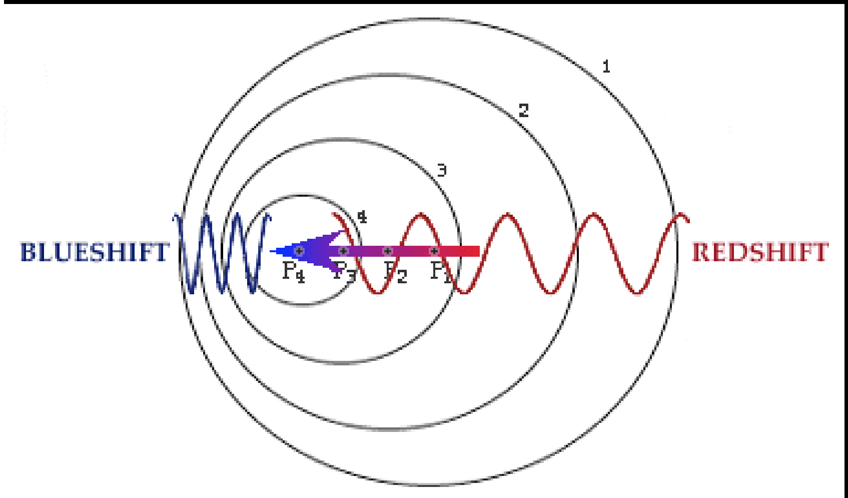
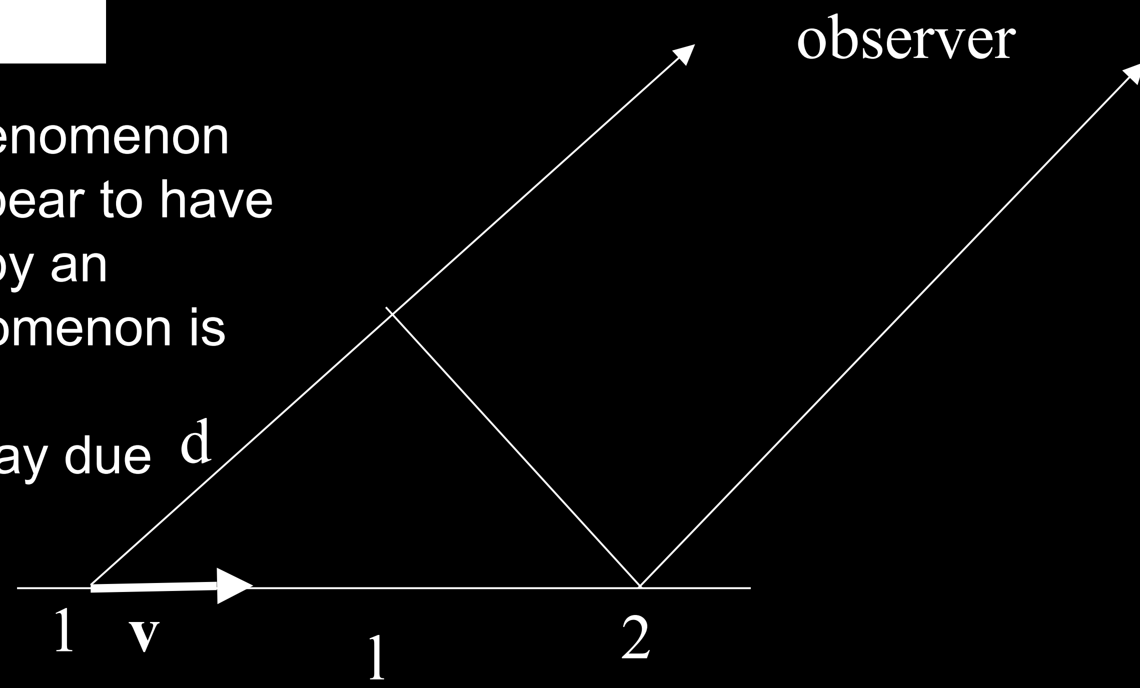
Doppler effect



The frequency of the radiation of an object moving towards an observer appears increased (blueshifted)
If the object moves away it appears decreased (redshifted)



Time dilation: any periodic phenomenon in the moving frame K' will appear to have a longer period when viewed by an observer in K where the phenomenon is 'at rest'
This in addition to the time delay due to light propagation makes the Doppler effect

