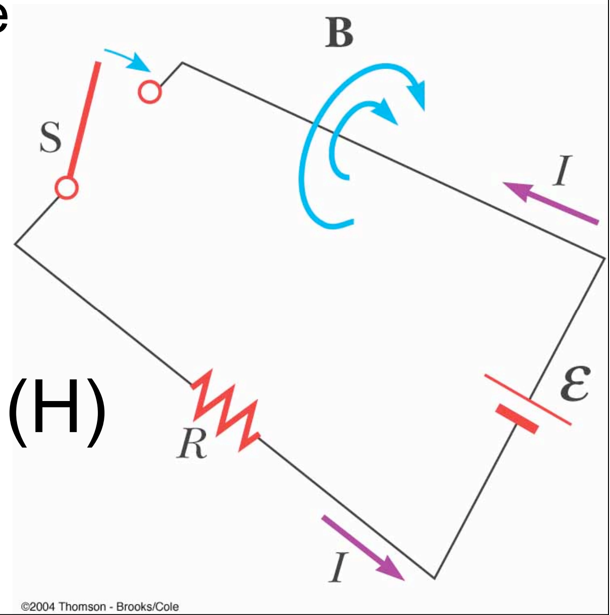
Self-inductance

- When the switch is closed, the current does not immediately reach its max value
- As the current increases, the magnetic flux through the loop due to this current also increases. The magnetic flux is proportional to I and the proportionality constant is called self-inductance

$$\phi_m = LI$$

- when I is changing, the magnetic flux varies due to the current changes, so an emf is induced in the circuit opposite to the emf of the battery



$$\varepsilon_L = -L\frac{dI}{dt}$$

The SI unit of inductance is the **henry** (H)

$$1\text{H} = 1\frac{\text{V}\cdot\text{s}}{\text{A}}$$

Self-inductance of a solenoid

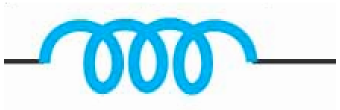
For a long tightly wound solenoid with n = N/ ℓ

$$\Phi_m = N\mu_0 nIA = \mu_0 n^2 IA\ell = LI$$

$$L = \mu_0 n^2 A\ell$$

A coil or solenoid has a large self-inductance and is called an **inductor**. The potential difference across an inductor is

$$\Delta V = \varepsilon - Ir = -L\frac{dI}{dt} - Ir$$



For an ideal inductor (r=0)

$$\Delta V = -L\frac{dI}{dt}$$

Mutual Inductance

- magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits
- The current in coil 1 with N_1 turns sets up a magnetic field some of which pass through coil 2
- Coil 2 has N_2 turns
- M_{12} = **mutual inductance** of coil 2 with respect to coil 1

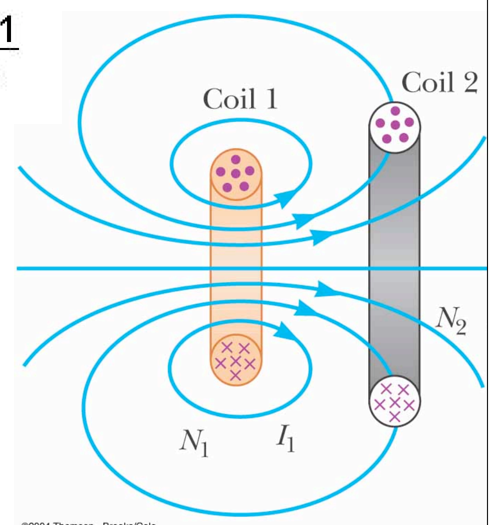
$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1}$$

- If I_1 varies with time
- Similarly for I_2

$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} = -M_{12} \frac{dI_1}{dt}$$

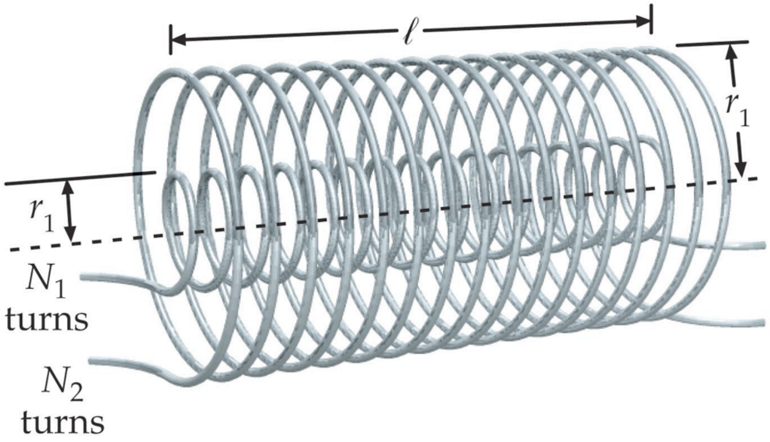
$$\varepsilon_1 = -M_{21} \frac{dI_2}{dt}$$

$$M_{12} = M_{21} = M$$



Example: 2 concentric solenoids

- Inside the inner solenoid $B_1 = \mu_0 n_1 I_1$
- Outside $B_1 = 0$, hence
- the flux of B_1 through the outer solenoid area is

