Electromagnetic Interactions of Radiation in Matter

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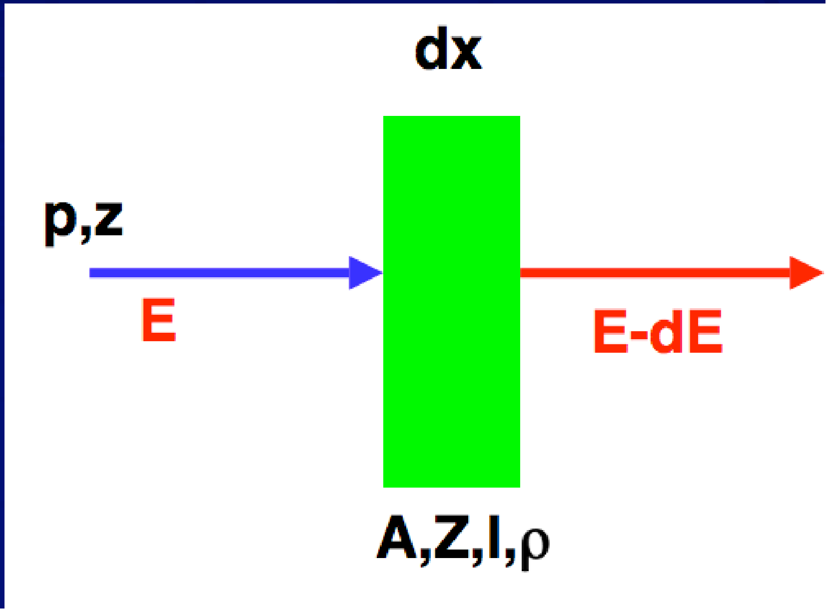
Energy loss of charged particles

Charged particles interact with a medium via em interactions by the exchange of photons. If the range of photons is short, the absorption of virtual photons constituting the field of the charged particle gives rise to ionization of the material. If the medium is transparent Cherenkov radiation can be emitted above a certain threshold. But also sub-threshold emission of electromagnetic radiation can occur, if discontinuities of the dielectric constant of the material are present (transition radiation). The emission of real photons by decelerating a charged particle in a Coulomb field is called bremsstrahlung.

Bethe-Block: mean energy loss in scattering of charged particles off electrons in a medium (MeV/g/cm^2)



Collision energy loss of massive charged particles textbook: Leroy-Rancoita

For an incoming particle of mass $m>m_e$, velocity $v = \beta c$ and charge z the energy loss by collision is given by the Bethe-Block formula

$$-\frac{dE}{dx} = \frac{2\pi n z^2 e^4}{mv^2}\left\{\ln\left[\frac{2mv^2 T_{\max}}{I^2(1-\beta^2)}\right] - 2\beta^2 - \delta - U\right\}$$

Where n = # of electrons/cm^3 of the material = $(Z\rho N_A)/A$ (A,Z=atomic weight and number)

I = 11.5 Z (eV) = mean excitation potential of the atoms in the material

T_{max} = max transferable energy from the incident particle to atomic electrons

$$T_{\max} = \frac{2m_e c^2 \beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

and for $m>>m_e$

$$T_{\max} = 2m_e c^2 \beta^2 \gamma^2$$

Exercise 4

δ = density effect correction

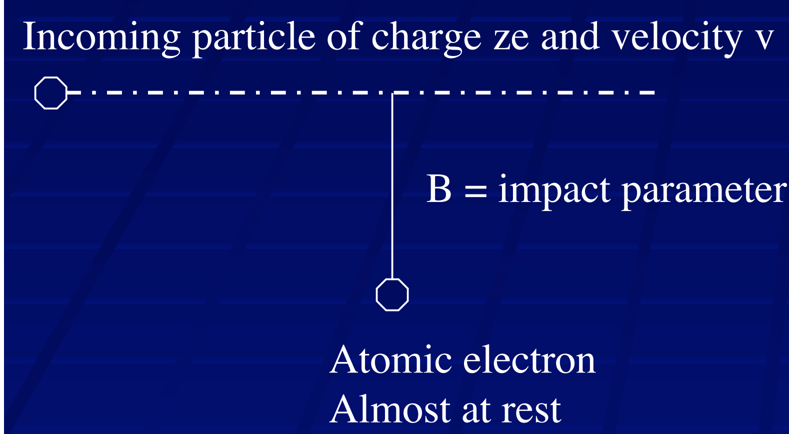
U = shell correction term (important for low kinetic energies of incoming particle related to the non-partecipation of inner shell (K, L, …) electrons