Time dilation again

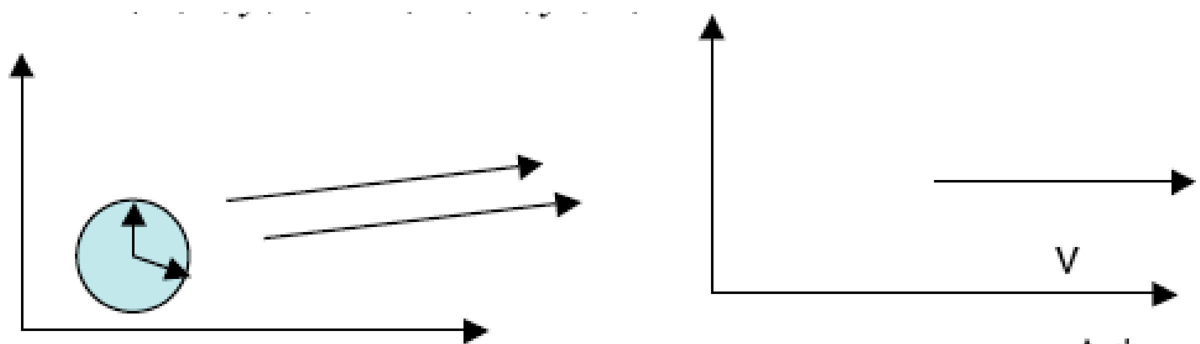
If K' is moving respect to K with velocity **v** parallel to x:
If the observer is at rest in K' and observes a clock at rest in K ($\Delta x = 0$) sending a periodic signal of period Δt, the period the observer will see is

$$\Delta t' = \gamma\Delta t - \gamma\beta\Delta x / c = \gamma\Delta t$$

The time dilation is the same if the clock is at rest in K' ($\Delta x'=0$) and the observer is measuring its period in K:

$$\Delta x' = 0 = \gamma(\Delta x - \beta c\Delta t) \Rightarrow \Delta x = \beta c\Delta t$$

$$\Delta t' = \gamma\Delta t - \gamma\beta\Delta x / c = \gamma\Delta t(1-\beta^2) = \gamma\Delta t / \gamma^2 = \Delta t / \gamma \Rightarrow \Delta t = \gamma\Delta t'$$



x1=x2, T=t2-t1

$$\Delta t' = \gamma(t_2 - t_1) - \gamma\beta(x_2 - x_1) / c$$

$$\Delta x' = \gamma(x_2 - x_1) - \gamma\beta c(t_2 - t_1)$$

Doppler effect

In the rest frame of the observer K the moving source emits one period of radiation as it moves from 1 to 2 at velocity v (hence emission of light at 1 and 2). The frequency of the emitted radiation in the source rest frame K' is ω'. The time taken taken to move from 1 to 2 in the observer frame is given by the time-dilation effect:

$$\Delta t = \frac{2\pi\gamma}{\omega'} \quad l = \mathrm{v}\Delta t \quad d = v\Delta t\cos\theta$$

