Pair production

If $h\nu > 2mc^2 \approx 1.02$ MeV the production of a pair of electron and positron becomes possible close to a charged massive object (eg. a nucleus) which takes away the amount of momentum needed to preserve momentum conservation during the interaction with the Coloumb field of the massive object itself. The probability of pair production is a slowly increasing function of energy while the Compton probability decreases rapidly with increasing energy

Exercise 3: calculate the threshold energy of the photon E_{th} = ?

Compton: For small energy the cross section reduces to Thompson one

$$\varepsilon \to 0 \Rightarrow \sigma_C \to \sigma_{Th} = \frac{8}{3}\pi r_e^2$$

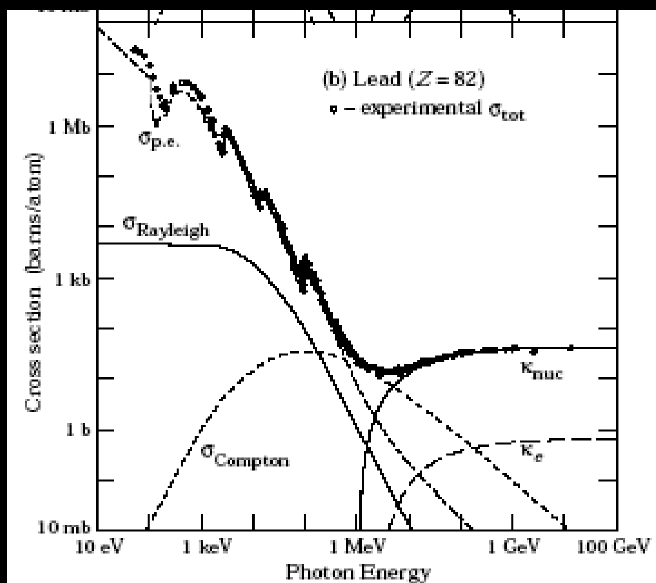
$$\varepsilon >> 1 \Rightarrow \sigma_C \to \frac{3}{8}\sigma_{Th}\frac{2\ln(2\varepsilon)+1}{\varepsilon}$$

$$\varepsilon = \frac{h\nu}{mc^2}$$

At high energy pair production dominates (Bethe-Heitler)

$$\sigma_{pair}(E) \approx \alpha Z^2 r_e^2 \ln E_\gamma$$

At low energies photoelectric absorption on atomic shells



Pair production

The differential cross section has the form

$$\frac{d\sigma}{dE} = \frac{A}{X_0 N_A}\left[1 - \frac{4}{3}x(1-x)\right]$$

$x = E/k$

Fractional energy transfer to the electron/positron (k incident γ energy)

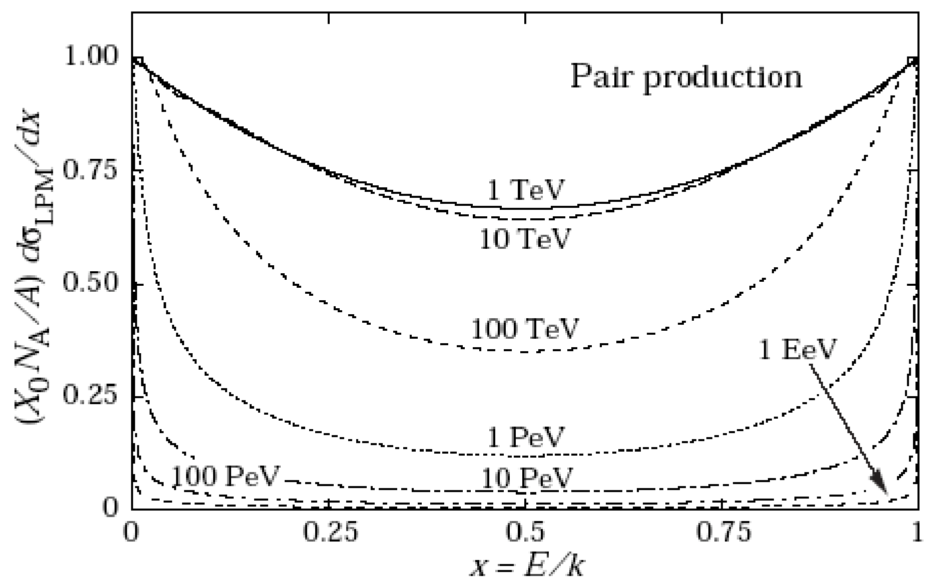
$$1 << \frac{h\nu}{m_e c^2} << \frac{1}{Z\alpha^{1/3}} \Rightarrow \sigma_{pair} = \alpha Z^2 r_e^2 \left[\frac{28}{9}\ln\left(\frac{2h\nu}{m_e c^2}\right) - \frac{218}{27}\right] m^2 atom^{-1}$$