For heavy particles dE/dx is called stopping power

Derivation of the formula

If the electron is at rest the transferred impulse is ortogonal to the particle direction of motion and the magnitude of the Coulomb force along the perpendicular direction is

$$F_{\perp} \approx \frac{ze^2}{b^2}$$



Since the interaction time is ≈ b/v the transferred momentum is

$$I_{\perp} \approx \int F_{\perp} dt \approx \frac{ze^2}{bv}$$

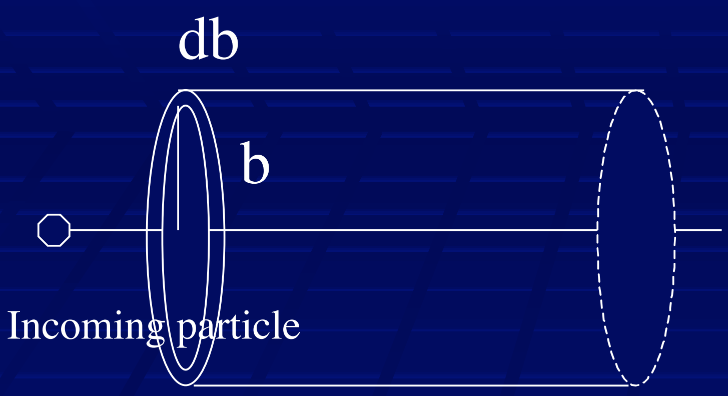
If relativistic corrections are accounted for

$$I_{\perp} = \frac{2ze^2}{bv}$$

And the electron kinetic energy

$$W = \frac{I_{\perp}^2}{2m_e} = \frac{2z^2e^4}{m_e b^2 v^2}$$

Energy loss by collisions



The number of electrons encountered between b and db is n(2πb)dbdx and the overall electron kinetic energy is

$$W_b = \frac{2z^2e^4}{m_e b^2 v^2} n 2\pi b db dx = \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b}dx$$

And the energy lost by the particle per unit path is

$$\frac{-dE_b}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b}$$

and integrating in the range of impact parameters:

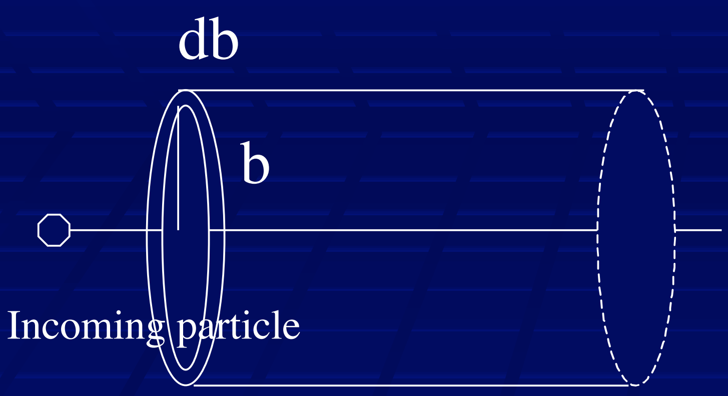
$$-\frac{dE}{dx} = \int_{b_{\min}}^{b_{\max}} \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2}\ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

b_{max}= collision time cannot exceed the typical period of bound electrons on their orbit $\tau \approx 1/\nu$ with ν mean frequency of excitation of electrons.

The region of space at the maximum field strength is relativistically contracted, hence $\tau \approx 1/\nu \approx b_{max}/\gamma v$ and introducing the mean excitation potential $I = h\nu \Rightarrow b_{max} = \gamma v h/I$.

b_{min} is evaluated considering the extent to which classical approach (not wave) can be adopted $b_{min} \approx h/p_e \Rightarrow b_{min} = h/(2m_e\gamma\beta c)$ where the max momentum the electron can acquire is $2m_e v$

Energy loss by collisions



$$-\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2}\ln\left(\frac{b_{max}}{b_{min}}\right)$$

$$T_{max} = 2m_e c^2\beta^2\gamma^2$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2}\ln\left[\left(\frac{vh\gamma}{I}\right)\left(\frac{2m\gamma\beta c}{h}\right)\right] = \frac{2\pi n z^2 e^4}{m_e v^2}\ln\left(\frac{2m\gamma^2 v^2}{I}\right)^2 = \frac{2\pi n z^2 e^4}{m_e v^2}\ln\left(\frac{2mv^2 T_{max}}{I^2(1-\beta^2)}\right)$$

This is the energy loss formula except for some correction terms

$$\frac{2\pi n z^2 e^4}{m_e v^2} = 2\pi N_A m_e c^2 r_e^2\left(z^2\rho\frac{Z}{A}\frac{1}{\beta^2}\right)$$