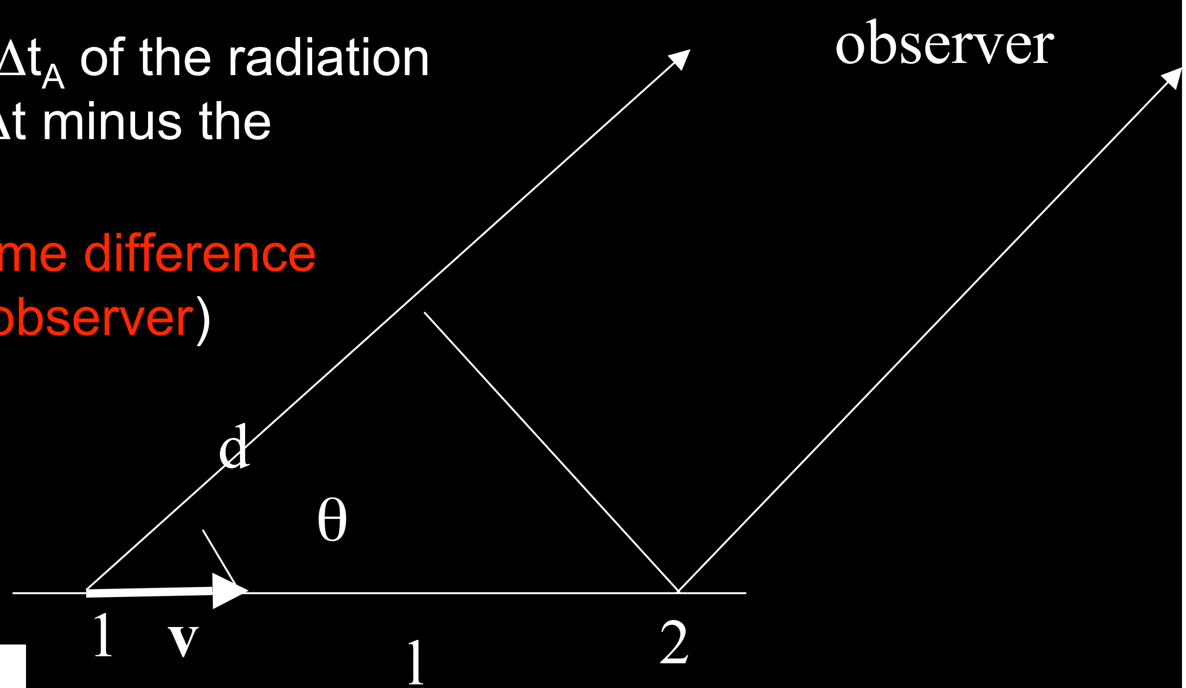
$$\Delta t = \gamma\Delta t'$$

The difference in arrival times Δt_A of the radiation emitted at 1 and 2 is equal to Δt minus the time taken for the radiation to propagate along d (apparent time difference between light received by the observer)

$$\Delta t_A = \Delta t - \frac{d}{c} = \Delta t\left(1 - \frac{\mathrm{v}}{c}\cos\theta\right)$$

And the observed frequency is

$$\omega = \frac{2\pi}{\Delta t_A} = \frac{2\pi}{\Delta t\left(1 - \frac{\mathrm{v}}{c}\cos\theta\right)} = \frac{\omega'}{\gamma\left(1 - \frac{\mathrm{v}}{c}\cos\theta\right)}$$



Relativistic Doppler formula

Doppler effect

The Doppler formula can be written as

$$\omega' = \omega\gamma\left(1 - \frac{v}{c}\cos\theta\right)$$

And the inverse is

$$\omega = \omega'\gamma\left(1 + \frac{v}{c}\cos\theta'\right)$$

For $\theta = 0$

$$\omega' = \omega\gamma\left(1 - \frac{v}{c}\right) = \omega\frac{(1-\beta)}{\sqrt{1-\beta^2}} = \omega\sqrt{\frac{(1-\beta)^2}{1-\beta^2}} = \omega\sqrt{\frac{1-\beta}{1+\beta}}$$

Doppler effect for light

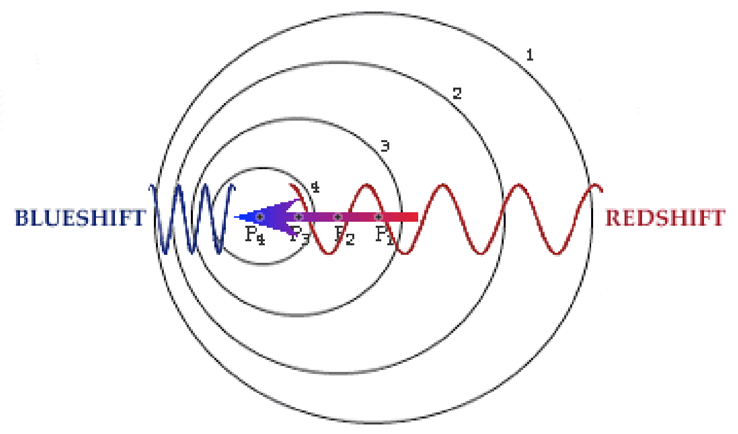
$$\nu_{observed} = \nu_{source}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

$\omega = \omega_{observed} > \omega' = \omega_{source}$ if source is approaching the observed frequency is larger than the source one and the wave length is squeezed

$\omega = \omega_{observed} < \omega' = \omega_{source}$ if source is departing the observed frequency is smaller than the source one and the wave length is stretched (redshift)

The factor γ^{-1} is a purely relativistic effect, the factor 1-(v/c)cosθ appears even classically.

Doppler effect



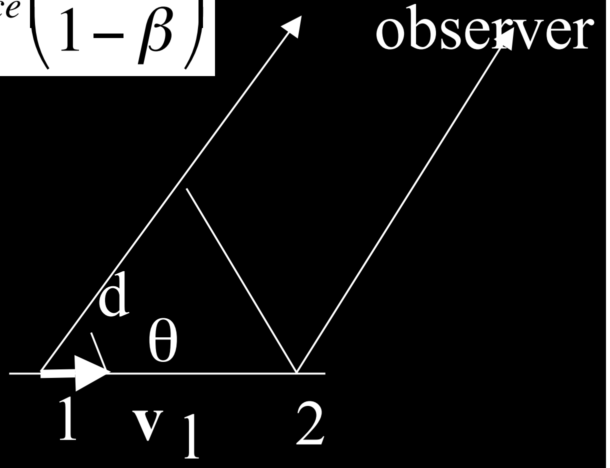
The frequency of the radiation of an object moving towards an observer is squeezed (blueshifted)
If the object moves away it is streched (redshifted)