No screening case

$$\frac{h\nu}{m_e c^2} >> \frac{1}{Z\alpha^{1/3}} \Rightarrow \sigma_{pair} = \alpha Z^2 r_e^2 \left[\frac{28}{9}\ln\left(\frac{183}{Z^{1/3}}\right) - \frac{2}{27}\right] m^2 atom^{-1}$$

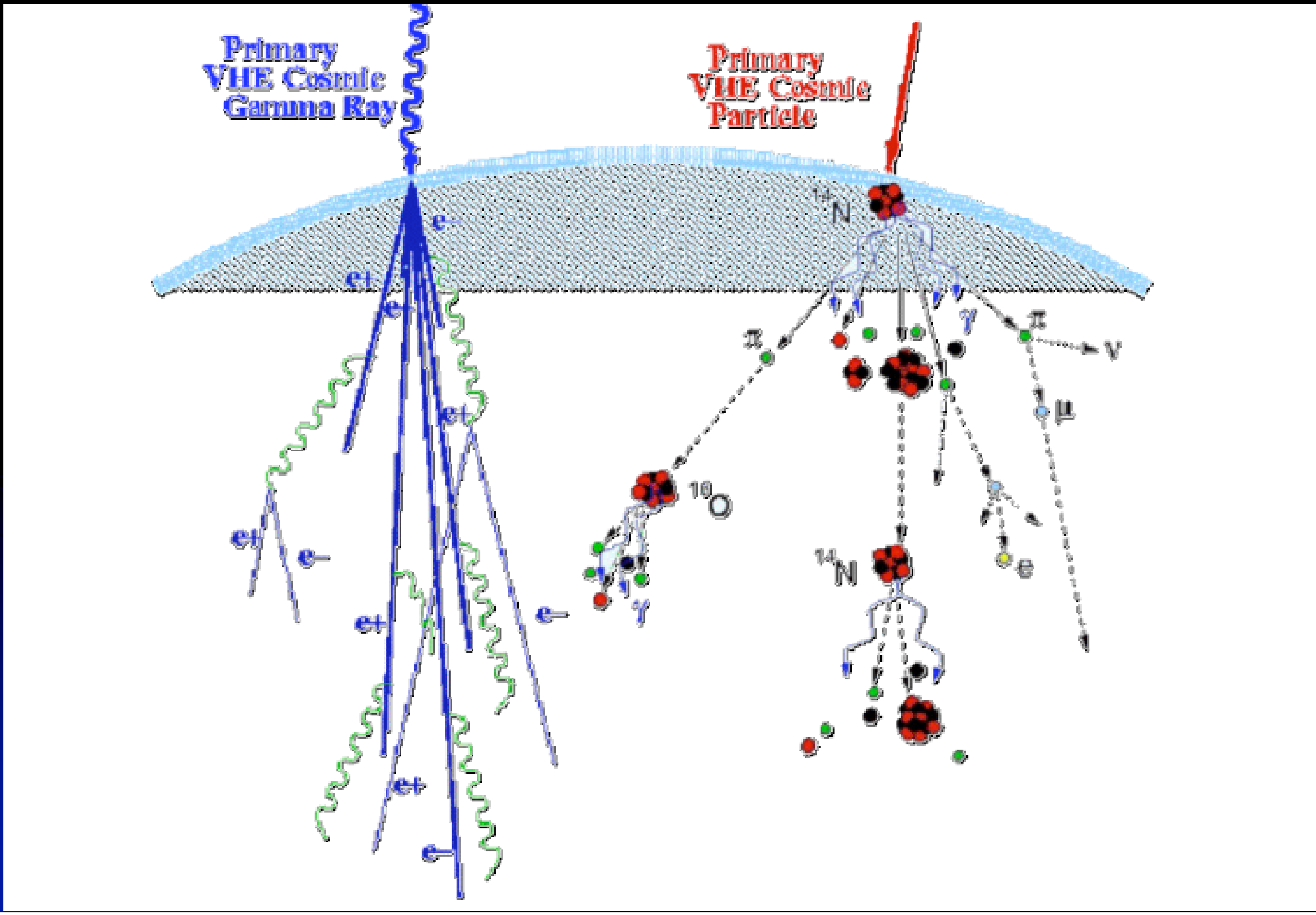
$$\sigma = \frac{7}{9}(A/X_0 N_A)\ .$$

Normalized pair prod cross section vs Fractional electron energy for photons of variouos energy



At high energies $\sigma_{pair} > \sigma_{Compton}$

Electromagnetic and hadronic showers



Electromagnetic showers

Cascades initiated by electrons or photons discovered by Blackett-Occhialini 1933. High energy electrons lose most of their energy through bremsstrahlung, producing high energy photons that in turn produce pairs and Compton electrons. These electrons and positrons radiate new photons which in turn undergo pair production or Compton scattering.

