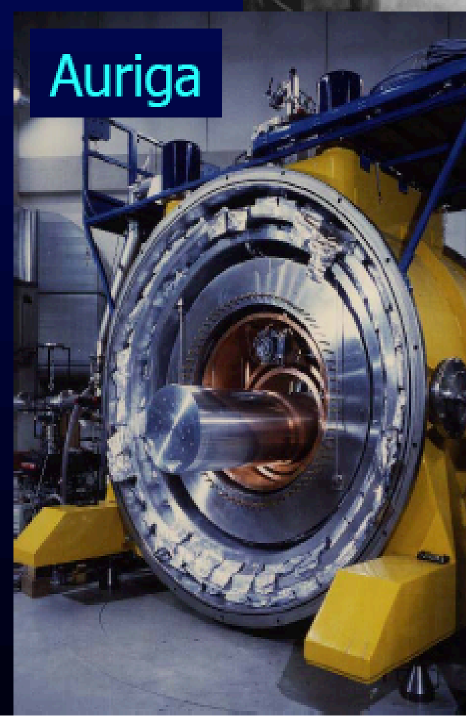
Also CR are a background.

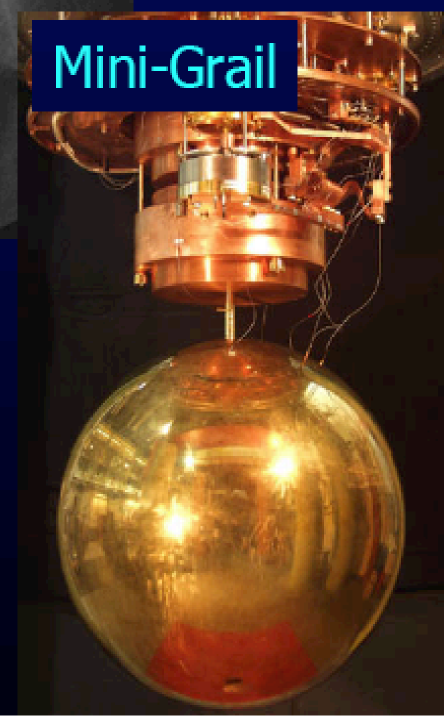
Detectors: antenna bars

http://sam.phys.lsu.edu/Overview/history.html

- A cryogenic aluminum or niobium bar with a transducer to measure the oscillations induced by the tidal interaction of GW and the bar
- Basically narrow band detectors with sensitivity around the bar's resonant frequency
- Several bars are currently in operation (3 in Europe, one in US, one in Australia)
- Sensitive to supernovae in our Galaxy - improved versions (spherical detectors) might help in probing neutron stars

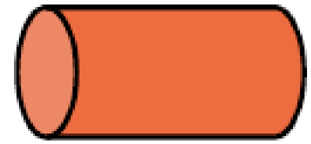
1966: Pioneering Antenna $h = 10^{-16}$

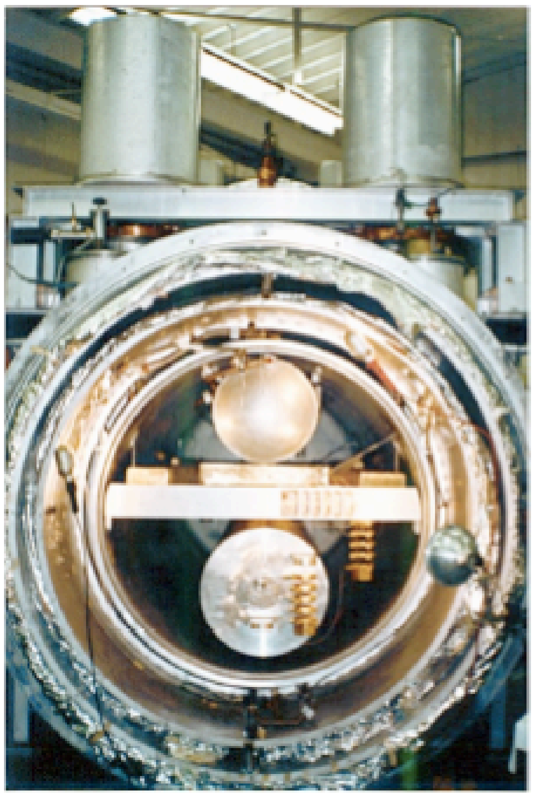




The world antenna bars in operation

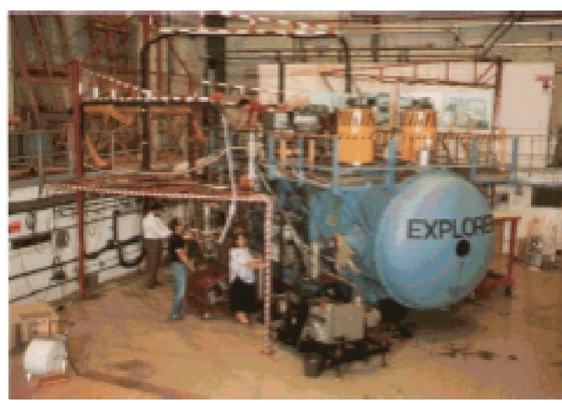
International Network of Bar Detectors
Now in Operation [~1000 Hz]