If the object moves towards the observer θ= 0

$$\omega' = \omega\gamma\left(1 - \frac{v}{c}\cos\theta\right)$$

$$\omega' = \omega\gamma\left(1 - \frac{v}{c}\right)$$

$$\omega_{observed} = \omega_{source}\left(\frac{1+\beta}{1-\beta}\right)$$



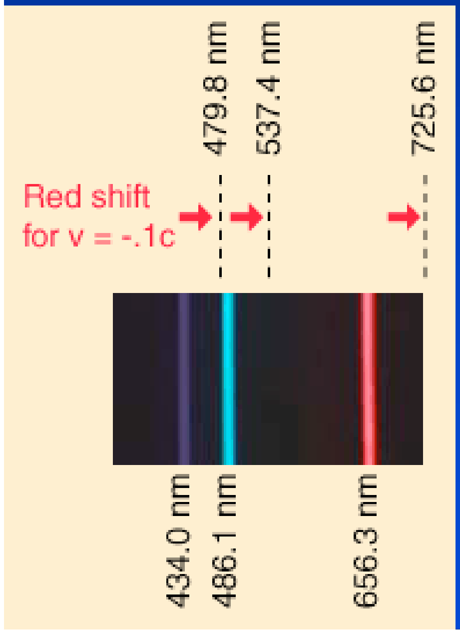
If it moves in the opposite direction θ = 180 deg

$$\omega' = \omega\gamma\left(1 + \frac{v}{c}\right)$$

$$\omega_{observed} = \omega_{source}\left(\frac{1-\beta}{1+\beta}\right)$$

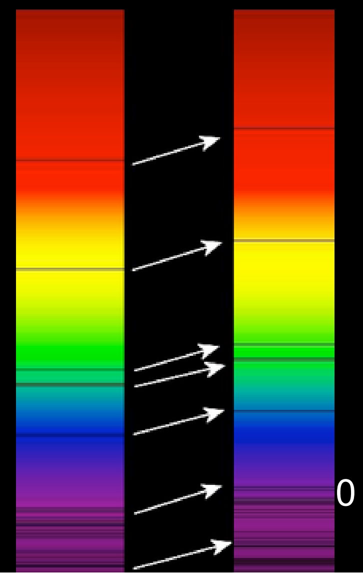
Doppler effect in astronomy

It is used in astronomy to measure the speed at which galaxies and stars are approaching or receding from us, ie their radial velocity.
Spectra exhibit absorption lines at well defined frequencies correlated with the energies required to excite electrons in various elements from 1 level to another. The spectral lines from an approaching object show a blue shift and receding ones a red shift so their spectrum differ compared to what obtained for steady sources. The redshift effect for far galaxies is caused by the expansion of the Universe.



H redshift

Redshift of spectral lines in the optical spectrum of a supercluster of distant galaxies as compared to the Sun



Exercises 1

1)

Calculate the minimum photon energy in the rest frame of the proton for direct pion production in the reaction $p\gamma \rightarrow \pi^+ + X$, given that the mass of the charged pion is 139.57 MeV. Baryon number and charge must be conserved in the reaction.

2)

Calculate the minimum projectile energy in the rest frame of the target proton for a pp reaction that produces a single π^0 (remember Baryon conservation!). The rest mass of the π^0 is 134.97 MeV, and the mass of the proton is $m_p = 938.27$ MeV.

3)

Demonstrate that the neutrino takes ~1/4 of the pion energy (rest mass) in the charged pion decay: $\pi^+ \rightarrow \mu^+ + \nu_\mu$, working in the rest frame of the pion. The muon mass is 105.66 MeV.

Exercise 2

An object emits a blob of material at speed v at an angle θ to the line-of-sight of a distant observer.