Since the bremsstrahlung and pair production cross sections become almost energy dependent for incoming electron and photon energies >>$mc^2/\alpha Z^{1/3}$, the radiation length emerges as typical unit length of which a reasonable approx is

$$X_0(g/cm^2) = \frac{716A}{Z(Z+1.3)\left(\ln\frac{183}{Z^{1/3}}+\frac{1}{8}\right)} = \frac{A}{N_A\sigma_{bremss}}$$

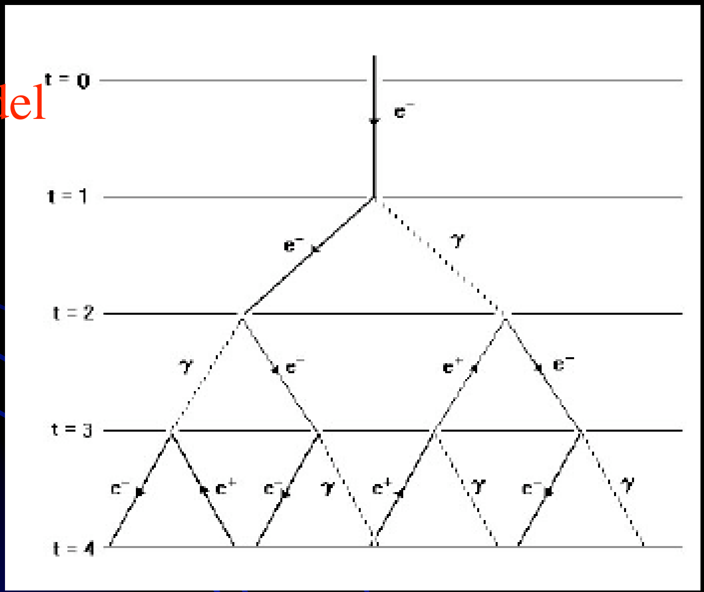
$L_p \sim 9/7 X_0$

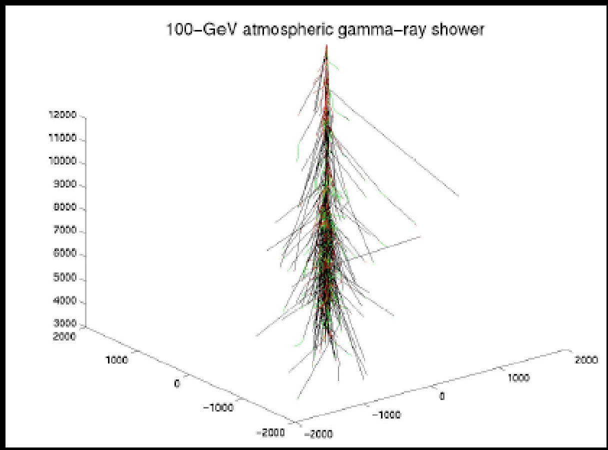
After reaching a maximum where the largest number of secondaries are created the cascade decays slowly and the multiplication process is almost stopped when the electron energy reaches the critical energy ε_c. Rossi - Greisen (1941)