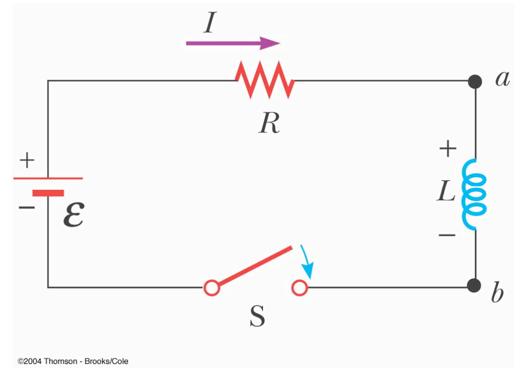


$$\Phi_{2,1} = N_2 B_1 (\pi r_1^2) = \mu_0 n_1 n_2 (\pi r_1^2) I_1 \ell \Rightarrow M_{2,1} = \Phi_{2,1} / I_1 = \mu_0 n_1 n_2 (\pi r_1^2) \ell$$

- The flux of B_2 through the inner solenoid is:

$$\Phi_{1,2} = N_1 B_2 (\pi r_1^2) = \mu_0 n_1 n_2 (\pi r_1^2) I_2 \ell \Rightarrow M_{1,2} = \Phi_{1,2} / I_2 = M_{2,1}$$

Energy in a magnetic field



$$\varepsilon - L\frac{dI}{dt} = RI$$

$$I\varepsilon = I^2 R + LI\frac{dI}{dt}$$

$I\varepsilon$ =rate of energy supplied by the battery

I^2R = rate at which energy dissipated by R

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$ = rate at which energy is stored in the inductor

$$U = \int_0^{I_f} LI\,dI = \frac{1}{2}LI_f^2$$

In a solenoid

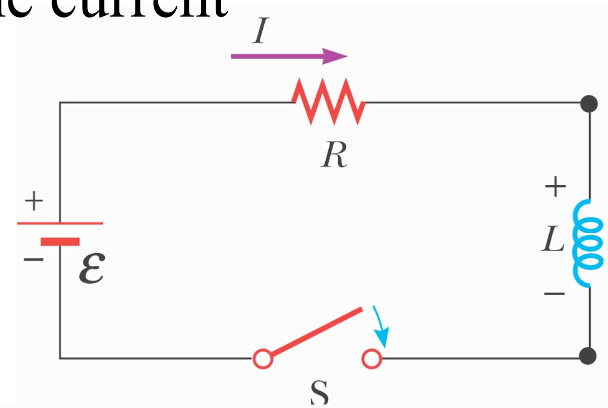
$$U = \frac{1}{2}\mu_o n^2 A\ell\left(\frac{B}{\mu_o n}\right)^2 = \frac{B^2}{2\mu_o}A\ell$$

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_o}$$

This applies to any region in which a magnetic field exists (not just the solenoid)

RL circuits

The inductor in the circuit opposes changes in the current

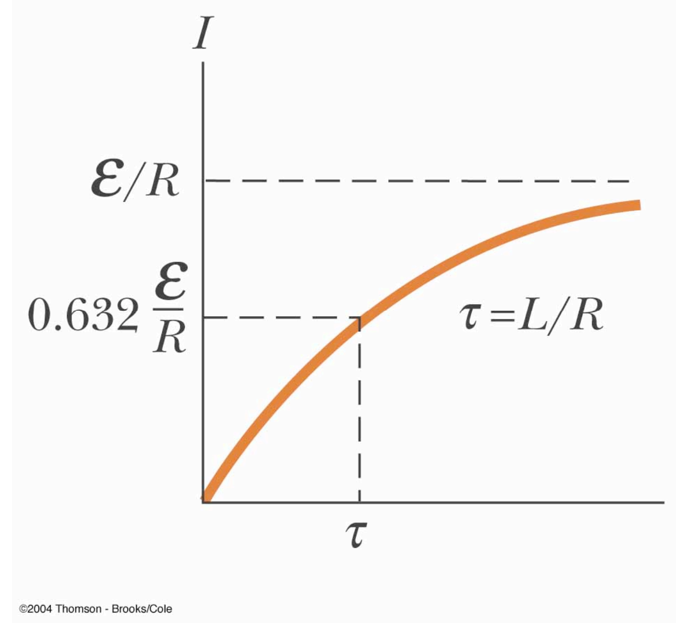


$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

At t=0 (switch open) I=0 and $\varepsilon = L\left.\frac{dI}{dt}\right|_{I=0} \Rightarrow \left.\frac{dI}{dt}\right|_{I=0} = \frac{\varepsilon}{L}$

At t=∞ dI/dt = 0 and I=ε/R

$$I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$



τ= *L* / *R* time constant = time required for I to reach 63.2% of its max value

RL circuit

- If the battery is excluded the current cannot go instantaneously to zero since there is L

$$-L\frac{dI}{dt} = RI$$

At t=0 $I=I_0$