1) Show that the apparent traverse velocity inferred by the observer is

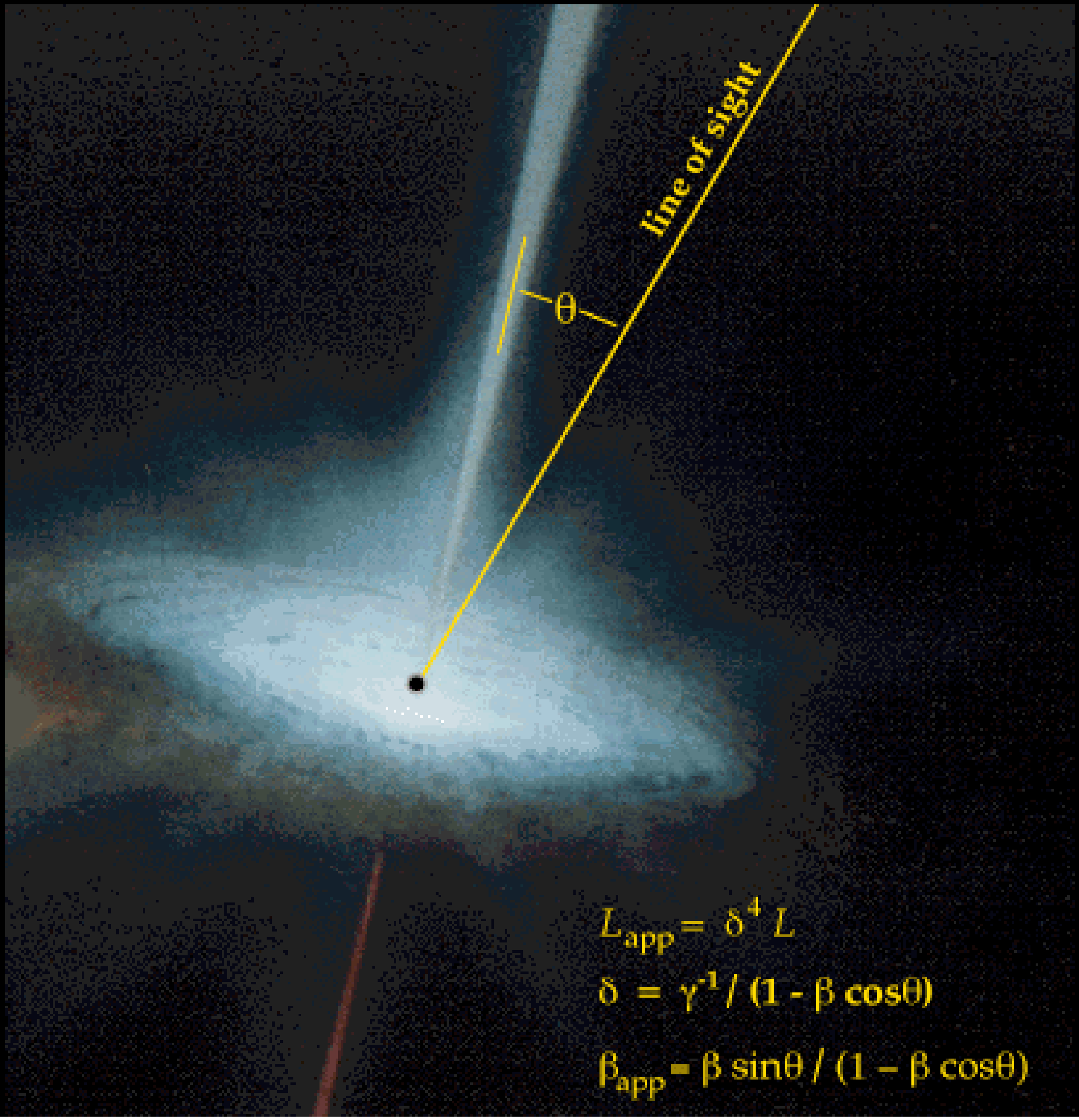
$$v_{app} = \frac{v \sin\theta}{1 - (v/c)\cos\theta}$$

2) Show that v_{app} can exceed c; find the angle for which v_{app} is maximum and show that this maximum is $v_{max} = \gamma v$