Electromagnetic showers

If E_0 is the energy of the incoming photon it generates after $1X_0$ a pair e^+e^- of equal energies. Both will emit a bremsstrahlung photon in $1X_0$. By continuing the process and assuming equal energy sharing the number of particles after t radiation lengths is $N(t) \approx 2^t$ and their energy will be $E(t) \approx E_0/2^t$. When $E \sim \varepsilon_c$ the multiplication stops and $t_{max} = \ln(E_0/\varepsilon_c)/\ln 2$. ε_c = critical energy where radiation and ionization loss are equal . $N_{max} = E_0/\varepsilon_c$=# of particles at max

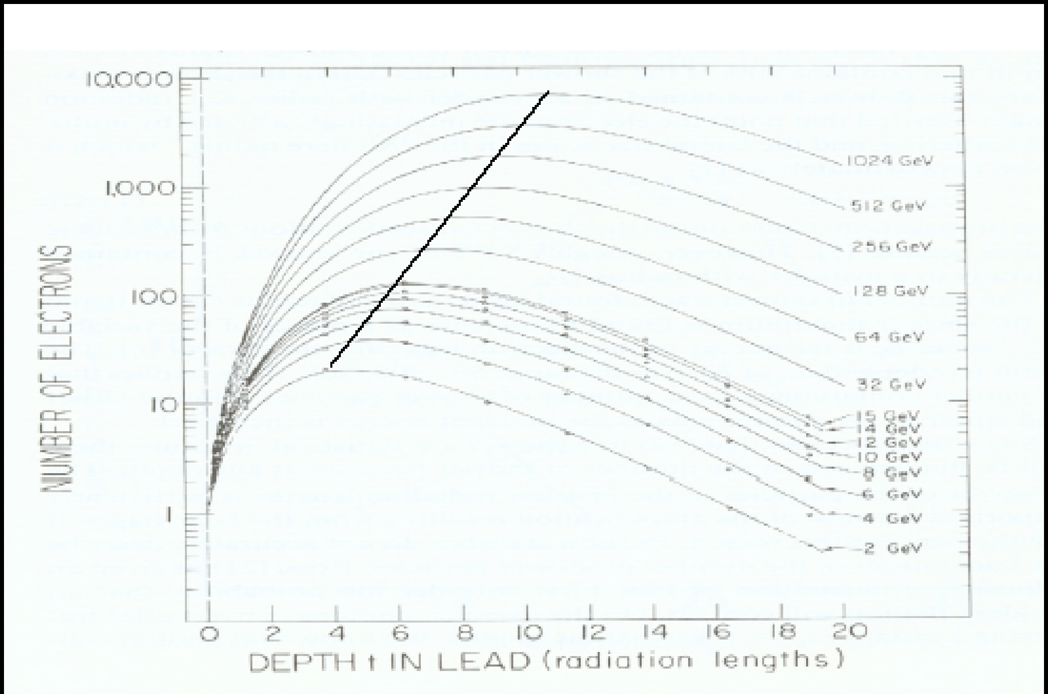
Toy model





At very high energies the Landau-Pomeranchuk-Midgal effect reduces the pair production and bremsstrahlung cross-section and cause significant elongation of showers (quantum interference between amplitudes from different scattering centers)

Longitudinal shower development



Number of electrons depth dependence for showers initiated by γs of various energies.