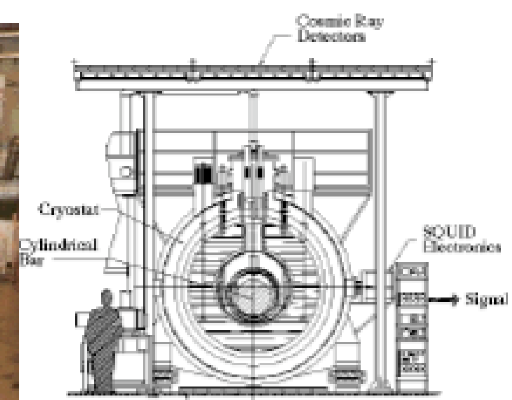



Louisiana State U. - Allegro


U. West Australia - Niobe

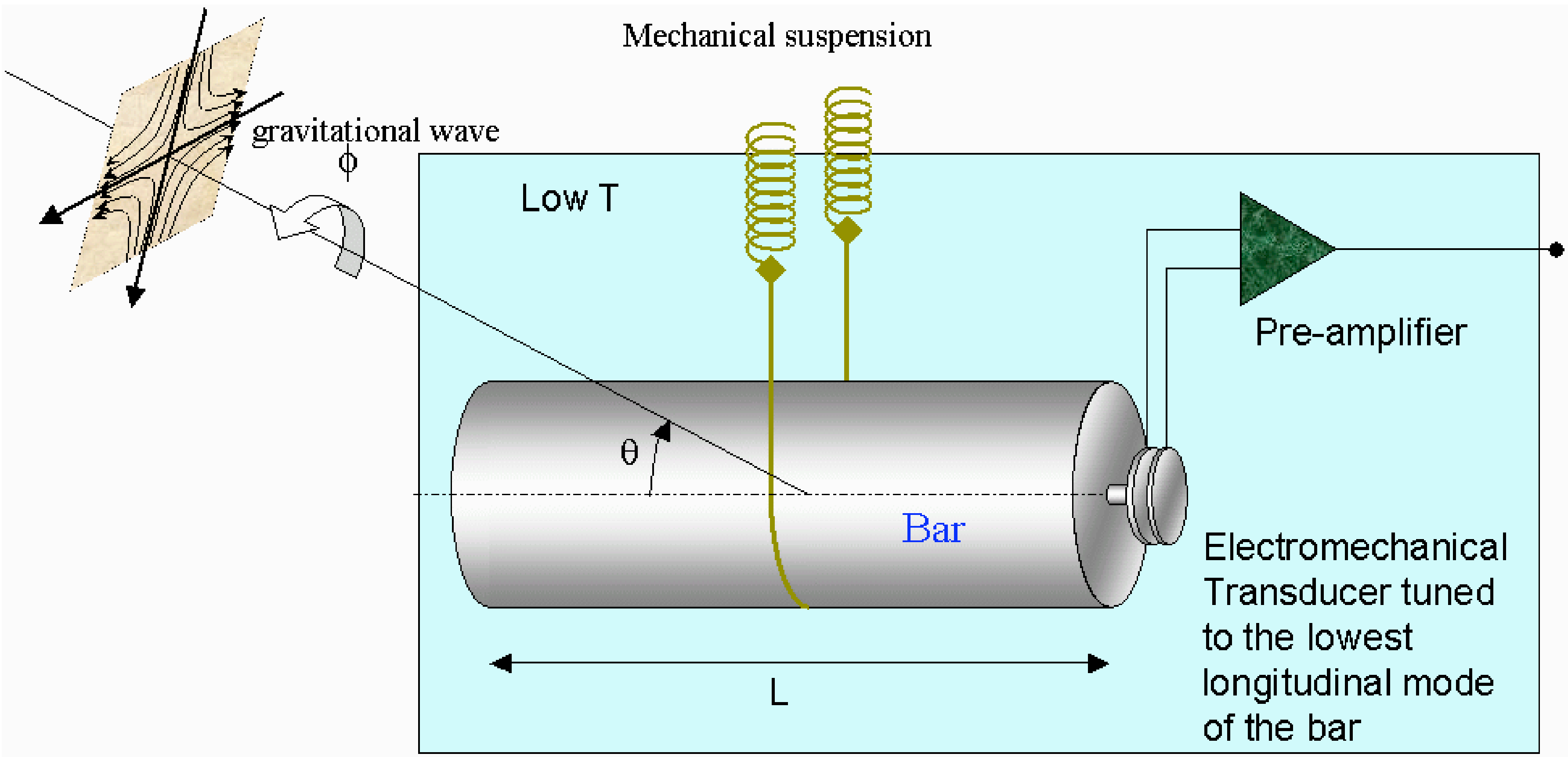

U. Padova - Auriga


CERN - Explorer



U. Rome - Nautilus

Scheme



http://www.auriga.lnl.infn.it/auriga/detector/overview.html

Transducer and the readout

The conversion of the mechanical vibration into an usable electromagnetic signal is accomplished by the transducer. Eg. A capacitive transducer is a plane plate capacitor with unperturbed capacitance C_0 biased at constant E_0 attached to 1 of the bar end faces. The other plate is fixed to a resonant body having the same frequency of the bar.

