MC techniques

Suggested readings:

http://pdg.lbl.gov/2005/reviews/monterpp.pdf

• MCs assume a random number generator which generates uniform statistically independent values in the interval [0,1)

•Inverse Transform Method: if the probability density function is f(x) with $-\infty<x<\infty$ its cumulative distribution function expressing the probability that $x \leq a$ is given by

$$F(a) = \int_{-\infty}^{a} f(x)\, dx$$

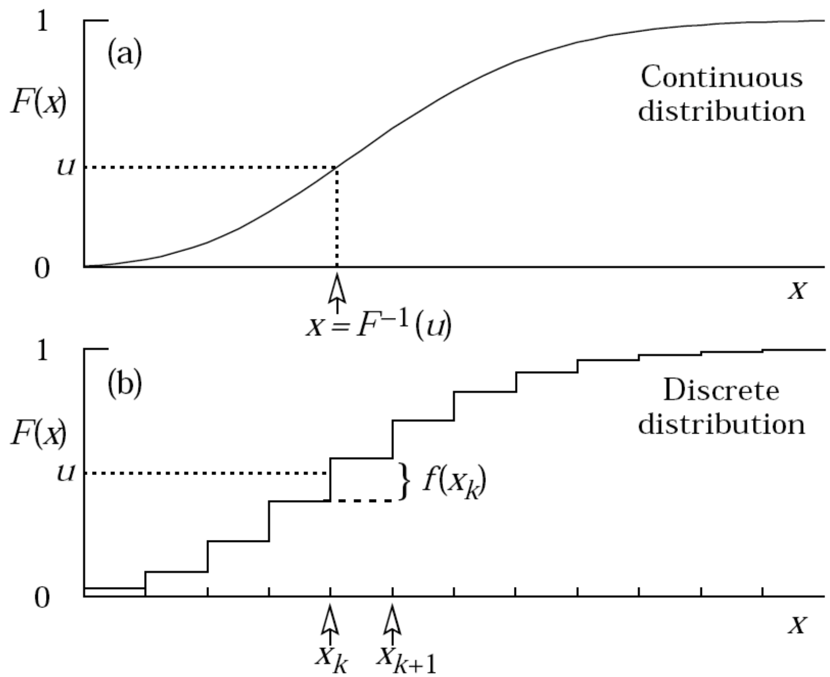
Notice this should be normalized to 1

If a is chosen with pdf f(a) then the integrated probability up to point a, F(a) is itself a random variable which will occur with uniform probability in [0,1]. We can find a unique x chosen from the pdf f for a given u if we set $u = F(x)$

Provided we can find an inverse function defined by

$$x = F^{-1}(u)$$

Inverse Transform Method



For a discrete function

$$F(x_{k-1}) < u \leq F(x_k) \equiv \text{Prob}\ (x \leq x_k) = \sum_{i=1}^{k} f(x_i)$$

Figure 33.1: Use of a random number u chosen from a uniform distribution (0,1) to find a random number x from a distribution with cumulative distribution function $F(x)$.

Acceptance-rejection technique

When F(x) is unknown or difficult to work out the inverse transform method cannot be applied. Let's suppose that we know enough f(x) so that we can enclose it in a shape h(x) (easily generated) that is C>1 times f(x). h(x) can be a normalized sum of uniform distributions. To generate f(x), first we generate a candidate x according to h(x) and then calculate f(x) and Ch(x). We generate randomly u and test if $uCh(x) \le f(x)$. If so we accept x, if not we reject x and retry.

The efficiency is the ratio of the areas 1/C.