$$t_{max}=\ln(E_0/\varepsilon_c)/\ln 2$$

$$N_{max} = E_0/\varepsilon_c$$

Electromagnetic showers

$t = x/X_0$

$y = E/E_c$

The longitudinal profile is described by

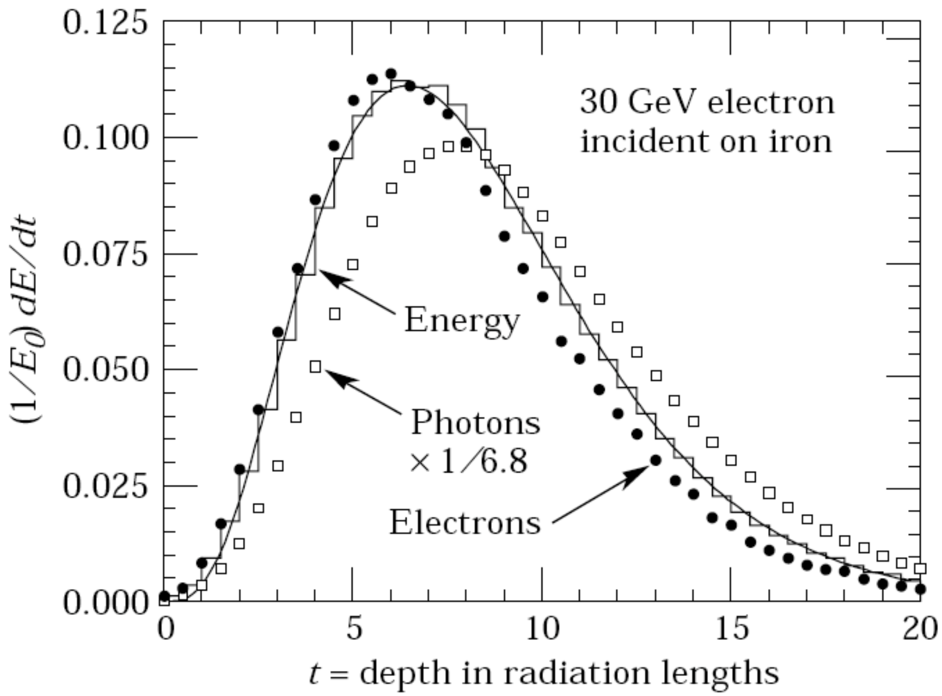
$$\frac{dE}{dt} = E_0\, b\, \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

$$t_{\rm max} = (a-1)/b = 1.0 \times (\ln y + C_j)\ , \qquad j = e, \gamma$$

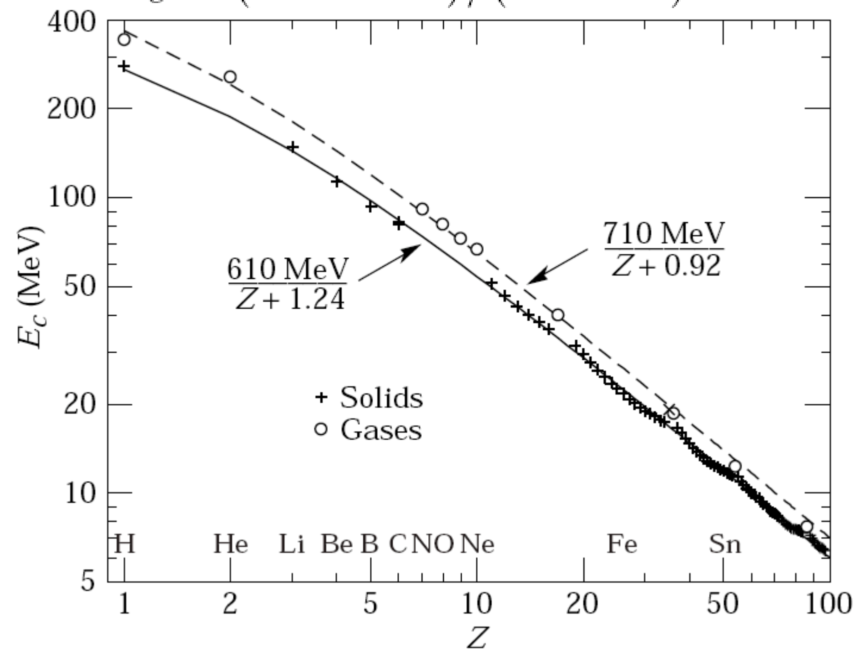
$C_e = -0.5$ $\quad C_\gamma = +0.5$ $\quad b \approx 0.5$

The traverse development is described by the Moliere radius $R_M = X_0\, E_s/E_c$