The bar-transducer system behaves as the system of two tuned harmonic oscillators coupled together: within a beat time the elastic energy of the main resonator is transferred to the lighter transducer producing a resonant plate displacement larger than the bar displacement by a factor equal to the square root of the bar-transducer effective mass ratio.

Any displacement x(t) of the bar end face produces a modulation of the transducer capacitance C(t): at first order in x(t) the voltage signal V(t) developed across the capacitor is:$V(t)=Q(t)/C(t) \sim E_0\, x(t) + q(t)/C_0$

Where q(t) is the time dependent transducer mass. The signal is then amplified by a SQUID (superconducting quantum interference device) for readout of the magnetic flux with low noise

Antenna characteristics

Characteristic	ALLEGRO	AURIGA	EXPLORER	NAUTILUS	NIOBE
Material	AL5056	AL5056	AL5056	AL5056	Nb
Mass (kg)	2296	2230	2270	2260	1500
Length (m)	3.0	2.9	3.0	3.0	2.9
$Q \times 10^6$	2	3	1.5	0.5	20
f_0 (Hz)	920 895	930 912	921 905	924 908	713 694
Δf (Hz)	1	1	9	1	1
Temperature (K)	4.2	0.2	2.6	0.1	5.0
Sensitivity ($\mathrm{Hz}^{-1/2}$)*	1×10^{-21}	2×10^{-22}	1×10^{-21}	2×10^{-22}	8×10^{-22}

The response of a bar of mass m, effective spring constant k, and damping term b to an external force F caused by the tidal acceleration of a gravitational wave is given by

$$G(f) = \frac{F/m}{\sqrt{\left(f_0^2 - 1/\tau_d^2 - (2\pi f)^2\right)^2 + (2\pi f b/m)^2}}, \qquad 28.$$

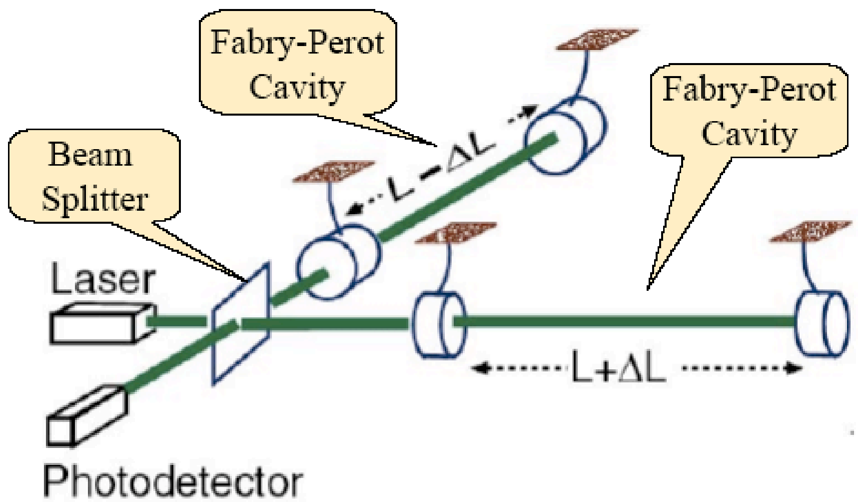
where f_0 is $\sqrt{(k/m) - (b^2/4m^2)}$ and where τ_d, the bar decay time, is $2m/b$. The bar's response at resonance is proportional to the bar's Q. To maximize the bar sensitivity, we seek the highest possible Q (equal to $2\pi f_0\, m/b$) and also seek to minimize the sensor and thermal noise.

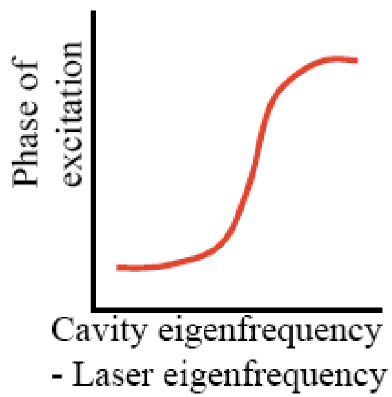
Detection with interferometers

Most promising technique use laser interferometry of the Michelson type
Light from a powerful laser is split into 2 long orthogonal paths and reflected against mirrors attached to test weights at the end of the arms. The 2 returning light beams are made interfere with each other creating interference fringes which would be stationary if the test bodies are at rest and their distance=const

A GW would cause a shift in the interference pattern. Cavities are resonant Fabry-Perot: light is allowed to travel many times through mirrors increasing the number of photons in the beams allowing a higher resolution Accuracy of 10^{-18} m can be achieved

- **Schematic description of detector:**





$$\Delta L = h\,L \lesssim 4\times10^{-16}\ \text{cm}$$

$h \lesssim 10^{-21}$, $L = 4$ km