Superluminal motion

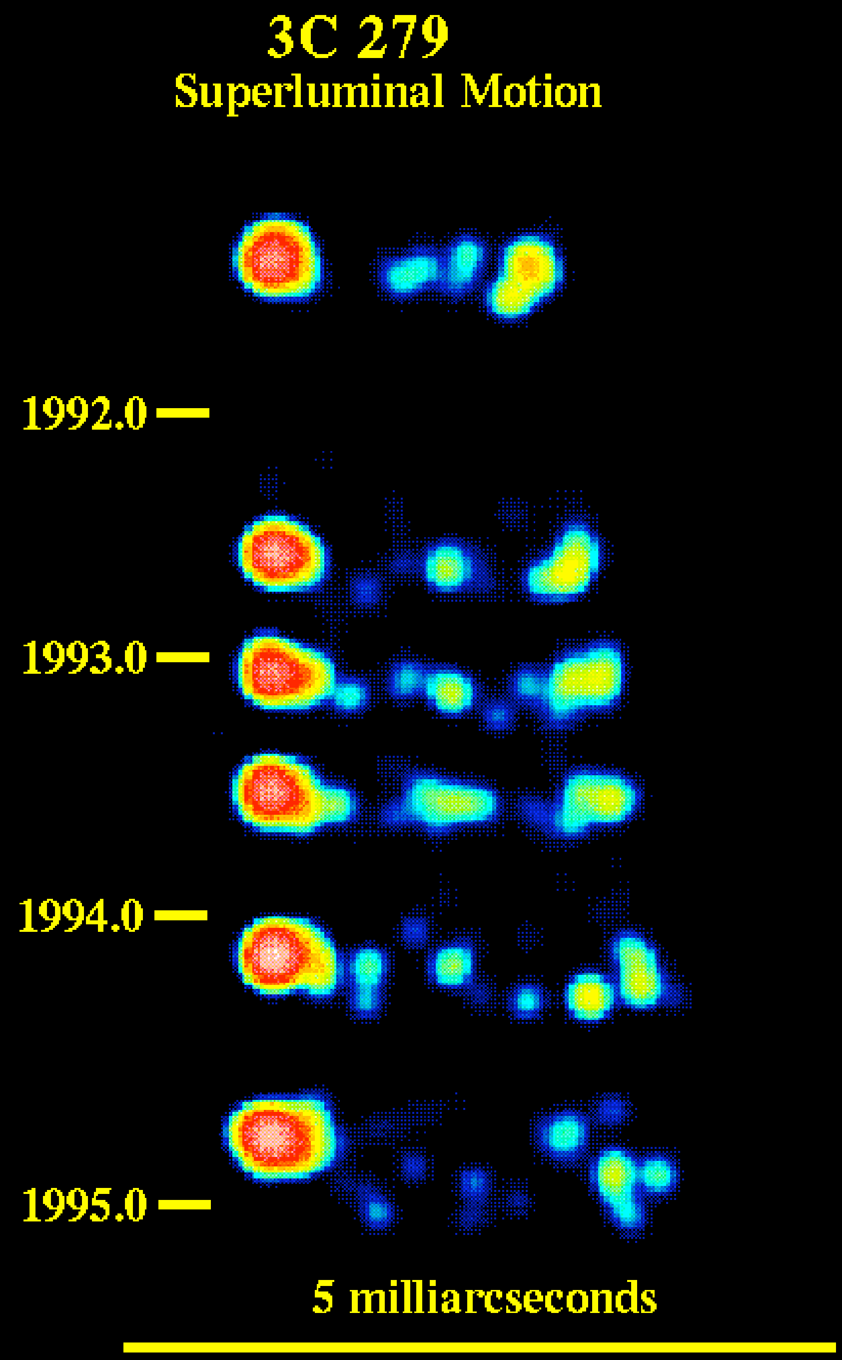
$$\omega_{obs} = \frac{\omega_{em}'}{\gamma(1-\beta\cos\theta)}$$

$$\beta_{app} = \frac{\beta\sin\theta}{(1-\beta\cos\theta)}$$



AGNs

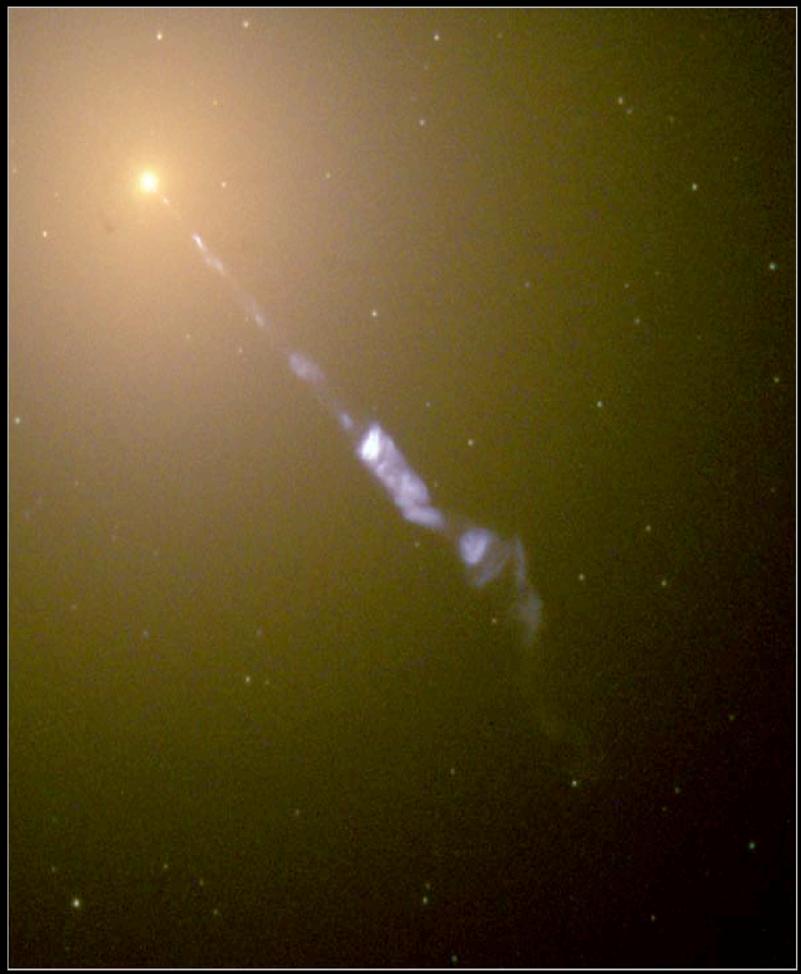
- Source: Wehrle et al., ApJ
- Bill Keel's WEB page: http://www.astr.ua.edu/keel/agn/
- Data at 22 GHz with resolution 0.2 marcsec (2 light years)
- 1 m arcsec= 10 light years
- BL Lacertae exhibit motion of materials along their jets faster than velocity of light (superluminal motion). The sources with superluminal motion are typically pointing towards us and bright for Doppler boosting. Here the prominent outer knots move at 4c