$E_s \approx 21$ MeV on average 10%(1%) outside cylinder of radius $R_M(3.5R_M)$



$E_c = (800\ \text{MeV})/(Z+1.2)$



Hadronic showers

Cascades initiated by hadronic interactions of nucleons or nuclei. In the first interaction about 1/2 of the energy is transferred to secondary mesons (charged and neutral - the ratio of $\pi\pm/\pi^0$~2). Secondary mesons can themselves interact and generate secondary hadronic cascades until hadron energy goes below interaction threshold (~hadron mass). Neutral pions induce em cascades due to their immediate decay with decay length $\pi^0 \to \gamma\gamma$

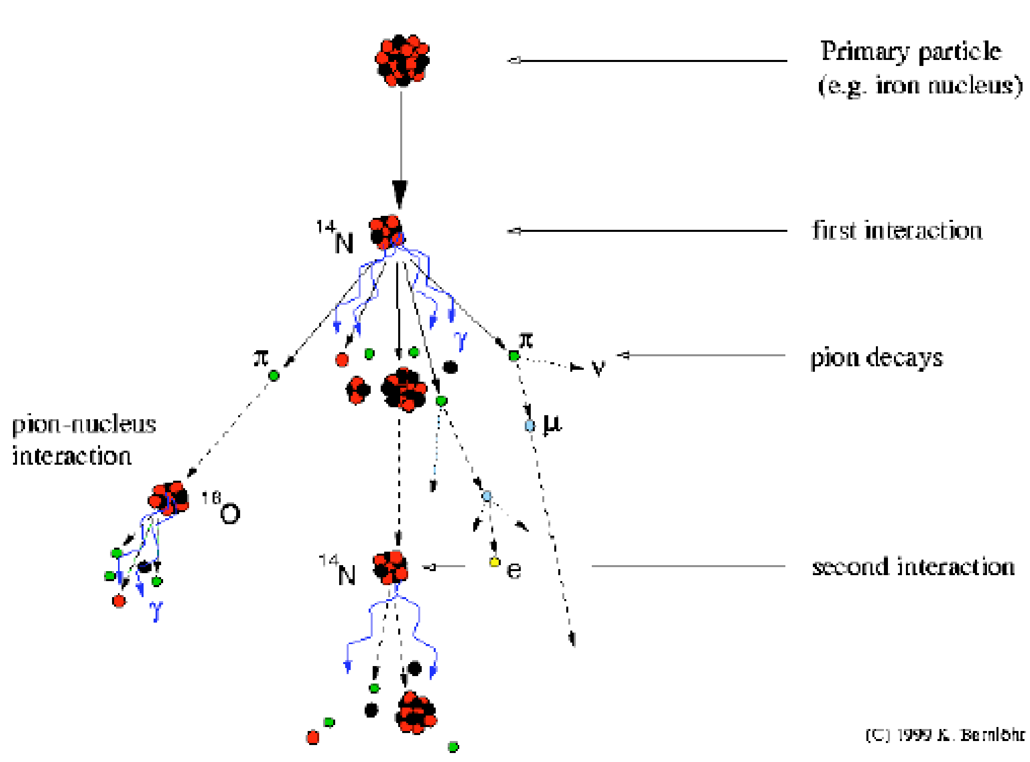
$$l_d = \gamma_{\pi^0} l_0 = \frac{E_{\pi^0}}{m_{\pi^0}} \times 2.51 \cdot 10^{-6} cm$$

High energy charged pions have large decay length hence they interact, lower energy ones decay into neutrinos and muons.