$n=(Z\rho N_A)/A$

where classical radius of electron $r_e = e^2/(m_e c^2)$

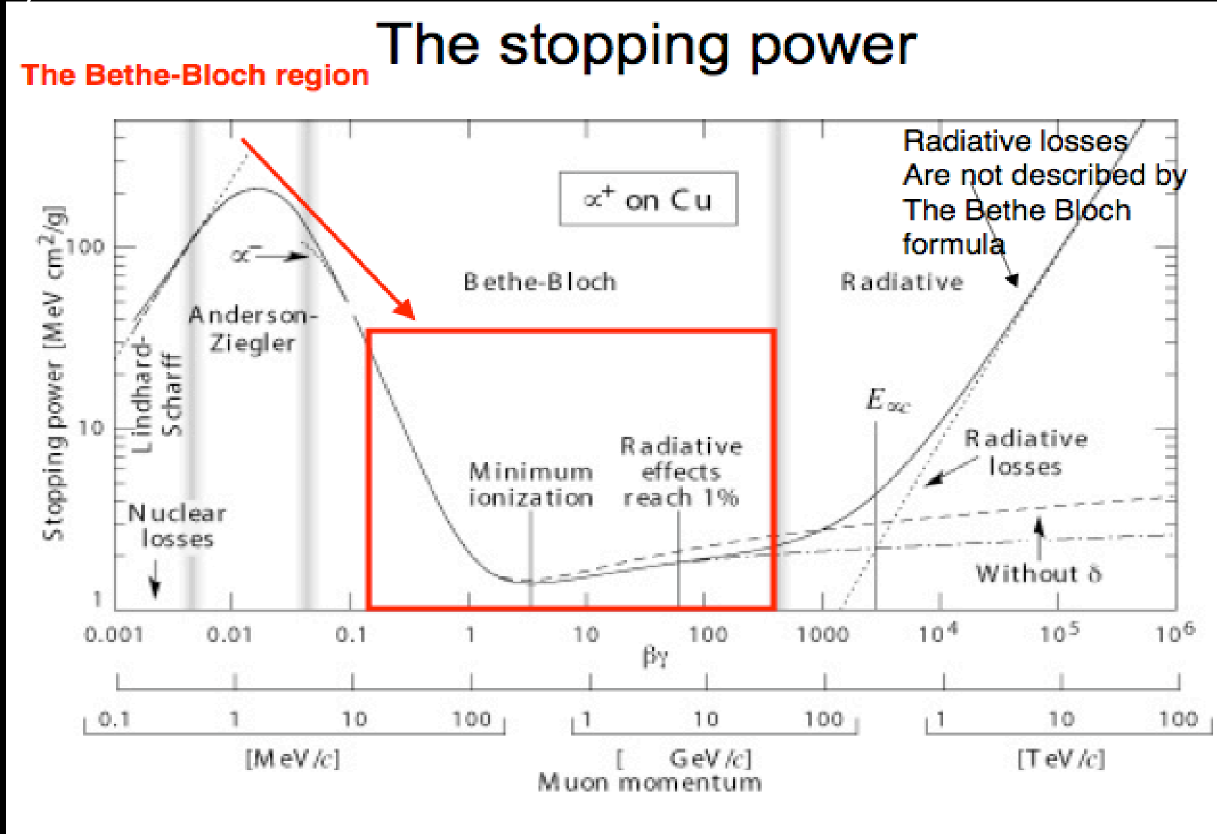
$$-\frac{dE}{dx} = 0.1535\left(z^2\rho\frac{Z}{A}\frac{1}{\beta^2}\right)\left\{\ln\left[\frac{2m_e v^2 T_{max}}{I^2(1-\beta^2)}\right] - 2\beta^2 - \delta - U\right\} MeV/cm$$

$$X = \rho x (g/cm^2) \Rightarrow \frac{dE}{dX} = \frac{1}{\rho}\frac{dE}{dx}\, in\left(MeV/g/cm^2\right)$$

Bethe-Block

Stopping power of positive muons in copper vs $\beta\gamma = p/Mc$. The slight dependence on M at highest energies through T_{max} can be used for PID but typically dE/dx depend only on β (given a particle and medium)

At low β $-dE/dx \propto 1/\beta^2$ decreases rapidly as β increases. At relativistic velocities $\beta \approx 1$ and reaches a min at $\beta\gamma \approx 3$ (a particle at the energy loss min is called mip). Beyond the min the energy loss increases logarithmically (due to the increase of T_{max} and b_{max}).



However as the range of distant collisions extends, the atoms close to the path of the particle will produce a polarization which results in reducing the electric field strength acting on electrons at large distances Density effect: δ/2

The relativistic rise depends on ln(βγ) but in the ultrarelativistic region only on lnγ hence on the particle mass (used for PID)