HST image of M87 jet

Superluminal motion has been detected also in optical

The M87 Jet



Hubble Heritage

PRC00-20 • Space Telescope Science Institute • NASA and The Hubble Heritage Team (STScI/AURA)

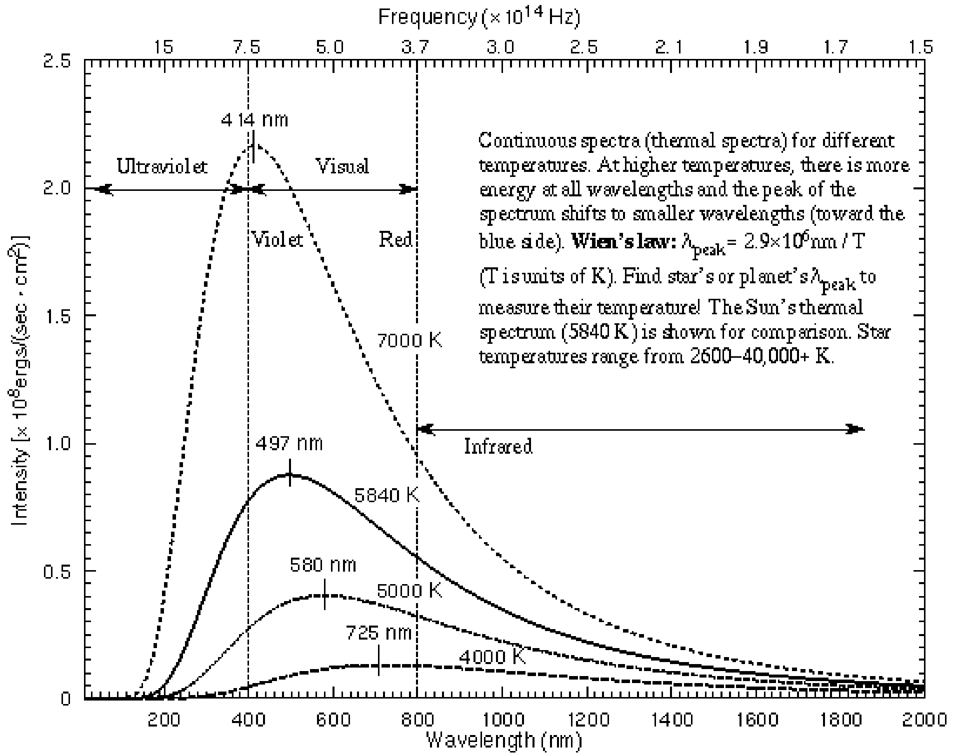
The GRBs and the fireball

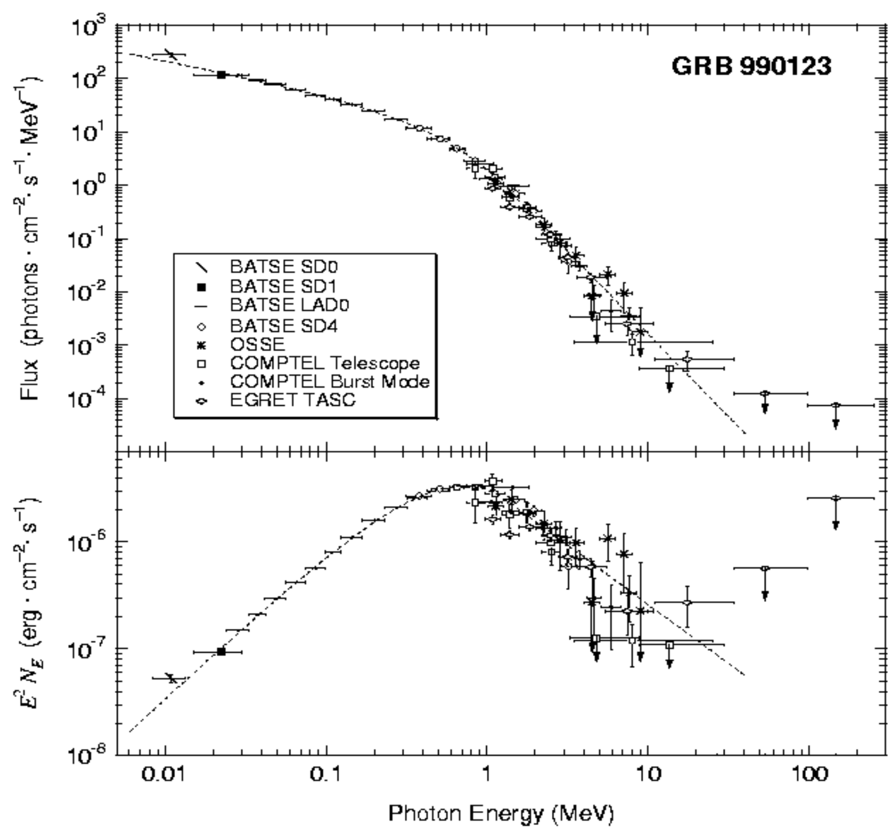
The fireball model was born to explain the ‘compactness problem’ of GRBs.