Int length > radiation length and also energy dependent

$\lambda n = 80\ g/cm^2$

$\lambda \pi = 120\ g/cm^2$



Hadronic showers

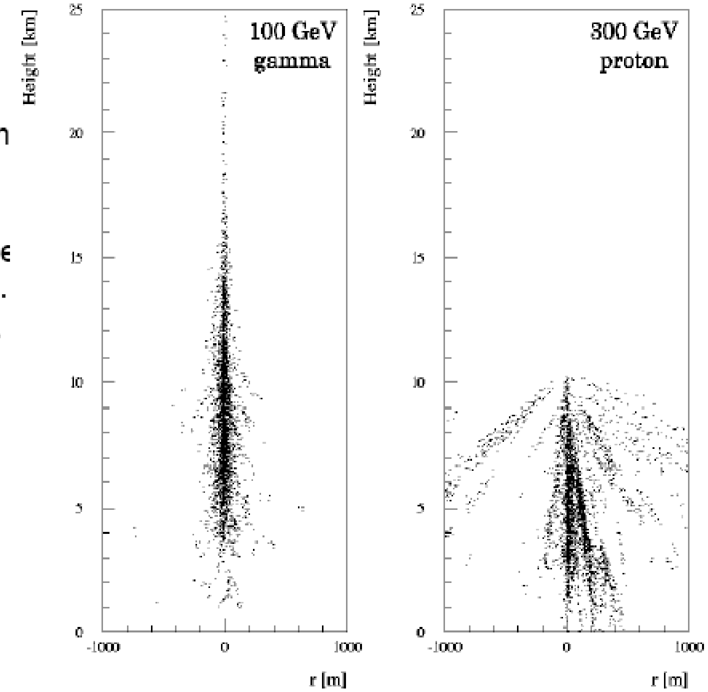
The shower hadronic component carries a large fraction of the primary particle energy deeper than an em cascade. But the secondary multiplicity in hadronic interactions is higher than the effective multiplicity of 2 secondaries in em interactions and this faster energy dissipation compensates for the longer interaction length. For a proton and a photon of 10^5 GeV the p initiated shower peaks earlier than the photon initiated one (max depths at 506 and 520 g/cm^2)

This is due to the higher secondary multiplicity in hadronic interactions

The p shower is not absorbed as easily as γ one

The toy model can be applied using the interaction length

- Electromagnetic showers develop more regularly, with less fluctuations.
- In the 300 GeV shower one can see jets of secondaries.
- At higther energies they are stronger forward boosted.
- The difference in structure can be used for particle identification.



Atmospheric Hadronic showers

Given a nucleon of energy E_0 (GeV) that interacts in the atmosphere at depth λ_N: in the interaction it loses $(1-K_{el})$ fraction of its energy and generates <m> secondary pions, 1/3 of which are neutral. The depth of the shower maximum can be written as the sum of the depths of the em showers induced by neutral pions and the int length of the primary nucleon:

$$X_{\max} = X_0 \ln\left[\frac{2(1-K_{el})E_0}{(\langle m\rangle/3)\varepsilon_0}\right] + \lambda_N(E_0)$$

And the number of electrons at maximum is:

$$N_e^{\max} = \frac{1}{2}\frac{\langle m\rangle}{3}\frac{(1-K_{el})E_0}{\varepsilon_0}$$

Splitting in 2 of π energy

π multiplicity

Eg. $\lambda_N = 80$ g/cm^2, <m> = 12, $K_{el} = 0.5$ $X_{max} = 500$ g/cm^2 for a 10^5 GeV proton

And $N_e^{max} = 8\ 10^4$ electrons

A longitudinal development parametrization (Gaisser)

$$N_e(X) = N_e^{\max}\left(\frac{X-X_1}{X_{\max}-\lambda}\right)^{\frac{X_{\max}-\lambda}{\lambda}} e^{-\left(\frac{X-X_1}{\lambda}\right)}$$

The depth of the 1st interaction X_1 is distributed as $\exp-(X_1/\lambda)$, main source of fluctuations

Atmospheric Hadronic showers

Number of muons related to number of decaying pions: $N_\mu = \left({E_0}/{\varepsilon_\pi} \right)^\beta$

Where β = 0.85 = fraction of multiplicity of charged pions=ln(2/3<m>)/ln<m>

ε_π = an effective energy = 20 GeV

For heavy nuclei the superposition model is a good approximation: the interaction of a nuclei of mass number A and total energy E_0 is equal to the interaction of A nucleons of energy E_0/A

And the shower depth is related to the nucleon one by

$$X^A_{\max} = X_0 \ln\left[\frac{2(1-K_{el})E_0/A}{(\langle m \rangle/3)\varepsilon_0} \right] + \lambda_N(E_0) \Rightarrow X^A_{\max} = X^p_{\max} - X_0 \ln A$$

And

$$N^A_\mu = A\left({E_0}/{A\varepsilon_\pi} \right)^\beta = A^{1-\beta} N^p_\mu$$

Showers generated by heavy nuclei generate more mouns than p showers

Inclined showers develop in more tenuous atmosphere where pions and kaons are more likely to decay than interact so contain more muons.