The observed variability of GRBs has a time scale ~ ms

If the initial energy is emitted by a source of radius R <c dt ~300 km the optical depth for pair production should be very high (γs do not emerge from the source) and the observed spectra should be thermal (typical of a black body) contrary to observations (power laws)

Solution: relativistic motion





The fireball model

$$\Delta t_{apparent} = \Delta t(1-\beta\cos\theta) \approx \frac{\Delta R_{source}}{c}(1-\beta\cos\theta)$$

$$\beta \approx 1 \Rightarrow \gamma^2 = \frac{1}{1-\beta^2} = \frac{1}{(1+\beta)(1-\beta)} \approx \frac{1}{2(1-\beta)}$$

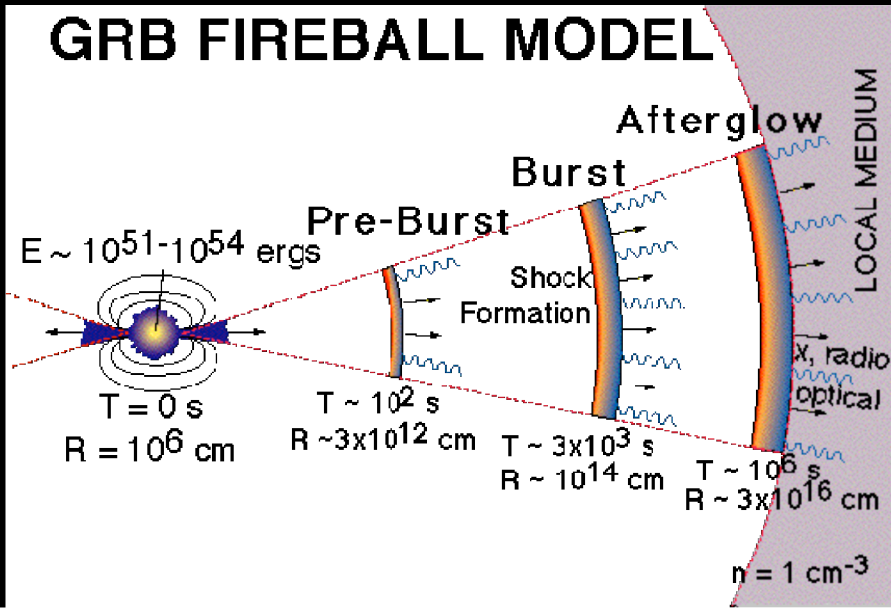
$$\theta \quad \text{small} \Rightarrow \cos\theta \approx 1 \Rightarrow \Delta t_{apparent} = \Delta t(1-\beta) \approx \frac{\Delta R_{source}}{2c\gamma^2}$$

$$\Delta R_{obs} \approx c\Delta t_{apparent} = \frac{c\Delta R_{source}}{2c\gamma^2} = \frac{\Delta R_{source}}{2\gamma^2}$$

The dimension of the source seen by the observer is reduced by γ^2 so the source can be much larger and the opacity is much lower

$\gamma \approx 1000$ for GRBs

A fireball (γ, $e^{\pm}$, p) forms due to the high energy density, that expands. When it becomes optically thin it emits the observed radiation through the dissipation of particle kinetic energy into relativistic shocks



Suggested readings

L. Bergstrom and A. Goobar, Cosmology and Particle Astrophysics (2nd edition), Springer 2004 cap 1-2

R. Hagedorn, Relativistic kinematics, W.A. Benjamin INC,1963 cap 1

G.B. Rybicki and A.P. Lightman, Radiative processes in astrophysics, Wiley & Sons, 1979

http://en.wikipedia.org/wiki/Special_Relativity

http://www.mathpages.com/rr/rrtoc.htm

Connecting quarks with the cosmos http://www.icecube.wisc.edu/~tmontaruli/connecting_quark_cosmos.pdf